Proximal methods for Poisson Intensity Cone-Beam Computerized and Positron Emission Tomography

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Outline

1. Recalls on CBCT and PET
2. Models
3. Methods
4. Results
   - CBCT : algorithms
   - CBCT : simulation results
   - CBCT : results on real data
   - PET : algorithms
   - PET : simulation results
   - PET : results on real data
5. Conclusion

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1. Recalls on CBCT and PET
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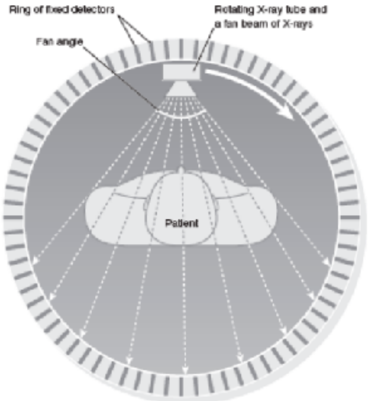
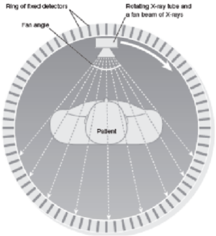
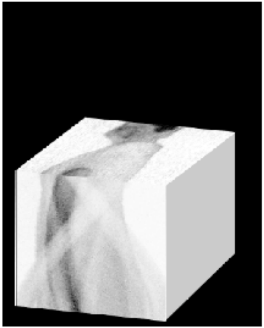
CBCT= *Cone Beam Computerized Tomography*

- Medical imaging modality providing anatomical information.
- CBCT-Scan principle : X-ray source + X-ray camera, the imaged object is in between.
- Cone-Beam geometry.



CBCT : data


Ring of fixed detectors
Rotating X-ray tube and
a fan beam of X-rays
Fan angle
Patient



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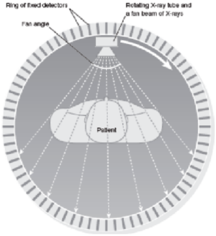
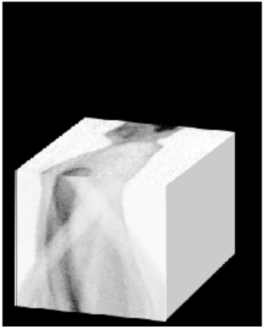
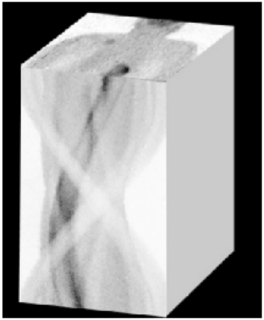
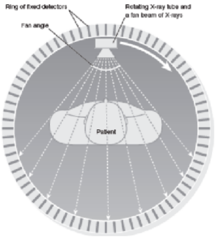






with more angles

CBCT : data



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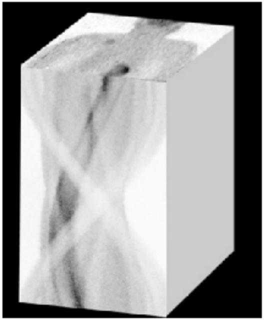
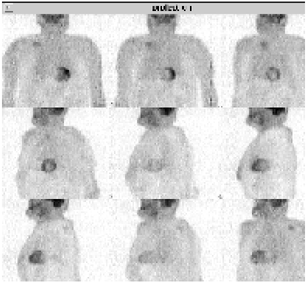
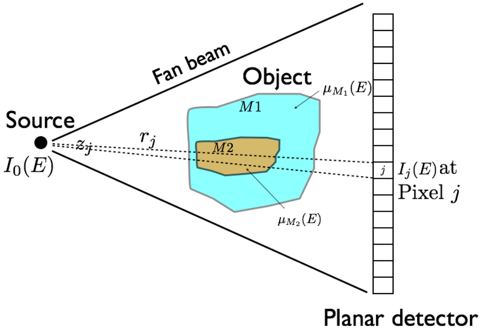




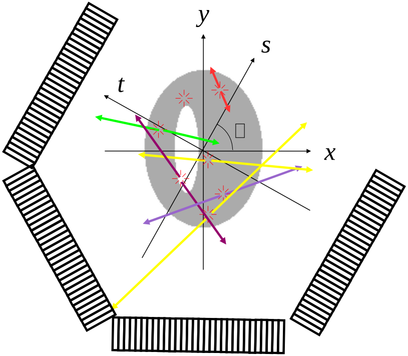
CBCT : X-ray absorption (or transmission) tomography

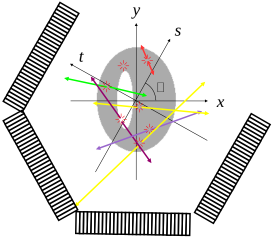


The Beer-Lambert law claims : $I_j(E) = z_j \exp\left[-\int_{r_j} \mu_E(n)dn\right]$ with

- $I_j(E)$ the number of photons received at pixel j.
- $\mu : n \mapsto \mu_E(n)$ the unknown absorption coefficient of tissue at point n in the beam r_j.
- $z_j(E)$ parameter proportional to the number of photons emitted by the X-ray source $I_0(E)$.

- Medical imaging modality providing metabolic information (measurement of the activity of an organ, etc.).
- PET-Scan principle : injection of a radiotracer and collection of emitted gamma rays by detectors.



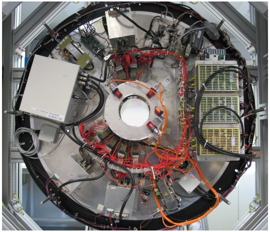


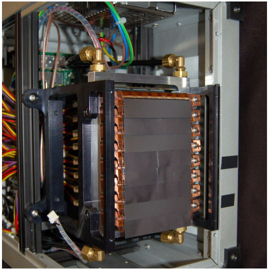
Measurement at pixels indexed by the line of response L :

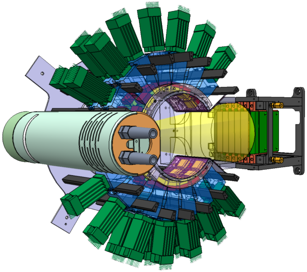
$$w_L = \int_L x(n) \exp\left(-\mu_{511}(n)\right) dn$$

- x is proportional to the unknown concentration of radiotracer activity at point n.
- $n \mapsto \mu_{511}(n)$ is the absorption coefficient at 511 keV at point n.

The developed demonstrator at CPPM : the ClearPET/XPAD







The developed demonstrator at CPPM : the ClearPET/XPAD

Technology

- Technological breakthrough : A 960 $\times$ 560 hybrid pixels detector.

  working in photon counting mode.
  $\hookrightarrow$ no charge integration thus no *dark noise*.
  $\hookrightarrow$ better SNR and better contrast at low statistics.
- Simultaneous CBCT and PET acquisition of the same field of view !

