Mutual Synchronization of Robots via Estimated State Feedback: A Cooperative Approach

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***Abstract*—In this paper, a controller that solves the problem of position synchronization of two (or more) robot systems, under a cooperative scheme, in the case when only position measurements are available, is presented. The synchronization controller consists of a feedback control law and a set of nonlinear observers. Coupling errors are introduced to create interconnections that render mutual synchronization of the robots. It is shown that the controller yields semiglobal exponential convergence of the synchronization closed-loop errors. Experimental results show, despite obvious model uncertainties, a good agreement with the predicted convergence.**

***Index Terms*—Cooperative systems, feedback systems, multiple manipulators, observers, synchronization.**

I. Introduction

TODAY, the developments in technology and the requirements on efficiency and quality in production processes have resulted in complex and integrated systems. In production processes such as manufacturing and automotive applications there is a high requirement on flexibility and maneuverability of the involved systems. In most of these processes the use of multicomposed systems is widely spread, and their variety in uses includes assembling, transporting, painting and welding. All these tasks require large maneuverability and manipulability of the executing systems, often even some of the tasks can not be carried out by a single system. In these cases, the use of multicomposed systems have been considered as an option.

In practice, many multicomposed systems work either under cooperative or under coordinated schemes. Synchronization, coordination, and cooperation are intimately linked subjects and very often they are used as synonyms to describe the same kind of behavior.

In the last century, synchronization received a lot of attention in the Russian scientific community since it was observed in balanced and unbalanced rotors and vibro-exciters [1]. Nowadays, there are several works related to synchronization of rotating bodies and electromechanical systems [2]–[5]. For mechanical systems synchronization is of great importance as soon as two machines have to cooperate. The cooperative behavior gives flexibility and maneuverability that cannot be achieved by an individual system, e.g., multifinger robot-hands, multirobot systems and multiactuated platforms [6], teleoperated master-slave systems [7]–[9].

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According to [2] synchronization may be defined as the mutual time conformity of two or more processes. Furthermore, based on the type of interconnections (interactions) in the system, different kinds of synchronization can be defined [10]. In case of disconnected systems that present synchronous behavior this is referred to as *natural synchronization*. When synchronization is achieved by proper interconnections in the systems, i.e., without any artificially introduced external action, then the system is referred to as *self-synchronized*. When there exist external actions (controls) and/or artificial interconnections then the system is called *controlled-synchronized*.

Depending on the formulation of the controlled synchronization distinction should be made between *internal (mutual) synchronization*, when all synchronized objects occur on equal terms, e.g., cooperative systems, and *external synchronization*, when one object is more powerful than the others and its motion can be considered as independent of the motion of the other objects, e.g., master–slave systems.

This paper focuses on controlled synchronization of robot systems, which nowadays are common and important systems in production processes. However, the general ideas developed here can be extended to more general mechanical systems, such as mobile robots, motors, etc. Robot manipulators are widely used in production processes where high flexibility, manipulability, and maneuverability are required. In tasks that cannot be carried out by a single robot, either because of the complexity of the task or limitations of the robot, the use of multirobot systems working in external synchronization, e.g., master-slave and coordinated schemes [8], [9], or mutual synchronization, e.g., cooperative schemes [6], have proved to be a good alternative. Coordinated and cooperative schemes are important illustrations of the same goal, where it is desired that two or more robot systems, either identical or different, work in synchrony [6], [11]. This can be formulated as a control problem that implies the design of suitable controllers to achieve the required synchronous motion.

The problem of synchronization of robotic systems seems to be a straightforward extension of classical tracking controllers, however it implies challenges that are not considered in the design of tracking controllers. The interconnections (interactions) between the robots imply control problems that are not considered in classical tracking controllers. However, the interconnections cannot be neglected since they are precisely what determine the synchronized behavior. The interconnections between the robots generate the flow of information necessary to guarantee the synchronous behavior. Also, problems can arise because of the particular structure of the robots, such as type of joints (rigid or elastic), kinematic pairs (prismatic, rotational, etc.), transmission elements (gears, belts). Furthermore, available equipment, such as, velocity and acceleration measuring capabilities, noise in the measurements and time delays, might cause other problems in robot synchronization.

In this paper, a synchronization controller based only on position measurements, that uses coupling errors to induce the mutual synchronization behavior, is proposed. The idea of coupling errors has been exploited by several authors, see for instance [12] and [13]. However, the mentioned works report only set point synchronization and, in addition, are based on full-state measurements.

The general setup in this paper is as follows. Consider a multirobot system formed by $p(p \geq 2)$ rigid joint robots together with a common desired trajectory for all of them, denoted by q_d, $\dot{q}_d$. Then, the mutual synchronization control problem can be formulated as to design interconnections and controllers $\tau_i(\cdot)$ for all the robots in the system, such that the angular positions and velocities $q_i, \dot{q}_i \in \mathbb{R}^n$ of the ith robot are synchronized with respect to $q_d, \dot{q}_d$ and $q_j, \dot{q}_j \in \mathbb{R}^n$, $(j = 1, \ldots, p, j \neq i)$. It is assumed that the dynamic model of each robot is known and free of uncertainties and modeling errors. The major constraint to design the synchronization controller is that only the angular positions q_i of all the robots are measured. This problem is solved by using nonlinear model-based observers. The estimated variables (velocities and accelerations) are used in a feedback loop, such that the feedback controller plus the observers, guarantee mutual synchronization of the multirobot system.

To clarify the proposed mutual synchronization controller first the case with all the state available is considered. Then the case of partial access to the state (only positions) is addressed.

The paper is organized as follows. The dynamic model of the robot is presented in Section II. A mutual synchronization controller for frictionless robots, assuming all measurements available is presented in Section III. Section IV presents a modified synchronization controller that considers nonlinear observers. A gain tuning procedure for the observers and feedback controller gains is given in Section V. Section VI presents a modified version of the synchronization controller when friction is considered. An experimental study with two robots with four degrees of freedom is presented in Section VII. The paper closes with some conclusions inSection VIII.

II. Dynamic Model of the Robot Manipulators

First, the dynamic model for frictionless robots is presented. Although, in some cases friction phenomena can straightforwardly be compensated, in general friction phenomena require special treatment. Therefore, a second model considering friction effects is also introduced.

Consider p frictionless rigid joint robots with n joints, i.e., with joint coordinates $q_i \in \mathbb{R}^n$, $i = 1, \ldots, p$. Assume that all the joints are rotational and fully actuated. The kinetic energy of the ith robot is given by $T(q_i, \dot{q}_i) = (1/2)\dot{q}_i^T M_i(q_i)\dot{q}_i$, with $M(q_i) \in \mathbb{R}^{n\times n}$ the symmetric, positive definite inertia matrix, and the potential energy due to gravity is denoted by $U_i(q_i)$. Hence, applying the Euler–Lagrange formalism [14], [15] the dynamic model of the ith robot manipulator is given by

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i, \quad i = 1, \ldots, p \tag{1}$$

where $g_i(q_i) = (\partial/\partial q_i)U_i(q_i) \in \mathbb{R}^n$ denotes the gravity forces, $C_i(q_i, \dot{q}_i) \in \mathbb{R}^n$ represents the Coriolis and centrifugal forces, and τ_i denotes the $[n \times 1]$ vector of torques.

Various mathematical models, both static and dynamic, have been proposed to describe a number of friction phenomena [16], [17]. Which model is more suitable for modeling and control purposes depends on the physical friction phenomena observed in the system, such as stiction, Stribeck effects, viscous friction, and on the velocity regime in which the system is supposed to work, i.e., slow, medium or high speed.

A major difficulty in static-friction models is the discontinuity that the Coulomb friction represents. The discontinuity at zero velocity may lead to nonuniqueness of the solution of the equation of motion, and numerical problems if such a model is used in simulations. A way to deal with the Coulomb discontinuity is to use approximations.

When friction forces $f_i(\dot{q}_i)$ are considered the model (1) changes to

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) + f_i(\dot{q}_i) = \tau_i \tag{2}$$

where as in [18], friction forces can be modeled as

$$\begin{aligned} f_i(\dot{q}_i) = B_{v,i}\dot{q}_i + B_{f1,i}\left(1 - \frac{2}{1 + e^{2w_{1,i}\dot{q}_i}}\right) \\ + B_{f2,i}\left(1 - \frac{2}{1 + e^{2w_{2,i}\dot{q}_i}}\right) \end{aligned} \tag{3}$$

where $B_{v,i}$ is the viscous friction coefficient and the remaining terms model the Coulomb and Stribeck friction. The coefficients $w_{1,i}$, $w_{2,i}$ determine the slope in the approximation of the sgn function in the Coulomb friction.

The dynamic models (1) and (2) possess structural properties which are useful along the stability analysis.

- The inertia matrix $M_i(q_i) \in \mathbb{R}^{n\times n}$ is symmetric and positive definite for all $q_i \in \mathbb{R}^n$.
- If the matrix $C_i(q_i, \dot{q}_i) \in \mathbb{R}^{n\times n}$ is defined using the Christoffel symbols [15], then the matrix $\dot{M}_i(q_i) - C_i(q_i, \dot{q}_i)$ is skew symmetric, i.e., for all $x \in \mathbb{R}^n$

$$x^T\left(\dot{M}_i(q_i) - C_i(q_i, \dot{q}_i)\right)x = 0. \tag{4}$$

- In addition, for the previous choice of $C_i(q_i, \dot{q}_i)$, it follows that

$$C_i(q_i, \dot{q}_i) = \begin{bmatrix} \dot{q}_i^T C_1(q_i) \\ \vdots \\ \dot{q}_i^T C_n(q_i) \end{bmatrix} \tag{5}$$

where $C_j(q_i) \in \mathbb{R}^{n\times n} j = 1, \ldots, n$ are symmetric matrices [19]. It follows that for any scalar α and for all q_i, x, y, $z \in \mathbb{R}^n$

$$\begin{aligned} C_i(q_i, x)y &= C_i(q_i, y)x \\ C_i(q_i, z + \alpha x)y &= C_i(q_i, z)y + \alpha C_i(q_i, x)y. \end{aligned} \tag{6}$$

- $M_i(q_i)$, $C_i(q_i, \dot{q}_i)$ and $g_i(q_i)$ are bounded with respect to q_i, [20], so for all $q \in \mathbb{R}^n$

$$0 < M_{m,i} \leq \|M_i(q_i)\| \leq M_{M,i}, \quad \|g_i(q_i)\| \leq g_{M,i} \quad (7)$$
$$\|C_i(q_i, x)\| \leq C_{M,i}\|x\| \quad \text{for all} \quad x \in \mathbb{R}^n. \quad (8)$$

