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Abstract

The paper consists in an empirical comparison of two automatic procedures of time series forecasting: the expert system called TSE-AX and the automatic procedure of TRAMO/SEATS. That comparison is based on the data of the M3-Competition, the latest of the M-Competitions launched by Spyros Makridakis and Michèle Hibon. TSE-AX was a competitor in the M3-Competition but an improved version has been used here. TRAMO-SEATS is a pair of programs initially aimed at seasonal decomposition of quarterly or monthly time series, using a signal extraction approach based on ARIMA modeling. We use only the automatic procedure within TRAMO. In this paper, we briefly describe the principle of each procedure before giving the results of that comparison. It is interesting that two procedures based on different strategies provide similar results, on the whole, and that TRAMO, which was not intended as a forecasting software package, appears as a very satisfactory forecasting solution.

Key words. Time series analysis, M3-Competition, Forecasting methods, TSE-AX, TRAMO-SEATS.

1. Introduction

Forecast errors can have harmful consequences and imply, for example, surplus production capacity, under-capacity, out of stock items or unsold goods. If it is impossible to eliminate them completely, reliability of the forecasts can however be increased by applying good principles resulting from research and practice (Armstrong, 2001). These should indicate which methods to rely on and specify the conditions for their optimal use.

To forecast data in economics and finance, several statistical and econometric methods are used such as regression models, multivariate analysis, decision theory or time series modeling. Among the later, we consider ARIMA processes made popular by Box and Jenkins who proposed a model building methodology composed of several stages (Box et al., 1994). ARIMA modeling is more difficult to use than other statistical forecasting techniques although, when implemented properly, it can be quite powerful and flexible. On the basis of Eurostat data bases, Fischer and Planas (2000) have argued that the so-called airline model can be used to fit a large number of series. Several algorithms of automated ARIMA modeling were developed in order to make the method more applicable and also available to a greater number of users. Most of these algorithms were implemented using expert systems technology. These systems make it possible to program the knowledge of an expert and to reproduce the reasoning carried out by the system. One of these expert systems for building univariate time series models is TSE-AX. Described by Mélard and Pasteels (2000), it is included in Time Series Expert 2.3. A slightly improved version 2.4 (Njimi et al., 2006) is used here for which Njimi et al. (2003) gave an early presentation. One of the recent features is the possibility to handle series other than monthly and quarterly, as illustrated by Azrak et al. (2004).

TSE-AX 2.3 was well ranked in the M3-Competition where the participating experts were asked to make forecasts beyond the available data (Makridakis and Hibon, 2000, Ord et al., 2000). Note that the real data corresponding to these forecasts were not available to the participants before making their forecasts and were not, therefore, used in developing their forecasting model.

In this paper we present an empirical comparison between TSE-AX 2.4 and the automatic procedure of TSW. TSW is a Windows version of TRAMO/SEATS with some modifications and additions, developed by Caporello et al. (2001) at the Banco de España. TRAMO-SEATS is a pair of programs initially aimed at seasonal decomposition of monthly or lower frequency time series, using a signal extraction approach based on ARIMA modeling. We use only the automatic procedure within TRAMO, which was not intended as a forecasting software package. That comparison is based on a subset of the series of the M3-Competition.

The contents of the paper are as follows. First, we give a description of TSE-AX. Second, we describe the automatic procedure of TSW (Gómez and Maravall, 2001). And finally we give the principle of the comparison between those two automatic procedures and some results.

2. Description of TSE-AX

The objective of TSE-AX (Mélard and Pasteels, 2000), is to build ARIMA models in an automated way, with and without an intervention analysis, but so that the user receives the intermediate and final results, and is informed of the quality of the final model. The system is adapted to several categories of users from beginners to experts. The later should use such a tool to save time, being qualified to assess the quality of the final model and possibly propose

an alternative model. Briefly, TSE-AX covers everything from the specification stage to the forecasting stage, given that the latter is immediate when a final model has been found. The user can specify his or her model building preferences: perform an intervention analysis or not, choose a specification strategy, etc.

