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A DOUBLE-STRUCTURE STRUCTURAL EQUATION MODEL FOR THREE-MODE DATA

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A Double-Structure Structural Equation Model for three-mode data

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Abstract

Structural equation models are commonly used to analyze two-mode data sets in which a set of objects is measured on a set of variables. The underlying structure within the object mode is evaluated using latent variables which are measured by indicators coming from the variable mode. When the objects are in addition measured under different conditions, three-mode data arise, and with this, the study of the correlational structure of two modes simultaneously may be of interest. In this paper we present a model which simultaneously uses a latent structure to handle two of the three modes of a data set. A study about emotion using a persons by situations by responses three-mode data set shows that the model is useful to study systematic differences across the entities of the persons mode and the situations mode.

Keywords: Structural equation model, three-mode data, individual differences, situational differences.

A Double-Structure Structural Equation Model for three-mode data

Structural equation models (SEM) are a popular tool in social sciences for investigating two-mode data arrays in which a set of objects (mode 1) is measured on a set of variables (mode 2). Examples of such two-mode data sets are abundant. As a specific example, that will be elaborated in the remainder of the paper, consider the case in which a group of persons is asked to answer a set of questions regarding emotional responses to a particular kind of situation. For example, various negative affect responses to a frustrating situation. In this case, the object mode consists of a random sample of persons and the variable mode are the emotional responses. The resulting data set is a two-dimensional array of persons by responses.

A SEM can then be used to explain the covariation between the manifest variables (i.e., the emotional responses) by making use of latent variables (and relationships between them). The latent variables are underlying individual difference variables.

It is not uncommon, however, that data can be collected regarding several situations. Following the previous example, consider that a group of P persons has been asked to answer a set of R questions regarding their emotions in S different situations. This kind of data are represented in a three-dimensional array and are referred to as three-mode data (e.g., Kroonenberg, 2005; Cattell, 1946, 1952) with persons and responses being extended with a third mode which is, in this case, the situation mode. More specifically for the data which will be analyzed in this paper, a set of persons is measured with respect to four emotional responses (frustration, tendency to act antagonistically, irritation, and anger) in a set of situations. Thus, the situations are considered to be the third mode of the data. Figure 1 shows a schematic representation of the three-dimensional data array in which the observation x_{prs} represents the response of person p ($p = 1, \ldots, P$) to question r ($r = 1, \ldots, R$) under situation s ($s = 1 \ldots, S$).

When confronted with this kind of data structure, the researcher may choose among several alternatives to analyze such data. A first option is simply to aggregate over the third mode (i.e., situation mode) to arrive to the more common two-mode data structure. Aggregation can be carried out by calculating some kind of summary statistic of the responses of a given person in the set of situations (e.g., mean, sum, median, etc.). However, aggregation has some severe disadvantages. After aggregation over the situation mode, one can only answer questions with respect to individual differences and correlations across individuals. Of course, to investigate the situational differences and the correlational structure on the situation side, one can always aggregate over the person mode, but the net result is two unrelated models. For example, when aggregating over situations, the questions asked concern whether persons who tend to be angry also tend to be irritated. When aggregating over persons, one might ask whether situations which tend to elicit anger also tend to elicit irritation. The relations between emotional responses in the two models can be similar or even identical but they don't need to be, neither on statistical grounds, nor on psychological grounds (see further).

Studying the correlational structure of two modes simultaneously instead of only studying a single mode, is new in behavioral research, but it is nevertheless reasonable and doable. In the context of the emotional response data set, it can be hypothesized that on the one hand, it is possible that the proneness of individuals to one response is positively correlated to the proneness to other responses because of general underlying traits related to negative and positive affect. On the other hand, an emotional response may be less correlated or even negatively correlated to similar other emotional responses when one looks at the responses from a situational perspective, for example because there is room for only one response at a time or because there are strong implicit or explicit situational rules. As a concrete example, the same persons may be inclined to both emotions of anger and guilt (but depending on the situations) whereas anger situations and guilt situations do not overlap (e.g., Vansteelandt, Van Mechelen, & Nezlek, 2005; Zelenski & Larsen, 2000).

Given that the interest is in both the individual differences structure and the situational differences structure, an approach is needed that allows one to model the structure at two sides of the data array simultaneously. This is exactly what the *double-structure structural equation model* (2sSEM) aims at. Basically, it will be assumed that, similarly as for persons, also the situations are sampled from a population (often called 'a universe'). In the next step, a joint model will be built for the individual differences and situational differences structure of the data. A preliminary version of the approach in this paper has been proposed by De Boeck & Smits (in press).

The paper is organized as follows. First, we introduce the data set we are going to analyze and formulate the basic research questions. Second, we introduce some necessary notation and briefly revise the ideas behind the regular SEM, and based on this, we formulate the 2sSEM. Then the issue of statistical inference is discussed. The model is evaluated using a three-mode data set of persons by situations by responses about four feelings. Finally, conclusions and a discussion will be presented.

Description of the data

A central question in the research of emotions pertains to the differences between irritation and anger (e.g., Averill, 1982; Frijda, Kuipers, & ter Schure, 1989; Van Coillie & Van Mechelen, 2006). Although the two emotions can be distinguished quite easily phenomenologically, it is still unclear in what ways they exactly differ. The componential theory in emotion research forms a fruitful framework to study emotions (Scherer, Schorr, & Johnstone, 2001). It is hypothesized that emotions are associated with differential componential patterns, and that the two emotions under consideration differ as to their componential nature.

The basic hypothesis underlying the application is that emotions rely on two components: a feeling component, and an action-tendency component, and that the weight of both may differ depending on the emotion. The more specific hypothesis is that anger is relatively more action-oriented than irritation, whereas irritation is relatively more feeling-based. Two corresponding components are considered for anger and irritation: the feeling of frustration, and an antagonistic action tendency. It is hypothesized that anger is primarily associated with an antagonistic action tendency, and to a lesser degree with frustration, whereas irritation is primarily associated with the feeling of frustration and to a lesser degree with an antagonistic action tendency.

The data in this paper stem from a sample of 679 Dutch speaking last-year high school students. They were presented with 11 hypothetical situations and were asked to indicate the degree to which they would experience frustration, a tendency to take antagonistic action, irritation and anger, on a 4-point scale, ranging from 0 (not) to 3 (very strong) (De Boeck & Smits, in press; Kuppens, Van Mechelen, Smits, De Boeck, & Ceulemans, in press). To simplify the model construction however, the original data were dichotomized recoding the values 0 and 1 as 0 and 2 and 3 as 1. The 11 situations are situations that may elicit anger or irritation. Examples of such situations are: “You find out that a friend is spreading gossip about you”, “A friend lets you down on an appointment to go out”, etc.

Some summary statistics of the data set are presented in Figure 2. In order to give an idea of the individual and situational differences, the three-mode data were aggregated first over the situation mode and second over the person mode. Aggregation consisted of computing the proportion 1-answers (called proportion of endorsements). Panel (a) shows histograms for each of the four responses of the proportion of endorsements in the 11 situations. As can be seen, on average there are 70% 1-responses in the situations, except for anger where the average proportion is larger. It is clear that the situations differ on all four responses. Panel (b) is constructed similarly, but shows the proportion of endorsements for each of the four responses for the 679 persons. From panel (b) it is clear that persons differ on all four responses. Many persons show negative affect on all the situations. The skewed distribution on the manifest level may seem a problem, but note that this is not necessarily a problem on the latent level, since a normal distribution on the latent level may appear as a skewed distribution on the manifest level, depending on the item parameters (situation and response parameters in this case). The main conclusion from the summary statistics is that for both, situations and persons, sufficiently large differences are found in order to investigate covariance structures.

The basic research questions regarding these data are as follows (See Figure 3 for a graphical representation):

1. What is the individual differences structure? If each emotional response (frustration, antagonistic action tendency, irritation, anger) is considered an indicator for a separate latent variable for which individuals can differ, how are these latent variables then related to one another? Is frustration the primary component for irritation (i.e., more important than the antagonistic action tendency) and is the tendency to act antagonistically the primary component for anger? (i.e., more important than frustration). In terms of Figure 3 the hypothesis implied by the first research question is that $a > b$, and that $d > c$.

2. What is the structure on the situation side? Is it the case also for situations that frustration is the primary component of irritation, and the antagonistic action tendency is the primary component of anger? In terms of Figure 3 the hypothesis is again that $a > b$, and $d > c$.

3. If the structures of the person side and the situation side are similar, as hypothesized in 1 and 2, are the population values for effects that link the four latent

variables the same for persons and situations? In terms of Figure 3, the hypothesis is that the standardized values for a, b, c, d, are identical for the structure of individual differences and the structure of the situational differences. The hypothesis is phrased in terms of standardized values, in order for the hypothesis to be independent of the variances.

