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A measure of market imperfection by frontier analysis *

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Abstract

In this paper, we propose an empirical method to measure the market imperfection and the bargaining power of the agents, by extending the methods of frontier analysis. A case study in the field of freight transport illustrates the proposed method.

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1 Introduction

Measuring the imperfection of a market is a challenging issue in industrial economics. In this note, we propose an empirical measure adapted to the case where the available data provide, for each contract of a "representative" sample, the price y_i of the transaction i and a vector z_i of "characteristics" of the transaction i. These characteristics are assumed to be a source of cost for the supply and a source of satisfaction for the demand. For instance, in freight transport, the characteristics z may involve: the distance, the weight, and/or volume, speed etc. Thus, we assume that each component of the vector z_i is measured in coordinates such that the partial derivatives of the demand utility function and of the supply cost function are strictly positive.

A set of virtual transactions defines a region in the (y, z)-space where one may accordingly define a *supply bid function*, $P_*(z)$, (resp. *demand bid function*, $P^*(z)$), namely the minimum (resp. maximum) price suppliers (resp. demanders) are ready to accept for a given characteristic z. In this context a supply (resp. demand) bid function may be viewed as a frontier of a set of acceptable contracts and therefore as an extension of the usual concept of supply (resp. demand) function.

Defining a precise concept of perfect competition in the (y, z)-space is not an easy task (see *e.g.* Drèze and Hagen (1978)). A pragmatic approach might be to consider that, possibly after following for free entry and exit in the market, a perfect competition would generate a *contract curve* $P(z)$, such that all the transactions would be operated at the points $(P(z), z)$ and $P_*(z) \leq P(z) \leq P^*(z)$ (*i.e.* the supply and the demand side would accept the price $P(z)$ of the characteristic z), the difference $P(z) - P_*(z)$ and $P^*(z) - P(z)$ representing the surplus of the supply and the demand respectively.

There are many causes for and many forms of imperfect competition but in all cases the observed prices of transactions would be distributed in the interval $[P_*(z), P^*(z)]$, without determining a curve $P(z)$. These imperfections may originate in the heterogeneity of the product - or the services - to be exchanged and/or in an imperfect information on the reliability, and other qualities, of the provider in the case of freight transport. Under such circumstances, the price of the transaction is a result, among other aspects, of the relative bargaining power of each partner. In other words, the contract (y, z) will be generated within a *contract set* bordered by the two bid functions; this region represents the set of feasible contracts.

The objective of this paper is to propose an empirical measure of market imperfection and an evaluation of the bargaining power of the two partners of a given contract. Our proposal makes use of ideas underlying the Data Envelopment Analysis (DEA), widely used in the estimation of efficiency frontier for the production process. In the next section, we first expose an economic analysis based on a set of virtual contracts and next propose an econometric estimation based on a sample of observed contracts. The last section illustrates the econometric estimation through real data provided by a survey conducted in the sector of freight transport in Belgium.

2 Market imperfection and bargaining power

Intuitively, one possible way to measure the market imperfection at a given characteristic z is to compute a distance between the *demand bid function* and the *supply bid function* (see Figure 1). When all observed contracts form a curve (the *contract curve*), all the demanders (resp. suppliers) pay (resp. receive) a same price for a given characteristic. This is an aspect of competitive equilibrium, in which case the bid functions are merged and represent the contract curve. If, on the contrary, the market is not perfectly competitive, we will expect to observe different prices for a same set of characteristics and the estimated bid functions are not any more merged. So, the longer is the distance between the *demand bid function* and the *supply bid function*, the more substantial is the market imperfection. Note that we are more interested in price variation for a same set of characteristics than in characteristics variation for a same price. The market imperfection may be graphically represented by the vertical distance between the two bid functions (see Figure 1).

