GOODNESS-OF-FIT TESTS FOR THE GENERALIZED PARETO DISTRIBUTION BASED ON TRANSFORMS

Dimitrios Konstantinides, Simos G. Meintanis

Department of Statistics and Acturial Science, University of the Aegean, Karlovassi, 832 00 Samos, Greece

and

Department of Economics, National and Kapodistrian University of Athens, 8 Pesmazoglou Street, 105 59 Athens, Greece

Abstract. The Generalized Pareto (GP) distribution is a very popular model for extreme events. The GP is a two–parameter model with a shape parameter α ($\alpha \in \mathbb{R}$) and a scale parameter c ($c > 0$). Its distribution function is $F(x) = 1 - (1 - (\alpha x/c))^{1/\alpha}$ and has support $x > 0$ (resp. $0 < x < c/\alpha$), if $a \leq 0$ (resp. $a > 0$). We write $\mathcal{GP}(\alpha, c)$ to denote the GP distribution with parameters α and c. In this article we develop a class of goodness–of–fit tests for the null hypothesis

$$H_0: \text{ The law of } X \text{ is } \mathcal{GP}(\alpha, c) \text{ for some } \alpha \in \mathbb{R} \text{ and } c > 0,$$

based on independent copies $X_1, \ldots, X_n$ of the random variable X. In doing so we transform $\{X_j\}_{j=1}^n$ into $\{Y_j\}_{j=1}^n$, where $Y_j = -(1/\alpha)\log(1 - (\alpha X_j/c))$. It is well known that under the null hypothesis H_0, $\{Y_j\}_{j=1}^n$ would follow a unit exponential distribution. Hence we may employ the methods in Henze and Meintanis (Statistics, 36 (2002), 147–161, and Communications in Statistics, 31 (2002), 1479–1497) on the transformed data $\{Y_j\}_{j=1}^n$, and test the null hypothesis of exponentiality instead. These methods utilize certain transforms of the empirical distribution function, such as the empirical Laplace transform and the empirical characteristic function, and have proved very competitive to the standard methods of Kolmogorov–Smirnov, Cramer–von Mises and Anderson–Darling. In practice one has to estimate the parameters α and c by consistent estimators in order to implement the tests. We consider several methods of estimation and compare the power of the proposed tests to the power of some standard methods referred to above.