It is neither a statistical necessity, nor a psychological necessity that the structure for the modes is similar or identical. For example, the structure may be one-dimensional for the persons, with all four feelings associated to only one and the same dimension (a general dimension), whereas for situations multidimensionality applies.

Double-structure structural equation model (2sSEM)

In the case of three-mode data, one needs a model that takes into account both individual and situational differences at the same time. To derive such a model, let us first start from the formulation of two separate SEMs, one for the study of individual differences and another one for the study of situational differences. Each model can be applied to a single slice of the three mode cube. For example, the model on the person side can be applied to a single situation and the model on the situation side can be applied to a single person. The better option is to apply the model to aggregated data sets, one of each type, as explained earlier. This option will be followed to explain the model.

Following Bollen (1989), a SEM encompasses two parts. The first part is the *measurement model* which contains the equations representing the link between the observed and latent variables. The second part is the *structural model* which contains the equations that summarize the relationship between the latent variables. We will explicitly write the measurement and structural model used for the study of both individual and situational differences. This allows us to introduce the necessary notation.

It is worth emphasizing that SEM is primarily designed to be used with continuous observed variables. This is why the 2sSEM is introduced using continuous observed variables. As pointed out earlier, the data that will be analyzed are dichotomous. However, the extension to a measurement model for dichotomous observed variables is straightforward and will be explained later. In preparation of formulating the 2sSEM, the models for describing individual differences and for describing situational differences will be formulated first.

Model for individual differences

After aggregating over the situation mode, the resulting data sets are rectangular arrays of dimension $P \times R$, suitable to be used in a SEM model.

Let $\boldsymbol{x}_p$ be an R-dimensional vector of observed variables measured on each person p $(p = 1, \ldots, P)$, $\boldsymbol{\theta}_p$ is a Q-dimensional vector of latent (person) variables, and $\boldsymbol{\epsilon}_p$ is an error term. In the LISREL (Jöreskog & Sörbom, 1996) formulation of a SEM, the relationship between observed and latent variables is modeled as follows

$$\begin{aligned} \boldsymbol{x}_p^{Ex} &= \boldsymbol{A}^{Ex}\boldsymbol{\theta}_p^{Ex} + \boldsymbol{\epsilon}_p^{Ex} \\ \boldsymbol{x}_p^{En} &= \boldsymbol{A}^{En}\boldsymbol{\theta}_p^{En} + \boldsymbol{\epsilon}_p^{En} \end{aligned} \tag{1}$$

where $\boldsymbol{\theta}_p$ has been partitioned into the Q_1-dimensional subvector $\boldsymbol{\theta}_p^{Ex}$ and the Q_2-dimensional subvector $\boldsymbol{\theta}_p^{En}$ with $Q_1 + Q_2 = Q$. Accordingly, $\boldsymbol{x}_p = (\boldsymbol{x}_p^{Ex}, \boldsymbol{x}_p^{En})$, and $\boldsymbol{\epsilon} = (\boldsymbol{\epsilon}_p^{Ex}, \boldsymbol{\epsilon}_p^{En})$, where subvectors with the superscript Ex are R_1-dimensional and refer to the exogeneous part of the model whereas vectors with the superscript En are R_2-dimensional and refer to the endogeneous part of the model, with $R_1 + R_2 = R$. $\boldsymbol{A}^{Ex}$ and $\boldsymbol{A}^{En}$ are $R_1 \times Q_1$ and $R_2 \times Q_2$ loading matrices, respectively. $\boldsymbol{\epsilon}_p^{Ex}$ is uncorrelated with $\boldsymbol{\theta}_p^{Ex}$, $\boldsymbol{\theta}_p^{En}$, and $\boldsymbol{\epsilon}_p^{En}$, and has a normal distribution $N(\mathbf{0}, \boldsymbol{\Psi}^{Ex})$. $\boldsymbol{\epsilon}_p^{En}$ is uncorrelated with $\boldsymbol{\theta}_p^{Ex}$, $\boldsymbol{\theta}_p^{En}$, and $\boldsymbol{\epsilon}_p^{Ex}$, and has a normal distribution $N(\mathbf{0}, \boldsymbol{\Psi}^{En})$. Equation (1) is known as the *measurement model.*

The interrelations between latent variables is modeled as

$$\boldsymbol{\theta}_p^{En} = \boldsymbol{\Delta}\boldsymbol{\theta}_p^{En} + \boldsymbol{\Omega}\boldsymbol{\theta}_p^{Ex} + \boldsymbol{\theta}_p^{*} \tag{2}$$

where $\boldsymbol{\Delta}$ and $\boldsymbol{\Omega}$ are $Q_2 \times Q_2$ and $Q_2 \times Q_1$ parameter matrices, respectively. $\boldsymbol{\theta}_p^{*}$ is a Q_2-dimensional vector of errors on the endogenous latent variables uncorrelated with $\boldsymbol{\theta}_p^{Ex}$. $\boldsymbol{\theta}_p^{Ex}$ follows a $N(\mathbf{0}, \boldsymbol{\Sigma}_\theta)$ distribution and $\boldsymbol{\theta}_p^{*}$ follows a $N(\mathbf{0}, \boldsymbol{\Sigma}_{\theta^*})$ distribution, with $\boldsymbol{\Sigma}_{\theta^*}$ typically being diagonal. Under this model, it is assumed that $(\boldsymbol{I} - \boldsymbol{\Delta})$ is nonsingular. Equation (2) is known as the *structural model.*

Each component of $\boldsymbol{\theta}_p^{Ex}$ and $\boldsymbol{\theta}_p^{En}$ represents a concept (feeling) to be studied. The superscripts *Ex* and *En* stand for *exogenous* and *endogenous*, respectively (see e.g., Bollen, 1989). The subscripts p indicate that person variables are concerned. Note that in Equation (1) an intercept, say, $\boldsymbol{\tau} = (\tau^{Ex}, \tau^{En})$, could have been added in the regression equations. However, if the variables are assumed to be deviation from their means, as it was assumed here, the intercept can be omitted.

An example. Suppose that one is interested in checking the hypothesis mentioned in the research question 1 (see Section 2). Then, frustration and the antagonistic action tendency are the exogeneous latent variables denoted by $\theta_{p,fru}$ and $\theta_{p,ant}$, respectively. Irritation and anger are the endogeneous latent variables denoted by $\theta_{p,irr}$ and $\theta_{p,ang}$, respectively; and no interrelations between these endogeneous latent variables are considered (i.e., $\boldsymbol{\Delta} = \mathbf{0}$). Then, following (1) the measurement model would read as

$$\left.\begin{aligned} x_{p,irr} &= \alpha_1\theta_{p,irr} + \epsilon_{p,irr} \\ x_{p,ang} &= \alpha_2\theta_{p,ang} + \epsilon_{p,ang} \end{aligned}\right\} \Leftrightarrow \boldsymbol{x}_p^{En} = \boldsymbol{A}^{En}\boldsymbol{\theta}_p^{En} + \boldsymbol{\epsilon}_p^{En} \tag{3}$$

$$\left.\begin{aligned} x_{p,fru} &= \alpha_3\theta_{p,fru} + \epsilon_{p,fru} \\ x_{p,ant} &= \alpha_4\theta_{p,ant} + \epsilon_{p,ant} \end{aligned}\right\} \Leftrightarrow \boldsymbol{x}_p^{Ex} = \boldsymbol{A}^{Ex}\boldsymbol{\theta}_p^{Ex} + \boldsymbol{\epsilon}_p^{Ex} \tag{4}$$

where $\boldsymbol{x}_p^{En} = (x_{p,irr}, x_{p,ang})$, $\boldsymbol{x}_p^{Ex} = (x_{p,fru}, x_{p,ant})$, $\boldsymbol{\epsilon}_p^{En} = (\epsilon_{p,irr}, \epsilon_{p,ang})$,

$\boldsymbol{\epsilon}_p^{Ex} = (\epsilon_{p,fru}, \epsilon_{p,ant})$, $\boldsymbol{A}^{En} = \text{diag}(\alpha_1, \alpha_2)$, and $\boldsymbol{A}^{Ex} = \text{diag}(\alpha_3, \alpha_4)$. In other words, the manifest x variables are used to *measure* the latent (unobserved) θs variables in the measurement model.