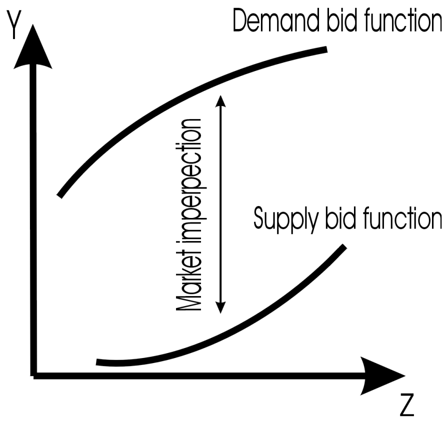

Figure 1: The market imperfection

2.1 Economic Analysis

Let $\xi = (\eta, \zeta) \in I\!R_+^{1+k}$ represents a contract with $\eta \in I\!R_+$ representing the price and $\zeta \in I\!R_+^k$ representing the vector of characteristics. We shall assume that the set of *virtual contracts*, $\mathcal{C} \in I\!R_+^{1+k}$ satisfies some requirements. Firstly, $\mathcal{C}$ is assumed to be convex, namely:

$$\forall\, \xi_i \in \mathcal{C} \text{ with } i = 1, 2 \text{ and } a \in [0, 1],\ a\xi_1 + (1 - a)\xi_2 \in \mathcal{C}$$

Secondly, as infinite contracts ($\eta \to \infty$ or $\zeta \to \infty$) are not meaningful, $\mathcal{C}$ is compact (closed and bounded). Thirdly, we assume that the contract set is *Demand Free Disposable* (DFD), *i.e.* if (η, ζ) is observed, it seems reasonable to assume that the demander would accept ζ for a lower η', or would accept η for an higher ζ', namely:

$$\mathcal{C} \text{ is DFD if } \forall (\eta, \zeta) \in \mathcal{C}, \quad \eta' \leq \eta \text{ and } \zeta' \geq \zeta \quad \text{then}, (\eta', \zeta') \in \mathcal{C}$$

If $\mathcal{C}$ does not satisfy the DFD property, we define its DFD-closure as:

$$\mathcal{C}_{DFD} = \cap \{\mathcal{A} \supset \mathcal{C} \mid \mathcal{A} \text{ is convex and DFD}\} \tag{1}$$

i.e. the smallest convex set $\mathcal{A}$ which satisfies the DFD property. Similarly, the contract set is assumed to be *Supply Free Disposable* (SFD), *i.e.* if (η, ζ) is observed, it seems reasonable to assume that the supplier would accept ζ for an higher η', or would accept η for a lower ζ', namely:

$$\mathcal{C} \text{ is SFD if } \forall (\eta, \zeta) \in \mathcal{C}, \quad \eta' \geq \eta \text{ and } \zeta' \leq \zeta \quad \text{then, } (\eta', \zeta') \in \mathcal{C}$$

If $\mathcal{C}$ does not satisfy the SFD property, we define its SFD-closure as:

$$\mathcal{C}_{SFD} = \cap \{\mathcal{A} \supset \mathcal{C} \mid \mathcal{A} \text{ is convex and SFD}\} \tag{2}$$

i.e. the smallest convex set $\mathcal{A}$ which satisfies the SFD property.
Remark : $\mathcal{C}_{DFD}$ and $\mathcal{C}_{SFD}$ are not bounded. Indeed, $\mathcal{C}_{DFD}$ (resp. $\mathcal{C}_{SFD}$) includes points where $\zeta \to \infty$ (resp. $\eta \to \infty$).

Finally, the last assumption supposes that $\mathcal{C}$ is Free disposable (FD), *i.e.* DFD and SFD. If $\mathcal{C}$ does not satisfy the FD property, we define its FD-closure as:

$$\mathcal{C}_{FD} = \mathcal{C}_{DFD} \cap \mathcal{C}_{SFD} \tag{3}$$

If one of the FD assumption were not satisfied, either the demand or the supply would refuse to conclude a contract. From now on, we shall assume that $\mathcal{C}$ is convex, compact and FD, so that $\mathcal{C} = \mathcal{C}_{FD}$.

The *supply bid function*, $P_* : \mathcal{C} \to I\!R_+$, representing the minimum price suppliers accept for the characteristic ζ, is defined in such a way that the points $(P_*(\zeta), \zeta)$ represent a lower frontier of $\mathcal{C}$, more precisely:

$$P_*(\zeta) = \inf\{\eta \mid (\eta, \zeta) \in \quad \mathcal{C}\} \tag{4}$$

Similarly, the *demand bid function*, $P^* : \mathcal{C} \to I\!R_+$, representing the maximum price demanders agree to pay for the characteristics ζ, is defined in such a way that the points $(P^*(\zeta), \zeta)$ represent an upper frontier of $\mathcal{C}$:

$$P^*(\zeta) = \sup\{\eta \mid (\eta, \zeta) \in \quad \mathcal{C}\} \tag{5}$$

The DFD (resp. SFD) property implies that the bid function $P^*(\zeta)$ (resp. $P_*(\zeta)$) is not decreasing (resp. increasing).
The market imperfection may be approached as the sum of the demand surplus, namely $S_D = P^*(\zeta) - P(\zeta)$, and the supply surplus, namely $S_S = P(\zeta) - P_*(\zeta)$; geometrically this is represented by a distance between the demand and the supply bid functions. In other words, the market imperfection is globally measured by the volume (in $I\!R_+^{1+k}$) delimited by the graphs of the two bid functions and this volume may be decomposed on the ζ-space by the (vertical) distance between $P^*(\zeta)$ and $P_*(\zeta)$.

