T E C H N I C A L
R E P O R T

0329

ELEMENTARY DERIVATIONS OF MULTIVARIATE NON-ADMISSIBILITY RESULTS

D. PAINDAVEINE

I A P S T A T I S T I C S
N E T W O R K

INTERUNIVERSITY ATTRACTION POLE

http://www.stat.ucl.ac.be/IAP

Elementary Derivations of Multivariate Non-admissibility Results

D. Paindaveine [1]

Département de Mathématique, I.S.R.O., and E.C.A.R.E.S., Université Libre de Bruxelles, Campus de la Plaine CP 210, B-1050 Bruxelles, Belgium

Abstract

We provide a simple proof of two recent non-admissibility results in multivariate location and serial problems. We also establish the Pitman non-admissibility of the classical Wilks' test in the problem of testing for multivariate independence.

Key words: Pitman non-admissibility, Rank-based inference, Chernoff-Savage results, Multivariate signs and ranks
1991 MSC: 62G20.

1 Introduction.

Chernoff and Savage (1958) established the amazing fact that the asymptotic relative efficiency (ARE) of the two-sample van der Waerden (i.e., normal-score) rank test with respect to the standard normal-theory competitor, namely the two-sample t-test, is never less than one. The Pitman non-admissibility of the two-sample t-test follows. This uniform dominance of van der Waerden rank tests over classical Gaussian tests holds under the whole class of location problems (one-sample and c-sample problems, ANOVA problems, and regression problems). Of course, this striking result had quite an impact on subsequent development of rank-based inference, an impact at least as important as that of the celebrated Hodges-Lehmann (1956) ".864 result" showing that the lower bound, still in location models, of the AREs of Wilcoxon tests (linear scores) with respect to the normal-theory competitors is .864.

Email address: `dpaindav@ulb.ac.be` (D. Paindaveine).
URL: `http://homepages.ulb.ac.be/~dpaindav` (D. Paindaveine).

[1] Research supported by a P.A.I. contract of the Belgian federal Government, and an A.R.C. contract of the Communauté française de Belgique.

Hallin and Paindaveine (2002a) developed optimal generalized signed-rank tests for the *multivariate* (elliptically symmetric) one-sample location problem, and extended the Chernoff-Savage result to that setup, showing that the classical Gaussian procedure—namely, the Hotelling (1931) T^2 test—is uniformly dominated by the van der Waerden version of their tests, and, consequently, is not admissible in the Pitman sense. In subsequent papers (see, e.g., Hallin and Paindaveine (2003c)), they developed similar rank tests for a variety of multivariate location testing problems (two-sample and c-sample problems, ANOVA problems, and regression problems), and showed that the AREs do coincide with those obtained in the one-sample problem, so that the Pitman non-admissibility of the corresponding classical Gaussian tests (two-sample Hotelling's test and multivariate F-tests) follows.

One should not believe, though, that this Chernoff-Savage result is some kind of miracle that is specific to location problems. Indeed, Hallin (1994) showed that the van der Waerden version of the serial rank tests proposed by Hallin and Puri (1994) also uniformly beats the corresponding everyday practice parametric Gaussian test. The serial rank tests under consideration here allow to test for randomness against serial dependence, to test the adequacy of an ARMA model, or to test linear restrictions on the parameter of an ARMA model (allowing, e.g., for order identification procedures). Hallin and Paindaveine (2002b, 2003a and b) generalized these tests to the multivariate setup (still in the elliptic case) and proved that the Pitman non-admissibility of the Gaussian procedures extends to the multivariate case.

In this paper, we give an elementary proof of the above multivariate location and serial non-admissibility results; the proof, which generalizes to the multivariate case the method used (a) in Gastwirth and Wolff (1968) to prove the univariate location Chernoff-Savage (1958) result and (b) in Hallin (1994) to establish its univariate serial counterpart, allows to avoid the variational arguments used in Hallin and Paindaveine (2002a and b). We also provide an elementary proof of an original non-admissibility result, showing that the van der Waerden version of the rank-score tests of multivariate independence, recently developed by Taskinen et al. (2003), uniformly dominates the classical Wilks (1935) test, which establishes the Pitman-non-admissibility of the latter. To the best of our knowledge, whether Wilks' test is Pitman-non-admissible or not was so far—*even in the univariate case*—an open problem.

