Constructive LMI Approaches for Anti-Windup Compensator Design for Systems Subject to Saturation

Sophie Tarbouriech

LAAS-CNRS, Toulouse, France

Main collaborations:

- J.-M. Gomes da Silva Jr (UFRGS, Brazil), M. Turner (University of Leicester, UK)
- G. Garcia (LAAS-CNRS), I. Queinnec (LAAS-CNRS), C. Prieur (LAAS-CNRS), Jean-Marc Biannic (ONERA)

General Outline

Organization details

- 12 curses of 1h15 = 6 slots of 3h (5, 6, 7, 13, 14, 15 May)

Curse 1	General introduction - 5 May
Curse 2	Context of stability - 5 May
Curse 3	Approximating the saturations - 6 May
Curses 4	Introduction on anti-windup strategy- 6 May
Curses 5, 6, 7	Solution via static anti-windup (1 loop) - 7 and 13 May
Curses 7, 8, 9	Solution via static Anti-windup (2 loops) - 13 and 14 May
Curses 9, 10, 11	Solution via dynamic anti-windup - 14 and 15 May
Curse 12	Concluding Remarks and Perspectives - 15 May

- Only the continuous-time framework but all the results could be presented in a discrete-time framework.
- Slides + list of references will be available on the WEB site.

Contents of part I

A simple example

☞ Consider the following example (balancing pointer)

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ -1 \end{bmatrix} u(t)$$

▷ Matrix A is unstable : $\sigma(A) = \{1; -1\}$.

☞ Consider the state feedback $K = \begin{bmatrix} 13 & 7 \end{bmatrix}$ $(u(t) = Kx(t))$

▷ Matrix $A + BK$ is asymptotically stable : $\sigma(A + BK) = \{-3; -4\}$

▷ The origin $x = 0$ is asymptotically stable for the closed-loop system

$$\dot{x}(t) = (A + BK)x(t)$$

▷ $\forall x(0) \in \Re^2$, $x(t, x(0)) \to 0$ as $t \to \infty$.