The modeling stage of TSE-AX consists in a succession of several phases. At the beginning, the user gives some information to the system like periodicity of the data and the sample to be used. The automated procedure starts with the *preliminary stage*, where interventions are selected, transformations are performed, and differences, regular and/or seasonal, are chosen to make sure that the series becomes stationary. In version 2.4, a new algorithm to select differences is used (Njimi et al., 2006). Next follows the *specification stage*, where an ARIMA model is identified using one of these three strategies: 'expert' (Mélard and Pasteels, 1998), 'autoregressive specification' (Mélard, 1990) and 'mixed' (Mélard and Pasteels, 2000) where a certain number of models are fitted and a choice among them is made. In version 2.4, we modified the algorithm to select the final model among a broader range of models (Njimi et al., 2006). The remaining stages are the *estimation stage*, where the final model is fitted, the *model checking stage*, were the adequacy of that model is investigated, and the *forecasting stage*.

There are more than twenty input commands that enable the user to customize the modeling strategy. They are concerned with the treatment of outliers by intervention analysis (several types are supported like additive outliers (AO) and level shift (LS)), the seasonal component, the Box-Cox transformation and difference operators. These commands are typically entered into a file and can act either on a single series or on a stream of series. Here, default values for all commands were used.

3. Description of TSW and its automatic procedure

TSW is a Windows interface that integrates the two programs TRAMO and SEATS. The software and its documentation are freely available at the address http://www.bde.es/. TRAMO, "Time series Regression with ARIMA noise, Missing values and Outliers" (Gómez and Maravall, 1994, 1996) is a program for fitting and forecasting of regression models with possibly non-stationary ARIMA errors and missing values. The program interpolates these values, identifies and corrects for several types of outliers, not only additive outliers (AO) and level shift (LS), but also temporary change (TC) and innovation outliers (IO), and estimates special effects such as trading day and Easter effects and, in general, intervention-variable type effects. SEATS, "Signal Extraction in ARIMA Time Series", is a program for extracting unobserved components in time series with the purpose to produce a seasonally adjusted series. TRAMO and SEATS are structured so as to be used together but TRAMO can be used alone. TRAMO preadjusts the series, and SEATS decomposes the linearized series into its stochastic components. The complete final component is equal to the stochastic one, plus the deterministic effect associated with that component, that has been removed in the preadjustment by TRAMO (for example, an AO outlier will be added to the irregular component, a LS outlier will be added to the trend-cycle, and so on).

The programs, TRAMO and SEATS, are fundamentally aimed at monthly or lower frequency time series. Although structured to meet the needs of an expert analyst, they can be reliably used in an entirely automatic manner on very large sets of time series. The main applications are seasonal adjustment, trend-cycle estimation, construction of leading indicators, interpolation, detection and correction of outliers, estimation of special effects, and quality control of data. It should be insisted that TRAMO-SEATS is not aimed at forecasting.

The automatic procedure of TSW requires the prior decision of whether or not to test for the presence of calendar effects and, if so, which specification for the trading day effect should be used. The different options are controlled by the parameter RSA, see Caporello et al. (2001). For example, for the RSA = 3 option, the program tests for the log/level specification, interpolates missing observations (if any), and performs automatic model identification and outlier detection. Three types of outliers are considered: additive outliers, transitory changes and level shifts; the level of significance is set by the program and depends on the length of the series. The full model is estimated by exact maximum likelihood, and forecasts of the series up to a two-year horizon are computed. For the alternative option RSA = 1, the program works like with RSA = 3 but the default Airline model is always used instead of an automatically obtained model.

Within SEATS, the model obtained by TRAMO is decomposed and optimal estimators and forecasts of the components are obtained, as well as their mean squared error. These components are the trend-cycle, and the seasonal, irregular and (perhaps) transitory components. If the model does not accept an admissible decomposition, it is replaced by a decomposable one.

4. Modeling methodology

Our analysis of the M3-Competition series is limited to yearly, quarterly and monthly series, i.e. 2829 series out of 3003. We recall that the 3003 series of the M3-Competition were selected on a quota basis to include various types of time series data (micro, industry, macro, etc.) and different time intervals between successive observations (yearly, quarterly, etc.), see Makridakis and Hibon (2000). Usually, yearly series are discarded because most of

them are too short to be modeled by ARIMA models with existing technology but here we kept them all. Series with an unknown time interval between successive observations are excluded because both automatic procedures, TSE-AX and TSW, require that information.

