

Decay of the probability of ruin under uncertain investments

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Abstract

The classical result of the Cramer-Lundberg risk model states that if the rate of premium, c , exceeds the average of the claims paid per unit time, $\lambda\mu$, then the probability of ruin of the insurance company decays exponentially fast as the initial capital $u \rightarrow \infty$. This asymptotic behavior of the probability of ruin is derived again by means of infinitesimal generators and Laplace transforms. Using the same tools, it is shown that the probability of ruin has an algebraic decay rate if the insurance company invests its capital in a risky asset with a price which follows a geometric Brownian motion. The latter result is shown to be valid not only for exponentially distributed claim amounts, as in Frolova et al. (2002), but, more generally, for any claim amount distribution that has a moment generating function defined in a neighborhood of the origin.

Keywords: ruin probability, Cramer-Lundberg model, risky asset, infinitesimal generator, Laplace transform.

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