1. Motivation (2)

☞ Consider now that control u is limited in amplitude

$$-5 \leq u \leq 5$$

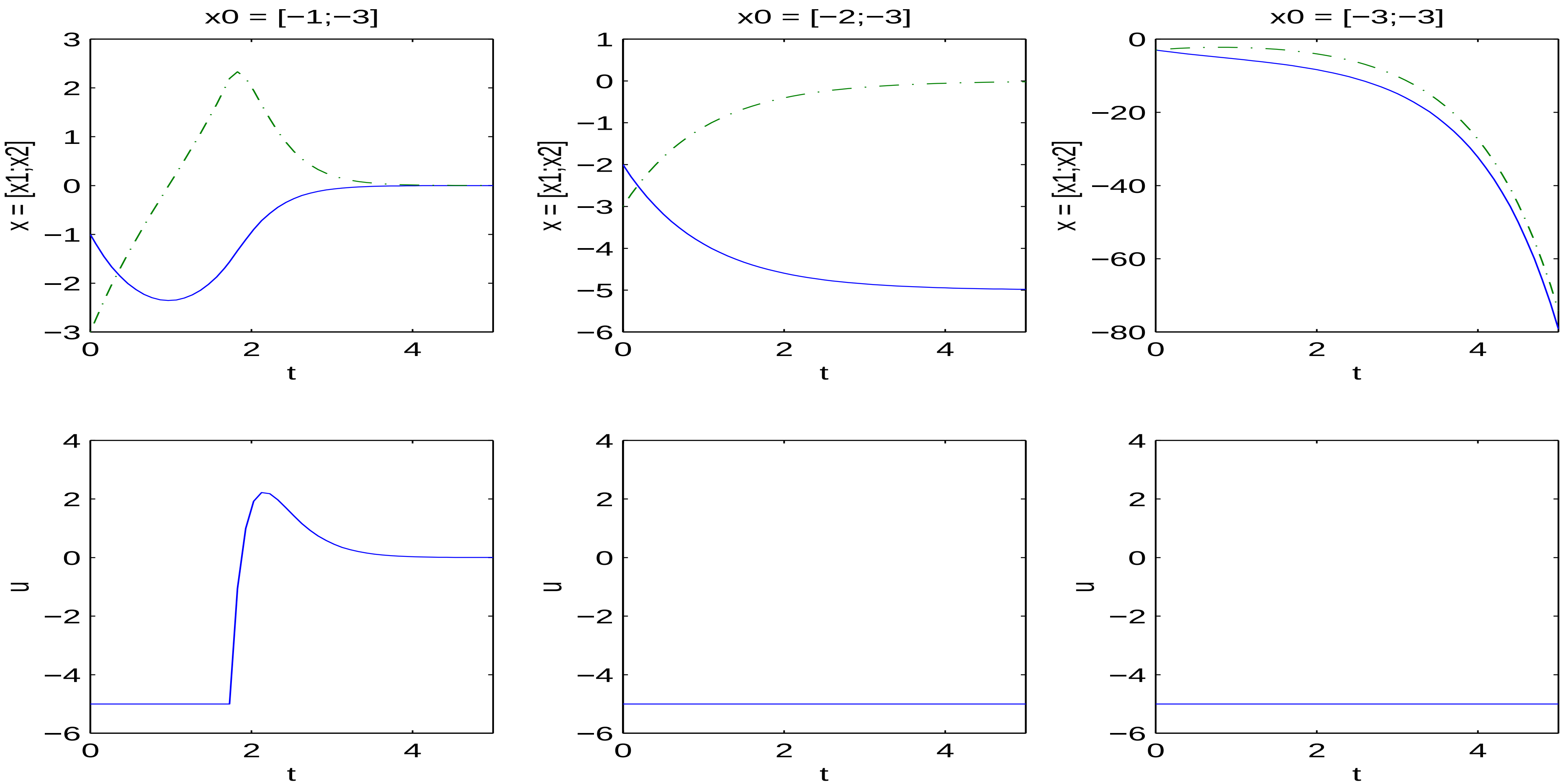
▷ The closed-loop system writes

$$\dot{x}(t) = Ax(t) + Bsat(Kx(t)) \quad \text{with} \quad sat(Kx(t)) = \begin{cases} 5 & \text{if} \quad Kx(t) > 5 \\ Kx(t) & \text{if} \quad |Kx(t)| \leq 5 \\ -5 & \text{if} \quad Kx(t) < -5 \end{cases}$$

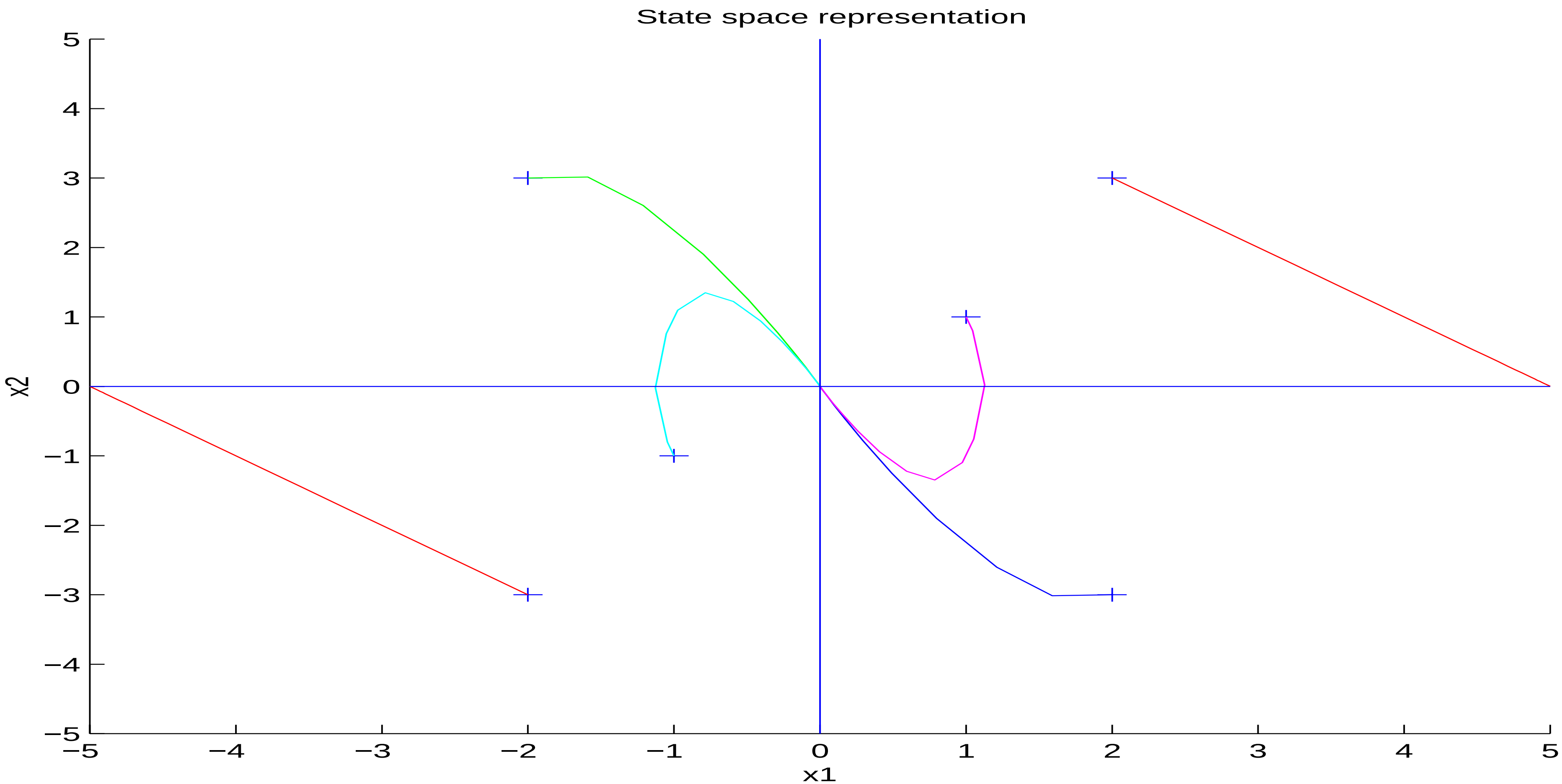
▷ There are two equilibrium points in plus of $x = 0$ when saturation is active:

$$x_{e1} = \begin{bmatrix} -5 \\ 0 \end{bmatrix} \quad \text{and} \quad x_{e2} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

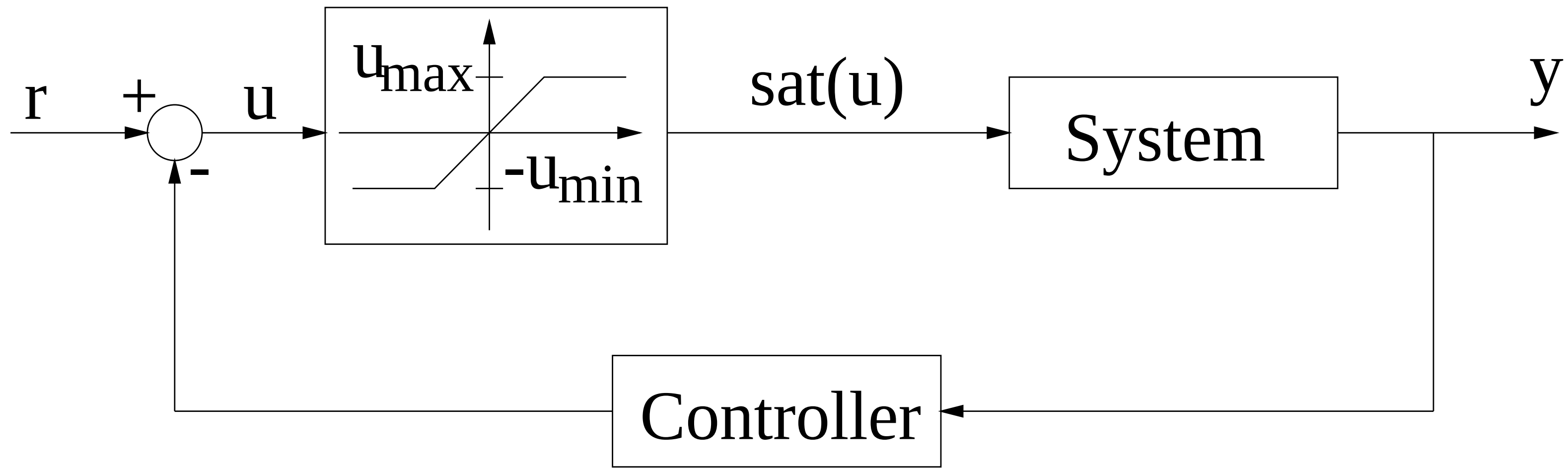
▷ There are some initial conditions $x(0)$ such that $x(t, x(0)) \not\to 0$ when $t \to \infty$.


x0 = [−1;−3]
x0 = [−2;−3]
x0 = [−3;−3]
x = [x1;x2]
u
t

1. Motivation (4)



2. Starting point (1)



☞ Saturations (position, rate, acceleration) occur in most systems for physical, technological and/or for security reasons

- ▷ In particular they are often present in actuator and/or sensor.
- ▷ They will be active as much as high performance level is required (quick rise time, large amplitude of setpoint or of the reference signal)
- ▷ Saturations may induce the occurrence of parasitic equilibrium points, limit circles, degradation of the performance or even instability, ...
- ▷ Examples: aircrafts, launchers, industrial processes, Tchernobyl, ...