For the treatment by TSE-AX, we have accepted the treatment of outliers by intervention analysis to avoid extreme values that would badly influence the various steps of the analysis: specification, estimation, test for adequacy, and forecasting. We also used the 'Mixed strategy' in the step of specification because that strategy is the most complete one (Njimi et al., 2003). For the treatment by TSW, we choose RSA = 3, as explained above. Even if the computational strategies of computation of TSE-AX and TSW are not the same, the choice of the option RSA = 3 is justified by the fact that this is the most general without calendar effects. This means that these two automatic procedures include a treatment of outliers and use exact maximum likelihood for estimation of the final model, and no pretest is made for the presence of trading day, leap year and Easter effects.

All fits are done in TSE-AX by exact maximum likelihood estimation whereas the Hannan-Rissanen estimation method is used in TSW for all fits except the final one which is performed by exact maximum likelihood estimation. Using the mixed strategy, TSE-AX fits at most 32 models. The number of models fitted by TSW is much larger but computations are faster.

5. Results of the comparison

As in the M-Competition, we selected a forecasting horizon (h) of maximum six years for yearly series, eight quarters for quarterly series, and eighteen months for monthly series. The symmetric mean absolute percentage error criterion (sMAPE) is used to analyze the performance of the two procedures. For each type of series, for each procedure and for each horizon, the average sMAPE is calculated. The results are summarized in Table 1 for yearly series, Table 2 for quarterly series, and Table 3 for monthly series (see Appendix). Tables 4-6 list the differences in terms of forecasting performance of TSW with respect to TSE-AX. They show average_sMAPE(TSE-AX) – average_sMAPE(TSW) using the results in Tables 1-3: a positive sign means that the accuracy of TSW is better than that of TSE-AX. Tables 7-9 give the resulting *p*-values from the means paired test, using the average results of the comparison between TSE-AX and TSW based on the sMAPE criterion for, respectively, yearly, quarterly and monthly data.

5.1 Yearly data

The results of the yearly series are shown in Tables 1, 4 and 7. For these series, the results suggest that TSW did worse than TSE-AX for macro series and did better in finance and other series.

Note that the differences in the forecasting performance (as far as the average sMAPE is concerned) between the two automatic procedures are small and the maximum of these differences across types and horizons is 1.91%. Note also that the results of these two methods appear better than those of the other competitors in the M3-Competition, but, of course, our analysis is ex post.

5.2 Quarterly data

The results of the quarterly series are shown in Tables 2, 5 and 8. For these series, the results suggest that TSW did worse than TSE-AX for finance and demographic series and did better in micro, macro and industrial series.

The differences in the forecasting performance between the two automatic procedures are small and the maximum of these differences across types and horizons is 2.52%.

5.3 Monthly data

The results of the monthly series are shown in Tables 3, 6 and 9. For these series, TSW did worse for finance, demographic and other series, and did better in micro, macro and industrial series. The differences in terms of forecasting performance are also small. Those differences did not exceed 1.4%.

5.4 Statistical significance of the results

Instead of just comparing averages of the sMAPE given by the two automatic procedures, we have performed statistical tests to compare their performance, using more precisely the paired *t*-test for every type of series and every horizon, except for the 'other' category for yearly series. Indeed the latter contains only 11 series, therefore the *p*-values are provided by the Wilcoxon signed rank test instead of the paired *t*-test. The *p*-value of each case (per type of series and horizon) is displayed in Tables 7-9. Small values imply that the null hypothesis of equal performance is rejected and thus the accuracies of the two automatic procedures are significantly different.

All values are greater than 10% except in microeconomic series and monthly industrial series. More precisely, the *p*-values are less than 5% for horizon 5 in yearly macro data, horizons 1 and 4 in quarterly micro data, for horizon 5 in monthly micro data and for horizons 3 and 4 in monthly industrial data. The winner is indicated in bold in Tables 1-3.

In all remainder cases, the *p*-values are not significant. Given the large number of tests performed, this suggests that the difference in the forecasting performance of the two automatic procedures is not different from zero.