- The diagonal positive–definite viscous-friction coefficient matrix $B_{v,i} \in \mathbb{R}^{n \times n}$ is bounded, i.e.,

$$0 < Bv_{m,i} \leq \|B_{v,i}\| \leq Bv_{M,i}.$$

III. Synchronization Controller Based on State Feedback

If the full state of all the robots in the multicomposed system is available, then one can propose a mutual synchronization controller τ_i for the ith robot, $i = 1, \ldots, p$, as

$$\tau_i = M_i(q_i)\ddot{q}_{ri} + C_i(q_i, \dot{q}_i)\dot{q}_{ri} + g_i(q_i) - K_{d,i}\dot{s}_i - K_{p,i}s_i \quad (9)$$

where $K_{p,i}$, $K_{d,i} \in \mathbb{R}^{n \times n}$ are positive definite gain matrices, $\dot{s}_i$, $s_i \in \mathbb{R}^n$ are the synchronization errors defined by

$$s_i := q_i - q_{ri}, \quad \dot{s}_i := \dot{q}_i - \dot{q}_{ri}. \quad (10)$$

To generate interactions between the robots and to guarantee the synchronous behavior, define the reference signals as

$$\begin{aligned} q_{ri} &= q_d - \sum_{j=1, j\neq i}^{p} K_{cp\_i,j}(q_i - q_j) \\ \dot{q}_{ri} &= \dot{q}_d - \sum_{j=1, j\neq i}^{p} K_{cv\_i,j}(\dot{q}_i - \dot{q}_j) \\ \ddot{q}_{ri} &= \ddot{q}_d - \sum_{j=1, j\neq i}^{p} K_{ca\_i,j}(\ddot{q}_i - \ddot{q}_j) \end{aligned} \quad (11)$$

where $K_{cp\_i,j}$, $K_{cv\_i,j}$, $K_{ca\_i,j} \in \mathbb{R}^{n \times n}$, $i, j = 1, \ldots, p$, are positive semi-definite diagonal matrices that define the interactions between the robots in the system.

Note that (10) and (11) define a tradeoff in the reference signals: each robot will be enforced to follow the desired common trajectory $q_d(t)$, while the robots should mutually synchronize. The second term in the right-hand side of q_{ri}, $\dot{q}_{ri}$, and $\ddot{q}_{ri}$ represents the "feedback" of the synchronization errors between the ith robot and the other robots in the system. The gains $K_{cp\_i,j}$, $K_{cv\_i,j}$, $K_{ca\_i,j}$ allow to weight the synchronization errors between the robots and the desired common trajectory. Therefore, these errors can be penalized; as a result priority to synchronicity between the robots or with respect to the common desired trajectory can be assigned. This is particularly important during transients when large errors can cause instability and/or compromise the synchronous behavior of the complete multi-robot system.

Assumption 1: For simplicity it is assumed that for all i, $j = 1, \ldots, p$ the coupling gains $K_{cp\_i,j}$, $K_{cv\_i,j}$, $K_{ca\_i,j}$ satisfy $K_{cp\_i,j} = K_{cv\_i,j} = K_{ca\_i,j} = K_{i,j}$

Define the partial synchronization errors between the ith and the jth robots in the multirobot system by

$$e_{i,j} = q_i - q_j, \quad \dot{e}_{i,j} = \dot{q}_i - \dot{q}_j \quad (12)$$

for all i, $j = 1, \ldots, p$, $i \neq j$, and for $j = i$ as

$$e_{i,i} = q_i - q_d, \quad \dot{e}_{i,i} = \dot{q}_i - \dot{q}_d. \quad (13)$$

Then $\dot{s}_i$, $s_i \in \mathbb{R}^n$ defined by (10) can be written as

$$s_i = e_{i,i} + \sum_{j=1, j\neq i}^{p} K_{i,j}e_{i,j}, \quad \dot{s}_i = \dot{e}_{i,i} + \sum_{j=1, j\neq i}^{p} K_{i,j}\dot{e}_{i,j}. \quad (14)$$

A. Stability Analysis

Substitution of (9) and (11) in the robot dynamics (1), and considering s_i, $\dot{s}_i$, given by (10), yields the synchronization closed-loop systems (for $i = 1, \ldots, p$)

$$M_i(q_i)\ddot{s}_i = -C_i(q_i, \dot{q}_i)\dot{s}_i - K_{d,i}\dot{s}_i - K_{p,i}s_i. \quad (15)$$

Note that all the couplings between the robots are modeled by s_i, $\dot{s}_i$, $\ddot{s}_i$, such that in these variables the synchronization-error dynamics for each robot in the system is decoupled (15).

The stability properties of the synchronization-error dynamics (15) are summarized in the following theorem.

Theorem 2: Consider the closed-loop system formed by the controller (9), the reference signals (11) and the robot dynamics (1). Then the synchronization errors s_i, $\dot{s}_i$ are globally asymptotically stable if the control gains $K_{d,i}$, $K_{p,i}$, $i = 1, \ldots, p$ are positive definite.

Proof: Define the vector $s = [s_1 \cdots s_p]$ and take as Lyapunov function

$$V(s, \dot{s}) = \sum_{i=1}^{p} \left\{ \frac{1}{2}\dot{s}_i^T M_i(q_i)\dot{s}_i + \frac{1}{2}s_i^T K_{p,i}s_i \right\}. \quad (16)$$

The matrix $V(s, \dot{s})$ is positive definite for all $K_{p,i} > 0$, s_i, $\dot{s}_i$, and $V(s, \dot{s}) = 0$ if and only if $s_i = 0$, $\dot{s}_i = 0$.

The time derivative of $V(s, \dot{s})$ along (15) is given by

$$\dot{V}(s, \dot{s}) = -\sum_{i=1}^{p} \dot{s}_i^T K_{d,i}\dot{s}_i. \quad (17)$$

Therefore, $\dot{V}(s, \dot{s})$ is negative semi-definite for all $K_{d,i} > 0$; thus the synchronization errors s_i, $\dot{s}_i$ are stable, but asymptotical stability cannot be concluded yet. By following very standard stability tools, e.g., Barbalat's lemma, it can be proved that the only invariant set that satisfies $\dot{V}(s, \dot{s}) = 0$ is the origin, see [19] and [20]. Therefore, it can be concluded that s_i, $\dot{s}_i$ are globally asymptotically stable. ■

Note that s_i, $\dot{s}_i$ are linear combinations of the partial synchronization errors. Therefore, it is still necessary to prove that s_i, $\dot{s}_i$ being asymptotically stable implies global-asymptotic synchronization between the robots.

Lemma 3: Consider the diagonally dominant matrix $M_c(K_{i,j}) \in \mathbb{R}^{(n \cdot p) \times (n \cdot p)}$, shown in (18) at the bottom of the next page) with $K_{i,j}$, i, $j = 1, \ldots, p$ the coupling matrices in q_{ri} (11), thus $M_c(K_{i,j})$ can be considered as the coupling matrix between the robots in the multicomposed system.

The matrix $M_c(K_{i,j})$ is nonsingular for all positive–semidefinite diagonal matrices $K_{i,j}$, $i, j = 1, \dots, p$. Moreover

$$M_c(K_{i,j})\begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_p \end{bmatrix} = \begin{bmatrix} q_d \\ q_d \\ \vdots \\ q_d \end{bmatrix} \Leftrightarrow \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_p \end{bmatrix} = \begin{bmatrix} q_d \\ q_d \\ \vdots \\ q_d \end{bmatrix} \tag{19}$$

holds for all positive semidefinite diagonal matrices $K_{i,j}$.

Proof: The proof follows from Gerschgorin's theorem about location of eigenvalues [21], see [4] and [5] for details. ■

To prove global-asymptotic synchronization of the multirobot system consider (14) and take the limit $t \to \infty$. By considering the definition of the synchronization errors (12), (13) the right-hand side of the implication (19) is obtained. Therefore, from Lemma 3 it follows that for all $i, j = 1, \dots, p$, $q_i \to q_d$, so that $q_i \to q_j$. In a similar way it can be proved that $\dot{q}_i \to \dot{q}_d$ as $t \to \infty$, so that $\dot{q}_i \to \dot{q}_j$ as $t \to \infty$.

Remark 4: The matrix $M_c(K_{i,j})$, (18), is nonsingular for all positive semidefinite diagonal coupling matrices $K_{i,j}$, $i, j = 1, \dots, p$, i.e., including $K_{i,j} = 0$, see Lemma 3. Therefore, partial and even master–slave synchronization of the robots in the system can be considered, i.e., some $K_{i,j} = 0$.

IV. Synchronization Controller Based on Estimated Variables

If only angular joint positions $q_i, i = 1, \dots, p$, are measured, then the synchronization controller (9), the reference signals $\dot{q}_{ri}$, $\ddot{q}_{ri}$, (11), and thus the synchronization error $\dot{s}_i$, (10), cannot be implemented. As an option the controller τ_i for the ith robot, (9), can be modified to

$$\tau_i = M_i(q_i)\hat{\ddot{q}}_{ri} + C_i(q_i, \hat{\dot{q}}_i)\hat{\dot{q}}_{ri} + g_i(q_i) - K_{d,i}\hat{\dot{s}}_i - K_{p,i}s_i \tag{20}$$

where $M_i(q_i)$, $C_i(q_i, \hat{\dot{q}}_i)$, $g_i(q_i)$ are defined as in (1), and $K_{p,i}$, $K_{d,i} \in \mathbb{R}^{n\times n}$ are positive–definite gain matrices. $\hat{\dot{s}}_i$ denotes the estimate of the velocity synchronization error $\dot{s}_i$, (10). $\hat{\dot{q}}_i$, $\hat{\dot{q}}_{ri}$ and $\hat{\ddot{q}}_{ri}$ are estimates of the angular velocity $\dot{q}_i$ and the reference signals $\dot{q}_{ri}$, $\ddot{q}_{ri}$ (11). They are given by

$$\hat{\dot{q}}_{ri} = \dot{q}_d - \sum_{j=1, j\neq i}^{p} K_{cv\_i,j}(\hat{\dot{q}}_i - \hat{\dot{q}}_j) \tag{21}$$

$$\hat{\ddot{q}}_{ri} = \ddot{q}_d - \sum_{j=1, j\neq i}^{p} K_{ca\_i,j}(\hat{\ddot{q}}_i - \hat{\ddot{q}}_j). \tag{22}$$