Challenges :

- Improvement of the quality of tomographic reconstructions.
  $\hookrightarrow$ introduction of *sparse* methods with non-differentiable constraints.
  $\hookrightarrow$ introduction of Poisson noise in the models.
- Dose reduction (X-ray exposure, radiotracers concentration).
- Application to bimodal and simultaneous data, to spectral or color X-ray data . . .

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Model for CBCT

$$I_j = z_j \exp\left[-\int_{r_j} \mu(n)dn\right] \Leftrightarrow y = z\exp(-A\mu)$$

- Discretization :
  - $y \in \mathbb{R}^n$ the measurements,
  - $\mu \in \mathbb{R}^m$ the unknown absorption coefficient,
  - $A \in \mathcal{M}(\mathbb{R}^m, \mathbb{R}^n)$ the system matrix ($n << m$, ill-conditioned).
  - z a constant.
- Noise model :
  - pure Poisson noise : $y_j \sim \mathcal{P}(z_j \exp(-[A\mu]_j))$.
  - no charge integration noise (*dark noise*, additive gaussien noise).

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Model for CBCT

$$I_j = \mathcal{P}(z_j \exp\left[-\int_{r_j} \mu(n)dn\right]) \Leftrightarrow y = \mathcal{P}(z \exp\left(-A\mu\right))$$

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Model for CBCT

$$y = \mathcal{P}(z \exp(-A\mu))$$

- Reconstruction problem : trade-off between the minimization of
  - the negative log-likelihood or data-fidelity :

$$\mathcal{L}_{CBCT}(\mu) = \sum_j y_j [A\mu]_j + z_j \exp(-[A\mu]_j)$$

  - and the regularization (priors on objects, for instance *sparsity priors*),
  - under the constraint : $\mu \geq 0$.

We consider the problem :

$$\hat{\mu} = \arg\min_{\mu \geq 0} \mathcal{L}_{CBCT}(\mu) + \lambda J_{reg}(\mu)$$

- Challenge : constraint + non-differentiability.

Model for PET

$$w_L = \int_L x(n) \exp\left(-\mu_{511}(n)\right) dn \Leftrightarrow y = Bx$$

- Discretization :
  - $y \in \mathbb{R}^n$ the measurements,
  - $x \in \mathbb{R}^m$ the unknown concentration of activity.
  - $B \in \mathcal{M}(\mathbb{R}^m, \mathbb{R}^n)$ the system matrix ($n << m$, ill-conditioned).

- Noise model :
  - pure Poisson noise : $x_j \sim \mathcal{P}\left([Bx]_j\right)$.

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PET model

$$y = \mathcal{P}(Bx)$$

- Reconstruction problem : trade-off between the minimization of
  - the negative log-likelihood or data-fidelity :

$$\mathcal{L}_{PET}(x) = \sum_j -y_j \ln([Bx]_j) + [Bx]_j$$

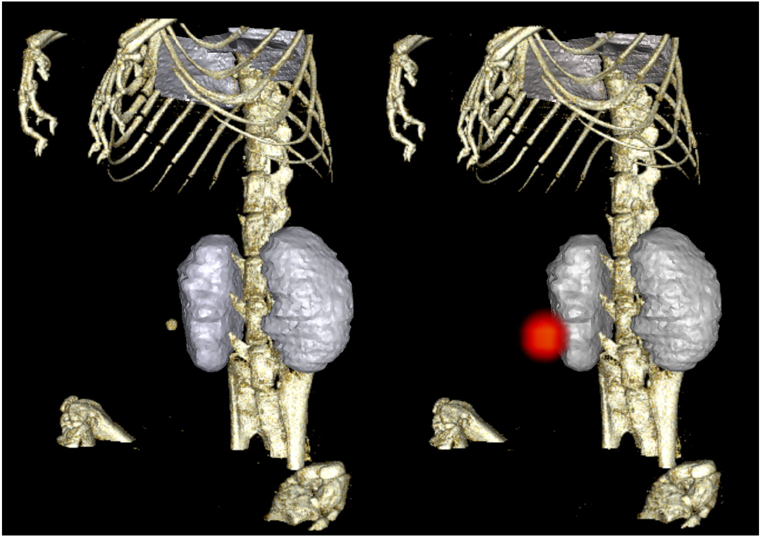
  - and the regularization (priors on objects, for instance *sparsity priors*),
  - under the constraint : $x \geq 0$.

We consider the problem :

$$\hat{x} = \arg\min_{x \geq 0} \mathcal{L}_{TEP}(x) + \lambda J_{reg}(x)$$

- Challenge : constraint + non-differentiability + ln.

Regularization choice



Regularization

- Total variation

$$J_{TV}(u) = \sum_{1 \leq i,j \leq N} |(\nabla u)_{i,j}|$$

  - sharp edges + homogeneous areas (cartoon).
  - non-differentiable.

- Sparse representation

$$J_{\ell^1,\Phi}(u) = \sum_{\lambda \in \Lambda} | < u, \phi_\lambda > |$$

  - sparsity in the dictionnary of $\Phi = \{\phi_\lambda\}$ (redundant).
  - non-differentiable.

- Regularized Total Variation

$$J_{TV}^{reg} = \sum_{1 \leq i,j \leq N} \sqrt{\alpha^2 + |(\nabla u)_{i,j}|^2}$$

Summing up ...

We consider the both following problems :

- CBCT

$$\hat{\mu} = \arg\min_{\mu \geq 0} \mathcal{L}_{CBCT}(\mu) + \lambda J_{reg}(\mu)$$

- TEP

$$\hat{x} = \arg\min_{x \geq 0} \mathcal{L}_{TEP}(x) + \lambda J_{reg}(x)$$

- Difficulties
  - \+ Convexity ;
  - \- Non-differentiability, constraints ;
  - \- logarithmic function for PET (similar to Kullback-Leibler divergence).

Summing up ...

We consider the both following problems :

- CBCT

$$\hat{\mu} = \arg\min_{\mu \geq 0} \left[\sum_j y_j [A\mu]_j + z_j \exp\left(-[A\mu]_j\right) \right] + \lambda J_{reg}(\mu)$$

- TEP

$$\hat{x} = \arg\min_{x \geq 0} \left[\sum_j -y_j \ln([Bx]_j) + [Bx]_j \right] + \lambda J_{reg}(x)$$

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- TEP

$$\hat{x} = \arg\min_{x} \left[\sum_j -y_j \ln([Bx]_j + \epsilon) + [Bx]_j \right] + \lambda J_{reg}(x) + \chi_{\mathcal{C}}(\mu)$$

- Difficulties
  - \+ Convexity ;
  - \- Non-differentiability, constraints ;
  - \- logarithmic function for PET (similar to Kullback-Leibler divergence).