Next section presents a computable version of the two bid functions $P_*(\zeta)$ and $P^*(\zeta)$ and is based on the following representations. For a given contract $(\eta, \zeta) \in \mathcal{C}$ let us define a *demand price multiplier* (resp. *supply price multiplier*) as a factor of dilatation (resp. of contraction) that would bring η up to the frontier $P^*(\zeta)$ (resp. $P_*(\zeta)$) :

$$\alpha : \mathcal{C} \to [1, \infty) \qquad \alpha(\eta, \zeta) = \sup\{a \mid a\eta \leq P^*(\zeta)\} \tag{6}$$

$$\beta : \mathcal{C} \to [0,1] \qquad \beta(\eta,\zeta) = \inf\{b \mid b\eta \geq P_*(\zeta)\} \tag{7}$$

By definition, we have : $\alpha(P^*(\zeta),\zeta) = \beta(P_*(\zeta),\zeta)$=1. The surpluses of a contract (η,ζ), as functions $S : \mathcal{C} \to I\!R_+$, may accordingly be represented as:

$$\text{demand surplus} : S_D(\eta,\zeta) = P^*(\zeta) - \eta = [\alpha(\eta,\zeta) - 1]\eta \tag{8}$$

$$\text{supply surplus} : S_S(\eta,\zeta) = \eta - P_*(\zeta) = [1 - \beta(\eta,\zeta)]\eta \tag{9}$$

The market imperfection, I : $\mathcal{C} \to I\!R_+$ may therefore be defined as :

$$I(\eta,\zeta) = [P^*(\zeta) - \eta] + [\eta - P_*(\eta)] = [\alpha(\eta,\zeta) - \beta(\eta,\zeta)]\eta \tag{10}$$

In the sequel we shall write $I(\zeta)$ instead of $I(\eta,\zeta)$ because the market imperfection of a contract (η,ζ) depends on ζ only; indeed $\forall\{(\eta_1,\zeta),(\eta_2,\zeta)\} \subset \mathcal{C}$, we have : $\alpha(\eta_1,\zeta)\eta_1 = \alpha(\eta_2,\zeta)\eta_2$ and $\beta(\eta_1,\zeta)\eta_1 = \beta(\eta_2,\zeta)\eta_2$ because definitions (6) and (7) imply : $\alpha(\eta,\zeta) = P^*(\zeta)\eta^{-1}$ and $\beta(\eta,\zeta) = P_*(\zeta)\eta^{-1}$. In other words, $I(\zeta)$ =$I(\eta,\zeta)$ provides a decomposition, on the ζ-space, of the volume generated by the set of virtual contracts.

In order to interpret the market imperfection from a bargaining power point of view, we define the *bargaining power* of each side of a contract:

$$\pi : \mathcal{C} \to [0,1] \quad \pi(\eta,\zeta) = \frac{\text{surplus of the contract}}{\text{market imperfection}}$$

More specifically,

$$\textit{demand bargaining power} : \pi_D(\eta,\zeta) = \frac{\alpha(\eta,\zeta) - 1}{\alpha(\eta,\zeta) - \beta(\eta,\zeta)} \tag{11}$$

$$\textit{supply bargaining power} : \pi_S(\eta,\zeta) = \frac{1 - \beta(\eta,\zeta)}{\alpha(\eta,\zeta) - \beta(\eta,\zeta)} \tag{12}$$

Thus $\pi_D(\eta,\zeta)$ (resp. $\pi_S(\eta,\zeta)$) gives a relative measure of the distance between the contract (η,ζ) and the *demand bid function* (resp. *supply bid function*). In the extreme cases, when a contract lies on the *supply bid function* (resp. *demand bid function*), $\pi_D = 1$ (resp. $\pi_S = 0$) and $\pi_S = 0$ (resp. π_D =1) and suggests that one of the agents has succeeded in fully taking over the market imperfection. Moreover, $\forall(\eta,\zeta) \in \mathcal{C} : \pi_S(\eta,\zeta) + \pi_D(\eta,\zeta) = 1$.