Where $\hat{\ddot{q}}_i$ corresponds to the derivative of the velocity estimate of the ith robot, i.e.,

$$\hat{\ddot{q}}_i = \frac{d}{dt}\hat{\dot{q}}_i$$

Then it follows that:

$$\hat{\dot{s}}_i = \hat{\dot{q}}_i - \hat{\dot{q}}_{ri}$$

or in terms of estimates for the partial synchronization errors

$$\hat{\dot{s}}_i := \hat{\dot{e}}_{i,i} + \sum_{j=1, j\neq i}^{p} K_{cv\_i,j}\hat{\dot{e}}_{i,j} \tag{23}$$

with $\hat{\dot{e}}_{i,i}$, $\hat{\dot{e}}_{i,j}$ given by

$$\begin{aligned} \hat{\dot{e}}_{i,j} &= \hat{\dot{q}}_i - \hat{\dot{q}}_j, \quad \text{for all } i, j = 1, \dots, p, \quad i \neq j \\ \hat{\dot{e}}_{i,i} &= \hat{\dot{q}}_i - \dot{q}_d, \quad \text{for } i = j. \end{aligned} \tag{24}$$

A. An Observer for the Joint Variables

An observer for estimating the joint variables q_i, $\dot{q}_i$ in the dynamic model of the ith robot given by (1), is given by

$$\begin{aligned} \frac{d}{dt}\hat{q}_i &= \hat{\dot{q}}_i + \mu_{i,1}\tilde{q}_i \\ \frac{d}{dt}\hat{\dot{q}}_i &= -M_i(q_i)^{-1}\left[C_i(q_i, \hat{\dot{q}}_i)\hat{\dot{q}}_i + g_i(q_i) - \tau_i\right] \\ &\quad + \mu_{i,2}\tilde{q}_i \end{aligned} \tag{25}$$

where $\hat{q}_i$, $\hat{\dot{q}}_i$, $(d/dt)\hat{\dot{q}}_i$ are estimates for q_i, $\dot{q}_i$, $\ddot{q}_i$, $\mu_{i,1}$, $\mu_{i,2} \in \mathbb{R}^{n\times n}$ are positive definite gain matrices, and the estimation errors $\tilde{q}_i$ and $\tilde{\dot{q}}_i$ are defined by

$$\tilde{q}_i := q_i - \hat{q}_i, \quad \tilde{\dot{q}}_i := \dot{q}_i - \hat{\dot{q}}_i. \tag{26}$$

Remark 5: Notice that τ_i, (20), depends on $\hat{\ddot{q}}_{ri}$ and thus on $(d/dt)\hat{\dot{q}}_i$, (25), which depends on τ_i. Therefore, when τ_i is substituted in (25), it results in an algebraic loop between the set of p observers. Nevertheless, this loop (25) can be solved by pure algebraic manipulation.

After substitution of τ_i (20) in (25), it follows that the second equation of (25) can be written as (for $i = 1, \dots, p$)

$$\begin{aligned} \left(I + \sum_{j=1, j\neq i}^{p} k_{i,j}\right)\frac{d}{dt}\hat{\dot{q}}_i - \sum_{j=1, j\neq i}^{p} k_{i,j}\frac{d}{dt}\hat{\dot{q}}_j &= \ddot{q}_d + \mu_{i,2}\tilde{q}_i \\ -M_i(q_i)^{-1}\left[C_i(q_i, \hat{\dot{q}}_i)\hat{\dot{s}}_i + K_{d,i}\hat{\dot{s}}_i + K_{p,i}s_i\right] \end{aligned} \tag{27}$$

$$M_c(K_{i,j}) = \begin{bmatrix} \left(I_n + \sum\limits_{j=1, j\neq 1}^{p} K_{1,j}\right) & -K_{1,2} & \cdots & -K_{1,p} \\ -K_{2,1} & \left(I_n + \sum\limits_{j=1, j\neq 2}^{p} K_{2,j}\right) & \cdots & -K_{2,p} \\ \vdots & & \ddots & \\ -K_{p,1} & -K_{p,2} & \cdots & \left(I_n + \sum\limits_{j=1, j\neq p}^{p} K_{p,j}\right) \end{bmatrix} \tag{18}$$

such that an implicit system of $n \cdot p$ differential equations is obtained.

Define the vectors $x, y \in \mathbb{R}^{n \cdot p}$ as

$$\begin{aligned} x &= \begin{bmatrix} \frac{d}{dt}\hat{\dot{q}}_1^T & \frac{d}{dt}\hat{\dot{q}}_2^T & \cdots & \frac{d}{dt}\hat{\dot{q}}_p^T \end{bmatrix}^T \\ y &= \begin{bmatrix} y_1^T & y_2^T & \cdots & y_p^T \end{bmatrix}^T \\ y_i &= \ddot{q}_d + \mu_{i,2}\tilde{q}_i - M_i(q_i)^{-1} \\ &\quad \times \left[C_i(q_i, \hat{\dot{q}}_i)\hat{\dot{s}}_i + K_{d,i}\hat{\dot{s}}_i + K_{p,i}s_i\right] \end{aligned}$$

then (27) can be written as

$$M_c(K_{i,j})x = y$$

with the matrix $M_c(K_{i,j})$ given by (18).

Since the matrix $M_c(K_{i,j})$ is nonsingular, see Lemma 3, (27) admits a unique solution, given by

$$x = M_c(K_{i,j})^{-1}y.$$

Thus, the observers (25) can be written as

$$\begin{aligned} \frac{d}{dt}\hat{q}_i &= \hat{\dot{q}}_i + \mu_{i,1}\tilde{q}_i \\ \frac{d}{dt}\hat{\dot{q}}_i &= f(\ddot{q}_d, q_j, \hat{\dot{q}}_j, \tilde{q}_j, e_j, \hat{\dot{e}}_j) \quad j = 1, \ldots, p \end{aligned} \tag{28}$$

for $i = 1, \ldots, p$, with $f(\cdot)$ a known nonlinear function.

B. Synchronization Closed-Loop Error Dynamics

Throughout the formulation of the error dynamics it is taken that Assumption 1 is satisfied. For simplicity in the stability analysis, the following assumption is introduced.

Assumption 6: The gains in the controller (20) and the observers (25) are a positive multiple of the unit matrix, i.e of the form $K = kI$ with k a positive scalar and I the identity matrix of appropriate dimensions. It is also assumed that the gain matrices at velocity and position level are equal for all the observers, i.e., $\mu_{i,1} = \mu_1 I$, $\mu_{i,2} = \mu_2 I$ for all $i = 1, \ldots, p$.

After substitution of the synchronization controller (20) and the observers (25), and by considering the properties (5) and (6) and the definitions of the partial synchronization errors (13), it follows that closed-loop synchronization-error dynamics is given by:

$$\begin{aligned} \ddot{\tilde{q}}_i &= M_i(q_i)^{-1}C_i(q_i, \dot{\tilde{q}}_i + \mu_1\tilde{q}_i) \\ &\quad \times (\dot{\tilde{q}}_i + \mu_1\tilde{q}_i - 2\dot{q}_d - 2\dot{e}_{i,i}) - \mu_1\dot{\tilde{q}}_i - \mu_2\tilde{q} \qquad (29) \\ M_i(q_i)\ddot{s}_i &= -C_i(q_i, \dot{e}_{i,i} + \dot{q}_d)\dot{s}_i - K_{d,i}\dot{s}_i - K_{p,i}s_i \\ &\quad - C_i(q_i, \dot{e}_{i,i} + \dot{q}_d - \dot{\tilde{q}}_i - \mu_1\tilde{q}_i) \\ &\quad \times \sum_{j=1, j\neq i}^{p} K_{i,j}(\dot{\tilde{q}}_i + \mu_1\tilde{q}_i - \dot{\tilde{q}}_j - \mu_1\tilde{q}_j) \\ &\quad - C_i(q_i, \dot{\tilde{q}}_i + \mu_1\tilde{q}_i)\left(\dot{q}_d - \sum_{j=1, j\neq i}^{p} K_{i,j}\dot{e}_{i,j}\right. \\ &\quad + K_{d,i}\left(\dot{\tilde{q}}_i + \mu_1\tilde{q}_i\right. \\ &\quad \left.\left. + \sum_{j=1, j\neq i}^{p} K_{i,j}(\dot{\tilde{q}}_i + \mu_1\tilde{q}_i - \dot{\tilde{q}}_j - \mu_1\tilde{q}_j)\right)\right) \\ &\quad + M_i(q_i)\sum_{j=1, j\neq i}^{p} K_{i,j} \\ &\quad \times \Big(M_i(q_i)^{-1}C_i(q_i, \dot{\tilde{q}}_i + \mu_1\tilde{q}_i) \\ &\qquad \times (\dot{\tilde{q}}_i + \mu_1\tilde{q}_i - 2\dot{q}_d - 2\dot{e}_{i,i}) \\ &\qquad - M_j(q_j)^{-1}C_j(q_j, \dot{\tilde{q}}_j + \mu_1\tilde{q}_j) \\ &\qquad \times (\dot{\tilde{q}}_j + \mu_1\tilde{q}_j - 2\dot{q}_d - 2\dot{e}_{j,j}) \\ &\qquad - \mu_1\dot{\tilde{q}}_i - \mu_2\tilde{q}_i + \mu_1\dot{\tilde{q}}_j + \mu_2\tilde{q}_j\Big). \qquad (30) \end{aligned}$$

Note that the second equation of the synchronization-error dynamics (30) corresponds to the synchronization error (15) with a disturbance that vanishes when $\tilde{q}_i = 0$, $\dot{\tilde{q}}_i = 0$. Therefore, it can be expected that if the estimation errors $\tilde{q}_i, \dot{\tilde{q}}_i$ asymptotically tend to zero, then the origin of the synchronization errors s_i, $\dot{s}_i$ is still an equilibrium point for (30).

C. Stability Analysis

The following assumption is required to prove stability of the synchronization closed-loop system.

Assumption 7: The common desired trajectory at velocity level $\dot{q}_d(t)$ is bounded by a positive scalar V_M such that

$$\sup_t \|\dot{q}_d(t)| = V_M < \infty \tag{31}$$

Based on the Assumptions 1, 6, and 7, the main result of the paper is formulated as follows.

Theorem 8: Consider a multirobot system formed by p rigid joint robots with dynamic models given by (1). Each robot in closed-loop with the controller (20), the reference signals (21), (22), and the observers (25). Introduce a positive scalar parameter η_0, which is defined and used throughout the proof. Then the p robots in the multicomposed system are semiglobally exponentially synchronized, i.e., for $i, j = 1, \ldots, p$, $q_i \rightarrow q_j$, $\dot{q}_i \rightarrow \dot{q}_j$ exponentially in a region that can be made arbitrarily large, if the scalar in the gains $K_{p,i}K_{d,i}$, μ_1, μ_2 are chosen such that for $i = 1, \ldots, p$

$$K_{p,i} > 0, \quad K_{d,i} > 0, \quad \eta_0 > 0 \tag{32}$$

$$\mu_1 > \max\{\mu_{1,1}^*, \ldots, \mu_{1,p}^*\} \tag{33}$$

$$\mu_2 > \max\left\{\frac{1}{M_{i,m}^2}\left(\eta_0^2 - \eta_0\mu_1 - 2V_M C_{i,M} \times \left(\mu_1 + \eta_0 M_{i,m}^{-1}\right)\right), \mu_{2,1}^*, \ldots, \mu_{2,p}^*\right\} \tag{34}$$

where $\mu_{i,1}^*$, $\mu_{i,2}^*$ are scalars given in the gain tuning procedure in Section V, and $\star_m$, $\star_M$ stand for the minimum and maximum eigenvalue of the matrix $\star$.