6. Closing comments and conclusion

In Section 3, we have provided a rough description of TSW and its module TRAMO in particular. TRAMO is aimed at building an ARIMA model for the signal extraction procedure within SEATS in order to obtain a seasonal decomposition of a series. Although TRAMO is not considered as a forecasting software package, it seemed interesting to investigate that complex procedure and its forecasting performance.

Having performed a comparison between the expert system TSE-AX and the automatic procedure of TSW on the yearly, quarterly and monthly series of the M3-Competition, it can be concluded that there is no statistically significant difference in the forecasting performance except perhaps for microeconomic data where TSW performs slightly better.

Even if the computational strategies of TSE-AX and TSW are not the same, they produce comparable forecasts on the whole. Therefore, given that TSE-AX was well assessed in the competition, that implies that TSW can produce good forecasts.

Again, this comparison illustrates the fact that the "use of statistically sophisticated or complex methods does not necessarily produce consistently more accurate forecasts" (Flores and Pearce, 2000).

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Appendix

Types of series	Methods	Horizon					
		1	2	3	4	5	6
Micro	TSE-AX	12.34	16.19	20.89	23.62	26.29	28.77
	TSW	11.45	15.66	21.07	23.59	26.74	28.75
Industrial	TSE-AX	9.60	11.81	14.05	16.69	18.59	20.45
	TSW	10.79	12.68	14.68	15.72	17.40	19.31
Macro	TSE-AX	2.40	3.68	4.92	5.82	6.51	7.09
	TSW	2.57	3.93	5.33	6.32	7.12	7.79
Finance	TSE-AX	17.16	20.59	22.06	24.37	27.00	29.09
	TSW	17.02	20.11	21.93	23.91	25.35	27.18
Demographic	TSE-AX	4.87	6.08	7.46	9.16	10.75	12.13
	TSW	5.36	6.33	7.55	8.66	10.07	11.28
Other	TSE-AX	16.57	19.17	21.80	22.71	22.02	22.47
	TSW	15.48	17.98	21.17	22.26	21.75	22.30

Table 1. Average symmetric MAPE for yearly data. Bold numbers are those significantly best at 5% in the sense of Section 5. 4.

Table 2. Average symmetric MAPE for quarterly data. Bold numbers are those significantly best at 5% in the sense of Section 5. 4.

Types of series	Methods	Horizon						
		1	2	3	4	5	6	8
Micro	TSE-AX	10.16	10.79	11.63	12.83	13.32	13.98	15.08
	TSW	7.64	8.64	9.44	10.56	11.08	11.85	13.25
Industrial	TSE-AX	6.63	7.74	8.10	8.29	8.85	9.91	11.22
	TSW	5.76	6.34	6.92	7.38	7.87	8.53	9.79
Macro	TSE-AX	2.63	3.01	3.52	3.96	4.45	4.87	5.89
	TSW	2.48	3.00	3.53	3.93	4.36	4.76	5.71
Finance	TSE-AX	5.08	8.79	10.14	11.81	13.41	15.08	16.84
	TSW	5.79	9.05	10.87	12.39	13.89	15.08	17.12
Demographic	TSE-AX	5.09	7.21	8.45	9.81	11.40	12.78	15.61
	TSW	6.86	8.84	10.02	11.55	13.27	14.52	17.25
Other	TSE-AX			N	Jo series			
	TSW							

Types of series	Methods	Horizon									
		1	2	3	4	5	6	8	12	15	18
Micro	TSW	24.60	23.81	24.24	24.41	23.86	23.68	23.51	23.81	25.31	26.48
	TSE-AX	25.19	24.56	24.87	25.51	25.28	24.74	24.40	24.36	25.31	26.50
Industrial	TSW	7.14	7.44	7.72	8.18	8.61	9.23	10.25	11.18	11.95	12.69
	TSE-AX	7.61	7.95	8.39	8.89	9.26	9.65	10.61	11.48	12.21	12.97
Macro	TSW	3.03	3.52	3.78	3.77	4.10	4.38	4.82	5.34	5.92	6.69
	TSE-AX	3.09	3.52	3.58	4.01	4.31	4.53	4.96	5.60	6.13	6.83
Finance	TSW	5.90	6.88	7.07	7.61	8.49	8.88	9.79	10.92	11.75	12.89
	TSE-AX	5.76	6.59	6.59	7.27	8.23	8.69	9.60	10.37	11.30	12.25
Demographic	TSW	2.55	2.82	3.30	4.13	4.91	5.35	6.10	6.98	7.56	8.44
	TSE-AX	2.62	2.98	3.33	4.09	4.80	5.16	5.71	6.52	7.01	7.78
Other	TSW	5.84	7.04	6.08	6.34	6.54	9.07	9.63	9.33	9.29	10.23
	TSE-AX	5.94	6.75	5.78	6.10	6.37	8.69	9.92	9.15	8.97	9.70