2.2 Econometric estimation

2.2.1 Introduction

Let us assume that the available data consists of n contracts $x_i = (y_i, z_i') \in I\!R_+^{1+k}$, $(i = 1, ..., n)$, where y_i represents the price of the transaction i and z_i is a vector of k characteristics. For notational convenience, these data are assembled in matrix form:

$$X = [x_i'] = [y_i \quad z_i'] = [y \quad Z]$$

where $X : n \times (1+k), y : n \times 1, Z : n \times k$. We shall also use the set of *observed contracts*, namely :

$$R_X = \{x_i \mid i = 1, ...n\} \subset I\!R^{k+1}$$

The objective is to estimate the frontiers of $\mathcal{C}$ and from this estimation to deduce the measures of market imperfection and of the bargaining powers. We take advantage of the formal similarity between the frontiers of $\mathcal{C}$ and the efficient frontier in production analysis, as initiated by Farrell (1957) and exposed, *e.g.* in Fried *et al.* (1993), Färe *et al.* (1994) or Simar and Wilson (2000, 2003). Thus, the set $\mathcal{C}$ is estimated by the FD-convex closure of R_X, namely :

$$\hat{\mathcal{C}} = [CoclR_X]_{FD} \quad \subset I\!R_+^{1+k} \tag{13}$$

where $CoclR_X = \{(\eta, \zeta') \in I\!R^{1+k} \mid \exists \gamma \in S_{[n-1]} : \eta = y'\gamma \quad \zeta = Z'\gamma\}$ and $S_{[n-1]}$ is the (n-1)-dimensional simplex, namely $S_{[n-1]} = \{\gamma \in I\!R_+^n \mid \sum_{i=1}^n \gamma_i = 1\}$.

2.2.2 P_* and P^* by linear programming

The two bid functions, $P_*(\zeta)$ and $P^*(\zeta)$, are estimated through the upper and lower frontier of $\hat{\mathcal{C}}$. Those frontiers are characterized by the efficient contracts for which $\alpha = 1$ or $\beta = 1$, respectively. Once those efficient contracts have been empirically identified, the demand and supply bid functions are constructed by linear interpolation while satisfying the free disposability condition.

For each observed contract, x_i, the value of the functions $\alpha(y_i, z_i)$ and $\beta(y_i, z_i)$ may be estimated by linear programming, namely :

$$\hat{\alpha}(y_i, z_i) = \sup\{a \mid \exists \gamma \in S_{[n-1]} : ay_i \leq y'\gamma, Z'\gamma \leq z_i\} \tag{14}$$

$$\hat{\beta}(y_i, z_i) = \inf\{b \mid \exists \gamma \in S_{[n-1]} : by_i \geq y'\gamma, Z'\gamma \geq z_i\} \tag{15}$$

Once α and β are estimated for each contract x_i, the market imperfection and the bargaining powers associated to each contract x_i can be estimated as follows :

$$\hat{I}(y_i, z_i) = [\hat{\alpha}(y_i, z_i) - \hat{\beta}(y_i, z_i)] \times y_i \tag{16}$$

$$\hat{\pi}_D(y_i, z_i) = \frac{\hat{\alpha}(y_i, z_i) - 1}{\hat{\alpha}(y_i, z_i) - \hat{\beta}(y_i, z_i)} \tag{17}$$

$$\hat{\pi}_S(y_i, z_i) = \frac{1 - \hat{\beta}(y_i, z_i)}{\hat{\alpha}(y_i, z_i) - \hat{\beta}(y_i, z_i)} \tag{18}$$

3 Illustration : the case of freight transport

The data are taken from a pilot survey of Belgian firms active in freight transport. Those data are gathered by face-to-face interviews during which the interviewee is asked to describe a typical shipment of goods (Beuthe *et al.* 2003). From this information, different attributes are obtained, namely: the cost of the transport (EUR/km.ton), the frequency defined as the number of trips executed per week, the reliability defined as the percentage of time the goods are delivered within the required time, the speed defined as the door to door transport time, including loading and unloading, divided by the distance, the safety defined as the percentage of time a loss (or damage) is not incurred and the flexibility defined as the percentage of time that unexpected demands are met by the carrier. This section summarizes a case study based on $n = 74$ observations; a more complete exposition may be found in Mouchart and Vandresse (2003).

