Contents


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% This small matlab demo tests the Binary Iterative Hard Thresholding algorithm
% developed in:
%
%  "Robust 1-bit CS via binary stable embeddings"
%  L. Jacques, J. Laska, P. Boufounos, and R. Baraniuk
%
% More precisely, using paper notations, two versions of BIHT are tested
% here on sparse signal reconstruction:
%
%  * the standard BIHT associated to the (LASSO like) minimization of
%
%        min || [ y o A(u) ]_- ||_1 s.t. ||u||_0 \leq K    (1)
%
%  * the (less efficient) BIHT-L2 related to
%
%        min || [ y o A(u) ]_- ||^2_2 s.t. ||u||_0 \leq K  (2)
%
% where y = A(x) := sign(Phi*x) are the 1-bit CS measurements of a initial
% K-sparse signal x in R^N; Phi is a MxN Gaussian Random matrix of entries
% iid drawn as N(0,1); [s]_-, equals to s if s < 0 and 0 otherwise, is applied
% component wise on vectors; "o" is the Hadamard product such that
% (u o v)_i = u_i*v_i for two vectors u and v.
%
% Considering the (sub) gradient of the minimized energy in (1) and (2),
% BIHT is solved through the iteration:
%
%     x^(n+1) = H_K( x^(n) - (1/M)*Phi'*(A(x^(n)) - y) )
%
% while BIHT-L2 is solved through:
%
%     x^(n+1) = H_K( x^(n) - (Y*Phi)' * [(Y*Phi*x^(n))]_-) )
%
% with Y = diag(y), H_K(u) the K-term thresholding keeping the K
% highest amplitude of u and zeroing the others.
%
% Authors: J. Laska, L. Jacques, P. Boufounos, R. Baraniuk
%          April, 2011
```

Important parameters and functions

```
N = 2000; % Signal dimension
M = 500;  % Number of measurements
K = 15;   % Sparsity

% Negative function [.]_-
neg = @(in) in.*(in <0);
```

Generating a unit K-sparse signal in R^N (canonical

basis)

```
x0 = zeros(N,1);
rp = randperm(N);
x0(rp(1:K)) = randn(K,1);
x0 = x0/norm(x0);
```

Gaussian sensing matrix and associated 1-bit sensing

```
Phi = randn(M,N);
A = @(in) sign(Phi*in);

y = A(x0);
```

Testing BIHT

```
maxiter = 3000;
htol = 0;

x = zeros(N,1);
hd = Inf;

ii=0;
while(htol < hd)&&(ii < maxiter)
        % Get gradient
        g = Phi'*(A(x) - y);

        % Step
        a = x - g;

        % Best K-term (threshold)
        [trash, aidx] = sort(abs(a), 'descend');
        a(aidx(K+1:end)) = 0;

        % Update x
        x = a;

        % Measure hammning distance to original 1bit measurements
        hd = nnz(y - A(x));
        ii = ii+1;
end

% Now project to sphere
x = x/norm(x);

BIHT_nbiter = ii;
BIHT_l2_err = norm(x0 - x)/norm(x0);
BIHT_Hamming_err = nnz(y - A(x));
```

Testing BIHT-l2

```
maxiter = 3000;
htol = 0;

x_l2 = Phi'*y;
x_l2 = x_l2/norm(x_l2);
hd = Inf;

% Update matrix (easier for computation)
```

```
cPhi = diag(y)*Phi;
tau = 1/M;

ii=0;
while (htol < hd) && (ii < maxiter)
        % Compute Gradient
        g = tau*cPhi'*neg(cPhi*x_l2);

        % Step
        a = x_l2 - g;

        % Best K-term (threshold)
        [trash, aidx] = sort(abs(a), 'descend');
        a(aidx(K+1:end)) = 0;

        % Update x_l2
        x_l2 = a;

        % Measure hammning
        hd = nnz(y - sign(cPhi*x));
        ii = ii+1;
end

%Now project to sphere
x_l2 = x_l2/norm(x_l2);