2. Starting point (2)

What can we do?

☞ Nothing, generally the engineering approach: oversize the actuators, limit the energy of the controller, and pray

▷ Necessity to study the closed-loop behavior analysis a posteriori

☞ Take the limitations of the actuators in the controller synthesis ***a priori***:

▷ Do not allow saturations: we impose a linear behavior for the closed-loop system.

▷ Saturations are allowed: the nonlinear behavior of the closed-loop system is taken into account.

☞ Modify the controller ***a posteriori***:

▷ Anti-windup approaches

2. Starting point (3)

☞ Then the objectives are:

▷ to evaluate the stability and the attainable performances for the closed-loop system.

▷ to determine regions corresponding to a safe closed-loop system behavior

- in which the closed-loop system behavior is linear avoidance of saturation;
- in which the closed-loop system behavior is nonlinear allowance of saturation.

☞ To do this, we have to develop efficient and performing tools of analysis and design.

2. Starting point (4)

Some references

- D.S. Bernstein and A.N. Michel (Eds.). *Special issue: saturating actuators*, Int. J. of Robust and Nonlinear Control, vol.5, no.5, 1995.
- S. Tarbouriech and G. Garcia (Eds.). *Control of uncertain systems with bounded inputs*, Lectures notes in Control and Information Sciences, vol.227, Springer Verlag, 1997.
- A.A. Stoorvogel and A. Saberi (Eds.). *Special issue: control problems with constraints*, Int. J. of Robust and Nonlinear Control, vol.9, no.10, 1999.
- T. Hu and Z. Lin. *Control systems with actuator saturation - Analysis and design*, Birkhäuser, Boston (USA), 2001.
- V. Kapila and K.M. Grigoriadis (Eds.). *Actuator saturation control*, Marcel Dekker, Inc., New York (USA), 2002.
- S. Tarbouriech, G. Garcia, A.H. Glattfelder (Eds.). *Advanced Control strategy Advanced Strategies in Control Systems with Input and Output Constraints*, Springer Verlag, LNCIS, vol.346, Berlin (Germany), 2007
- See papers from E.D. Sontag, H.J. Sussmann, A.R. Teel, A. Saberi, Z. Lin, A.A. Stoorvogel, F. Jabbari, R. Suarez, M. Morari, D.S. Bernstein, R. De Santis, S.M. Meerkov, J.M. Gomes da Silva Jr., I. Postlethwaite, S. Tarbouriech, G.Garcia, T. Alamo, E. Camacho, J-M. Biannic, ...

3. Actuator saturation (1)

☞ In the sequel we will study different types of saturations and their associate modelling.

▷ Saturation in amplitude

▷ Rate saturation

▷ Limited integrator

☞ We will also present different types of actuators often present in practice (aircraft).

