

Short Course: Multiscale statistical approach with applications in astronomy and solar physics

Véronique Delouille

February 4, 2005

Abstract

The objective of this short course is to study several multiscale statistical tools and to show how they are used to solve practical problems arising in astrophysics and in solar physics. After having motivated the use of multiscale approach in astronomy, we study in more details a few problems.

Detection of Gamma Ray Bursts We first present the Haar wavelet transform, and see how it can be used to detect Gamma Ray Bursts (GRB). These are commonly associated with star formations and supernovae, but their precise origin is still unknown. The intensity profile underlying Gamma Ray Bursts can be modeled as an inhomogeneous Poisson process observed on a background made of Poisson noise. We study the method proposed by Kolaczyk [3] that aims at removing Poisson noise. This procedure is based on the Haar transform and on a thresholding scheme specifically tailored to Poisson noise.

Analysis of solar images and introduction to fractal models The discrete wavelet transform (such as the Haar transform) is useful to *denoise* signals. However, when the aim is to *analyze* images, it is sometimes useful to consider the Continuous Wavelet Transform, since it gives information about the signal at all desired scales and locations. The Extreme ultraviolet Imaging Telescope (EIT) on board the SoHO mission provides since 1996 images of the solar atmosphere in four different passbands. It is of interest to analyze these images using signal processing methods in order to, e.g., automatically detect solar events. In the second course, we show how the scale measure associated to the Mexican Hat wavelet transform allows to build pertinent times series from a solar images dataset. We describe three applications of the use of the scale measure as presented in [1]. Finally, we introduce fractal models and show their potential to emulate images of the solar corona.

Multiscale polynomial estimation The third course will be devoted to a new non-parametric methodology introduced in [5] that relies on a *multiscale* signal decomposition based on *polynomials*. The corresponding estimator uses a penalized likelihood, and may be tailored to estimate either the intensity function of a Poisson process or a density. We show some optimality properties of this estimator.

Detection of clusters in a galaxy We show how nonparametric kernel density estimation allows to separate different clusters in a galaxy. Indeed, the presence of a cluster in a data sample is indicated by a peak in the probability function underlying the data. The method presented in [4] provides a kernel density estimator adapted to this problem, and tests for the presence of modes in the density function. We compare this method with other procedures described in [2].

Table of Contents

- 1 Introduction: motivating examples for multiscale decompositions
- 2 Haar wavelet transform for detection of Gamma Ray Bursts
 - 2.1 Haar wavelet transform
 - 2.2 Thresholding in presence of Gaussian noise
 - 2.3 Thresholding in presence of Poisson noise
 - 2.4 Estimation of an intensity function. Application to Gamma Ray Burst detection
 - 2.5 Generalization to the two-dimensional case. Application to the estimation of Intensity Maps in astronomy
- 3 Continuous wavelet transform for analysis of solar images
 - 3.1 Solar images and the EIT/SoHO telescope
 - 3.2 Continuous wavelet transform and scale measure
 - 3.3 Characteristic scale
 - 3.4 Detection of eruptions on the Sun
 - 3.5 Segmentation of the solar corona
 - 3.6 Introduction to fractal analysis
- 4 Multiscale likelihood estimation
 - 4.1 Piecewise Polynomial Approximations
 - 4.2 Likelihood factorization
 - 4.3 Denoising using Maximum Penalized Likelihood estimators
 - 4.4 Properties of the estimator
 - 4.5 Application to the estimation of an intensity function
- 5 Kernel estimation for cluster analysis of Galaxy
 - 5.1 Estimation of the probability density function
 - 5.2 Identification of clusters
 - 5.3 Generalization to the multivariate case

6 Further readings

- J.-L. Starck and F. Murtagh. *Astronomical Image and Data Analysis*. Springer, 2002.
- Rebecca Willett. Multiscale analysis for intensity and density estimation. Master's thesis, Rice University, 2002.

References

- [1] V. Delouille, J. de Patoul, J.-F. Hochedez, L. Jacques, and J.-P. Antoine. Wavelet spectrum analysis of solar EUV images: method and applications to network characteristic scale evolution, flare nowcasting, and extraction of active regions with EIT/SoHO. Submitted to *Solar Physics: Topical issue for Signal Image processing*. Available at <ftp://omaftp.oma.be/pub/astro/verodelo/ScaleMeasure-Delouille-VF1.ps>, February 2005.
- [2] D. Fadda, E. Slezak, and A. Bijaoui. Density estimation with non-parametric methods. *Astronomy and Astrophysics Supplement Series*, 127:335–352, January 1998.
- [3] E.D. Kolaczyk. Wavelet shrinkage estimation of certain Poisson intensity signals using corrected thresholds. *Statistica Sinica*, 9:119–135, 1998.
- [4] A. Pisani. A non-parametric and scale-independent method for cluster analysis - part one - the univariate case. *Monthly Notices of the Royal Astronomical Society*, 265:706, 1993. Available at <ftp://omaftp.oma.be/pub/astro/verodelo/Pisani1996.pdf>
- [5] R. Willett and R. Nowak. Multiscale likelihood analysis and image reconstruction. In *Wavelets X, SPIE meeting*, 2003. Available at <ftp://omaftp.oma.be/pub/astro/verodelo/Willett2003.pdf>