The corresponding structural model is

$$\left.\begin{aligned} \theta_{p,irr} &= \omega_1 \theta_{p,fru} + \omega_2 \theta_{p,ant} + \theta^*_{p,irr} \\ \theta_{p,ang} &= \omega_3 \theta_{p,fru} + \omega_4 \theta_{p,ant} + \theta^*_{p,ang} \end{aligned}\right\} \Leftrightarrow \boldsymbol{\theta}_p^{En} = \boldsymbol{\Omega}\boldsymbol{\theta}_p^{Ex} + \boldsymbol{\theta}_p^* \tag{5}$$

The relations among the latent variables are accomplished by this structural model. Looking at the structural model, one can now see whether irritation is primarily related with frustration, more so than with the antagonistic action tendency, and whether the reverse is true for anger.

Finally, note that the model is not identified because there is only one indicator for each latent variable, which is a problem for SEM. However, as one will see, the problem vanishes when the 2sSEM will be defined. The solution is not found by using the situation specific variables as S indicators variables for each latent variable, because in that way the situational structure does not appear. Although all data would be used simultaneously, the variation that is modeled with the various situational responses as indicator variables would still be only variation between individuals.

More details about rules for identification and estimation of SEMs can be found in Bollen (1989).

Model for situational differences

After aggregating over the person mode, a SEM can be used to investigate the correlational structure of situations. The SEM is exactly the same model that was used to investigate the correlational structure of persons, including the model assumptions. The only difference relies in the notation we use for situation latent variables.

Let $\boldsymbol{\beta}_s = (\boldsymbol{\beta}_s^{Ex}, \boldsymbol{\beta}_s^{En})$ be a T-dimensional vector of latent (situation) variables. $\boldsymbol{\beta}_s^{Ex}$

is a T_1-dimensional subvector and $\boldsymbol{\beta}_s^{En}$ is a subvector of dimension T_2 with $T_1 + T_2 = T$. The subscripts s indicate that situation variables are concerned. The measurement model is

$$\begin{aligned} \boldsymbol{x}_s^{En} &= \boldsymbol{\Gamma}^{En}\boldsymbol{\beta}_s^{En} + \boldsymbol{\epsilon}_s^{En} \\ \boldsymbol{x}_s^{Ex} &= \boldsymbol{\Gamma}^{Ex}\boldsymbol{\beta}_s^{Ex} + \boldsymbol{\epsilon}_s^{Ex} \end{aligned} \tag{6}$$

where, as before, subvectors with the superscript Ex are R_1-dimensional whereas subvectors with the superscript En are R_2-dimensional. $\boldsymbol{\Gamma}^{Ex}$ and $\boldsymbol{\Gamma}^{En}$ are $R_1 \times T_1$ and $R_2 \times T_2$ loading matrices, respectively.

The corresponding structural model is

$$\boldsymbol{\beta}_s^{En} = \boldsymbol{\Upsilon}\boldsymbol{\beta}_s^{En} + \boldsymbol{\Lambda}\boldsymbol{\beta}_s^{Ex} + \boldsymbol{\beta}_s^{*} \tag{7}$$

with the $\boldsymbol{\Upsilon}$ and $\boldsymbol{\Lambda}$ being $T_2 \times T_2$ and $T_2 \times T_1$ parameter matrices, respectively, describing interrelations among latent situation variables, and $\boldsymbol{\beta}_s^{*}$ a T_2-dimensional vector of errors on the endogenous latent situation variables.

An example. Suppose that one is now interested in checking the hypothesis mentioned in the research question 2 (see Section 2). A very similar example like the one given above can be thought for the situational differences part. After the model is fitted, one can see, for example, whether irritation is primarily feeling based, whereas anger would be primarily action based.

A disadvantage of considering two different models at a time, one for the study of the person side of the data, and another for the study of the situation side of the data, is that the modeling of individual and situational differences occurs in two separate and unrelated models. When three-mode data are available, one may compare the two structures in a natural way considering both individual differences and situational differences together, because they are derived from the same set of data. This requires a model including both aspects and a simultaneous estimation of the two structures.

The 2sSEM

In order to present a general formulation of the model, assume that there are Q latent person variables and T latent situation variables. The following notation will be used. The observed random variable x_{prs} is used to denote to which extent person p exhibits response r in situation s. The index r may take four values in our data set, and these values correspond with frustration (*fru*), antagonistic action tendency (*ant*), irritation (*irr*), and anger (*ang*). Following a convention in psychometrics, θ refers to person effects and β to situation effects. The effect of responses will be represented by τ. From the context it will be clear whether an effect is fixed or random.

Measurement model. Following (1) and (6), the *measurement* model we use is

$$x_{prs} = \sum_{q=1}^{Q} \alpha_{rq}\theta_{pq} + \sum_{t=1}^{T} \gamma_{rt}\beta_{st} + \tau_r + \epsilon_{prs} = \boldsymbol{\alpha}_r^\top \boldsymbol{\theta}_p + \boldsymbol{\gamma}_r^\top \boldsymbol{\beta}_s + \tau_r + \epsilon_{prs} \tag{8}$$

where α_{rq} is the weight of the q-th $(q = 1, \ldots, Q)$ latent person variable for response r and γ_{rt} is the weight of the t-th $(t = 1, \ldots, T)$ latent situation variable for response r. τ_r is assumed to be a fixed general effect for response r, and ϵ_{prs} is an error term.

The measurement model can also be written in matrix form. First, collect all responses pertaining to the same person-situation combination in the $R \times 1$ vector $\boldsymbol{x}_{ps}$. The row vectors of weights ($\boldsymbol{\alpha}_r'$ and $\boldsymbol{\gamma}_r'$) can be stacked below each other in the weight matrices $\boldsymbol{A}$ and $\boldsymbol{\Gamma}$. The model now becomes:

$$\boldsymbol{x}_{ps} = \boldsymbol{A}\boldsymbol{\theta}_p + \boldsymbol{\Gamma}\boldsymbol{\beta}_s + \boldsymbol{\tau} + \boldsymbol{\epsilon}_{ps} \tag{9}$$

with

$$\begin{pmatrix} x_{p1s} \\ \vdots \\ x_{prs} \\ \vdots \\ x_{pRs} \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \cdots & \alpha_{1Q} \\ \vdots & \ddots & \vdots \\ \alpha_{r1} & \cdots & \alpha_{rQ} \\ \vdots & \ddots & \vdots \\ \alpha_{R1} & \cdots & \alpha_{RQ} \end{pmatrix} \begin{pmatrix} \theta_{p1} \\ \vdots \\ \theta_{pq} \\ \vdots \\ \theta_{pQ} \end{pmatrix} + \begin{pmatrix} \gamma_{11} & \cdots & \gamma_{1T} \\ \vdots & \ddots & \vdots \\ \gamma_{r1} & \cdots & \gamma_{rT} \\ \vdots & \ddots & \vdots \\ \gamma_{R1} & \cdots & \gamma_{RT} \end{pmatrix} \begin{pmatrix} \beta_{s1} \\ \vdots \\ \beta_{st} \\ \vdots \\ \beta_{sT} \end{pmatrix} + \begin{pmatrix} \tau_{1} \\ \vdots \\ \tau_{r} \\ \vdots \\ \tau_{R} \end{pmatrix} + \begin{pmatrix} \epsilon_{p1s} \\ \vdots \\ \epsilon_{prs} \\ \vdots \\ \epsilon_{pRs} \end{pmatrix} \quad (10)$$

Note that for the specific set of models considered for the data set discussed in this article, one has $Q = R$ and $T = R$. However, it is perfectly possible to formulate a model in which $Q < R$ and $T < R$, and also $Q \neq T$. For reasons of simplicity, the measurements parts of the models fitted in this paper will have loading matrices that are known and binary. In this case, $\alpha_{rq} = 1$ when $r = q$ and 0 otherwise, and, correspondingly, $\gamma_{rt} = 1$ when $r = t$, and 0 otherwise.

Note that now one has multiple indicators for θ_{pr}, because x_{pr1} to x_{prS} are indicators, and similarly one has multiple indicators for β_{sr}, because x_{1rs} to x_{Prs} are indicators.

(Double-structure) Structural model. For the structural model, again a partition into endogeneous and exogeneous latent variables is made, both, for latent person variables and for latent situation variables: $\boldsymbol{\theta}_p = (\boldsymbol{\theta}_p^{En}, \boldsymbol{\theta}_p^{Ex})$ and $\boldsymbol{\beta}_s = (\boldsymbol{\beta}_s^{En}, \boldsymbol{\beta}_s^{Ex})$.