The paper is organized as follows. In Section 2, we define the notation to be used in the sequel and describe the elliptically symmetric location and serial problems we will consider. In Section 3, we state the two non-admissibility results proved in Hallin and Paindaveine (2002a and b), and give the announced elementary proof. In Section 4, we establish the Pitman non-admissibility of Wilks' test of multivariate independence. Several final comments are given in Section 5.

2 Elliptically symmetric serial and location problems.

We will discuss non-admissibility issues for several multivariate *elliptically symmetric* testing problems. Recall that the distribution of the k-random vector $\mathbf{X}$ is said to be elliptically symmetric with "parameters" $\mathbf{\Sigma}$ and f (we will denote this distribution by $\mathcal{E}_k(\mathbf{\Sigma}, f)$), iff its pdf is given by

$$\underline{f}_{\mathbf{\Sigma};f}(\mathbf{x}) := c_{k,f}\,(\det\mathbf{\Sigma})^{-1/2} f\left(\left(\mathbf{x}^T\mathbf{\Sigma}^{-1}\mathbf{x}\right)^{1/2}\right), \quad \mathbf{x} \in \mathbb{R}^k, \tag{1}$$

for some symmetric positive definite real $k \times k$ matrix $\mathbf{\Sigma}$ and some function $f : \mathbb{R}_0^+ \longrightarrow \mathbb{R}^+$ such that $f > 0$ a.e. and $\mu_{k-1;f} := \int_0^\infty r^{k-1} f(r)dr < \infty$ ($c_{k,f}$ is a normalization factor depending on the dimension k and f).

The *shape parameter* $\mathbf{\Sigma}$ determines the orientation and shape of the associated equidensity contours. Since every testing problem we will consider is invariant under affine transformations, the AREs under study will not depend on the value of $\mathbf{\Sigma}$, and consequently, we will restrict, without loss of generality, to spherical distributions (for which $\mathbf{\Sigma}$ coincides with the k-dimensional identity matrix $\mathbf{I}_k$).

Under $\mathcal{E}_k(\mathbf{I}_k, f)$, the *radial density* f determines the distribution of $\|\mathbf{X}\|$. More precisely, the pdf of $\|\mathbf{X}\|$ is $\tilde{f}_k(r) := (\mu_{k-1;f})^{-1}\, r^{k-1} f(r)\, I_{[r>0]}$ (I_A stands for the indicator function of the set A). In the sequel, we denote by $\tilde{F}_k$ the distribution function associated with $\tilde{f}_k$.

To guarantee that (1) is a density, we need to assume that $\mu_{k-1;f} < \infty$. When discussing non-admissibility issues of the various parametric Gaussian procedures (which require the underlying distribution to have a finite variance), we will restrict to radial densities satisfying the stronger condition $\mu_{k+1;f} := \int_0^\infty r^{k+1} f(r)dr < \infty$, under which second-order moments associated with the distribution in (1) are finite. One can associate with each radial density f the *density type of* f defined as the class $\{f_a,\, a > 0\}$, where $f_a(r) := f(ar)$, for all $r > 0$. By affine-invariance, one could restrict to couples of parameters of the form $(\mathbf{\Sigma}, f) = (\mathbf{I}_k, f_{a_0})$ for which the variance of the associated elliptical distributions is equal to $\mathbf{I}_k$. However, it will be convenient in the sequel to consider all possible radial densities, so that we will only fix $\mathbf{\Sigma} = \mathbf{I}_k$. Some mild smoothness conditions on f—that we will throughout assume to be fulfilled—are required to derive the AREs we will consider. We refer to Hallin and Paindaveine (2002a and b) for details.

The radial density f is said to be Gaussian iff $f = \phi_a$ for some $a > 0$, where $\phi(r) := \exp(-r^2/2)$. Under $\mathcal{E}_k(\mathbf{I}_k, \phi)$, the pdf of $\|\mathbf{X}\|$ is $\tilde{\phi}_k(r) := (2^{(k-2)/2}\Gamma(k/2))^{-1} r^{k-1}\phi(r)\, I_{[r>0]}$ ($\Gamma(.)$ stands for the Euler gamma function), and we will denote by $\tilde{\Phi}_k$ the associated cdf. Under $\mathcal{E}_k(\mathbf{I}_k, \phi)$, the distribution of $\|\mathbf{X}\|^2 = (\tilde{\Phi}_k^{-1}(U))^2$ (throughout U will stand for a random variable that is uniformly distributed on $]0,1[$) is χ_k^2, so that the cdf of $\|\mathbf{X}\|$ is simply given by $\tilde{\Phi}_k(r) = \Psi_k(r^2)$, where Ψ_k denotes the distribution function of the χ_k^2 distribution.

