Optimal reinsurance and dividend distribution in a compound-Poisson process: Viscosity solutions and numerical methods.

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Abstract: It is usual to value a company as the expected present value of the net dividend pay out stream. We consider an insurance company in which the management has the possibility of controlling the risk exposure by reinsurance and deciding the distribution of dividends pay-outs. Our aim is to maximize the cumulative expected discounted dividend pay outs assuming the reserve of the insurance company follows a compound-Poisson process. We consider the usual reinsurance contracts as well as a general contract in the cheap and non-cheap cases.

In the setting of diffusion approximation of the Cramér-Lundberg process, for instance in Asmussen, Hjgaard and Taksar (2000), the optimal return function V satisfy $V(0) = 0$ and it is concave, twice continuously differentiable and always comes from an optimal barrier strategy. So V is the only classical solution of the corresponding Hamilton-Jacobi-Bellman equation with boundary condition $V(0) = 0$. As Gerber (1969) suggested in the non-reinsurance case and we showed in Azcue-Muler (2004) this is not the case in the compound-Poisson setting since there is not a natural boundary condition, the optimal return function V could be not differentiable at some points and the optimal dividend strategy could be non-barrier and so the return function is not concave. We solved this problem showing that the optimal value function is a viscosity solution of the associated HJB equation.

We present a numerical scheme for the approximation of both the optimal value function and the optimal strategy and compare numerical results for different reinsurance contracts and claim distributions.

Keywords: compound-Poisson process, dividend pay-outs, insurance, reinsurance, Hamilton-Jacobi-Bellman equation, viscosity solution, risk control, dynamic programming principle, numerical approximations.

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