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Proximal operator

Let F be a convex proper function to minimize.

- We recall that the subgradient of F, denoted by ∂F, is defined by :

$$\partial \mathcal{F}(x) = \{p \in X \ \text{ such that } \ \mathcal{F}(y) \geq \mathcal{F}(x) + \langle p, y - x \rangle \ \forall y\}$$

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- For any $h > 0$ the following problem always has a unique solution :

$$\arg\min_{y} \frac{1}{2}\|x - y\|^2 + h\mathcal{F}(y)$$

- This solution is given by :

$$y = (I + h\partial F)^{-1}(x) = \mathrm{prox}_{h\mathcal{F}}(x)$$

The mapping $(I + h\partial F)^{-1}$ is called the *proximal* operator.

Examples

$$\mathrm{prox}_{h\mathcal{F}}(x) = \arg\min_y \frac{1}{2}\|x-y\|^2 + h\mathcal{F}(y)$$

- When $\mathcal{F}$ is the indicator function of some closed convex set $\mathcal{C}$, i.e. :

$$\mathcal{F}(x) = \chi_{\mathcal{C}}(x) = \begin{cases} 0 \text{ if } x \in \mathcal{C} \\ +\infty \text{ otherwise} \end{cases}$$

  $\hookrightarrow$ then $\mathrm{prox}_{h\mathcal{F}}(x)$ is the orthogonal projection of x onto $\mathcal{C}$.

- When $\mathcal{F}(x) = \|x\|_{\dot{B}^1_{1,1}}$,

  $\hookrightarrow$ then $\mathrm{prox}_{h\mathcal{F}}(x)$ is the soft wavelet shrinkage of x with parameter h.

- When $\mathcal{F}(x) = J_{TV}(x)$

  $\hookrightarrow$ then $\mathrm{prox}_{h\mathcal{F}}(x) = x - hP_{hK}(x)$, with P_{hK} orthogonal projection onto hK, and $K = \{\mathrm{div}\, g \;/\; |g_{i,j}| \leq 1 \;\forall i,j\}$.

- Let F and G be two convex functions

$$\arg\min_x \mathcal{F}(x) = \arg\min_x \underbrace{F(x)}_{\text{gradient L-Lipschitz}} + \underbrace{G(x)}_{\text{simple}}$$

Forward-backward splitting

Init. $x_0 \in X$.

Loop $x_{k+1} = \mathrm{prox}_{hG}(x_k - h\nabla F(x_k))$

- Convergence ensured provided $h \leq 1/L$.
- For objective functions, convergence speed of order $1/k$.

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Proximal algorithms : accelerated Forward-Backward

- Let F and G be two convex functions

$$\arg\min_x \mathcal{F}(x) = \arg\min_x \underbrace{F(x)}_{\text{gradient L-Lipschitz}} + \underbrace{G(x)}_{\text{simple}}$$

- Nesterov (2005) and Beck-Teboule (2009) showed that convergence could be accelerated.

FISTA algorithm

Init. $x_0 \in X;\ y_1 = x_0;\ t_1 = 1.$

Loop $\begin{cases} x_k = & \operatorname{prox}_{hG}(y_k - h\nabla F(y_k)) \\ t_{k+1} = & \frac{1+\sqrt{1+4t_k^2}}{2} \\ y_{k+1} = & x_k + \frac{t_k - 1}{t_{k+1}}(x_k - x_{k-1}) \end{cases}$

- Convergence ensured provided $h \leq 1/L$.
- For objective functions, convergence speed of order $1/k^2$.

Primal-dual proximal algorithm : Chambolle-Pock

- Let F and G be two convex functions, X and Y two finite-dimensional real vector spaces, $K : X \to Y$ continuous linear operator.

$$\arg\min_x \mathcal{F}(x) = \arg\min_x \underbrace{F(Kx)}_{\text{F simple, K linear}} + \underbrace{G(x)}_{\text{simple}}$$

- We remind the definition of the Legendre-Fenchel conjugate of F :

$$F^*(y) = \max_{x \in X} (\langle x, y \rangle - F(x)) \tag{1}$$

- The associated saddle point problem is :

$$\min_{x \in X} \max_{y \in Y} (\langle Kx, y \rangle + G(x) - F^*(y))$$

$\hookrightarrow$ Arrow-Urwicz method (ascent in y, descent in$x$).

- Notice that F and G can be non smooth.

Chambolle-Pock algorithm

Init. $\tau, \sigma > 0$, $(x_0, y_0) \in (X \times Y)$, $\bar{x}_0 = x_0$.

$$\text{Loop} \left\{ \begin{array}{l} y_{n+1} = \mathrm{prox}_{\sigma F^*}(y_n + \sigma K \bar{x}_n) \\ x_{n+1} = \mathrm{prox}_{\tau G}(x_n - \tau K^* y_{n+1}) \\ \bar{x}_{n+1} = 2x_{n+1} - x_n \end{array} \right.$$

- Convergence is provided if $\tau\sigma\|K\|^2 < 1$.
- In terms of objective functions, the convergence speed is of order $1/k$ if both F and G are non smooth functions, and of order $1/k^2$ if at least one has smoothness properties.

Dealing with non-simple functions

How to compute the prox. of $G = J_{TV} + \chi_{\mathcal{C}}$ with $\mathcal{C}$ a closed non empty convex set ? The FISTA algorithm can solve this problem.

Constrained Total Variation

$$\min_{u \in \mathcal{C}} J_{TV}(u) + \frac{1}{2\lambda}\|f - u\|^2 \qquad (2)$$

Proposition :

Let us set :

$$h(v) = -\|H_{\mathcal{C}}(f - \lambda \mathrm{div}\, v)\|^2 + \|f - \lambda \mathrm{div}\, v\|^2$$

with $H_{\mathcal{C}}(u) = u - P_{\mathcal{C}}(u)$ and $P_{\mathcal{C}}(u)$ is the orthogonal projection of u onto $\mathcal{C}$. Let us define :

$$\tilde{v} = \arg\min_{\|v\| \leq 1} h(v).$$

Then the solution of problem (2) is given by :

$$u = P_{\mathcal{C}}(f - \lambda \mathrm{div}\, \tilde{v})$$

Dealing with non-simple functions (2)

For some general ℓ^1 constrained regularization $G = J_{\ell^1,\Phi} + \chi_{\mathcal{C}}$ with $\mathcal{C}$ a closed non empty convex set, K a continuous linear operator.