The first problem we consider is the problem of testing for multivariate (elliptical) randomness against (elliptical) VARMA dependence. More precisely, we want to test the null hypothesis that the k-variate sample $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n$ is the realization of an i.i.d. process; under the alternative, the sample is generated by some non-trivial VARMA process of the form

$$\mathbf{X}_t - \sum_{i=1}^{p} \mathbf{A}_i \mathbf{X}_{t-i} = \boldsymbol{\varepsilon}_t + \sum_{i=1}^{q} \mathbf{B}_i \boldsymbol{\varepsilon}_{t-i},$$

where $\mathbf{A}_1, \ldots, \mathbf{A}_p, \mathbf{B}_1, \ldots, \mathbf{B}_q$ are $k \times k$ real matrices and where the $\boldsymbol{\varepsilon}_t$'s are i.i.d. elliptically symmetric k-vectors. Hallin and Paindaveine (2002b) proposed multivariate signed-rank procedures for this problem, and showed that the ARE, under radial density f, of the van der Waerden version of their tests, with respect to the classical parametric Gaussian procedure (a multivariate Portmanteau test), is given by

$$\mathrm{ARE}_{k,f}^{(\mathrm{ser})}(\phi_{vdW}/\phi_{\mathcal{N}}) = \frac{1}{k^4}\Big[D_k(\phi, f)\Big]^2 \Big[C_k(\phi, f)\Big]^2,$$

where we let

$$C_k(\phi, f) := \mathrm{E}\Big[\tilde{\Phi}_k^{-1}(U)\, \varphi_f(\tilde{F}_k^{-1}(U))\Big], \quad \text{and} \quad D_k(\phi, f) := \mathrm{E}\Big[\tilde{\Phi}_k^{-1}(U)\, \tilde{F}_k^{-1}(U)\Big];$$

see Hallin and Paindaveine (2002b).

The second problem we will consider is the multivariate elliptically symmetric one-sample location problem, for which, on the basis of the sample $\mathbf{X}_i = \boldsymbol{\theta} + \boldsymbol{\varepsilon}_i$, $i = 1, \ldots, n$, where the $\boldsymbol{\varepsilon}_i$'s are i.i.d. elliptically symmetric k-vectors, one wants to test $\boldsymbol{\theta} = \boldsymbol{\theta}_0$ against $\boldsymbol{\theta} \neq \boldsymbol{\theta}_0$, for some fixed k-vector $\boldsymbol{\theta}_0$. The classical parametric Gaussian test for this problem is Hotelling's T^2 test. Hallin and Paindaveine (2002a) defined optimal signed-rank competitors of Hotelling's test. The ARE of the van der Waerden version of their tests, with respect to Hotelling's test, under radial density f, is given by

$$\mathrm{ARE}_{k,f}^{(\mathrm{loc})}(\phi_{vdW}/\phi_{\mathcal{N}}) = \frac{1}{k^3}\, D_k(f) \Big[C_k(\phi, f)\Big]^2,$$

where

$$D_k(f) := \mathrm{E}\Big[\Big(\tilde{F}_k^{-1}(U)\Big)^2\Big];$$

see Hallin and Paindaveine (2002a).

In the next section, we give an elementary proof showing that both above families of AREs are uniformly larger than 1, showing that the classical Gaussian procedures are not admissible in the Pitman sense. In the multivariate case, this result was first established in Hallin and Paindaveine (2002a and b) for the location and serial case, respectively.

3 An elementary proof of two recent non-admissibility results.

We start with the non-admissibility of the parametric Gaussian (everyday practice) procedure allowing to test for multivariate randomness, i.e., the multivariate Portmanteau test (see Hallin and Paindaveine (2002b)).