The two-fold *structural* model can now be defined as follows:

$$\begin{aligned} \boldsymbol{\theta}_p^{En} &= \boldsymbol{\Delta}\boldsymbol{\theta}_p^{En} + \boldsymbol{\Omega}\boldsymbol{\theta}_p^{Ex} + \boldsymbol{\theta}_p^{*} \\ \boldsymbol{\beta}_s^{En} &= \boldsymbol{\Upsilon}\boldsymbol{\beta}_s^{En} + \boldsymbol{\Lambda}\boldsymbol{\beta}_s^{Ex} + \boldsymbol{\beta}_s^{*} \end{aligned} \quad (11)$$

The vector of exogenous latent person variables $\boldsymbol{\theta}_p^{Ex}$ follows a Q_2-variate normal distribution with a mean vector of all zeros and a variance-covariance matrix Σ_θ. Independently of the exogenous variables, the vector of error terms, $\boldsymbol{\theta}_p^{*}$, follows a

Q_1-variate normal distribution with mean vector of all zeros and diagonal variance-covariance matrix Σ_{θ^*}. The model at the situation side can be explained in a completely analogous manner. Note that in Equation (11) a structure is imposed on both, the person latent variables and the situation latent variables and both, θs and βs, are modeled as random effects.

The standard graphical way of representing the SEM is by mean of a path diagram (e.g., Bollen, 1989). This graphical pictorial representation of a system of simultaneous equations can be transferred to the 2sSEM as well. Straight one-headed arrows represent the directional influence between the variables connected by the arrows. Curved two-headed arrows represent correlations between variables. In the application, we will make use of the graphical representation to introduce several models; see Figure 4. The observed variables and the error terms are not shown.

The model for dichotomous data. A latent threshold formulation will be used to make the model suitable for analyzing binary data. Consider a random variable Y_{prs} logistically distributed with mean $\boldsymbol{\alpha}_r^\top \boldsymbol{\theta}_p + \boldsymbol{\gamma}_r^\top \boldsymbol{\beta}_s + \tau_r$ and the scale parameter equal to one (or equivalently, assume that ϵ_{prs} in (8) follows a standard logistic distribution). This continuous variable is considered to be latent, and it is mapped into the manifest binary variable X_{prs} in the following way: $X_{prs} = 1$ if $Y_{prs} > 0$ and $X_{prs} = 0$ otherwise. More precisely,

$$\begin{aligned} E(X_{prs} \mid \theta_p, \beta_s) = \Pr(X_{prs} = 1 \mid \theta_p, \beta_s) = & \quad \pi_{prs} \\ = & \quad \Pr(Y_{prs} > 0 \mid \theta_p, \beta_s) \\ = & \quad \frac{\exp(\boldsymbol{\alpha}_r^\top \boldsymbol{\theta}_p + \boldsymbol{\gamma}_r^\top \boldsymbol{\beta}_s + \tau_r)}{1+\exp(\boldsymbol{\alpha}_r^\top \boldsymbol{\theta}_p + \boldsymbol{\gamma}_r^\top \boldsymbol{\beta}_s + \tau_r)} \end{aligned} \tag{12}$$

such that

$$\log\left(\frac{\pi_{prs}}{1-\pi_{prs}}\right) = \text{logit}(\pi_{prs}) = \eta_{prs} = \boldsymbol{\alpha}_r^\top \boldsymbol{\theta}_p + \boldsymbol{\gamma}_r^\top \boldsymbol{\beta}_s + \tau_r \tag{13}$$

Note that the resulting response model is a nonlinear mixed model (NLMM) specified by a bilinear predictor η_{prs}, a logit link, and a distribution from the exponential family, say, the Bernoulli distribution of the Xs. If the loadings are assumed to be fixed, as will be the case in this paper, the model is more specifically a generalized linear mixed model (GLMM). Note also that now the τs parameters can be more clearly interpreted as an indication of the probability (on the logistic scale) of showing each of the r ($r = 1, \ldots, R$) responses if all latent variables are set to zero.

Statistical inference

Bayesian estimation

The 2sSEM is a model with random effects for two of the three modes in the data. Therefore, it is a crossed-random effects model (e.g., Snijders & Bosker, 1999; Janssen, Schepers, & Peres, 2004). The estimation of the parameters of such a model is not trivial (Tuerlinckx, Rijmen, Verbeke, & De Boeck, 2006). The reason is that the likelihood contains an integral of a very high dimension (at minimum as high as the product of the number of elements of the modes with random effects). The well known Gauss-Hermite quadrature method becomes unfeasible, because of the high dimensionality of the integrals in the likelihood function. Also Laplace approximations tend to break down because the dimensionality of the integral grows as a function of the sample size. (e.g., Shun & McCullagh, 1995; Shun, 1997). Possible alternative solutions, are Monte-Carlo and Quasi-Monte Carlo methods for numerical integration (e.g., González, Tuerlinckx, De Boeck, & Cools, 2006), and Bayesian methods, which is the approach taken in this paper. De Boeck & Smits (in press) solved the problem by fitting a nonlinear mixed model with only random effects at the person side, and with fixed β's being a function of other fixed β's. Unfortunately, by assuming situation parameters (β's) to be fixed effects, one does not fit a genuine 2sSEM, but only a fixed-effects approximation.

Bayesian inference is based on the posterior distribution of all parameters of interest given the observed data. Let $\boldsymbol{\vartheta}$ be the vector of parameters of interest and $\boldsymbol{x}$ the observed data. If $\boldsymbol{X} \mid \boldsymbol{\vartheta} \sim f(\boldsymbol{x} \mid \boldsymbol{\vartheta})$, then Bayes' theorem leads to the following relation

$$f(\boldsymbol{\vartheta} \mid \boldsymbol{x}) \propto f(\boldsymbol{x} \mid \boldsymbol{\vartheta}) f(\boldsymbol{\vartheta}) \tag{14}$$

which shows that the posterior density is proportional to the likelihood function times the prior. The idea is then to obtain a sample from the posterior distribution and use the posterior mean and posterior confidence intervals to summarize the distribution of each parameter of interest.

In the particular case of the 2sSEM for a three-mode data set with persons, situations and responses, $\boldsymbol{\vartheta} = (\boldsymbol{\theta}_p, \boldsymbol{\theta}^*_p, \boldsymbol{\beta}_s, \boldsymbol{\beta}^*_s, \boldsymbol{\Delta}, \boldsymbol{\Omega}, \boldsymbol{\Upsilon}, \boldsymbol{\Lambda}, \boldsymbol{\tau}, \boldsymbol{\Sigma}_\theta, \boldsymbol{\Sigma}_\beta, \boldsymbol{\Sigma}_{\theta^*}, \boldsymbol{\Sigma}_{\beta^*})$. For the intercepts τs and the loading and effect parameters in $\boldsymbol{\Delta}, \boldsymbol{\Omega}, \boldsymbol{\Upsilon}$, and $\boldsymbol{\Lambda}$, normal diffuse priors were chosen. For the covariance matrices $\boldsymbol{\Sigma}_\theta, \boldsymbol{\Sigma}_\beta, \boldsymbol{\Sigma}_{\theta^*}$, and $\boldsymbol{\Sigma}_{\beta^*}$, inverse-Wishart prior distributions were chosen. Finally, the distribution for person and situation latent variables are given by the model assumptions and are multivariate normal distributions with zero vector mean and covariance matrices $\boldsymbol{\Sigma}_\theta$ and $\boldsymbol{\Sigma}_\beta$, respectively.

Using the relation in (14), the posterior distribution is then given by the product of the likelihood

$$f(\boldsymbol{x} \mid \boldsymbol{\vartheta}) = \left(\prod_{p=1}^{P} \prod_{r=1}^{R} \prod_{s=1}^{S} \pi_{prs}^{x_{prs}} [1 - \pi_{prs}]^{1-x_{prs}} \right) \tag{15}$$

and the prior

$$\begin{aligned} f(\boldsymbol{\vartheta}) &= \prod_{p=1}^{P} N(\boldsymbol{\theta}_p; \mathbf{0}, \boldsymbol{\Sigma}_\theta) \prod_{p=1}^{P} N(\boldsymbol{\theta}^*_p; \mathbf{0}, \boldsymbol{\Sigma}_{\theta^*}) \prod_{s=1}^{S} N(\boldsymbol{\beta}_s; \mathbf{0}, \boldsymbol{\Sigma}_\beta) \prod_{s=1}^{S} N(\boldsymbol{\beta}^*_s; \mathbf{0}, \boldsymbol{\Sigma}_{\beta^*}) \\ &\times \prod_{r=1}^{R} p(\delta_r) \prod_{r=1}^{R} p(\omega_r) \prod_{r=1}^{R} p(\upsilon_r) \prod_{r=1}^{R} p(\lambda_r) \prod_{r=1}^{R} p(\tau_r) \times p(\boldsymbol{\Sigma}_\theta) p(\boldsymbol{\Sigma}_\beta) p(\boldsymbol{\Sigma}_{\theta^*}) p(\boldsymbol{\Sigma}_{\beta^*}) \end{aligned} \tag{16}$$

where $N(\boldsymbol{Z}; \boldsymbol{\mu}, \boldsymbol{\Sigma})$ means that the random vector $\boldsymbol{Z}$ is distributed as a multivariate normal distribution with vector mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. Note that independence of the priors is assumed.