Table 3. Average symmetric MAPE for monthly data. Bold numbers are those significantly best at 5% in the sense of Section 5. 4.

Table 4. Average_sMAPE(TSE-AX) – Average_sMAPE(TSW): yearly data. Bold numbers are those significantly best at 5% in the sense of Section 5. 4.

Types of series	Horizon					
	1	2	3	4	5	6
Micro	0.89	0.53	-0.18	0.04	-0.45	0.02
Industrial	-1.19	-0.87	-0.63	0.97	1.18	1.14
Macro	-0.17	-0.25	-0.41	-0.51	-0.62	-0.70
Finance	0.14	0.48	0.13	0.47	1.65	1.91
Demographic	-0.50	-0.25	-0.09	0.50	0.68	0.86
Other	1.10	1.19	0.63	0.45	0.27	0.17

Table 5. Average_sMAPE(TSE-AX) – Average_sMAPE(TSW): quarterly data. Bold numbers are those significantly best at 5% in the sense of Section 5. 4.

Types of series	Horizon									
	1	2	3	4	5	6	8			
Micro	2.52	2.14	2.19	2.27	2.24	2.13	1.83			
Industrial	0.86	1.41	1.18	0.91	0.99	1.38	1.43			
Macro	0.15	0.01	-0.01	0.03	0.10	0.10	0.18			
Finance	-0.71	-0.26	-0.73	-0.58	-0.48	0.00	-0.28			
Demographic	-1.77	-1.62	-1.57	-1.74	-1.87	-1.74	-1.64			
Other		No series								

Types of series	Horizon									
	1	2	3	4	5	6	8	12	15	18
Micro	0.58	0.74	0.63	1.10	1.42	1.06	0.89	0.55	0.00	0.02
Industrial	0.47	0.51	0.67	0.72	0.65	0.42	0.36	0.31	0.26	0.29
Macro	0.06	0.01	-0.19	0.24	0.21	0.15	0.14	0.26	0.21	0.14
Finance	-0.13	-0.29	-0.47	-0.34	-0.26	-0.19	-0.20	-0.54	-0.45	-0.65
Demographic	0.07	0.15	0.03	-0.05	-0.11	-0.19	-0.39	-0.46	-0.56	-0.66
Other	0.10	-0.28	-0.31	-0.24	-0.17	-0.38	0.29	-0.19	-0.32	-0.53

Table 6. Average_sMAPE(TSE-AX) – Average_sMAPE(TSW): monthly data. Bold numbers are those significantly best at 5% in the sense of Section 5. 4.

Table 7. Resulting p-values from means paired test. Results of the comparison between TSE-AX and TSW based on the sMAPE criterion: yearly data

Types of series	Horizon	Horizon								
	1	2	3	4	5	6				
Micro	0.41	0.64	0.88	0.98	0.77	0.99				
Industrial	0.14	0.29	0.46	0.32	0.25	0.33				
Macro	0.37	0.25	0.10	0.06	0.04	0.05				
Finance	0.93	0.78	0.95	0.82	0.48	0.44				
Demographic	0.12	0.47	0.83	0.43	0.34	0.29				
Other	0.08	0.32	0.41	0.41	0.72	0.61				

Table 8. Resulting p-values from means paired test. Results of the comparison between TSE-AX and TSW based on the sMAPE criterion: quarterly data