Theorem 1 *For all radial density f such that $\mu_{k+1;f} < \infty$ and all positive integer k, we have $\mathrm{ARE}^{(\mathrm{ser})}_{k,f}(\phi_{vdW}/\phi_{\mathcal{N}}) \geq 1$, where equality holds iff f is Gaussian.*

Let us now give the elementary proof we propose for this result, which allows to avoid variational arguments, such as those used in Hallin and Paindaveine (2002b). The proof, which is based on the method developed in Gastwirth and Wolff (1968), does only make use of Jensen's inequality and—*in the multivariate case*—of the arithmetic-harmonic inequality.

Proof of Theorem 1. First rewrite the functional $f \mapsto C_k(\phi, f)$ as

$$\begin{aligned}
C_k(\phi,f) &= \int_0^\infty \tilde{\Phi}_k^{-1}(\tilde{F}_k(r))\,\varphi_f(r)\,\tilde{f}_k(r)\,dr \\
&= \frac{1}{\mu_{k-1;f}}\int_0^\infty \tilde{\Phi}_k^{-1}(\tilde{F}_k(r))\,(-f'(r))\,r^{k-1}\,dr \\
&= \int_0^\infty \left[\frac{\tilde{f}_k(r)}{\tilde{\phi}_k(\tilde{\Phi}_k^{-1}(\tilde{F}_k(r)))} + (k-1)\frac{\tilde{\Phi}_k^{-1}(\tilde{F}_k(r))}{r}\right]\tilde{f}_k(r)\,dr,
\end{aligned}$$

where the last equality follows by integrating by parts. Applying successively Jensen's inequality (with convex function $g(x) = 1/x$) and the arithmetic-harmonic mean inequality, one obtains

$$\begin{aligned}
C_k(\phi,f) &\geq \left\{\int_0^\infty \left[\frac{\tilde{f}_k(r)}{\tilde{\phi}_k(\tilde{\Phi}_k^{-1}(\tilde{F}_k(r)))} + (k-1)\frac{\tilde{\Phi}_k^{-1}(\tilde{F}_k(r))}{r}\right]^{-1}\tilde{f}_k(r)\,dr\right\}^{-1} \\
&\geq k^2\left\{\int_0^\infty \left[\frac{\tilde{\phi}_k(\tilde{\Phi}_k^{-1}(\tilde{F}_k(r)))}{\tilde{f}_k(r)} + (k-1)\frac{r}{\tilde{\Phi}_k^{-1}(\tilde{F}_k(r))}\right]\tilde{f}_k(r)\,dr\right\}^{-1}. \quad (2)
\end{aligned}$$

Now, integrating by parts again yields

$$\begin{aligned}\int_0^\infty \tilde{\phi}_k(\tilde{\Phi}_k^{-1}(\tilde{F}_k(r)))\,dr &= -\int_0^\infty r\,\frac{\tilde{\phi}_k'(\tilde{\Phi}_k^{-1}(\tilde{F}_k(r)))}{\tilde{\phi}_k(\tilde{\Phi}_k^{-1}(\tilde{F}_k(r)))}\,\tilde{f}_k(r)\,dr\\ &= \int_0^\infty r\left[\tilde{\Phi}_k^{-1}(\tilde{F}_k(r)) - \frac{k-1}{\tilde{\Phi}_k^{-1}(\tilde{F}_k(r))}\right]\tilde{f}_k(r)\,dr.\end{aligned}$$

Substituting in (2), we obtain

$$C_k(\phi, f) \geq k^2\left\{\int_0^\infty r\ \tilde{\Phi}_k^{-1}(\tilde{F}_k(r))\,\tilde{f}_k(r)\,dr\right\}^{-1} = k^2\left[D_k(\phi, f)\right]^{-1},$$

which establishes the inequality in Theorem 1.