In order to obtain a sample from the posterior distribution, different iterative methods belonging to the class of Markov Chain Monte Carlo (MCMC) techniques (e.g., Gelman, Carlin, Stern, & Rubin, 1995) have been developed. WinBUGS (Spiegelhalter, Thomas, Best, & Lunn, 2003) is a software program for Bayesian analysis of complex statistical models using MCMC techniques. After providing a likelihood and prior distribution, the program automatically draws a sample of all parameters from their posterior distribution. Once convergence has been reached, parameter estimates can be obtained and inferences can be made.

All models in this paper were fitted with the WinBUGS software. Convergence was assessed using the CODA R package (Best, Cowles, & Vines, 1997) which implements standard convergence criteria (e.g., Cowles & Carlin, 1996). The main WinBUGS code that was used is shown in the Appendix.

Bayesian model selection and model checking

Different models for the data will be fitted. For model selection, the deviance information criterion (DIC) (Spiegelhalter, Best, Carlin, & Van der Linde, 2002) will be used which is calculated as a compromise between model fit and the number of effective parameters in the model. DIC is defined as

$$DIC = \overline{D(\boldsymbol{\vartheta})} + p_D \tag{17}$$

where

$$\overline{D(\boldsymbol{\vartheta})} = E_{\boldsymbol{\vartheta}|\boldsymbol{x}}\{-2\log f(\boldsymbol{x} \mid \boldsymbol{\vartheta})\}$$

is the posterior mean of the deviance, used to measure model fit and,

$$p_D = E_{\boldsymbol{\vartheta}|\boldsymbol{x}}\{-2\log f(\boldsymbol{x} \mid \boldsymbol{\vartheta})\} + 2\log f(\boldsymbol{x} \mid \widehat{\boldsymbol{\vartheta}})\}$$

is an estimate of the effective number of parameters. Lower values of the criterion indicate models with a better fit (for more details, see Spiegelhalter et. al, 2002).

After a model is selected, it is recommended to evaluate whether it fits the data in a global way. For this purpose use is made of the sample from the posterior distribution via posterior predictive checks (PPC) (e.g., Gelman et al., 1995). The main idea behind PPC is that "if the model fits, then replicated data generated under the model should look similar to observed data" (Gelman et al., 1995, p.165). The way to measure the discrepancy between some features of the model and data is to use a test quantity or discrepancy measure $T(\boldsymbol{x}, \boldsymbol{\vartheta})$ which is calculated on the data and may depend on the model parameters. When the test quantity $T(\cdot, \cdot)$ does not depend on the model parameters, then it is called a test statistic and it is denoted by $T(\boldsymbol{x})$. Then, lack of fit can be assessed by the tail-area probability or p-value of the test quantity. Note that the PPC procedure allows one to choose any test quantity which can be used to check a particular relevant aspect of the model. In practice, given a discrepancy measure $T(\boldsymbol{x}, \boldsymbol{\vartheta})$, the posterior predictive check p-value is calculated as follows:

1. Draw a vector $\boldsymbol{\vartheta}^{(k)}$ from the posterior distribution.
2. Simulate a replicated data set $\boldsymbol{x}_{rep}^{(k)}$ from $f(\boldsymbol{x} \mid \boldsymbol{\vartheta}^{(k)})$.
3. Calculate $T(\boldsymbol{x}, \boldsymbol{\vartheta}^{(k)})$ and $T(\boldsymbol{x}_{rep}^{(k)}, \boldsymbol{\vartheta}^{(k)})$
4. Repeat 1 to 3 K times.
5. Count the proportion of replicated data sets $\boldsymbol{x}_{rep}^{(k)}$ for which $T(\boldsymbol{x}_{rep}^{(k)}, \boldsymbol{\vartheta}^{(k)}) \geq T(\boldsymbol{x}, \boldsymbol{\vartheta}^{(k)})$.

Note that when a test statistic $T(\boldsymbol{x})$ is used, step 3 does not require a model estimation at each iteration k, but only the calculation of $T(\boldsymbol{x})$, and $T(\boldsymbol{x}_{rep}^{(k)})$ for each replicated data set.

Another useful tool to look at is the graphical display. In the case of discrepancy measures, one can plot $T(\boldsymbol{x}_{rep}^{(k)}, \boldsymbol{\vartheta}^{(k)})$ and $T(\boldsymbol{x}, \boldsymbol{\vartheta}^{(k)})$ against each other. In the case of test statistics one can compare $T(\boldsymbol{x})$ with $T(\boldsymbol{x}_{rep}^{(1)}, \ldots, \boldsymbol{x}_{rep}^{(K)})$ by localizing $T(\boldsymbol{x})$ in the frequency distribution of $T(\boldsymbol{x}_{rep})$.

Application

Model

Six different models will be fitted to the data. For convenience of notation, let us represent frustration, antagonistic action tendency, irritation, and anger with indices 1, 2, 3 and 4, respectively. Frustration and the antagonistic action tendency are considered as the exogeneous latent variables and, irritation and anger are considered as the endogeneous latent variables. Thus, $\boldsymbol{\theta}_p^{Ex} = (\theta_{p1}, \theta_{p2})$, $\boldsymbol{\theta}_p^{En} = (\theta_{p3}, \theta_{p4})$, $\boldsymbol{\beta}_s^{Ex} = (\beta_{s1}, \beta_{s2})$, and $\boldsymbol{\beta}_s^{En} = (\beta_{s3}, \beta_{s4})$.

The six models have the same measurement model but differ with respect to their structural part. In general, the model equations can be written as

$$\eta_{prs} = \theta_{pr} + \beta_{sr} + \tau_r \qquad \text{Measurement model} \tag{18}$$

and

$$\begin{aligned} \boldsymbol{\theta}_p^{En} &= \boldsymbol{\Omega}\boldsymbol{\theta}_p^{Ex} + \boldsymbol{\theta}_p^{*} \qquad \text{Structural model} \\ \boldsymbol{\beta}_s^{En} &= \boldsymbol{\Lambda}\boldsymbol{\beta}_s^{Ex} + \boldsymbol{\beta}_s^{*} \end{aligned} \tag{19}$$

with $p = 1, \ldots, 679,\ r = 1, \ldots, 4$, and $s = 1, \ldots, 11$. As already mentioned, the loading matrices in the measurement model do not contain any free parameters but are fixed instead. For every response, there is a separate latent variable on the person and on the situation side, so that the loadings in Equation (8) equal 1 if $q = r$ and $t = r$ and 0 otherwise with $Q = T = 4$. We assume that there is no effect of latent endogeneous variables on one another; neither on the person side (i.e., $\boldsymbol{\Delta} = \boldsymbol{0}$), nor on the situation side (i.e., $\boldsymbol{\Upsilon} = \boldsymbol{0}$) (see Equation (11)). Finally, as stated earlier, it is assumed that

$\boldsymbol{\theta}_p^{Ex} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_\theta)$, $\boldsymbol{\beta}_s^{Ex} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_\beta)$, $\boldsymbol{\theta}_p^* \sim N(\mathbf{0}, \boldsymbol{\Sigma}_{\theta^*})$, and $\boldsymbol{\beta}_s^* \sim N(\mathbf{0}, \boldsymbol{\Sigma}_{\beta^*})$, with both $\boldsymbol{\Sigma}_{\theta^*}$ and $\boldsymbol{\Sigma}_{\beta^*}$ being diagonal matrices and in addition with an equal variance for both θs such that, $\boldsymbol{\Sigma}_{\theta^*} = \sigma^2_{\theta^*} I$ and an equal variance for both βs, such that $\boldsymbol{\Sigma}_{\beta^*} = \sigma^2_{\beta^*} I$, respectively, where I is the identity matrix. A model allowing different variances was also fitted, but this did not result in a better fit. Note that, because of the zero correlations among the residuals, the model is not a saturated SEM.

The different structural models can be most easily presented by means of their graphical representation. Figure 4 shows the representation for three models (the other three models that will be fitted will be derived from these). Model 1 is the most general model in which both irritation and anger are determined by frustration and an antagonistic tendency. Model 2 is the extreme form of the earlier mentioned hypothesis, with irritation based exclusively on frustration, and anger based exclusively on the antagonistic action tendency. Model 3 is the opposite, with irritation exclusively based on the action tendency and anger exclusively based on feelings. The corresponding parameter matrices $\boldsymbol{\Omega}$ and $\boldsymbol{\Lambda}$ in the structural model are shown in Table 1. The three remaining models are constructed by restricting the parameter matrices of the person and situation side to be equal ($\boldsymbol{\Omega} = \boldsymbol{\Lambda}$). These models will be denoted as Models 1r, 2r and 3r, with r denoting the equality restriction in question.

