LARGE DEVIATIONS AND RUIN PROBABILITIES FOR SOLUTIONS TO STOCHASTIC RECURRENCE EQUATIONS WITH HEAVY-TAILED INNOVATIONS

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Abstract. In this paper we consider the stochastic recurrence equation $Y_t = A_t Y_{t-1} + B_t$ for an iid sequence of pairs (A_t, B_t) of non-negative random variables, where we assume that B_t is regularly varying with index $\kappa > 0$ and $EA_t^\kappa < 1$. We show that the stationary solution (Y_t) to this equation has regularly varying finite-dimensional distributions with index κ. This implies that the partial sums $S_n = Y_1 + \cdots + Y_n$ of this process are regularly varying. In particular, the relation $P(S_n > x) \sim c_1 \, n \, P(Y_1 > x)$ as $x \to \infty$ holds for some constant $c_1 > 0$. For $\kappa > 1$, we also study the large deviation properties $P(S_n - ES_n > x)\,, \quad x \geq x_n$, for some sequence $x_n \to \infty$) whose growth depends on the heaviness of the tail of the distribution of Y_1. We show that the relation $P(S_n - ES_n > x) \sim c_2 \, n \, P(Y_1 > x)$ holds uniformly for $x \geq x_n$ and some constant $c_2 > 0$. Then we apply the large deviation result to derive bounds for the ruin probability $\psi(u) = P(sup_{n\geq 1}([S_n - ES_n] - \mu\, n) > u)$ for any $\mu > 0$. We show that $\psi(u) \sim c_3 \, u \, P(Y_1 > u) \, \mu^{-1} (\kappa - 1)^{-1}$ for some constant $c_3 > 0$. In contrast to iid regularly varying Y_t's, when the above results hold with $c_1 = c_2 = c_3 = 1$ the constants c_1, c_2 and c_3 are different from 1.

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