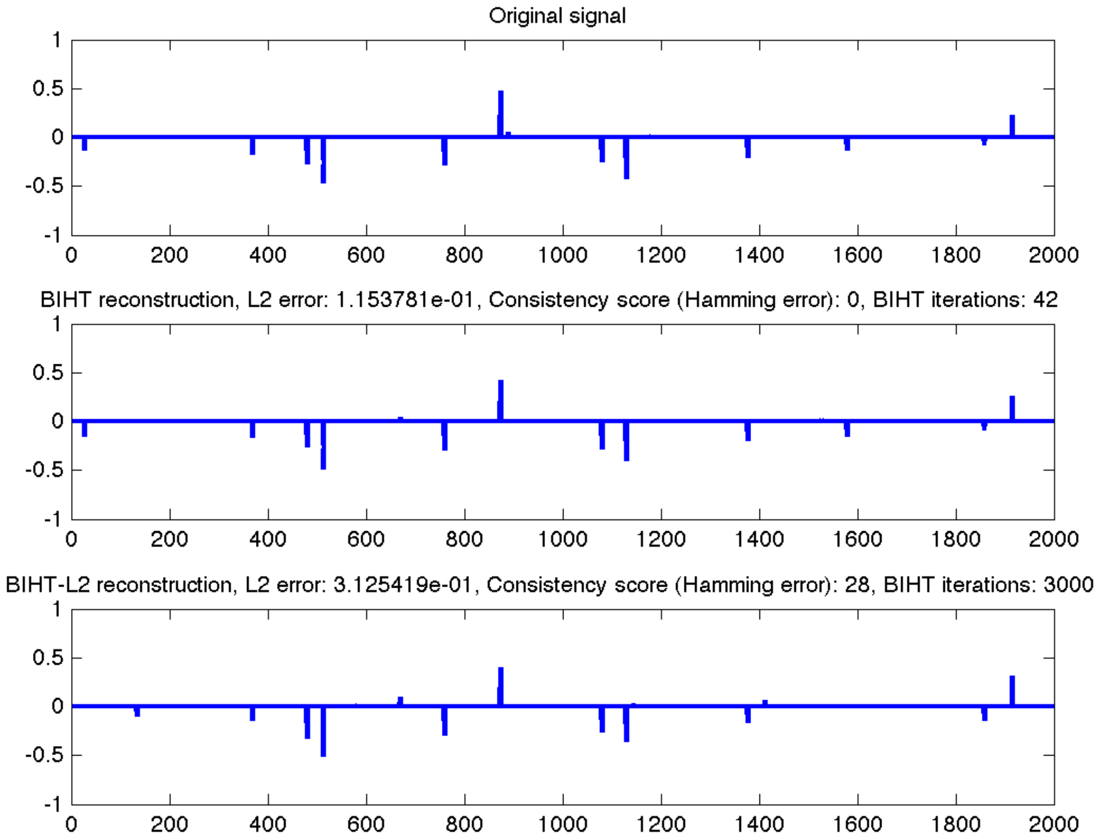
BIHTl2_nbiter = ii;
BIHTl2_l2_err = norm(x0 - x_l2)/norm(x0);
BIHTl2_Hamming_err = nnz(y - A(x_l2));
```

Plotting results

```
figure;
subplot(3,1,1);
plot(x0, 'linewidth', 2);
title('Original signal')
ylim([-1 1]);

subplot(3,1,2);
plot(x, 'linewidth', 2);
title(sprintf('BIHT reconstruction, L2 error: %e, Consistency score (Hamming error
    BIHT_l2_err, BIHT_Hamming_err, BIHT_nbiter));
ylim([-1 1]);

subplot(3,1,3);
plot(x_l2, 'linewidth', 2);
title(sprintf('BIHT-L2 reconstruction, L2 error: %e, Consistency score (Hamming er
    BIHTl2_l2_err, BIHTl2_Hamming_err, BIHTl2_nbiter));
ylim([-1 1]);
```


Original signal
BIHT reconstruction, L2 error: 1.153781e-01, Consistency score (Hamming error): 0, BIHT iterations: 42
BIHT-L2 reconstruction, L2 error: 3.125419e-01, Consistency score (Hamming error): 28, BIHT iterations: 3000
1
0.5
0
-0.5
-1
0
200
400
600
800
1000
1200
1400
1600
1800
2000