Estimation and model selection

Five chains were ran starting from different initial values for the parameters of interest in each of the fitted models. Each chain was run with 5000 iterations, discarding the first half of each chain as a burn-in stage. For the inferences and model checking we accepted every fifth draw of the remaining 2500 draws of each of the five chains. This helps to reduce autocorrelation and then to avoid dependence between subsequent draws. Thus, the final sample size of iterations was 2500 and the results that will be reported are based on it.

As a convergence diagnostics the $\widehat{R}$ (Gelman & Rubin, 1992) was used, which is implemented in CODA. Also, graphical tools implemented in CODA to check the mixing and autocorrelation of the chains were used.

When estimating the model under the parametrization in (18) and (19) a poor mixing of the chains for the τ parameters was obtained. This is because the τs (fixed effect) parameters are involved as a term in sums of random effects. In other words, the τs appear in the mean likelihood contribution for each person p and each situation s. Then, a strong correlation between τs and all the random effects arises. To solve this problem we used hierarchical centering (e.g., Gelfand, Sahu, & Carlin, 1995; Browne, 2004). The idea is to consider the τ parameters as the mean of one of the set of random effects. Thus, the model was re-parameterized as follows

$$\eta_{prs} = \theta_{pr} + \widetilde{\beta}_{sr} \quad \text{with} \quad \widetilde{\beta}_{sr} = \beta_{sr} + \tau_r \tag{20}$$

and

$$\begin{aligned} \boldsymbol{\theta}_p^{En} &= \boldsymbol{\Omega}\boldsymbol{\theta}_p^{Ex} + \boldsymbol{\theta}_p^{*} \\ \widetilde{\boldsymbol{\beta}}_s^{En} &= \boldsymbol{\Lambda}\widetilde{\boldsymbol{\beta}}_s^{Ex} + \boldsymbol{\beta}_s^{*} \end{aligned} \tag{21}$$

and where we now assume that $\begin{pmatrix} \widetilde{\beta}_{s1} \\ \widetilde{\beta}_{s2} \end{pmatrix} \sim N\left(\begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix}, \boldsymbol{\Sigma}_{\beta}\right)$ and $\begin{pmatrix} \beta^*_{s3} \\ \beta^*_{s4} \end{pmatrix} \sim N\left(\begin{pmatrix} \tau_3 \\ \tau_4 \end{pmatrix}, \boldsymbol{\Sigma}_{\beta^*}\right)$

Note that under this reparametrization, the values for τ_1 and τ_2 remain the same as before the reparametrization. However, given that $\widetilde{\beta}_{s1}$ and $\widetilde{\beta}_{s2}$ play a role in $\widetilde{\beta}_{s3}$ and $\widetilde{\beta}_{s4}$ the values for τ_3 and τ_4 can no longer be interpreted as explained before (as an indication of the probability on the logistic scale showing response 3 and 4, respectively). This interpretation now applies to $\tilde{\tau}_3$ and $\tilde{\tau}_4$, respectively.

$$\widetilde{\tau}_3 = \lambda_1\tau_1 + \lambda_2\tau_2 + \tau_3 \quad \text{, and} \quad \widetilde{\tau}_4 = \lambda_3\tau_1 + \lambda_4\tau_2 + \tau_4 \tag{22}$$

After reparametrization, all the $\widehat{R}$ statistics were less than or equal to 1.03 for all the models, so that convergence seems established (Gelman et. al, 1995).

Table 2 shows the obtained DIC values for all the fitted models. From these results, it is clear that Models 1 and 1r are to be preferred above the others. The model with the lowest DIC, and then the preferred model, was Model 1r in which the person and situation structure are restricted to be equal both qualitatively and quantitatively. It must be noted however that the difference in DIC values seems to be relatively small.

Model checking

Two types of tests were carried out. The first test checks a particular aspect of our model, while the second is a general omnibus test of goodness-of-fit. More precisely, given that our main interest is to evaluate the correlation structure of the four traits (frustration, antagonistic action tendency, irritation, and anger), the correlation between the sum scores over persons, $Corr(x_{+rs}, x_{+r's})$, and over situations, $Corr(x_{pr+}, x_{pr'+})$ for each of the constructs ($r = 1, \ldots, 4$) was used as a test statistic. The second test, which assesses the model fit in a global way, is the χ^2 discrepancy measure (Gelman et al, 1995). In the context of the application, the Pearson χ^2 measure is defined as

$$\chi^2(x, \boldsymbol{\vartheta}) = \sum_{p=1}^{P}\sum_{r=1}^{R}\sum_{s=1}^{S} \frac{(x_{prs} - E(x_{prs} \mid \boldsymbol{\vartheta}))^2}{Var(x_{prs} \mid \boldsymbol{\vartheta})} \tag{23}$$

When using x_{+rs}, the estimated p-values varied between 0.25 and 0.52 with a median of 0.36, whereas when using x_{pr+}, the estimated p-values varied between 0.08 and 0.97 with a median of 0.84. Considering a reasonable range for the p-value to be the interval 0.05 and 0.95 (Gelman et al, 1995) it can be seen that the model fits the data.

The more extreme values of the p-values when using x_{pr+} are a consequence of the smaller number of situations considered.

Figure 5 shows an scatter plot of the χ^2 test quantity evaluated for the observed and the replicated data. The posterior p-value is also reported in the figure and it is calculated as the proportion of points above the diagonal line (i.e., the proportion of observed values that is larger than the replicated ones). From Figure 5 it can be concluded that the model fits the data also reasonably well in a global way.

Results

Table 3 shows the posterior means and standard deviations of the parameters in Model 1r. Figure 6 contains the graphical representation of the structural part of Model 1r. The numbers refer to the corresponding estimated parameter values.

For the latent person structure, it can be seen that frustration and the antagonistic action tendency both have an effect on irritation and anger. On the contrary, for irritation, the feeling of frustration is more important than the antagonistic tendency, as hypothesized, and for anger, the antagonistic tendency is more important than the feeling of frustration again as hypothesized. For irritation, the difference is very small (0.49 versus 0.46); but for anger it is much larger (0.54 versus 0.30). For persons, the correlation between the exogeneous latent person variables is moderately high and positive. On the situation side, the correlation between the exogeneous latent situation variables is much lower but still positive. The posterior standard deviation for correlations on the situation side is larger than the one on the situation side due to a smaller sample of situations.

The τ parameters can be interpreted as an indicator of the probability of showing each of the four responses, when all latent variables are set to zero (on the logistic scale). Stated otherwise, for an average person, in an average situation, the probability of endorsing response r, equals:

$$\pi_{p1s} = \frac{\exp(1.56)}{1+\exp(1.56)} = 0.83, \quad \pi_{p2s} = \frac{\exp(1.51)}{1+\exp(1.51)} = 0.82 \tag{24}$$
$$\pi_{p3s} = \frac{\exp(1.74)}{1+\exp(1.74)} = 0.85, \quad \pi_{p4s} = \frac{\exp(1.80)}{1+\exp(1.80)} = 0.86$$

Then, for instance, the probability of being angry or feeling irritated is slightly larger than the probability of feeling frustrated and feeling an antagonistic action tendency.

It can be concluded that the leading hypotheses of the study are confirmed. The set of questions before mentioned in Section 2, can now be answered in the following way. Regarding Question 1, one should look at the $\mathbf{\Omega}$ matrix, and conclude that irritation seems to be slightly more feeling based but the difference is negligible, whereas anger is clearly more action based. Regarding Question 2, one should look at the $\mathbf{\Lambda}$ matrix and, in this particular case, the conclusions coincide with the ones stated for the person side because the best fitted model is the one in which $\mathbf{\Omega}$ and $\mathbf{\Lambda}$ matrices are assumed to be equal. Regarding Question 3, the conclusion is that the equality of the two structures, the one of individual differences and the one of situational differences, is not contradicted by the data.

Discussion and Conclusion

An extension of the regular SEM, called double-structure structural equation model (2sSEM), has been presented. The model is shown to be useful for the study of underlying structures in two of the the three modes of a three-mode data. A study of emotions using a persons by situations by responses three-mode data set shows that the model can be used to assess individual differences and situational differences simultaneously without the necessity of using two separate and unrelated models. Using a common model for both structures can be advantageous in the sense that no loss of information is implied.

A special feature of the data we consider here is that they are binary after dichotomization. A person judges a response to occur in a situation or not. The model developed is thus in the first place an extension of a nonlinear mixed model for binary data. However, the model can be easily adapted for ordered-category data (rating-scale data), and for continuous data.