Now, for the equality to hold, Jensen's inequality and the arithmetic-harmonic inequality need to be degenerate, i.e., we need to have

$$\frac{\tilde{f}_k(r)}{\tilde{\phi}_k(\tilde{\Phi}_k^{-1}(\tilde{F}_k(r)))} + (k-1)\frac{\tilde{\Phi}_k^{-1}(\tilde{F}_k(r))}{r} = c,\ \forall\, r > 0, \tag{3}$$

and

$$\frac{\tilde{\phi}_k(\tilde{\Phi}_k^{-1}(\tilde{F}_k(r)))}{\tilde{f}_k(r)} = \frac{r}{\tilde{\Phi}_k^{-1}(\tilde{F}_k(r))},\ \forall\, r > 0, \tag{4}$$

respectively. Equation (3) can be rewritten

$$r^{1-k}\left[r^{k-1}\,\tilde{\Phi}_k^{-1}(\tilde{F}_k(r))\right]' = c,\ \forall\, r > 0,$$

and holds iff $r^{k-1}\,\tilde{\Phi}_k^{-1}(\tilde{F}_k(r)) = ar^k + b$, for all $r > 0$, for some real numbers a, b. Since the limit of $r^{k-1}\,\tilde{\Phi}_k^{-1}(\tilde{F}_k(r))$ as r goes to 0 is 0, we must have $b = 0$. This implies that $\tilde{\Phi}_k^{-1}(\tilde{F}_k(r)) = ar$, for all $r > 0$, that is, $\tilde{\Phi}_k^{-1}(u) = a\tilde{F}_k^{-1}(u)$ for all $0 < u < 1$, which means that f is Gaussian (with arbitrary scale).

Finally, (4) is successively equivalent to

$$\begin{aligned}\tilde{f}_k(\tilde{F}_k^{-1}(u))\tilde{F}_k^{-1}(u) &= \tilde{\phi}_k(\tilde{\Phi}_k^{-1}(u))\,\tilde{\Phi}_k^{-1}(u),\ \forall\, 0 < u < 1,\\ \Leftrightarrow \left[\tilde{F}_k^{-1}(u)\right]'/\tilde{F}_k^{-1}(u) &= \left[\tilde{\Phi}_k^{-1}(u)\right]'/\tilde{\Phi}_k^{-1}(u),\ \forall\, 0 < u < 1,\\ \Leftrightarrow \tilde{F}_k^{-1}(u) &= a\tilde{\Phi}_k^{-1}(u),\ \forall\, 0 < u < 1,\end{aligned}$$

so that (4) holds iff f is Gaussian (still with arbitrary scale). □

An important corollary is the non-admissibility of Hotelling's T^2 test for the multivariate one-sample location problem. More precisely, we have the following :

Corollary 1 *For all radial density f such that $\mu_{k+1;f} < \infty$ and all positive integer k, we have $\mathrm{ARE}^{(\mathrm{loc})}_{k,f}(\phi_{vdW}/\phi_{\mathcal{N}}) \geq 1$, where equality holds iff f is Gaussian.*

Proof of Corollary 1. Cauchy-Schwarz inequality yields

$$\Big[D_k(\phi, f)\Big]^2 \leq D_k(\phi)D_k(f) = k\,D_k(f), \tag{5}$$

so that $\mathrm{ARE}^{(\mathrm{loc})}_{k,f}(\phi_{vdW}/\phi_{\mathcal{N}}) \geq \mathrm{ARE}^{(\mathrm{ser})}_{k,f}(\phi_{vdW}/\phi_{\mathcal{N}})$ for all radial density f. Consequently, the result follows from Theorem 1. Equality holds iff we have equality in (5), i.e., iff $\tilde{F}_k^{-1}(u) = a\tilde{\Phi}_k^{-1}(u)\;\forall\, 0 < u < 1$ for some $a > 0$, that is, iff f is Gaussian (with arbitrary scale). □

4 Pitman non-admissibility of Wilks' test of multivariate independence.

We now consider the problem of testing for multivariate independence. More precisely, consider a sample of i.i.d. $(k+l)$-random vectors $(\mathbf{X}_1^T, \mathbf{Y}_1^T)^T$, $(\mathbf{X}_2^T, \mathbf{Y}_2^T)^T, \ldots, (\mathbf{X}_n^T, \mathbf{Y}_n^T)^T$. We want to test the null hypothesis of independence between the k-subvector $\mathbf{X}_1$ and l-subvector $\mathbf{Y}_1$. We restrict to the elliptic version of this problem, for which the distributions of the subvectors are elliptically symmetric. In the sequel, f (resp. g) will stand for the radial density of $\mathbf{X}_1$ (resp. of $\mathbf{Y}_1$). Since the problem is invariant under block-diagonal affine transformations, we still restrict to spherically symmetric marginal distributions.