Constrained sparse representation

$$\min_{u \in \mathcal{C}} \|Ku\|_1 + \frac{1}{2\lambda}\|f - u\|^2 \tag{3}$$

Proposition :

Let us define

$$h_K(v) = -\|H_{\mathcal{C}}(f + \lambda K^* v)\|^2 + \|f + \lambda K^* v\|^2$$

with $H_{\mathcal{C}}(u) = u - P_{\mathcal{C}}(u)$ and $P_{\mathcal{C}}(u)$ is the orthogonal projection of u onto $\mathcal{C}$. Let us define :

$$\tilde{v} = \arg\min_{\|v\| \leq 1} h_K(v)$$

The solution of problem (3) is given by :

$$u = P_{\mathcal{C}}(f + \lambda K^* \tilde{v})$$

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CBCT : algorithms

- Problem

$$\hat{\mu} = \arg\min_{\mu} \underbrace{\sum_j y_j [A\mu]_j + z_j \exp\left(-[A\mu]_j\right)}_{\text{F : L-Lipschitz gradient}} + \underbrace{\lambda J_{reg}(\mu) + \chi_{\mathcal{C}}(\mu)}_{\text{G : simple}}$$

- Forward-backward splitting
  - $x_{k+1} = prox_{hG}(x_k - h\nabla F(x_k))$.
  - Prox. computation (constraint + regularization) : FISTA.

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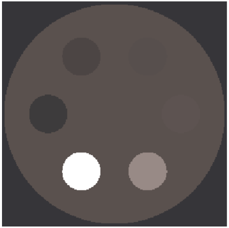
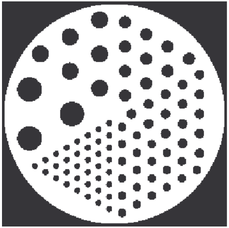
Simulated data

- Phantoms, i.e objects to reconstruct :

Zubal



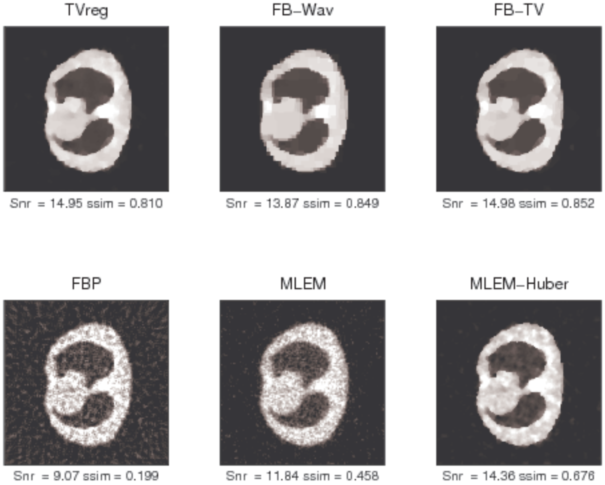
Contrast



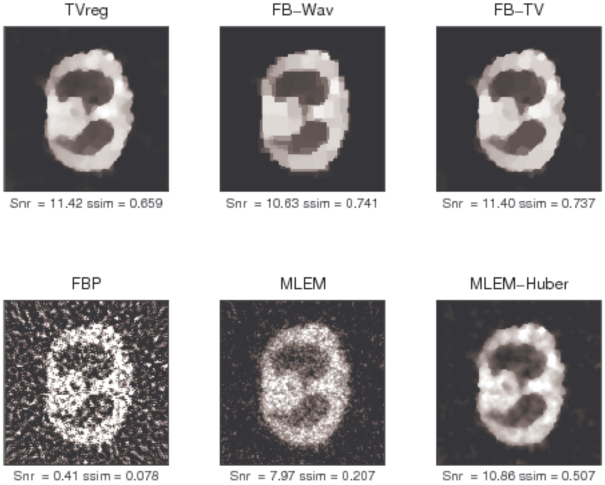
Resolution

- Quantitative criteria
  - $SNR(I,T) = 10\log_{10}\left(\frac{\mathrm{mean}(I^2)}{\mathrm{mean}(|I-T|^2)}\right)$
  - $SSIM(I,T) = \mathrm{mean}_w\left(\frac{(2\mathrm{mean}(I_w)\mathrm{mean}(T_w)+a)(2\,\mathrm{cov}(I_w,T_w)+b)}{\mathrm{mean}(I_w)^2+\mathrm{mean}(T_w^2+a)(\mathrm{var}(I_w)+\mathrm{var}(T_w)+b)}\right)$
  - $CNR(I) = \frac{|\mathrm{mean}(I_{in})-\mathrm{mean}(I_{out})|}{\sqrt{\mathrm{var}(I_{in})+\mathrm{var}(I_{out})}}$

CBCT Zubal : $z = 1e3$ photons



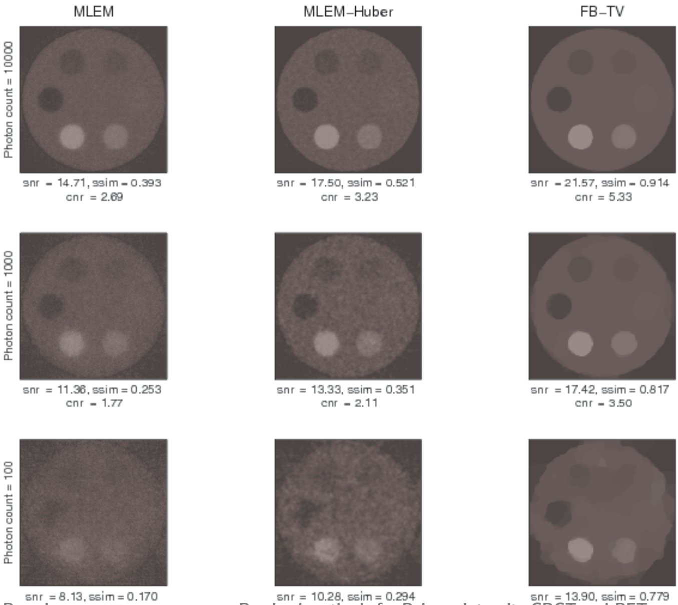
CBCT Zubal : $z = 1e2$ photons



CBCT Zubal : $z = 1e3$ and $z = 1e2$

Photon count	Algorithm	SNR	SSIM	λ	nb. iter.	time (s)
$1e3$	TVreg	15.06	0.808	200	300	36
	FB-Wav	14.06	0.826	25	300	110
	FB-TV	15.10	0.845	200	300	85
	FBP	9.08	0.201	-	-	0.09
	MLEM	11.86	0.462	-	43	14
	MLEM-H	14.52	0.680	7e5	752	
$1e2$	TVreg	11.34	0.625	80	300	32
	FB-Wav	10.62	0.695	10	300	110
	FB-TV	11.35	0.690	80	300	78
	FBP	0.44	0.076	-	-	0.07
	MLEM	7.90	0.200	-	17	5.67
	MLEM-H	10.78	0.489	3.5e4	605	