Other techniques for the analysis of three-mode data have been proposed and used earlier such as three-mode factor analysis (Tucker, 1966), three-mode principal component analysis (Kroonenberg, 1983), for continuous data, and, the three-way hierarchical classes models (e.g., Ceulemans & Van Mechelen, 2005; Ceulemans, Van Mechelen, & Leenen, 2003; Leenen, Van Mechelen, De Boeck, & Rosenberg, 1999), for binary data. All of them refer to decomposition models that reduce each of the three modes of the data to a few components. The main similarities and differences between these techniques and the 2sSEM will be the subject of future study.

In this paper, the focus is on data sets of a particular kind: a fully crossed three-dimensional data array with binary data on persons, situations, and emotional responses. Of course, other kinds of three-mode can be thought of. For instance, suppose that a sample of persons has been asked to rate another sample of persons on a set of personality items. The items can be thought of measuring latent variables. The structure of the raters is not necessarily equal to the structure of the rated persons. One could even argue that the equality of both structures is suspicious since then the former might be a mere reflection of beliefs from the part of the raters.

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Appendix

WinBUGS code used to fit selected model 1r. Note that in order to fit the other models showed in this paper, the modification of the code is straightforward. A brief explanation of the code is given below.

```
model;
{
 for(i in 1:n){
   for( j in 1:J) {
         logit(p[i,j,1]) <- theta[i,1] + beta[j,1]
         x[i,j,1] ~ dbern(p[i,j,1])
         x.rep[i,j,1] ~ dbern(p[i,j,1])
    t[i,j,1]<-pow(x[i,j,1]-p[i,j,1],2)/(p[i,j,1]*(1-p[i,j,1]))
    t.rep[i,j,1]<-pow(x.rep[i,j,1]-p[i,j,1],2)/(p[i,j,1]*(1-p[i,j,1]))
         logit(p[i,j,2]) <- theta[i,2] + beta[j,2]
         x[i,j,2] ~ dbern(p[i,j,2])
         x.rep[i,j,2] ~ dbern(p[i,j,2])
    t[i,j,2]<-pow(x[i,j,2]-p[i,j,2],2)/(p[i,j,2]*(1-p[i,j,2]))
    t.rep[i,j,2]<-pow(x.rep[i,j,2]-p[i,j,2],2)/(p[i,j,2]*(1-p[i,j,2]))
         logit(p[i,j,3]) <- w[1]*theta[i,1] +w[2]*theta[i,2]+theta3[i]+w[1]*beta[j,1]+w[2]*beta[j,2]+beta3[j]
         x[i,j,3] ~ dbern(p[i,j,3])
         x.rep[i,j,3] ~ dbern(p[i,j,3])
    t[i,j,3]<-pow(x[i,j,3]-p[i,j,3],2)/(p[i,j,3]*(1-p[i,j,3]))
    t.rep[i,j,3]<-pow(x.rep[i,j,3]-p[i,j,3],2)/(p[i,j,3]*(1-p[i,j,3]))
         logit(p[i,j,4]) <- w[3]*theta[i,1] +w[4]*theta[i,2]+theta4[i]+w[3]*beta[j,1]+w[4]*beta[j,2]+beta4[j]
         x[i,j,4] ~ dbern(p[i,j,4])
         x.rep[i,j,4] ~ dbern(p[i,j,4])
    t[i,j,4]<-pow(x[i,j,4]-p[i,j,4],2)/(p[i,j,4]*(1-p[i,j,4]))
    t.rep[i,j,4]<-pow(x.rep[i,j,4]-p[i,j,4],2)/(p[i,j,4]*(1-p[i,j,4]))
}
 theta[i,1:2] ~ dmnorm(mu.theta[],R[,])
 theta3[i]~dnorm(0,tt)
 theta4[i]~dnorm(0,tt)
}
 tt ~ dgamma(0.0001,0.0001)
 vt<-1/tt
```

```
#30#  mu.theta[1]<-0
#31#  mu.theta[2]<-0
#32#  R[1:2,1:2]~dwish(O[,],2)
#33#  O[1,1]<-0.001
#34#  O[2,1]<-0
#35#  O[1,2]<-0
#36#  O[2,2]<-0.001
#37#  for( j in 1 : J) {
#38#     beta[j,1:2] ~ dmnorm(mu.beta[],E[,])
#39#     beta3[j] ~ dnorm(tau[3],tb)
#40#     beta4[j] ~ dnorm(tau[4],tb)
#41#  }
#42# tb ~ dgamma(0.0001,0.0001)
#43# vb<-1/tb
#44# mu.beta[1]<-tau[1]
#45# mu.beta[2]<-tau[2]
#46# E[1:2,1:2]~dwish(G[,],2)
#47# G[1,1]<-0.001
#48# G[2,1]<-0
#49# G[1,2]<-0
#50# G[2,2]<-0.001
#51# for(k in 1:4){
#52# tau[k]~ dnorm(0,0.0001)
#53# w[k]~ dnorm(0,0.0001)
#54# }
#55# SigmaT[1:2,1:2]<-inverse(R[1:2,1:2])
#56# SigmaB[1:2,1:2]<-inverse(E[1:2,1:2])
#57# CorrT<-SigmaT[1,2]/sqrt(SigmaT[1,1]*SigmaT[2,2])
#58# CorrB<-SigmaB[1,2]/sqrt(SigmaB[1,1]*SigmaB[2,2])
#59# tau3<-w[1]*tau[1]+w[2]*tau[2]+tau[3]
#60# tau4<-w[3]*tau[1]+w[4]*tau[2]+tau[4]
#61# Tobs<-sum(t[,,])
#62# Trep<-sum(t.rep[,,])
#63# p.value <- step(Trep-Tobs)
}
```

From lines 1 to 23 the likelihood of the model in equations (18) and (19) is specified.

From 24 to 54, the prior distributions for the parameters are specified. From 55 to the end of the code, covariance matrices, correlations, reparametrized values of τ (see Equation (22)) and the p-values are obtained.

Author Note

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	Model 1	Model 2	Model 3
$\mathbf{\Omega}$	$\begin{pmatrix} \omega_1 & \omega_2 \\ \omega_3 & \omega_4 \end{pmatrix}$	$\begin{pmatrix} \omega_1 & 0 \\ 0 & \omega_4 \end{pmatrix}$	$\begin{pmatrix} 0 & \omega_2 \\ \omega_3 & 0 \end{pmatrix}$
$\mathbf{\Lambda}$	$\begin{pmatrix} \lambda_1 & \lambda_2 \\ \lambda_3 & \lambda_4 \end{pmatrix}$	$\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_4 \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda_2 \\ \lambda_3 & 0 \end{pmatrix}$

Table 1

Parameter matrices in the structural model for the three 2sSEM estimated models

	DIC	p_D
Model 1	25211.80	1456.53
Model 1r	25209.50	1451.83
Model 2	25335.80	1440.09
Model 2r	25337.80	1444.47
Model 3	25424.60	1454.99
Model 3r	25423.10	1457.20

Table 2

DIC and p_D values for the three fitted models (lower values are preferable)

Model 1r					
	Value	sd		Value	sd
$\omega_1 = \lambda_1$	0.30	0.04	$\sigma_{\theta_{p1},\theta_{p2}}$	2.19	0.20
$\omega_2 = \lambda_2$	0.54	0.05	$\sigma^2_{\theta_{p2}}$	2.70	0.24
$\omega_3 = \lambda_3$	0.49	0.04	$\sigma^2_{\beta_{s1}}$	0.66	0.36
$\omega_4 = \lambda_4$	0.46	0.05	$\sigma_{\beta_{s1},\beta_{s2}}$	0.17	0.27
τ_1	1.56	0.26	$\sigma^2_{\beta_{s2}}$	0.71	0.42
τ_2	1.51	0.27	$\rho_{\theta_{p1},\theta_{p2}}$	0.68	0.03
$\widetilde{\tau}_3$	1.74	0.21	$\rho_{\beta_{s1},\beta_{s2}}$	0.25	0.28
$\widetilde{\tau}_4$	1.80	0.23	$\sigma^2_{\theta^*}$	0.51	0.06
$\sigma^2_{\theta_{p1}}$	3.93	0.34	$\sigma^2_{\beta^*}$	0.10	0.04

Table 3

Results for Model 1r

Figure Captions

Figure 1. Three-mode data. The observation x_{prs} represents the response of person p to question r under situation s.

Figure 2. Proportion of endorsements for each of the four responses: (a) Situation side, (b) Person side

Figure (a).

Figure (b).