Proof: First a Lyapunov function and conditions for positive definitiveness are presented, then its derivative along the closed-loop error dynamics (29), (30) is bounded and sufficient conditions for negative definiteness are formulated.

Lyapunov function: Consider the synchronization-error dynamics given by (29) and (30). Define the vectors s, $\widetilde{q}$ as

$$s = [s_1 \quad \cdots \quad s_p], \quad \widetilde{q} = [\widetilde{q}_1 \quad \cdots \quad \widetilde{q}_p] \tag{35}$$

and take as a Lyapunov function

$$V(\dot{s}, \dot{\widetilde{q}}, s, \widetilde{q}) = \sum_{i=1}^{p} V_i(\dot{s}_i, \dot{\widetilde{q}}_i, s_i, \widetilde{q}_i) \tag{36}$$

where for $i = 1, \ldots, p$

$$V_i(\dot{s}_i, \dot{\widetilde{q}}_i, s_i, \widetilde{q}_i) = V_{i,1}(\dot{s}_i, s_i) + V_{i,2}(\dot{\widetilde{q}}_i, \widetilde{q}_i) \tag{37}$$

$$V_{i,1}(\dot{s}_i, s_i) = \frac{1}{2}\dot{s}_i^T M_i(q_i)\dot{s}_i + \frac{1}{2}s_i^T K_{p,i} s_i \tag{38}$$

$$V_{i,2}(\dot{\widetilde{q}}_i, \widetilde{q}_i) = \frac{1}{2}\begin{bmatrix}\dot{\widetilde{q}}_i^T & \widetilde{q}_i^T\end{bmatrix}\begin{bmatrix} M_i(q_i) & \eta_i(\widetilde{q}_i)I_n \\ \eta_i(\widetilde{q}_i)I_n & \mu_2 + \beta_i I_n\end{bmatrix} \times \begin{bmatrix}\dot{\widetilde{q}}_i \\ \widetilde{q}_i\end{bmatrix} \tag{39}$$

with $\eta_i(\widetilde{q}_i)$ and β_i defined as

$$\eta_i(\widetilde{q}_i) = \frac{\eta_0}{1 + \|\widetilde{q}_i\|} \tag{40}$$

$$\beta_i = \eta_0\mu_1 + 2V_M C_{i,M}\left(\mu_1 + \eta_0 M_{i,m}^{-1}\right) - \mu_2(1 - M_{i,m}) \tag{41}$$

where Assumptions 6 and 7 have been used. The scalars $C_{i,M}$, $M_{i,m}$ are the bounds of the Coriolis term and the inertia matrix, η_o is a positive scalar.

Note that $\eta_i(\widetilde{q}_i)$ is bounded by

$$0 < \eta_i(\widetilde{q}_i) < \eta_0.$$

Moreover, it follows that:

$$\dot{\eta}_i\dot{\widetilde{q}}_i^T\widetilde{q}_i = -\eta_i\left(\frac{\widetilde{q}_i^T\dot{\widetilde{q}}_i}{1 + \|\widetilde{q}_i\|}\right)\dot{\widetilde{q}}_i^T\widetilde{q}_i \leq \eta_i\|\dot{\widetilde{q}}_i\|^2 .$$

$$\dot{\eta}_i\dot{\widetilde{q}}_i^T\widetilde{q}_i \leq \eta_0\|\dot{\widetilde{q}}_i\|^2$$

Notice that $V_{i,1}(\dot{s}_i, s_i)$ in (37) corresponds to the Lyapunov function (16), which has been used for the case of synchronization based on full state measurements. $V_{i,1}(\dot{s}_i, s_i)$ is a sum of quadratic positive terms, therefore, it is positive definite. On the other hand a sufficient condition for positive definiteness of $V_{i,2}(\dot{\widetilde{q}}_i, \widetilde{q}_i)$ is given by

$$\mu_2 > \frac{1}{M_{i,m}^2}\left(\eta_0^2 - (\eta_0 + 2V_M C_{i,M})\mu_1 - 2V_M C_{i,M}\eta_0 M_{i,m}^{-1}\right) \tag{42}$$

with $M_{i,m}$ the minimum eigenvalue of the matrix $M_i(q_i)$.

Define the coupled synchronization error s_c as

$$s_c = [\dot{s} \quad \dot{\widetilde{q}} \quad s \quad \widetilde{q}] \tag{43}$$

then the Lyapunov function (36) satisfies

$$P_m\|s_c(t)\|^2 \leq V(s_c(t)) \leq P_M\|s_c(t)\|^2 \tag{44}$$

for some positive scalar P_m, P_M.

Time derivative of the Lyapunov function: Consider the vector $\widetilde{q}$ defined by (35). Then along the error dynamics (29), (30) the time derivative of the Lyapunov function (36) has an upper bound given by, (see Appendix I)

$$\dot{V}_i(\dot{s}_i, \dot{\widetilde{q}}_i, s_i, \widetilde{q}_i) \leq \|\Phi_{3,i}\| - \left[\|\dot{s}_i\| \quad \|\dot{\widetilde{q}}\| \quad \|\widetilde{q}\|\right] M_{vi}\left[\|\dot{s}_i\| \quad \|\dot{\widetilde{q}}\| \quad \|\widetilde{q}\|\right]^T \tag{45}$$

with $\|\Phi_{3,i}\|$ an upper bound of $\Phi_{3,i}$, which is given by

$$\begin{aligned}\|\Phi_{3,i}\| = &\left(2\|\dot{s}_i\| + \|\dot{\widetilde{q}}\| + \mu_1\|\widetilde{q}\|\right)\left(\|\dot{\widetilde{q}}\| + \mu_1\|\widetilde{q}\|\right) \\ &\times\Bigg[C_{i,M}\left(\|\dot{\widetilde{q}}\| + \eta_0 M_{i,m}^{-1}\|\widetilde{q}\|\right) \\ &+ \|\dot{s}_i\|\sum_{j=1,j\neq i}^{p} K_{i,j}\left(C_{i,M} + M_{i,m}M_{j,m}^{-1}C_{j,M}\right)\Bigg] \\ &+ C_{i,M}\|\dot{s}_i\|\left(\|\dot{\widetilde{q}}\| + \mu_1\|\widetilde{q}\|\right) \\ &\times\left(\|\dot{s}_i\| + 2\left(\|\dot{s}_i\| + \|\dot{\widetilde{q}}\| + \mu_1\|\widetilde{q}\|\right)\sum_{j=1,j\neq i}^{p} K_{i,j}\right)\end{aligned} \tag{46}$$

and the matrix M_{vi} given by

$$M_{vi} = \begin{bmatrix} M_{vi,11} & M_{vi,12} & \mu_1 M_{vi,12} \\ M_{vi,12} & M_{vi,22} & 0 \\ \mu_1 M_{vi,12} & 0 & M_{vi,33}\end{bmatrix} \tag{47}$$

$$M_{vi,11} = K_{di} \tag{48}$$

$$M_{vi,12} = -\frac{1}{2}\Bigg[K_{di} - V_M C_{i,M} + 2V_M\sum_{j=1,j\neq i}^{p} K_{i,j} \times\left(M_{i,m}M_{j,m}^{-1}C_{j,M} - C_{i,M}\right)\Bigg] \tag{49}$$

$$M_{vi,22} = M_{i,m}\mu_1 - 2\eta_0 - \frac{1}{2}M_{i,pM} + 2V_M C_{i,M} \tag{50}$$

$$M_{vi,33} = \eta_0\left(\mu_2 + 2\mu_1 V_M C_{i,M} M_{i,M}^{-1}\right). \tag{51}$$

From Sylvester's criterion about positive definiteness of a matrix based on the principal minors, see [22], it follows that the matrix $M_{v,i}$, given by (47) is positive–definite if:

$$\eta_0 > 0, \quad K_{di} > 0 \tag{52}$$

$$\mu_{1,i}^* > M_{i,m}^{-1}\left(\frac{M_{vi,12}^2}{M_{vi,11}} + 2\eta_0 + \frac{1}{2}M_{i,pM} - 2V_M C_{i,M}\right) \tag{53}$$

$$\mu_{2,i}^* > \frac{\mu_1^2 M_{vi,12}^2 M_{vi,22}}{\eta_0\left(M_{vi,11}M_{vi,22} - M_{vi,12}^2\right)} - 2\mu_1 V_M C_{i,M} M_{i,M}^{-1}. \tag{54}$$

Remark 9: In (53), (54) $\mu_{1,i}^*$, $\mu_{2,i}^*$, are equal to μ_1, μ_2, the notation is used to emphasize that for each matrix M_{vi}, $i = 1, \ldots, p$, a different μ_1, μ_2 is obtained.

Consider the vector s_c defined by (43), then $\dot{V}_i(\dot{s}_i, \dot{\widetilde{q}}_i, s_i, \widetilde{q}_i)$ given by (45) results in

$$\dot{V}_i(\dot{s}_i, \dot{\widetilde{q}}_i, s_i, \widetilde{q}_i) \leq \|s_c|^2\left(-M_{vi,m} + \alpha\|s_c\|\right) \tag{55}$$

where $M_{vi,m}$ is the minimum eigenvalue of the matrix M_{vi}, $i = 1, \ldots, p$, so that $M_{vi,m}$ is positive if the conditions (52), (53), and (54) are satisfied.