CBCT Contrast with 60 projections



CBCT Contrast with 60 projections

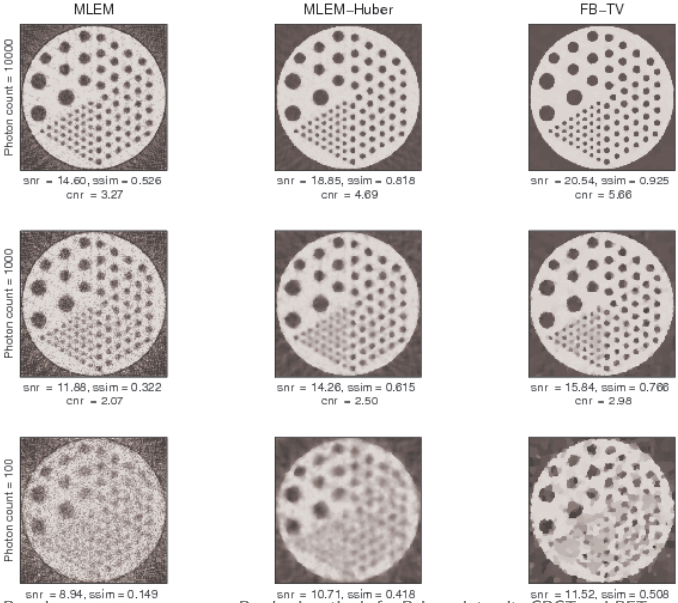
photon count	Algorithm	CNR	SSIM	SNR
$1e4$	FB-Wav	4.18	0.911	20.09
	FB-TV	5.33	0.914	21.57
	MLEM-H	3.23	0.521	17.50
$1e3$	FB-Wav	2.96	0.839	17.01
	FB-TV	3.50	0.817	17.42
	MLEM-H	2.11	0.351	13.33
$1e2$	FB-Wav	2.08	0.779	12.93
	FB-TV	2.34	0.779	13.90
	MLEM-H	1.46	0.294	10.28

CBCT Contrast : influence of number of projections

	Nb. angles	90			60		
Photon count	Algorithm	CNR	SSIM	SNR	CNR	SSIM	SNR
1e3	TVreg	3.17	0.793	17.15	3.00	0.743	16.95
	FB-Wav	3.18	**0.851**	**17.68**	2.96	**0.839**	17.01
	FB-TV	**3.93**	0.831	17.65	**3.50**	0.817	**17.42**
	FBP	0.82	0.046	3.57	0.65	0.033	2.09
	MLEM	1.95	0.274	11.87	1.77	0.253	11.36
	MLEM-H	2.27	0.337	13.49	2.11	0.351	13.33

	Nb. angles	30		
1e3	TVreg	2.78	0.728	**15.46**
	FB-Wav	2.61	**0.802**	15.39
	FB-TV	**3.36**	0.756	15.25
	FBP	0.44	0.017	-0.79
	MLEM	1.53	0.218	10.33
	MLEM-H	2.04	0.393	13.47

CBCT Resolution with 60 projections



Outline

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2. Models
3. Methods
4. Results
   - CBCT : algorithms
   - CBCT : simulation results
   - CBCT : results on real data
   - PET : algorithms
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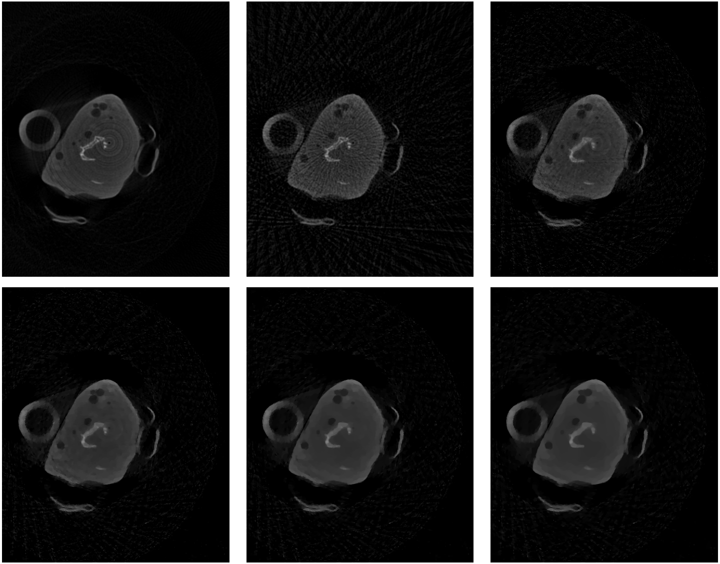
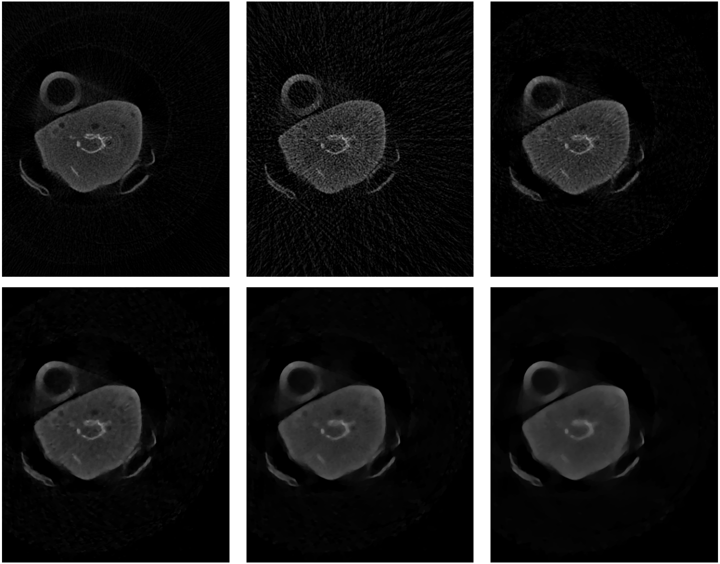
5. Conclusion

10000 photons per pixel, 60 projections ($\lambda = 15; 25; 40$)



1000 photons per pixel, 60 projections ($\lambda = 15; 25; 40$)



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- Problem

$$\hat{x} = \arg\min_x \underbrace{\sum_j -y_j \ln([Bx]_j + \epsilon) + [Bx]_j}_{\text{F : L-Lipschitz gradient}} + \underbrace{\lambda J_{reg}(x) + \chi_{\mathcal{C}}(\mu)}_{\text{G : simple}}$$

- Forward-backward splitting
  - $x_{k+1} = prox_{hG}(x_k - h\nabla F(x_k))$.
  - Prox. computation (constraint + regularization) : FISTA.
  - $\epsilon > 0$!!!