Taskinen et al. (2003) recently proposed rank-score competitors of the classical Gaussian procedure, namely Wilks' test. Considering quite specific local alternatives (see Taskinen et al. (2003) for details), they showed that the ARE, under radial density f, of the van der Waerden version of their tests with respect to Wilks' test is given by

$$\mathrm{ARE}^{(\mathrm{ind})}_{f,k;g,l}(\phi_{vdW}/\phi_{\mathcal{N}}) = \frac{1}{4k^2l^2}\Big(D_k(\phi, f)C_l(\phi, g) + D_l(\phi, g)C_k(\phi, f)\Big)^2.$$

Some numerical values of the AREs are provided in Table 1. All these values are larger or equal than 1, and seem to be equal to 1 only if the marginals are both Gaussian. This is actually an empirical verification of the following result which establishes the Pitman-non-admissibility of Wilks' test.

Corollary 2 *For all integers $k, l \geq 1$ and all radial densities f, g such that $\mu_{k+1;f} < \infty$ and $\mu_{l+1;g} < \infty$, we have $\mathrm{ARE}^{(\mathrm{ind})}_{f,k;g,l}(\phi_{vdW}/\phi_{\mathcal{N}}) \geq 1$, where equality holds iff f and g do coincide and are Gaussian.*

Proof of Corollary 2. The proof is based on the decomposition

$$\Big(D_k(\phi, f)C_l(\phi, g) + D_l(\phi, g)C_k(\phi, f)\Big)^2 = A_{f,k;g,l} + B_{f,k;g,l},$$

where we let

$$A_{f,k;g,l} := 4\, D_k(\phi, f)C_k(\phi, f)D_l(\phi, g)C_l(\phi, g), \text{ and}$$

$$B_{f,k;g,l} := \Big(D_k(\phi, f)C_l(\phi, g) - D_l(\phi, g)C_k(\phi, f)\Big)^2.$$

It directly follows from $D_k(\phi, f)C_k(\phi, f) \geq k^2$ (see Theorem 1) that

$$\mathrm{ARE}^{(\mathrm{ind})}_{f,k;g,l}(\phi_{vdW}/\phi_{\mathcal{N}}) \geq \frac{1}{4k^2l^2}\, A_{f,k;g,l} \geq 1. \tag{6}$$

Let us now show that equality holds iff f and g are Gaussian. For the equality to hold, we need to have $A_{f,k;g,l} = 4k^2l^2$ and $B_{f,k;g,l} = 0$. From Theorem 1, $A_{f,k;g,l} = 4k^2l^2$ implies that both f and g are Gaussian ($f = \phi_a$ and $g = \phi_b$, say). Now, since $D_k(\phi, \phi_a) = a^{-1}D_k(\phi) = a^{-1}k$ and $C_k(\phi, \phi_a) = aC_k(\phi) = aD_k(\phi) = ak$, we have $B_{\phi_a,k;\phi_b,l} = k^2l^2((b/a) - (a/b))^2$, which is equal to zero iff $a = b$. Consequently, equality holds iff $f = g = \phi_a$, for some $a > 0$. $\square$

5 Final comments.

We would like to stress that the non-admissibility results in Theorem 1 and Corollary 1 are not confined to the problem of testing for multivariate randomness and to the multivariate one-sample location problem, respectively. Indeed, in a series of papers, Hallin and Paindaveine (2003a, b, and c) extended their generalized signed-rank tests to more complicated models culminating in the multivariate general linear model with VARMA errors. The problems that can be dealt with are either associated with *simple* null hypotheses or with null hypotheses that are *linear restrictions* on the parameter of that very general model. Hallin and Paindaveine (2003c) showed that the AREs of the van der Waerden version of their tests with respect to normal-theory competitors are convex linear combinations of the AREs obtained in the one-sample location case and the problem of testing for randomness. Consequently, the Pitman non-admissibility of parametic Gaussian procedures extends to that very broad class of problems, which contains problems of high practical relevance, such as multivariate Durbin Watson problems, ANOVA problems, the problem of testing the orders of a VARMA series, etc.

References

[1] Chernoff, H., Savage, I. R., 1958. Asymptotic normality and efficiency of certain nonparametric tests, *Ann. Math. Stat.* 29, 972-994.