The coefficient α is determined by $\|\Phi_{3,i}\|$ and is given by

$$\begin{aligned}\alpha = & C_{i,M}(1+\mu_1)\left(1+2(2+\mu_1)\sum_{j=1,j\neq i}^{p} K_{i,j}\right) \\ & + (3+\mu_1)(1+\mu_1) \\ & \times \left[C_{i,M}\left(1+\eta_0 M_{i,m}^{-1}\right)\right. \\ & \left. + \sum_{j=1,j\neq i}^{p} K_{i,j}\left(M_{i,m}M_{j,m}^{-1}C_{j,M}+C_{i,M}\right)\right] \end{aligned} \tag{56}$$

The minimum eigenvalue of M_{vi}, i.e., $M_{vi,m}$, is proportional to the gain μ_2, while α is independent of μ_2. Therefore, by increasing μ_2 it can be ensured that for $i = 1, \ldots, p$

$$\dot{V}_i(\dot{s}_i, \dot{\tilde{q}}_i, s_i, \tilde{q}_i) \leq \|s_c\|^2 \left(-M_{vi,m} + \alpha\|s_c\|\right) < 0.$$

Thus, the matrix $\dot{V}(\dot{s}, \dot{\tilde{q}}, s, \tilde{q})$, given by (45), is negative definite. Moreover, it follows that there exist a positive scalar κ, such that $\dot{V}$ has an upper bound given by

$$\dot{V} \leq -\kappa\|s_c\|^2 \quad \text{for all} \quad t \geq .0$$

From the above equation and (44), it is concluded that there exist some constants m^*, $\rho > 0$, such that

$$\|s_c(t)\|^2 \leq m^* e^{-\rho t}\,\|s_c(0)\|^2 \quad \text{for all} \quad t \geq 0$$

and thus by the definition of s_c given by (43), it follows that the synchronization closed-loop errors are semiglobally exponentially stable with convergence region β_c given by

$$\beta_c = \left\{ s_c \quad | \quad \|s_c\| < \frac{M_{vi,m}}{\alpha}\sqrt{\frac{P_m}{P_M}} \right\}. \tag{57}$$

Since the synchronization error s_c is exponentially stable, it follows that s, and s_i, for $i = 1, \ldots, p$, are exponentially stable too. The proof that the partial synchronization errors $e_{i,j}$, i, $j = 1, \ldots, p$ are exponentially stable, and thus the robots in the multicomposed system are semiglobally exponentially synchronized, follows along the same lines as in the case of available full state measurements, Section III. ■

V. Gain Tuning Procedure

The gain tuning procedure to ensure the stability results stated in Theorem 8 can be summarized as follows.

1) Determine the bounds of the physical parameters $M_i(q_i)$, $C_i(q_i, \dot{q}_i)$, $\dot{M}_i(q_i)$.
2) Determine the bound of the common desired trajectory at velocity level $\dot{q}_d$, i.e., V_M.
3) Choose positive–semidefinite coupling gains $K_{i,j}$ for i, $j = 1, \ldots, p$, $j \neq i$.
4) Choose the scalars on the gains $K_{p,i}$, and $K_{d,i}$, for $i = 1, \ldots, p$, and the auxiliary scalar η_0, to be positive.
5) For $i = 1, \ldots, p$ determine the value $\mu_{1,i}^*$, that is given by (53), and take μ_1 as the maximum of all $\mu_{1,i}^*$.
6) For $i = 1, \ldots, p$ determine the value $\mu_{2,i}^*$, that is given by (54), and take μ_2 as the maximum of all $\mu_{2,i}^*$ and

$$\frac{1}{M_{i,m}^2}\left(\eta_0^2 - \eta_0\mu_1 - 2V_M C_{i,M}\left(\mu_1 + \eta_0 M_{i,m}^{-1}\right)\right).$$

VI. Friction Compensation

Considering the dynamic model of the robot manipulators with friction effects given by (2), it follows that the feedback controller τ_i (20) can be modified as:

$$\begin{aligned}\tau_{if} &= \tau_i + f_i(\hat{\dot{q}}_i) \\ &= M_i(q_i)\hat{\ddot{q}}_{ri} + C_i(q_i, \hat{\dot{q}}_i)\hat{\dot{q}}_{ri} + g_i(q_i) \\ &\quad - K_{d,i}\hat{\dot{s}}_i - K_{p,i}s_i + f_i(\hat{\dot{q}}_i)\end{aligned} \tag{58}$$

with the friction compensation term $f_i(\hat{\dot{q}}_i)$ given by

$$\begin{aligned}f_i(\hat{\dot{q}}_i) = B_{v,i}\hat{\dot{q}}_i &+ B_{f1,i}\left(1 - \frac{2}{1+e^{2w_{1,i}\hat{\dot{q}}_i}}\right) \\ &+ B_{f2,i}\left(1 - \frac{2}{1+e^{2w_{2,i}\hat{\dot{q}}_i}}\right)\end{aligned} \tag{59}$$

and $\hat{\dot{q}}_i$ the estimated for the velocity of the ith robot, that is obtained by the observer (28).

The stability analysis of the closed-loop system formed by the synchronization controller (20), the observers (28), and the robots in the multicomposed system, follows in the same way as for frictionless rigid joint robots. To support this note that the friction model (3) implies that

$$\left\|f_i(\hat{\dot{q}}_i) - f_i(\dot{q}_i)\right\| \leq B_{v,iM}\|\tilde{\dot{q}}_i\| + 2B_{f1,iM} + 2B_{f2,iM} \tag{60}$$

with $\tilde{\dot{q}}_i$ the velocity estimation error, and $B_{v,iM}$, $B_{f1,iM}$, $B_{f2,iM}$ the maximum eigenvalue of the coefficient matrices $B_{v,i}$, $B_{f1,i}$, $B_{f2,i}$.

Because the friction effects appear as an additive term in the robot dynamics (2) and the feedback controller (58), it follows that the difference $f_i(\hat{\dot{q}}_i) - f_i(\dot{q}_i)$ appears in the synchronization closed-loop error. Then by considering the Lyapunov function (36) it follows that the bound (60) appears in the bound of the derivative of the Lyapunov function (63). Then following the same steps as in the frictionless robot case, semiglobal exponential stability of the closed-loop synchronization error can be concluded.

VII. Experimental Case Study

The proposed mutual synchronization controller τ_{if}, given by (58), has been implemented on a four degree of freedom multirobot system formed by two identical robots manufactured by the Center for Manufacturing Technology (CFT) Philips Laboratory. Comparison between simulated and experimental results has been reported in [4] and [5], although for the sake of brevity only experimental results are reported here.

A. The CFT Robot

The CFT robot is a Cartesian basic elbow configuration robot. It consists of a two links arm on a rotating and translational

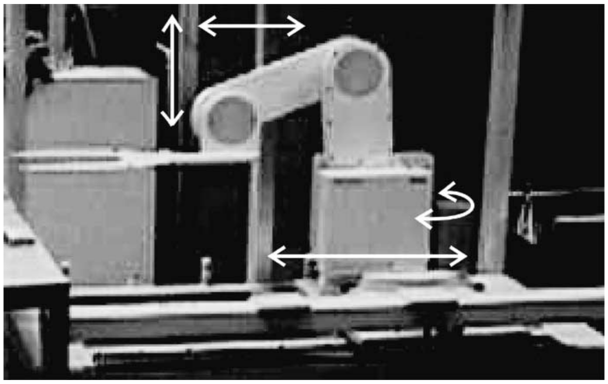


Fig. 1. The CFT-transposer robot.

base, see Fig. 1. It has four degrees of freedom in the Cartesian space, denoted by x_{ci} $(i = 1, \ldots, 4)$, and seven degrees of freedom in the joint space, denoted by q_j $(j = 1, \ldots, 7)$, and is actuated by 4-dc brushless servomotors. Although the robot has seven degrees of freedom in the joint space, three of them are kinematically constrained $\{q_3, q_6, q_7\}$. Therefore, the robot can be represented in the joint space by four degrees of freedom $\{q_1, q_2, q_4, q_5\}$ actuated by four servomotors.

The four Cartesian degrees of freedom are rotation, up and down, forward and backward of the arm, forward and backward of the whole robot, see Fig. 1. The robot is equipped with encoders with a resolution of 2000 PPR, which results in an accuracy of ±0.5 [mm]. A more detailed description of the structure of the robot can be found in [4], [5], and [23].

For implementation of the controllers and communication to the robots, the experimental setup is equipped with a DS1005 dSPACE system, with a processor PPC750, a clock of 480 MHz and a bus clock of 80 MHz. Throughout the experiments the sampling frequency was set to 2 kHz.

B. Joint Space Dynamics

The multirobot system is formed by two structurally identical transposer robots, so that they have the same kinematic and dynamic model. However, the physical parameters of the robots, such as masses, inertias, friction coefficients are different for both robots.

Hereafter, the notation q_i, for $i = 1, 2$ refers to the ith robot in the multicomposed system, rather than to the jth joint, $j = 1, 2, 4, 5$, in the ith robot. From the Euler–Lagrange approach and the Denavit–Hartenberg parameters procedure, the dynamics of the transposer robots are given by (2), with $q_i = [q_{i,1} \quad q_{i,2} \quad q_{i,4} \quad q_{i,5}]^T$ the vector of generalized coordinates of robot i, $M(q_i) \in \mathbb{R}^{4\times 4}$ the symmetric, positive definite inertia matrix, $g(q_i) \in \mathbb{R}^4$ denotes the gravity forces, $C(q_i, \dot{q}_i)\dot{q}_i \in \mathbb{R}^4$ represents the Coriolis and centrifugal forces, $f(\dot{q}_i) \in \mathbb{R}^4$ are the forces due to friction effects, and $\tau_i = [\tau_{i,1} \quad \tau_{i,2} \quad \tau_{i,4} \quad \tau_{i,5}]^T$ is the vector of external torques. The parameters in the matrices $M(q_i)$, $C(q_i, \dot{q}_i)$ and the gravity vector $g(q_i)$ can be found in [4], [5], and [23].

C. Experimental Results

The robots in the multirobot system are identified as robot 1 (R1) and robot 2 (R2). The synchronization controllers τ_{1f}, τ_{2f} are implemented according to (58), with τ_1, τ_2 as in (20). The friction compensation term $f_i(\widehat{\dot{q}}_i)$ is a function of the estimated velocity $\widehat{\dot{q}}_i$, that is obtained by the observer (28).