- Problem

$$\hat{x} = \arg\min_x \sum_j -y_j \ln([Bx]_j) + [Bx]_j + \lambda J_{reg}(x) + \chi_{\mathcal{C}}(x)$$

$\hookrightarrow$ rewritten as :

$$\hat{x} = \arg\min_x F(Kx) + G(x)$$

- Assumptions : F and G proper, convex, l.s.c, F and G non-differentiable, K linear and continuous.
- Chambolle-Pock algorithm (primal-dual) :

$$\begin{cases} y_{n+1} = prox_{\sigma F^\star}(y_n + \sigma K\bar{x}_n) \\ x_{n+1} = prox_{\tau G}(x_n - \tau K^* y_{n+1}) \\ \bar{x}_{n+1} = 2x_{n+1} - x_n \end{cases}$$

PET algorithm : Chambolle-Pock version 1

- Problem

$$\hat{x} = \arg\min_x \underbrace{\sum_j -y_j \ln([Bx]_j) + [Bx]_j}_{F(Kx),\ K=B} + \underbrace{\lambda J_{reg}(x) + \chi_{\mathcal{C}}(x)}_{G(x)}$$

- Assumptions : F and G proper, convex, l.s.c, F and G non differentiable, K linear and continuous.

- Algorithme de Chambolle-Pock (primal-dual) :

$$\begin{cases} y_{n+1} = prox_{\sigma F^\star}(y_n + \sigma K \bar{x}_n) \\ x_{n+1} = prox_{\tau G}(x_n - \tau K^* y_{n+1}) \\ \bar{x}_{n+1} = 2x_{n+1} - x_n \end{cases}$$

PET algorithm : Chambolle-Pock version 2

- Problem

$$\hat{x} = \arg\min_x \underbrace{\sum_j -y_j \ln([Bx]_j) + [Bx]_j + \chi_{\mathcal{C}}(Bx)}_{G(x)} + \underbrace{\lambda J_{reg}(x)}_{\substack{F(Kx)\\ F=\|.\|_1,\ K=\nabla}} + \chi_{\mathcal{C}}(x)$$

- Assumptions : F and G proper, convex, l.s.c, F and G non differentiable, K linear and continuous.

- Chambolle-Pock algorithm (primal-dual) :

$$\begin{cases} y_{n+1} = prox_{\sigma F^\star}(y_n + \sigma K \bar{x}_n) \\ x_{n+1} = prox_{\tau G}(x_n - \tau K^* y_{n+1}) \\ \bar{x}_{n+1} = 2x_{n+1} - x_n \end{cases}$$

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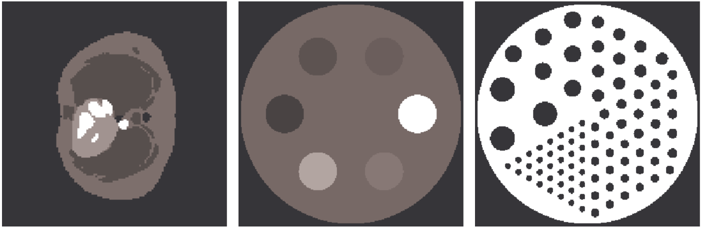
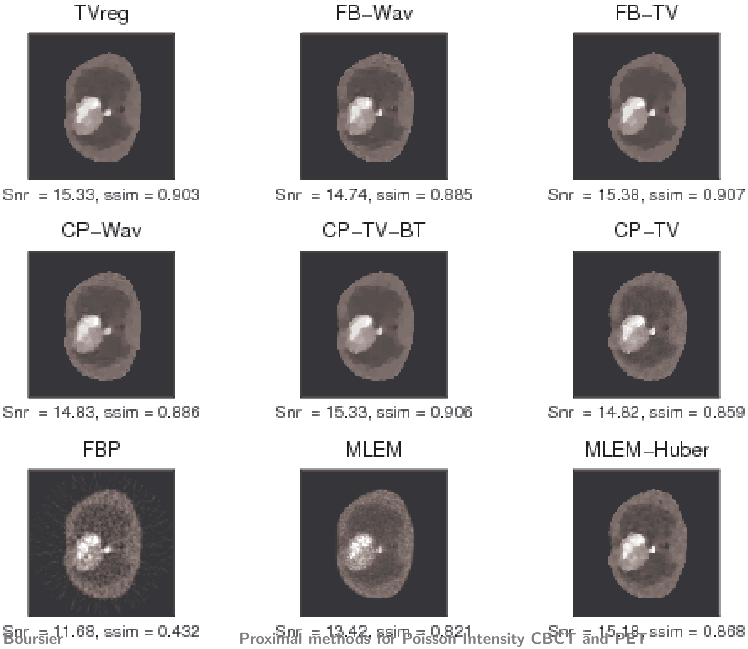
- Phantoms, i.e objects to reconstruct :



Zubal Contrast Resolution

- Quantitative criteria
  - $SNR(I,T) = 10\log_{10}\left(\frac{\mathrm{mean}(I^2)}{\mathrm{mean}(|I-T|^2)}\right)$
  - $SSIM(I,T) = \mathrm{mean}_w\left(\frac{(2\mathrm{mean}(I_w)\mathrm{mean}(T_w)+a)(2\,\mathrm{cov}(I_w,T_w)+b)}{\mathrm{mean}(I_w)^2+\mathrm{mean}(T_w^2+a)(\mathrm{var}(I_w)+\mathrm{var}(T_w)+b)}\right)$
  - $CNR(I) = \frac{|\mathrm{mean}(I_{in})-\mathrm{mean}(I_{out})|}{\sqrt{\mathrm{var}(I_{in})+\mathrm{var}(I_{out})}}$