Types of series	Horizon							
	1	2	3	4	5	6	8	
Micro	0.02	0.06	0.06	0.04	0.05	0.06	0.11	
Industrial	0.39	0.16	0.22	0.34	0.34	0.22	0.21	
Macro	0.26	0.91	0.94	0.82	0.56	0.59	0.43	
Finance	0.23	0.69	0.13	0.15	0.26	0.99	0.69	
Demographic	0.15	0.21	0.23	0.21	0.28	0.35	0.51	
Other	No series							

Horizon									
1	2	3	4	5	6	8	12	15	18
0.48	0.26	0.31	0.08	0.02	0.05	0.09	0.33	0.99	0.98
0.18	0.12	0.04	0.04	0.05	0.20	0.31	0.73	0.48	0.49
0.84	0.98	0.64	0.41	0.53	0.69	0.72	0.49	0.58	0.72
0.77	0.52	0.15	0.30	0.45	0.60	0.64	0.31	0.46	0.32
0.74	0.40	0.85	0.80	0.55	0.30	0.10	0.19	0.15	0.13
0.69	0.21	0.23	0.41	0.57	0.28	0.64	0.75	0.63	0.54
	Horizon 1 0.48 0.18 0.84 0.77 0.74 0.69	Horizon 1 2 0.48 0.26 0.18 0.12 0.84 0.98 0.77 0.52 0.74 0.40 0.69 0.21	Horizon 1 2 3 0.48 0.26 0.31 0.18 0.12 0.04 0.84 0.98 0.64 0.77 0.52 0.15 0.74 0.40 0.85 0.69 0.21 0.23	Horizon 1 2 3 4 0.48 0.26 0.31 0.08 0.18 0.12 0.04 0.04 0.84 0.98 0.64 0.41 0.77 0.52 0.15 0.30 0.74 0.40 0.85 0.80 0.69 0.21 0.23 0.41	Horizon 1 2 3 4 5 0.48 0.26 0.31 0.08 0.02 0.18 0.12 0.04 0.04 0.05 0.84 0.98 0.64 0.41 0.53 0.77 0.52 0.15 0.30 0.45 0.74 0.40 0.85 0.80 0.55 0.69 0.21 0.23 0.41 0.57	Horizon 1 2 3 4 5 6 0.48 0.26 0.31 0.08 0.02 0.05 0.18 0.12 0.04 0.04 0.05 0.20 0.84 0.98 0.64 0.41 0.53 0.69 0.77 0.52 0.15 0.30 0.45 0.60 0.74 0.40 0.85 0.80 0.55 0.30 0.69 0.21 0.23 0.41 0.57 0.28	Horizon 1 2 3 4 5 6 8 0.48 0.26 0.31 0.08 0.02 0.05 0.09 0.18 0.12 0.04 0.04 0.05 0.20 0.31 0.84 0.98 0.64 0.41 0.53 0.69 0.72 0.77 0.52 0.15 0.30 0.45 0.60 0.64 0.74 0.40 0.85 0.80 0.55 0.30 0.10 0.69 0.21 0.23 0.41 0.57 0.28 0.64	Horizon 1 2 3 4 5 6 8 12 0.48 0.26 0.31 0.08 0.02 0.05 0.09 0.33 0.18 0.12 0.04 0.04 0.05 0.20 0.31 0.73 0.84 0.98 0.64 0.41 0.53 0.69 0.72 0.49 0.77 0.52 0.15 0.30 0.45 0.60 0.64 0.31 0.74 0.40 0.85 0.80 0.55 0.30 0.10 0.19 0.69 0.21 0.23 0.41 0.57 0.28 0.64 0.75	Horizon 1 2 3 4 5 6 8 12 15 0.48 0.26 0.31 0.08 0.02 0.05 0.09 0.33 0.99 0.18 0.12 0.04 0.04 0.05 0.20 0.31 0.73 0.48 0.84 0.98 0.64 0.41 0.53 0.69 0.72 0.49 0.58 0.77 0.52 0.15 0.30 0.45 0.60 0.64 0.31 0.46 0.74 0.40 0.85 0.80 0.55 0.30 0.10 0.19 0.15 0.69 0.21 0.23 0.41 0.57 0.28 0.64 0.75 0.63

Table 9.Resulting p-values from means paired test. Results of the comparison between TSE-AX and TSW based on the sMAPE criterion: monthly data