TABLE I
COEFFICIENTS OF THE DESIRED TRAJECTORY $x_{cj,d}(t)$, $j = 1, \ldots, 4$

$a_{i,j}$	$i = 0$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$s_{f,j}$
$j = 1$ [m]	-0.1343	-0.02	-0.015	-0.005	-0.005	1.0
$j = 2$ [m]	0.2766	0.05	0.03	-0.03	0.02	1.0
$j = 3$ [rad]	2.4	0.2	0.1	-0.05	0.05	1.0
$j = 4$ [m]	-0.265	0.15	0.05	-0.03	0.02	0.25

TABLE II
CONTROL AND COUPLING GAINS IN THE MUTUAL SYNCHRONIZATION CONTROLLER

$i = 1, 2$	$K_{p,i}$	$K_{d,i}$	$\mu_{i,1}$	$\mu_{i,2}$	$K_{1,2}$
joint q_1	10000	600	500	20000	1
joint q_2	10000	300	600	55000	2
joint q_4	10000	300	600	55000	1
joint q_5	10000	300	600	55000	2

TABLE III
INITIAL CONDITIONS FOR ROBOTS 1 AND 2 AND OBSERVER (28)

	$q_1(0)$ [m]	$q_2(0)$ [rad]	$q_4(0)$ [rad]	$q_5(0)$ [rad]
robot 1	-0.06	-0.1	-1.0	2.115
robot 2	-0.079	0.2	-1.09	2.06
	$\widehat{q}_1(0)$ [m]	$\widehat{q}_2(0)$ [rad]	$\widehat{q}_4(0)$ [rad]	$\widehat{q}_5(0)$ [rad]
robot 1 and 2	-0.08	0.0	-0.9899	1.9805

The desired common trajectory for the robots $q_d(t)$ is obtained by transformation of a desired trajectory given in Cartesian coordinates $x_{cj,d}(t)$, $j = 1, \ldots, 4$, that is given by

$$x_{cj,d}(t) = a_{0,j} + a_{1,j}\sin(2s_f\pi\omega t) + a_{2,j}\sin(4s_f\pi\omega t) + a_{3,j}\sin(6s_f\pi\omega t) + a_{4,j}\sin(8s_f\pi\omega t) \quad (61)$$

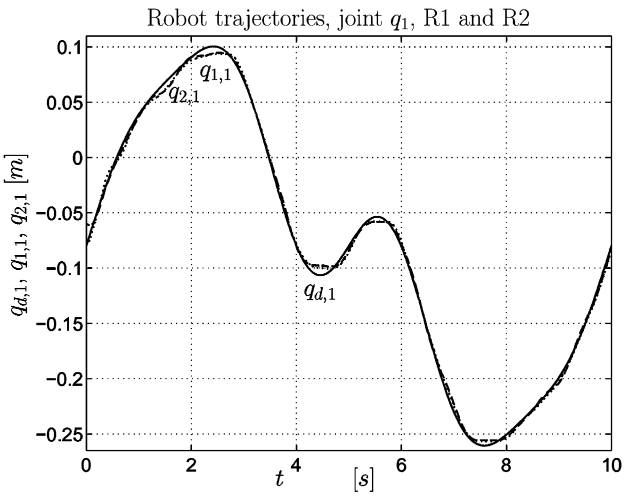
with the coefficients $a_{i,j}$, $i = 0, \ldots, 4$, $j = 1, \ldots, 4$ given in Table I.

The fundamental frequency of the desired common trajectory $x_{cj,d}(t)$ given by (61) is set as $\omega = 0.4$ Hz. The joint space desired trajectory $q_d(t)$ is obtained by transformation of the desired Cartesian trajectories $x_{cj,d}(t)$, $j = 1, \ldots, 4$ by considering the inverse kinematics model of the CFT robot.

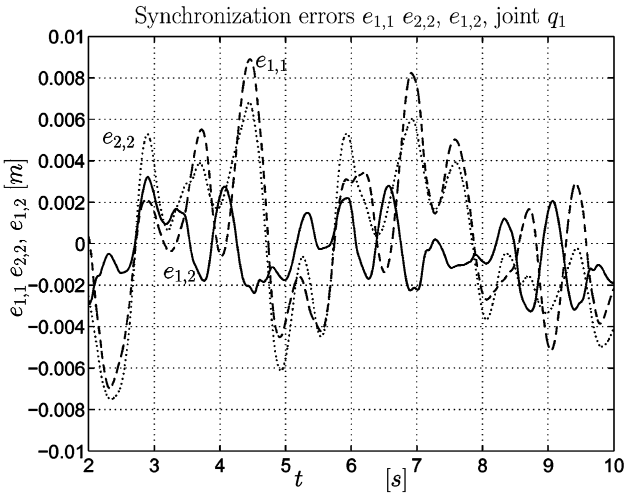
The controller and observer gains for the robots R1 and R2 are set equal for the corresponding joints. For the coupling gains symmetric synchronization is chosen, i.e., the coupling gains between the robot are equal, $K_{1,2} = K_{2,1}$. After a series of experiments in order to decrease the synchronization position errors $e_{i,i}$ and $e_{i,j}$, $i, j = 1, 2$, the gains on the synchronization controller (58) and the coupling gains $K_{i,j}$ were set as listed in Table II.

The initial position of the links and the initial conditions in the observers (28) were chosen as in Table III. The robots start from a steady state, therefore, the joint velocity $\dot{q}_i(0)$ and the estimated joint velocity $\widehat{\dot{q}}_i(0)$ are all equal to zero.

Figs. 2–5 show for the robots R1, R2, $i = 1, 2$, and the joints $j = 1, 2, 4, 5$, the desired common trajectory $q_d(t)$ (solid), the position trajectories $q_{i,j}$, and the synchronization errors $e_{1,1} = q_{1,j} - q_d$ (dashed), $e_{2,2} = q_{2,j} - q_d$ (dotted), and $e_{1,2} = q_{1,j} - q_{2,j}$ (solid), after the transient period has finished.

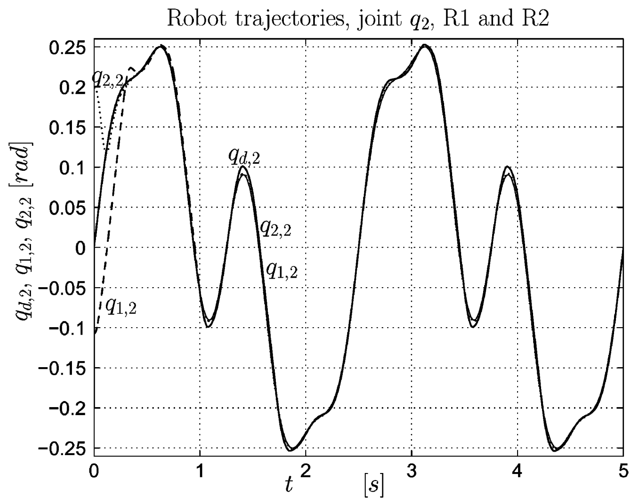


(a)

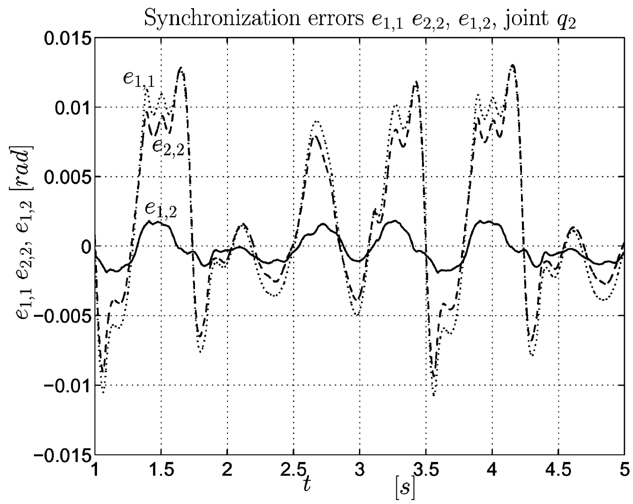


(b)

Fig. 2. Position trajectories and synchronization errors for joint q_1. (a) Positions: desired $q_{d,1}$, solid; $q_{1,1}$, dashed; and $q_{2,1}$, dotted. (b) Errors: $e_{1,1}$, dashed; $e_{2,2}$, dotted; and $e_{1,2}$, solid.



(a)



(b)

Fig. 3. Position trajectories and synchronization errors for joint q_2. (a) Positions: desired $q_{d,2}$, solid; $q_{1,2}$, dashed; and $q_{2,2}$, dotted. (b) Errors: $e_{1,1}$, dashed; $e_{2,2}$, dotted; and $e_{1,2}$ solid.

From Figs. 2–5 it is concluded that synchronization between the robots is achieved, and that bounded synchronization errors are obtained. Particularly Fig. 5 shows that the synchronization errors $e_{1,1}$, $e_{2,2}$ are penalized in order to minimize the coupled synchronization error $e_{1,2}$. Thus mutual synchronization between the robots is favored over synchronization between the robots and q_d.

1) Comparison Between Synchronization and Tracking Controllers: In this section the proposed controller (58), (20), is compared with a tracking controller, that is, each system aims at tracking the common desired trajectory without any mutual coupling. The system considered for the comparison study is the joint $q_{i,5}$ of the robots R1 ($i = 1$) and R2 ($i = 2$). When the coupling gains are set equal to zero, i.e., $K_{i,j} = 0$ the synchronization controller (20) becomes the tracking controller proposed by Paden and Panja [24] but based on estimated velocities. To compare the mutual synchronization controller, the gains are taken as $K_{1,2} = K_{2,1} = 100$, whereas in the uncoupled case these are set equal to zero.

Fig. 6 shows the position trajectories $q_{d,5}$, $q_{1,5}$ and $q_{2,5}$ for the coupled and uncoupled cases, while the synchronization errors are shown in Fig. 7. In the coupled case the coupling error $e_{1,2} = q_{1,5} - q_{2,5}$ converges faster than the errors with respect to $q_{d,5}$, i.e., $e_{1,1} = q_{1,5} - q_{d,5}$, $e_{2,2} = q_{2,5} - q_{d,5}$, this proves the mutual synchronization behavior. Meanwhile for the uncoupled case, the errors $e_{1,1}$, $e_{2,2}$ converge faster than $e_{1,2}$, since there is not any interaction between them.

Fig. 8 presents the estimation error for the joint positions [observer (28)]. Notice that the mutual synchronization controller yields smaller estimation errors than the uncoupled case. The reference signals (11) constrain the variety in which the synchronization errors evolve and at the same time give more information to the observer (28), such that better convergence in the estimation is achieved.

VIII. Concluding Remarks and Discussion

The proposed synchronizing controller yields semiglobal exponential mutual synchronization of a multirobot system. The ideas behind the synchronization controller (20), the observer (25), and the reference signals (11) are quite general; thus they

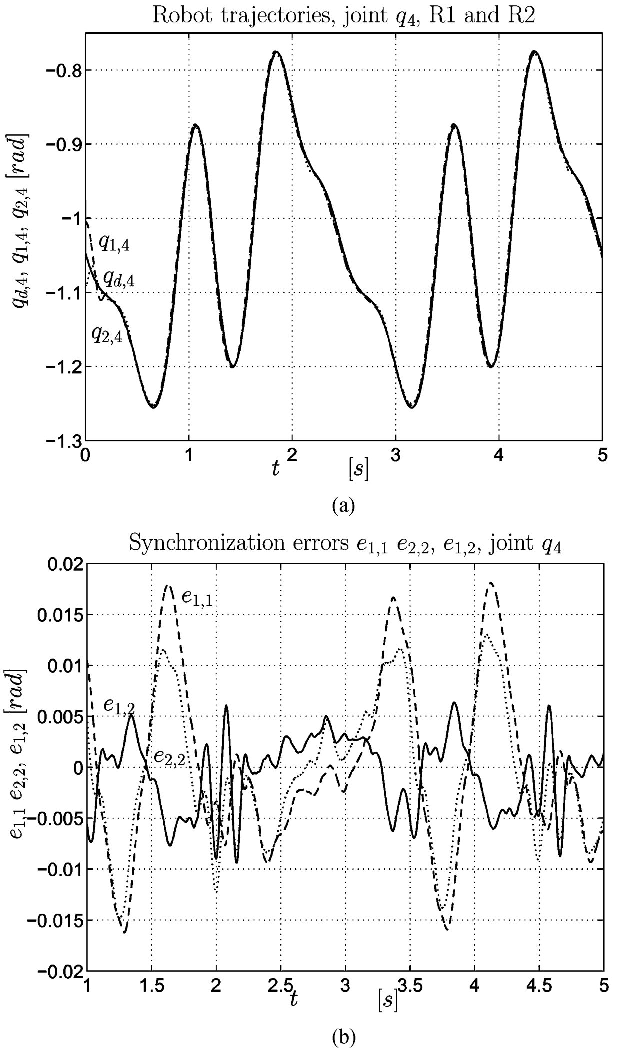


Fig. 4. Position trajectories and synchronization errors for joint q_4. (a) Positions: desired $q_{d,4}$, solid; and $q_{1,4}$, dashed; $q_{2,4}$, dotted. (b) Errors: $e_{1,1}$, dashed; $e_{2,2}$, dotted; and $e_{1,2}$ (solid).