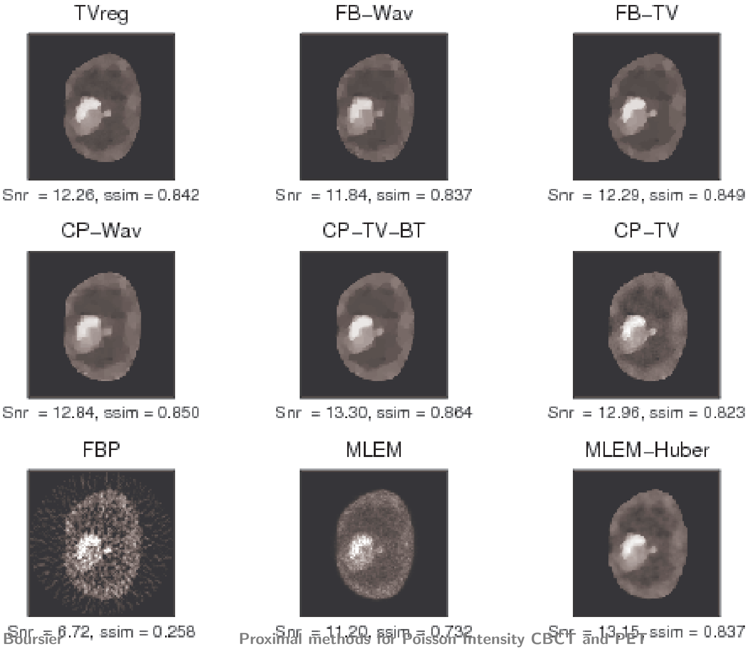
PET Zubal, fcount = 500 000



PET Zubal, fcount = 500 000

Algorithm	SNR	SSIM	λ	nb. iter.	time (s)
TVreg	15.33	0.902	0.70	200	10
FB-Wav	14.77	0.889	0.10	150	89
FB-TV	15.37	0.905	0.70	100	62
CP-Wav	14.68	0.885	0.10	80	63
CP-TV-BT	15.32	0.905	0.70	80	63
CP-TV	14.84	0.860	0.70	400	266
SPIRAL	15.17	0.905	0.70	100	76
FBP	11.59	0.429	-	-	0.04
MLEM	13.38	0.819	-	17	2
MLEM-H	15.22	0.866	0.9/0.25	267	46

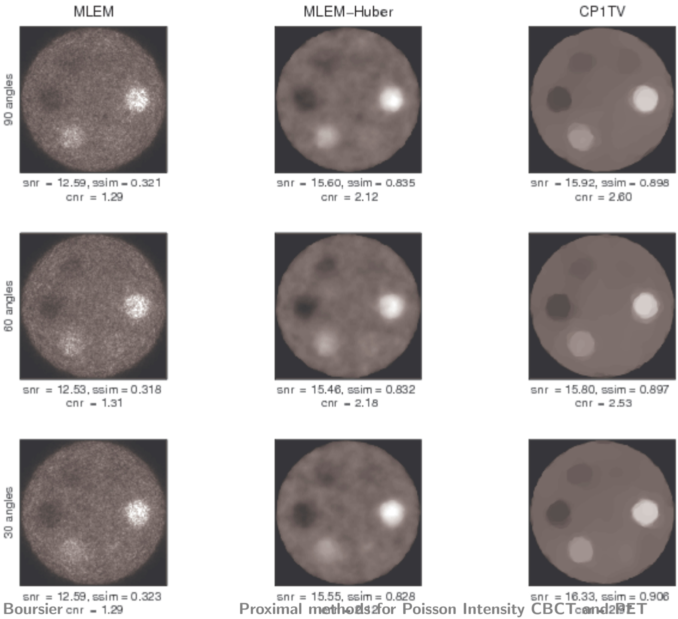
PET Zubal, fcount = 100 000



PET Zubal, fcount = 100 000

Algorithm	snr	ssim	λ	nb. iterations	time (s)
TVreg	12.12	0.841	0.40	200	13
FBwav	11.55	0.834	0.0625	150	89
FB-TV	12.14	0.847	0.40	100	68
CPwav	11.65	0.835	0.0625	50	40
CP-TV-BT	13.13	0.862	0.40	50	46
CP-TV	12.86	0.823	0.40	100	78
SPIRAL	11.77	0.841	0.40	100	86
FBP	6.66	0.254	-	-	0.08
MLEM	11.06	0.731	-	10	2
MLEM-H	12.92	0.837	0.8/0.25	278	58

PET Contrast for $fcount = 2e5$



PET Contrast

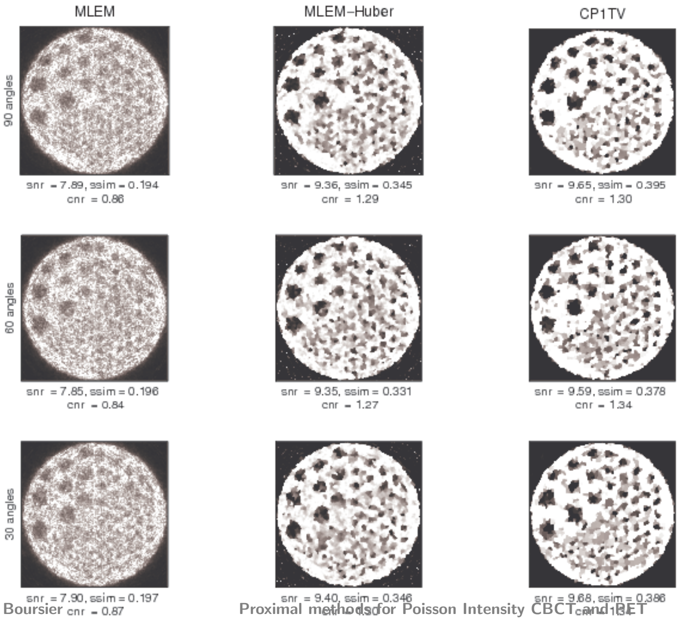
$fcount = 2e5$

N. angles	30			60			90		
Algo	CNR	SSIM	SNR	CNR	SSIM	SNR	CNR	SSIM	SNR
CP-TV-BT	2.60	0.898	15.92	2.53	0.897	15.80	2.97	0.906	16.33
MLEM	1.29	0.321	12.59	1.31	0.318	12.53	1.29	0.323	12.59
MLEM-H	2.12	0.835	15.60	2.18	0.832	15.46	2.12	0.828	15.55

$fcount = 1e5$

N. angles	30			60			90		
Algo	CNR	SSIM	SNR	CNR	SSIM	SNR	CNR	SSIM	SNR
CP-TV-BT	2.64	0.900	16.11	2.55	0.897	15.84	2.72	0.901	15.92
MLEM	1.62	0.405	14.00	1.59	0.418	13.91	1.67	0.428	14.14
MLEM-H	2.67	0.842	17.50	2.42	0.837	17.24	2.59	0.842	17.39