Figure 3. Graphical representation of the research questions

Figure 4. Three different models

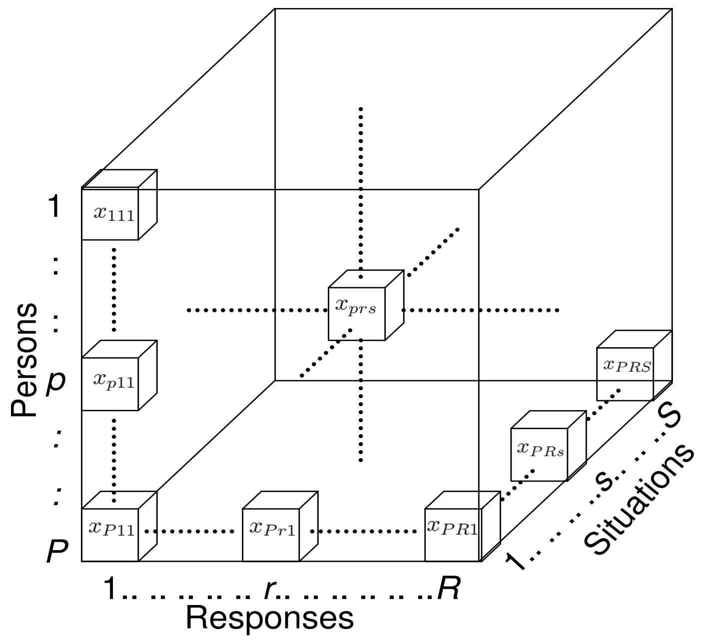
Figure (a). Model 1

Figure (b). Model2

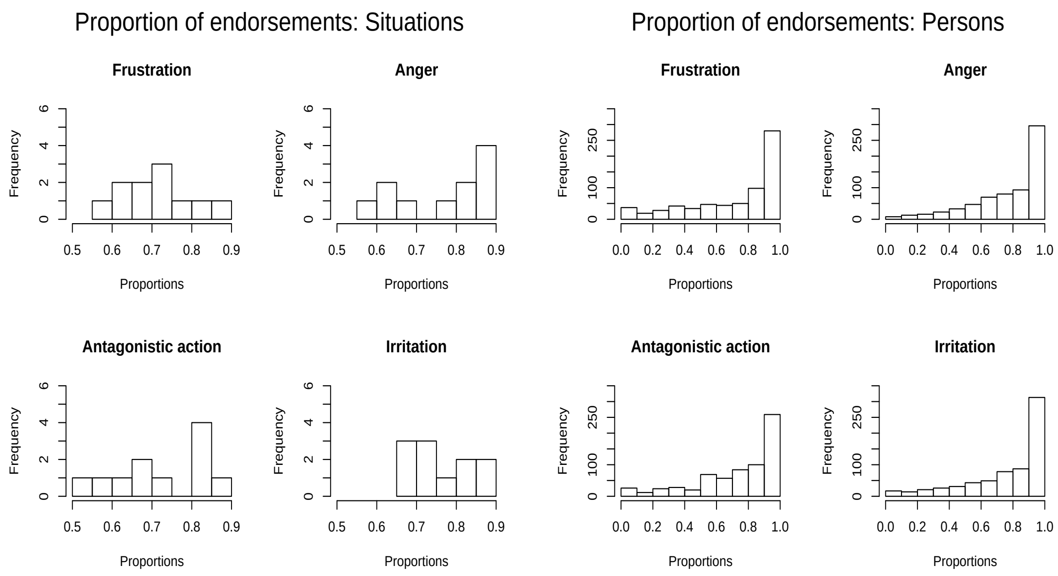
Figure (c). Model 3

Figure 5. χ^2 values for the observed verus the replicated data based on 2500 simulations from the posterior distribution of $(\boldsymbol{x}_{rep}, \boldsymbol{\vartheta})$.

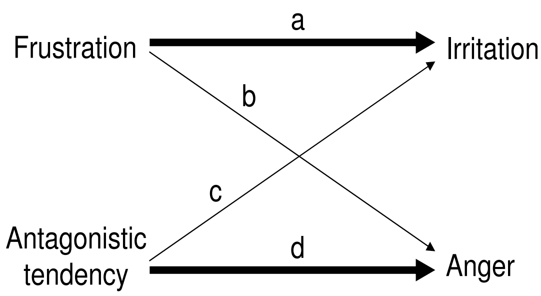
Figure 6. Results for Model 1r

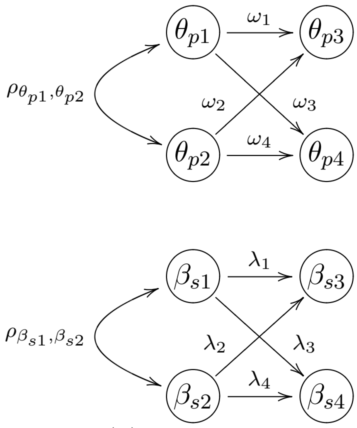


Persons
1
p
P
1.. .. .. r.. .. ..R
Responses
1.. .. .. .. s.. .. ..S
Situations
x_{111}
x_{p11}
x_{P11}
x_{Pr1}
x_{PR1}
x_{prs}
x_{PRs}
x_{PRS}

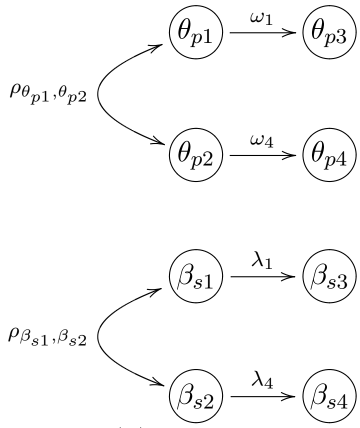

Proportion of endorsements: Situations
Frustration
Anger
Antagonistic action
Irritation
Frequency
Proportions
0.5 0.6 0.7 0.8 0.9
0 2 4 6
Proportion of endorsements: Persons
Frustration
Anger
Antagonistic action
Irritation
Frequency
Proportions
0.0 0.2 0.4 0.6 0.8 1.0
0 100 250

(a) (b)

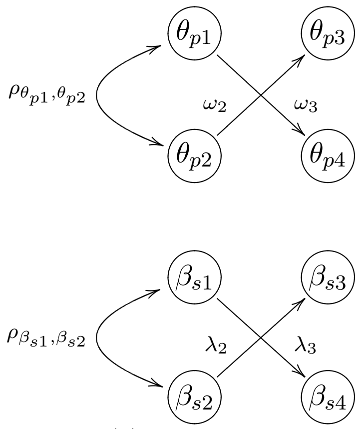

Frustration
a
Irritation
b
c
Antagonistic tendency
d
Anger



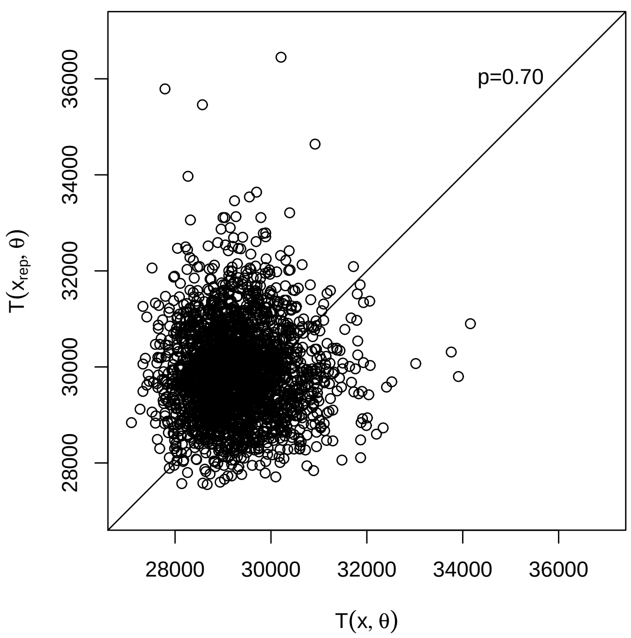
(a) Model 1

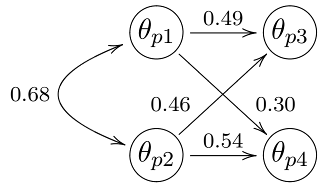


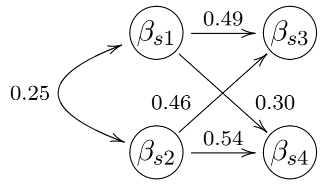
(b) Model2



(c) Model 3


p=0.70
T(x, θ)
T(x_rep, θ)
28000
30000
32000
34000
36000


θ_{p1}
θ_{p2}
θ_{p3}
θ_{p4}
0.68
0.49
0.46
0.30
0.54


β_{s1}
β_{s2}
β_{s3}
β_{s4}
0.25
0.49
0.46
0.30
0.54