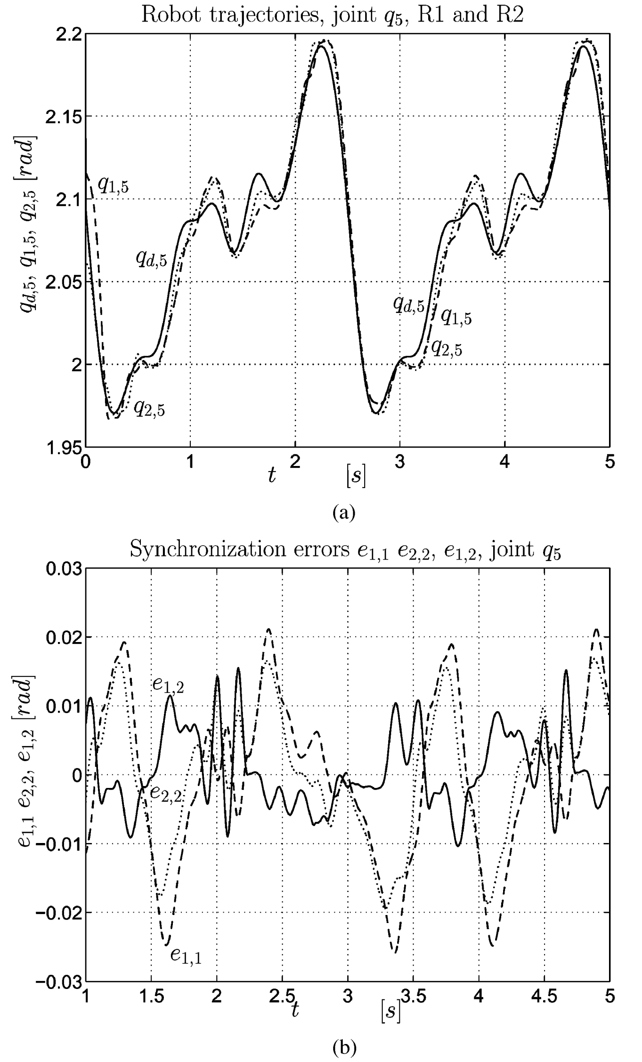


Fig. 5. Position trajectories and synchronization errors for joint q_5. (a) Positions: desired $q_{d,5}$, solid; $q_{1,5}$, dashed; and $q_{2,5}$, dotted. (b) Errors: $e_{1,1}$, dashed; $e_{2,2}$, dotted; and $e_{1,2}$, solid.

can be extended to other systems different from robot manipulators, such as communication systems, mobile robots, coupled generators, multisatellite systems.

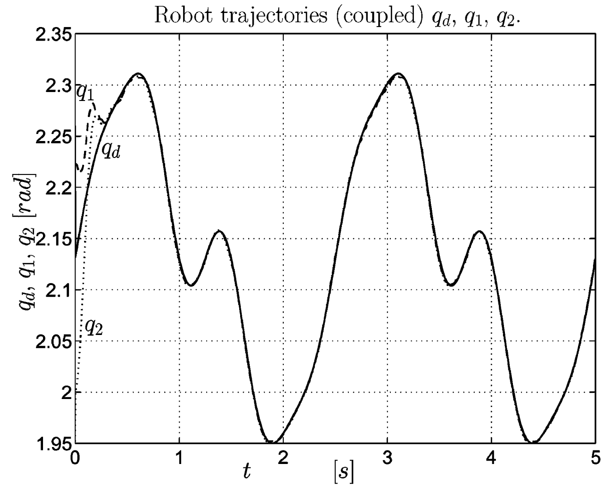
The proposed mutual synchronization controller provides a systematic way of proving stability of the synchronization closed-loop system. On the one hand, the predicted region of stability and the outcome of the proposed gain tuning procedures are quite conservative. Thus, there exist more freedom on the initial errors and the controller gains than the predicted ones. On the other hand, even without knowledge of the bounds implied in (33), (34), the synchronization closed-loop system can be made exponentially stable, by selecting the control gains large enough. However, high gains are not desirable in practical applications since they may amplified the noise in the position measurements.

The controller and observers (20), (25) are model based, nevertheless the stability analysis allows a straightforward robustness analysis for parametric uncertainties. At the same time on-line adaptation of the robot parameters can be considered, [12].

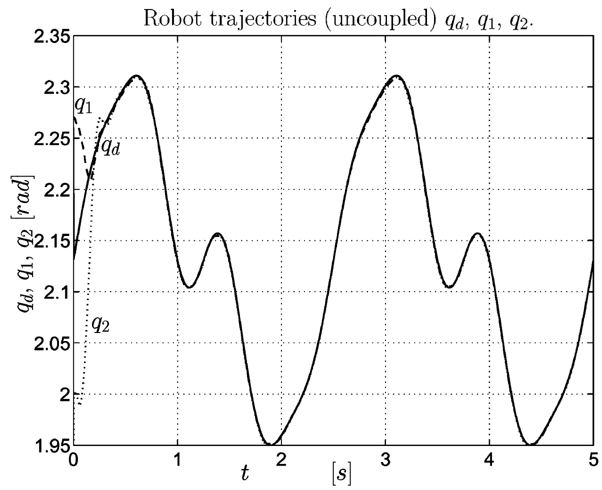
The advantage of the proposed mutual synchronization scheme over traditional tracking controllers lies in the ability to control the relationships between the position and velocities of all the robots in the system. In other words, the proposed controller regulates not only the convergence of the position and velocity of the robots to the common desired trajectory, but also how these errors converge between them, which greatly improves the performance during transients. The reference signals (11) give a clear insight in the tradeoff between the synchronization errors between the robots and with respect to the common desired trajectory. The mutual synchronization behavior induced by the reference signals (11) is particularly useful during transients or sudden disturbances on the robots such as unknown payloads.

APPENDIX I
PROOF OF THEOREM 8

In this appendix, some details behind the proof of the Theorem 8 are presented.

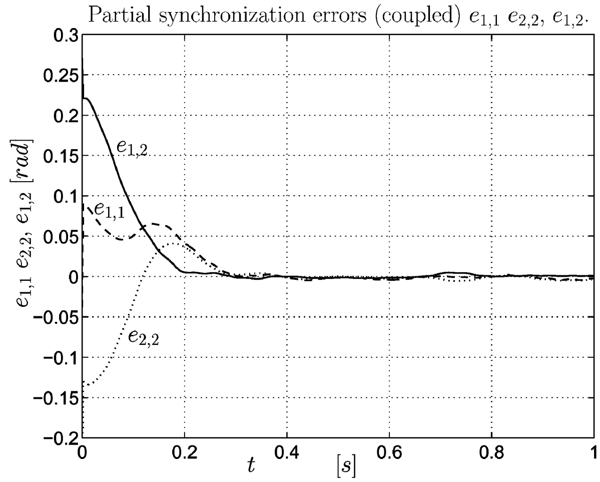


(a)

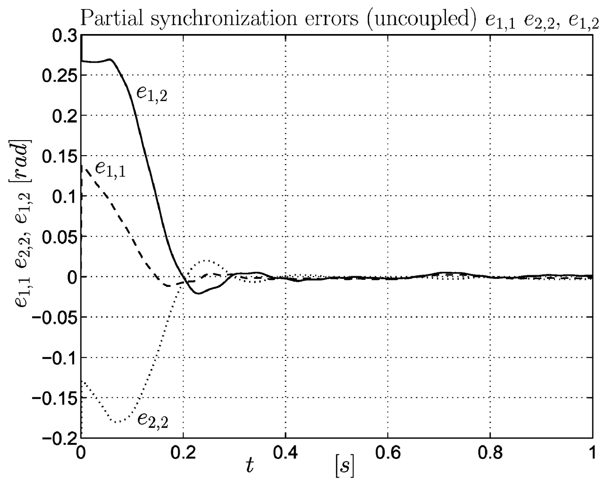


(b)

Fig. 6. Position trajectories: desired $q_{d,5}$, solid; $q_{1,5}$, dashed; and $q_{2,5}$, dotted. (a) Coupled case (mutual synchronization). (b) Uncoupled case (tracking).



(a)



(b)

Fig. 7. Partial synchronization errors: $e_{1,1}$, dashed; $e_{2,2}$, dotted; and $e_{1,2}$, solid. (a) Coupled case (mutual synchronization). (b) Uncoupled case (tracking).