PET Resolution for *fcount* = 2e5



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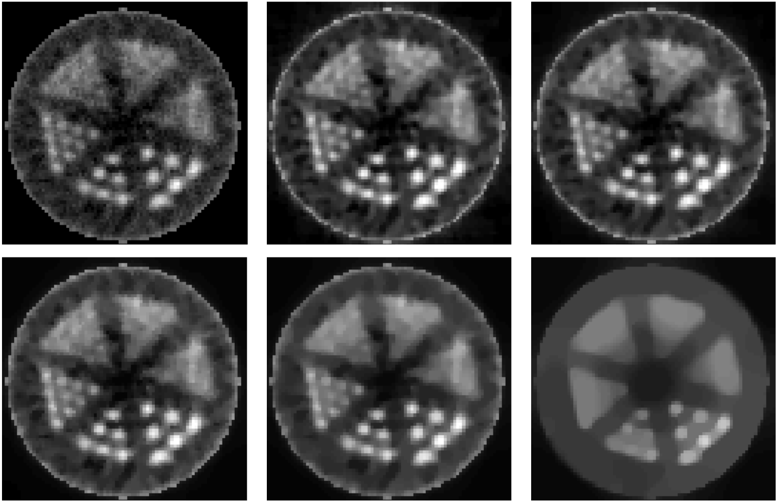
PET : real data



(80 projections, FBP then FB-TV $\lambda = 0.5; 1; 5; 10; 50$)

Outline

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Conclusion

- Demonstrator of simultaneous PET and CBCT (based on hybrid pixel technology) have been developed.
- Proposed algorithm are adapted to the physics of acquisition (photon-couting mode) and also to the pure Poisson noise.
- Efficiency and robustness of the algorithms have been proved on synthetic data and for a low number of projection angles.
- Results obtained on real data are very encouraging and confirm this trend.
- Perspectives : 3D implementation on GPU (work in progress), simultaneous acquisitions.

Thank you for your attention !

Questions ?

PET

- radiotracer attached to a molecule that will be absorbed by some organs, depending of their function

$\longrightarrow$ radioactive decay emits a positron, which annihilates with an electron after a very short time, and this yields... two gamma rays radiation of 511 keV and opposite direction.
Rings of detectors are supposed to detect them.
parallel-beam geometry

- Possible absorption of photons when crossing the body

State of the art : quick non exhaustive review

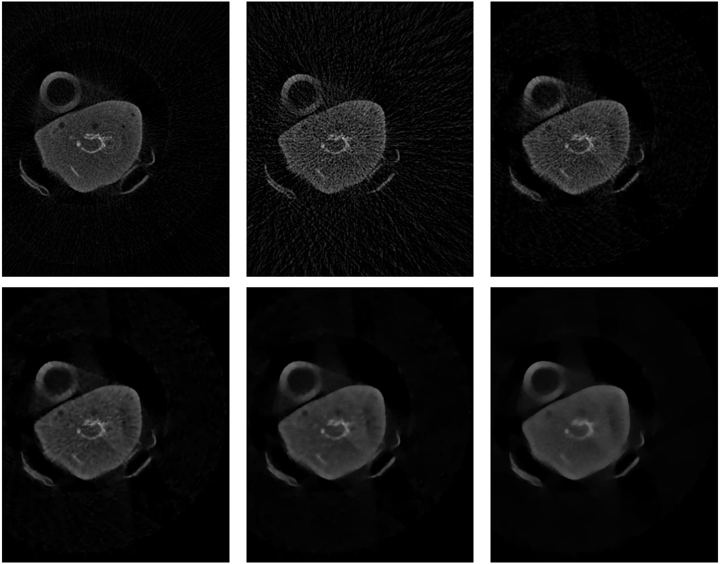
- some algorithms to recover CBCT and PET images viewed as Poisson noisy data
  - Filtered backprojection for Cone Beam geometry : FDK algorithm (Feldkamp and all 1984...)
  - EM algorithm and variants (Shepp and Vardi 1982, Lange and Carson 1984, Hudson and Larkin 1994...)
  - Regularization of EM type algorithms : quadratic surrogate functions (De Pierro 1994, Fessler and all 1998...), Huber (Chlewicki and all 2004...), TV (Harmany and all 2011...)

$\longrightarrow$ technics closed to the ones used in convex optimization

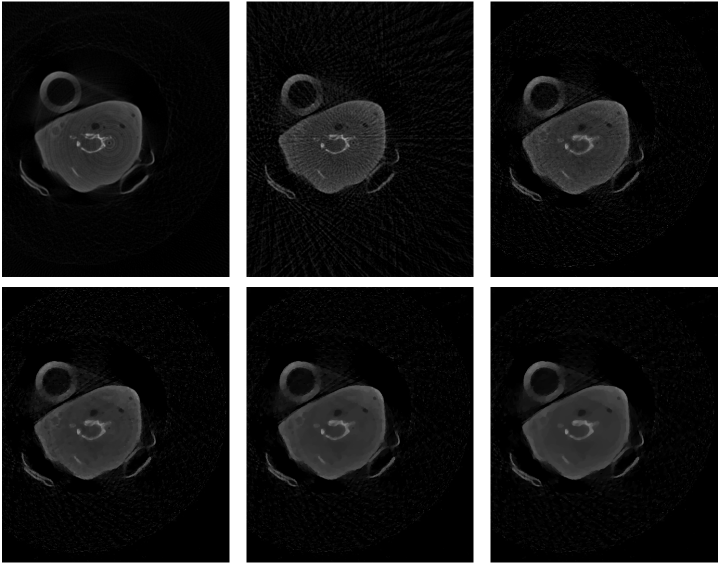
State of the art : quick non exhaustive review

- Forward backward splitting (Combettes-Wajs 2005) after using an Anscombe transform to go back to Gaussian noise applied in the setting of Deconvolution problems with Poisson noisy data (Dupé et al 2009)
- Alternative Direction Method of Multipliers in the context of poissonian image reconstruction (Figueiredo 2010)
- PPXA algorithm applied in the context of dynamical PET (Pustelnik et al 2010)
- Primal dual algorithm using TV regularization in the context of blurred Poisson noisy data (Bonettini and Ruggiero 2010)
- ...

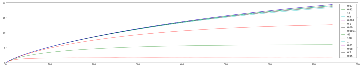
600 photons per pixel, 60 projections ($\lambda = 15; 25; 40$)



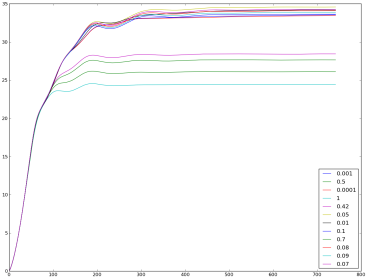
15000 photons per pixel, 60 projections ($\lambda = 15; 25; 40$)



Acceleration en 3D



Non accelerated



Accelerated