Figure 2 provides a graph of the values of the two bid functions; on the abcissa, the labels of the contracts (ranging from 1 to n) and on the ordinates, the value of β, ranging from 0 to 1 and the value of $1/\alpha$, ranging from 0.003 to 1. The very same empirical finding may also be obtained from Figure 3 providing, in the form of box-plots, the distribution of the market imperfection and the bargaining power of each agent.

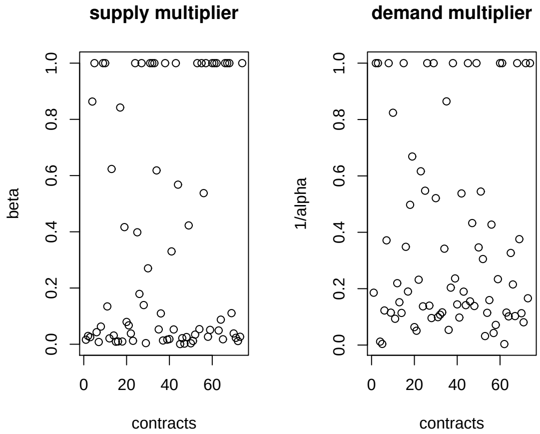

Figure 2: The demand and supply price multipliers

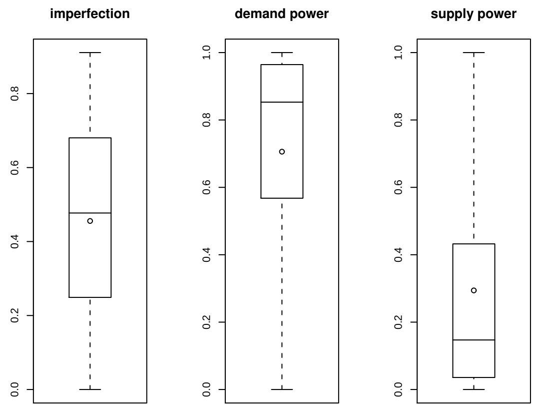

Figure 3: The market imperfection and bargaining powers

When interpreting these numerical results, one should keep in mind that the demand and supply price multipliers and the bargaining power are unit-free coefficients, *i.e.* they are invariant under a change of units of the variables y_i or z_i whereas the measure of market imperfection ranging from 0 to 0.91 stands in the same unit as the price variable.

From the observation of the box-plots, we observe that 50% of the demand (resp. supply) bargaining power is greater (resp. lower) than 0.85 (resp. 0.15). This suggests that freight transport in Belgium is dominated by the demand. Furthermore, on average, the imperfection on the market lies around 0.45 EUR/km.ton. This would mean that, on average, the space of negocation for each firm turns around 0.45 EUR/km.ton.

4 Conclusion

The paper proposes an empirical method for a measure of the market imperfection along with a measure of the bargaining power of the supply and the demand sides. The proposed method is based on an extension of the literature on the measurement of productive efficiency. Market imperfection is approached as a nonunicity of the price corresponding to a specific characteristic of the good (or the service). Working on the product space "price $\times$ characteristic" implies an hedonic approach in which the objective functions of the agents are defined on a set of characteristics whereas the price of the good is defined on the quantities.

From an econometric method point of view, this paper is in line with an exploratory data analysis approach . Indeed neither a precise economic model of market imperfection nor a precise sampling model have been specified. The main motivation for such an approach is that, in the field of freight transport (in Belgium), there is a consensus among field experts that "the market is anything but perfect" but, at the same time, field experts are also interested in trying to evaluate, even roughly, how far from perfect competition operates the market and how asymmetrically, with respect to the demand and supply, operates the market (*i.e.* whether the bargaining powers displays, or not, a major concentration around the value 0.5). That no precise economic model is specified a priori also means that no explanation of the market imperfection is implied. In particular, interpreting $\hat{\mathcal{C}}$ as an estimation of the support of the joint distribution of (y, z) involves no exogeneity assumption (neither on the characteristics z, nor on the price y).

In the illustration presented above, the space of negociation between supply and demand might reflect a non-observable heterogeneity of the transport service and/or of the firms and might also reflect an imperfect information among agents. The method suggested in this paper may be used as a first step, to be completed by a wider data collection oriented toward a more structural economic modelling.

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