Extremes and dependence, a copula approach.

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Keyword

Copulas; Extreme Value Theory; Limiting Distributions; Tail Dependence;

Summary

Taking care of dependencies becomes therefore important in order to extend standard models towards a more efficient risk management. It is (intuitively) clear that the probabilistic mechanism governing the interactions between random variables is completely described by their joint distribution. On the other hand, in most applied situations, the joint distribution may be unknown or difficult to estimate such that only the marginals are known (estimated or fixed a priori). To tackle this problem a flexible and powerful approach consists in trying to determine the joint distribution by means of copulas.

The reverse side of the medal of the copula approach is that it is usually difficult to chose the appropriate copula for the problem at hand. Often, the only possibility is to start with some guess such as a parametric family of copulas and then try to fit the parameters. As a consequence, the models obtained may suffer a certain degree of arbitrariness. Some remedy to this is provided by models for (bivariate) conditional joint extremes, where limiting results along the lines of the Central Limit Theorem or the Fisher-Tippett Theorem of extreme value theory for sums, respectively maxima, of random variables are obtained (see Draisma et al. (1996, 1997, 2003) and Ledford et al. (1996)).

Bust instead of focusing on tail coefficients, or some limiting results based on the spectral measure, we will study here some “copula-convergence theorems”. These theorems reflect a distributional approach to the modelling of dependencies in the tails and provide natural descriptions of multivariate extremal events. A further advantage of this kind of results is that they also allow to better face the problem of the lack of data which is typical for rare events. In fact, there are situations where the knowledge of the limiting dependence structure reduces the issue of modelling tail events to fitting only few parameter solely (see Juri et al. (2003) or Charpentier et al. (2004)).

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