A. Time Derivative of the Lyapunov Function $V(\dot{s}, \dot{\tilde{q}}, s, \tilde{q})$

The time derivative of the Lyapunov function (36) along the error dynamics (29), (30) is given by

$$\dot{V}(\dot{s}, \dot{\tilde{q}}, s, \tilde{q}) = \sum_{i=1}^{p} \left\{ \dot{V}_{i,1}(\dot{s}_i, s_i) + \dot{V}_{i,2}(\dot{\tilde{q}}_i, \tilde{q}_i) \right\} \tag{62}$$

with $\dot{V}_{i,1}(\dot{s}_i, s_i), \dot{V}_{i,2}(\dot{\tilde{q}}_i, \tilde{q}_i)$ the time derivative of (38) and (39). From the synchronization closed-loop error dynamics (29), (30), and by the properties of the Coriolis term, it follows that:

$$\begin{aligned}
\dot{V}_{i,1}(\dot{s}_i, s_i) = & -\dot{s}_i^T K_{d,i} \dot{s}_i - \dot{s}_i^T C_i(q_i, \dot{\tilde{q}}_i + \mu_1 \tilde{q}_i) \\
& \times \left(\dot{q}_d - \sum_{j=1, j\neq i}^{p} K_{i,j} \dot{e}_{i,j} \right) \\
& - \dot{s}_i^T C_i(q_i, \dot{e}_{i,i} + \dot{q}_d - \dot{\tilde{q}}_i - \mu_1 \tilde{q}_i) \\
& \times \sum_{j=1, j\neq i}^{p} K_{i,j} (\dot{\tilde{q}}_i + \mu_1 \tilde{q}_i - \dot{\tilde{q}}_j - \mu_1 \tilde{q}_j) + \dot{s}_i^T K_{d,i} \\
& \times \Bigg(\dot{\tilde{q}}_i + \mu_1 \tilde{q}_i + \sum_{j=1, j\neq i}^{p} K_{i,j} \\
& \times (\dot{\tilde{q}}_i + \mu_1 \tilde{q}_i - \dot{\tilde{q}}_j - \mu_1 \tilde{q}_j) \Bigg) \\
& + \dot{s}_i^T M_i(q_i) \sum_{j=1, j\neq i}^{p} K_{i,j} \\
& \times \Big[M_i(q_i)^{-1} C_i(q_i, \dot{\tilde{q}}_i + \mu_1 \tilde{q}_i) \\
& \quad \times (\dot{\tilde{q}}_i + \mu_1 \tilde{q}_i - 2\dot{q}_d - 2\dot{e}_{i,i}) \\
& \quad - \mu_1 \dot{\tilde{q}}_i - \mu_2 \tilde{q}_i + \mu_1 \dot{\tilde{q}}_j + \mu_2 \tilde{q}_j \\
& \quad - M_j(q_j)^{-1} C_j(q_j, \dot{\tilde{q}}_j + \mu_1 \tilde{q}_j) \\
& \quad \times (\dot{\tilde{q}}_j + \mu_1 \tilde{q}_j - 2\dot{q}_d - 2\dot{e}_{j,j}) \Big]
\end{aligned}$$

$$\begin{aligned}
\dot{V}_{i,2}(\dot{\tilde{q}}_i, \tilde{q}_i) = & \Big(-\mu_1 \dot{\tilde{q}}_i - \mu_2 \tilde{q} + M_i(q_i)^{-1} C_i(q_i, \dot{\tilde{q}}_i + \mu_1 \tilde{q}_i) \\
& \times (\dot{\tilde{q}}_i + \mu_1 \tilde{q}_i - 2\dot{q}_d - 2\dot{e}_{i,i}) \Big) \\
& \times \left(\dot{\tilde{q}}_i^T M_i(q_i) + \eta_i(\tilde{q}_i) \tilde{q}_i^T \right) \\
& + \frac{1}{2} \dot{\tilde{q}}_i^T \dot{M}_i(q_i) \dot{\tilde{q}}_i + \eta_i(_i) \dot{\tilde{q}}_i^T \dot{\tilde{q}}_i + \dot{\eta}_i(\tilde{q}_i) \dot{\tilde{q}}_i^T \tilde{q}_i \\
& + \tilde{q}_i^T (\mu_2 + \beta_i I_n) \dot{\tilde{q}}_i.
\end{aligned}$$

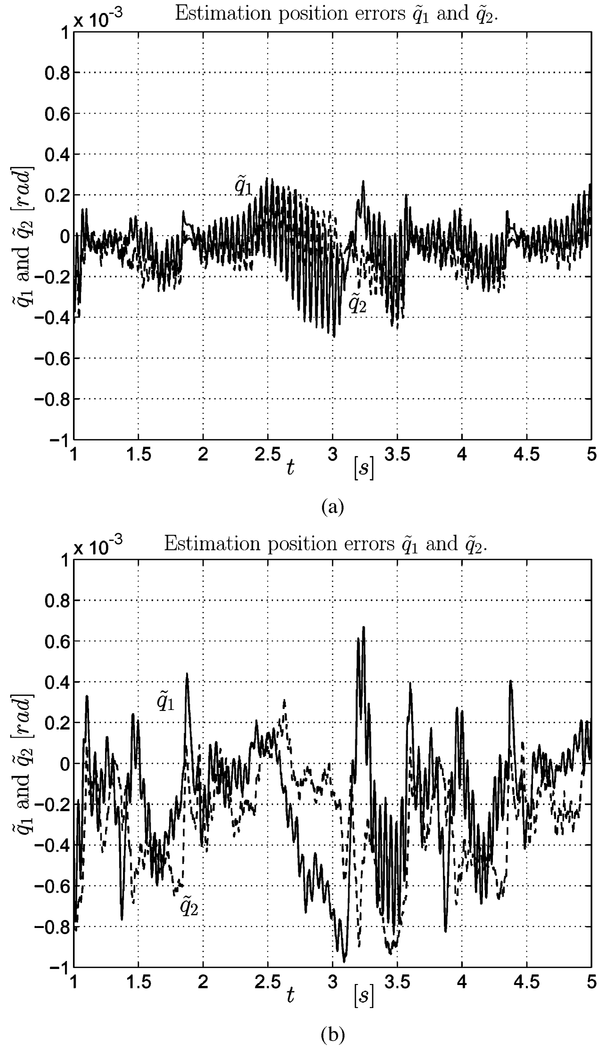


Fig. 8. Estimation joint position errors: $\tilde q_{1,5}$, solid; $\tilde q_{2,5}$, dashed. (a) Coupled case (mutual synchronization). (b) Uncoupled case (tracking).

After a long but straightforward computation and by considering Assumptions 1, 6, and 7, the properties of the robot dynamics, and the bound of $\dot M_i(q_i)$ given by

$$M_{i,pm}\|\dot q_i\| \le \left\|\dot M_i(q_i)\right\| \le M_{i,pM}\|\dot q_i\|$$

it follows that $\dot V_i(\dot s_i, \dot{\tilde q}_i, s_i, \tilde q_i)$ in (62) is bounded as:

$$\begin{aligned}
\dot V_i(\dot s_i, \dot{\tilde q}_i, s_i, \tilde q_i) \le{} & \left(2\eta_0 - M_{i,m}\mu_1 + \frac{1}{2}M_{i,pM} - 2V_M C_{i,M}\right) \\
& \times \|\dot{\tilde q}_i\|^2 - K_{di}\|\dot s_i\|^2 \\
& - \eta_0\left(\mu_2 + 2V_M C_{i,M} M_{i,m}^{-1}\mu_1\right)\|\tilde q_i\|^2 \\
& + \left(\beta_i - \eta_0\mu_1 - 2V_M C_{i,M}\left(\mu_1 + \eta_0 M_{i,m}^{-1}\right)\right. \\
& \quad \left. + \mu_2(1 - M_{i,m})\right)\|\dot{\tilde q}_i\|\|\tilde q_i\| \\
& + \left[\sum_{j=1, j\neq i}^{p} K_{i,j}(K_{di} - M_{i,m}\mu_1 - 3V_M C_{i,M})\right. \\
& \quad \left. + K_{di} - V_M C_{i,M}\right]\|\dot s_i\|\|\dot{\tilde q}_i\| \\
& + \left[\mu_1(K_{di} - V_M C_{i,M})\right. \\
& \quad + \sum_{j=1, j\neq i}^{p} K_{i,j}(K_{di}\mu_1 - M_{i,m}\mu_2 \\
& \quad \left. - 3V_M C_{i,M}\mu_1)\right] \\
& \times \|\dot s_i\|\|\tilde q_i\| + (M_{i,m}\mu_1 - K_{di} + V_M C_{i,M}) \\
& \times \|\dot s_i\| \sum_{j=1, j\neq i}^{p} K_{i,j}\|\dot{\tilde q}_j\| \\
& + 2V_M M_{i,m}\|\dot s_i\| \\
& \times \sum_{j=1, j\neq i}^{p} K_{i,j}\left(M_{j,m}^{-1} C_{j,M}\|\dot{\tilde q}_j\|\right) \\
& + 2\mu_1 V_M M_{i,m}\|\dot s_i\| \\
& \times \sum_{j=1, j\neq i}^{p} K_{i,j}\left(M_{j,m}^{-1} C_{j,M}\|\tilde q_j\|\right) \\
& + (M_{i,m}\mu_2 - K_{di}\mu_1 + \mu_1 V_M C_{i,M}) \\
& \times \|\dot s_i\| \sum_{j=1, j\neq i}^{p} K_{i,j}\|\tilde q_j\| + \Phi_{3,i}
\end{aligned} \tag{63}$$

with $\Phi_{3,i}$ given by

$$\begin{aligned}
\Phi_{3,i} ={} & C_{i,M}\|\dot s_i\|\left(\|\dot{\tilde q}_i\| + \mu_1\|\tilde q_i\|\right)\left(\sum_{j=1, j\neq i}^{p} K_{i,j}\|\dot e_{i,j}\|\right) \\
& + \left(\|\dot{\tilde q}_i\| + \eta_0 M_{i,m}^{-1}\|\tilde q_i\|\right) C_{i,M}\left(\|\dot{\tilde q}_i\| + \mu_1\|\tilde q_i\|\right) \\
& \times \left(\|\dot{\tilde q}_i\| + \mu_1\|\tilde q_i\| - 2\|\dot e_{i,i}\|\right) \\
& - \left(\|\dot e_{i,i}\| - \|\dot{\tilde q}_i\| - \mu_1\|\tilde q_i\|\right) \times C_{i,M}\|\dot s_i\| \\
& \times \sum_{j=1, j\neq i}^{p} K_{i,j}\left(\|\dot{\tilde q}_i\| - \|\dot{\tilde q}_j\| + \mu_1\left(\|\tilde q_i\| - \|\tilde q_j\|\right)\right) \\
& + M_{i,m}\|\dot s_i\| \\
& \times \sum_{j=1, j\neq i}^{p} K_{i,j}\left(M_{i,m}^{-1} C_{i,M}\left(\|\dot{\tilde q}_i\| + \mu_1\|\tilde q_i\|\right)\right. \\
& \quad \times \left(\|\dot{\tilde q}_i\| + \mu_1\|\tilde q_i\| - 2\|\dot e_{i,i}\|\right) \\
& \quad - M_{j,m}^{-1} C_{j,M} \\
& \quad \times \left(\|\dot{\tilde q}_j\| + \mu_1\|\tilde q_j\|\right) \\
& \quad \left. \times \left(\|\dot{\tilde q}_j\| + \mu_1\|\tilde q_j\| - 2\|\dot e_{j,j}\|\right)\right).
\end{aligned} \tag{64}$$

From the definition of β_i in (41), the coefficient of the bilinear term $\|\dot{\tilde q}_i\|\|\tilde q_i\|$ in (63) is equal to zero. Therefore, from $\tilde q$ in (35) the bound (63) results in (45).

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