[2] Gastwirth, J. L., Wolff, S. S., 1968. An elementary method for obtaining lower bounds on the asymptotic power of rank tests, *Ann. Math. Stat.* 39, 2128-2130.

[3] Hallin, M., 1994. On the Pitman-nonadmissibility of correlogram-based methods, *J. Times Series Anal.* 15, 607-612.

[4] Hallin, M., Paindaveine, D., 2002a. Optimal tests for multivariate location based on interdirections and pseudo-Mahalanobis ranks, *Ann. Stat.* 30, 1103-1133.

[5] Hallin, M., Paindaveine, D., 2002b.Optimal procedures based on interdirections and pseudo-Mahalanobis ranks for testing multivariate elliptic white noise against ARMA dependence, *Bernoulli* 8, 787-816.

[6] Hallin, M., Paindaveine, D., 2003a. Rank-based optimal tests of the adequacy of an elliptic VARMA model. Submitted.

[7] Hallin, M., Paindaveine, D., 2003b. Asymptotic linearity of serial and nonserial multivariate signed rank statistics. Submitted.

[8] Hallin, M., Paindaveine, D., 2003c. Affine-invariant aligned rank tests for the multivariate general linear model with VARMA errors. Submitted.

[9] Hallin, M., Puri, M. L., 1994. Aligned rank tests for linear models with autocorrelated error terms, *J. Multivariate Anal.* 50, 175-237.

[10] Hodges, J. L., Lehmann, E. L., 1956. The efficiency of some nonparametric competitors of the t-test, *Ann. Math. Stat.* 27, 324-335.

[11] Hotelling, H., 1931. The generalization of Student's ratio, *Ann. Math. Stat.* 2, 360-378.

[12] Taskinen, S., Kankainen, A., Oja, H., 2003. Rank scores tests of multivariate independence. Submitted.

[13] Wilks, S. S., 1935. On the independence of k sets of normally distributed statistical variables, *Econometrica* 3, 309-326.

		ν_k							ν_k				
l	ν_l	3	4	6	12	∞	l	ν_l	3	4	6	12	∞
1	3	1.378	1.295	1.266	1.281	1.339	4	3	1.430	1.332	1.292	1.298	1.348
	4	1.293	1.190	1.141	1.135	1.167		4	1.336	1.223	1.167	1.156	1.183
	6	1.267	1.144	1.078	1.054	1.067		6	1.294	1.165	1.096	1.069	1.080
	12	1.285	1.141	1.058	1.019	1.016		12	1.295	1.149	1.064	1.024	1.020
	∞	1.343	1.174	1.072	1.017	1.000		∞	1.343	1.174	1.072	1.017	1.000
2	3	1.400	1.311	1.277	1.289	1.343	6	3	1.448	1.345	1.301	1.304	1.351
	4	1.311	1.204	1.152	1.144	1.174		4	1.353	1.236	1.177	1.163	1.189
	6	1.277	1.152	1.085	1.060	1.072		6	1.306	1.175	1.103	1.075	1.085
	12	1.289	1.144	1.060	1.021	1.017		12	1.300	1.153	1.068	1.027	1.023
	∞	1.343	1.174	1.072	1.017	1.000		∞	1.343	1.174	1.072	1.017	1.000
3	3	1.417	1.323	1.286	1.294	1.346	10	3	1.471	1.361	1.312	1.311	1.353
	4	1.325	1.214	1.161	1.150	1.179		4	1.375	1.252	1.190	1.173	1.196
	6	1.286	1.159	1.091	1.065	1.076		6	1.323	1.188	1.114	1.084	1.092
	12	1.292	1.146	1.062	1.023	1.019		12	1.308	1.159	1.073	1.032	1.027
	∞	1.343	1.174	1.072	1.017	1.000		∞	1.343	1.174	1.072	1.017	1.000

Table 1
AREs of the van der Waerden version of Taskinen et al. (2003) rank-score tests for multivariate independence with respect to Wilks' test, under standard multivariate Student (with 3, 4, 6, and 12 degrees of freedom) and standard Gaussian densities, respectively, for subvector dimensions $k = 2$ and $l = 1, 2, 3, 4, 6$, and 10.