Statistical questions in a random field based forward interest rate model

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There are several different methods in financial mathematics to construct models for forward interest rate curves of financial markets. Based on such a model, bond processes, (interest rate) derivatives can be priced. In our talk we focus on Heath-Jarrow-Morton type forward rate structures. Our motivation is based on the papers Kennedy [**?**], Goldstein [**?**] and Santa Clara and Sornette [**?**], in which the authors proposed different generalisations of the Heath-Jarrow-Morton (HJM) forward rate model (see [**?**]) such that the forward rate processes (with different time-to-maturity) are driven by a random field, i.e. for each $x \in \mathbb{R}_+$ we have

$$d_t f(t,x) = \alpha(t,x)dt + \sigma(t,x)d_t Z(t,x),$$

where $f(t,x)$ is the forward rate at time t with time to maturity x, where the random field Z is assumed to satisfy certain further conditions.

In the talk we shall study discrete time forward rate models driven by random field, which where proposed in Gáll, Pap and Zuijlen [**?**], [**?**]. We consider Gaussian random field cases, in particular, a Gaussian AR field. After showing the necessary no-arbitrage (drift) conditions we derived for such models and the role of the market price of risk parameters, we turn to the maximum likelihood estimation of the parameters. Despite the lack of explicit solutions in many cases we can show the (joint) asymptotic normality of the estimators. We also show our results on the strong consistency of some parameters (like the volatility), but only the weak consistency of some other parameters of the model. (Our earlier results on the volatility estimation can be found in [**?**].) Finally we discuss the numerical steps and difficulties occurring in the application of the results in real data.

References

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