▷ First order saturated actuator

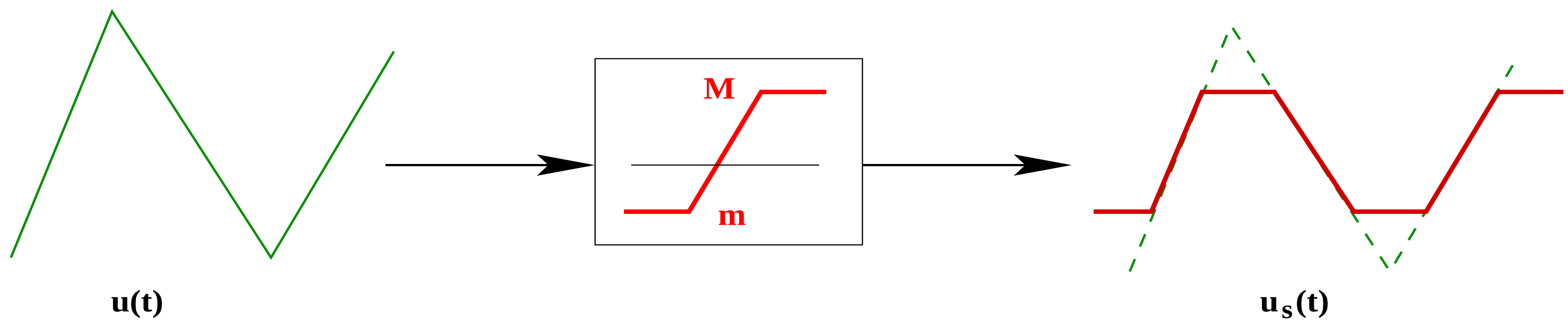
▷ Second order saturated actuator

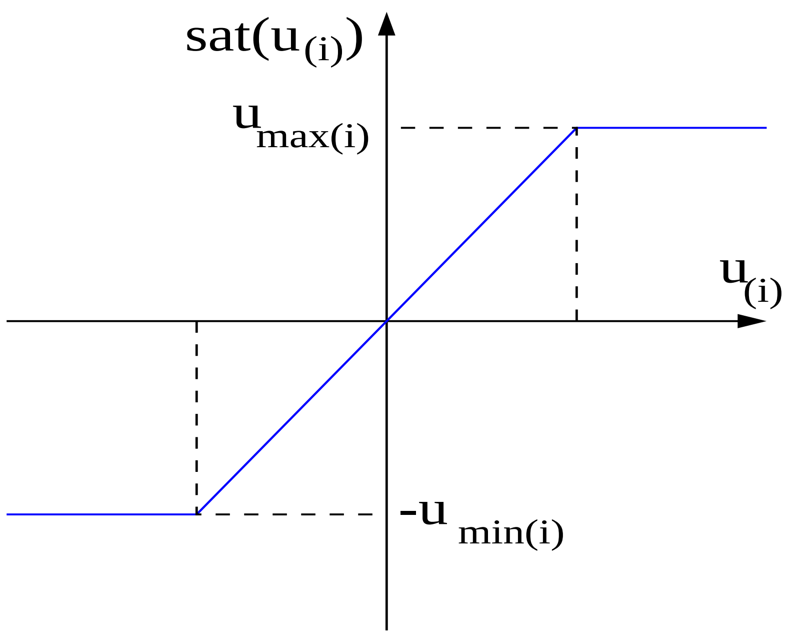
▷ Saturated actuator with higher order dynamics

3. Actuator saturation (2)

Saturation in amplitude (position)

☞ The amplitude saturation, possibly dissymmetrical, limits the amplitude of the input signal

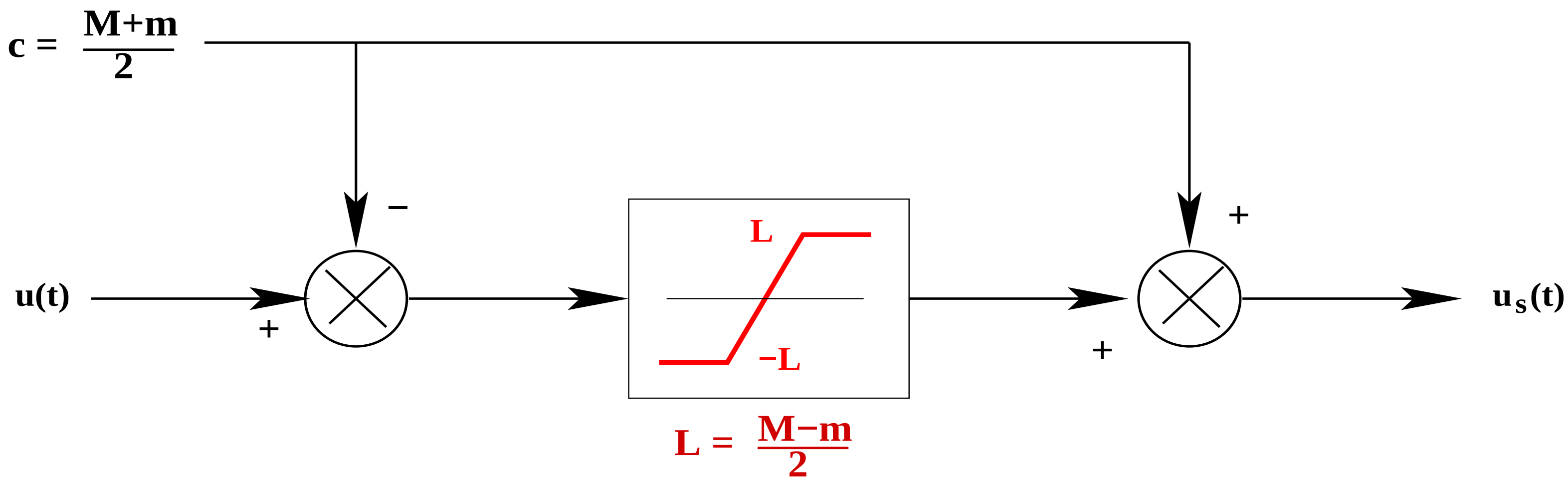




▷ The function saturation is defined by: $sat(u_{(i)}) =$

$$\begin{cases} u_{max(i)} & \text{if} \quad u_{(i)} > u_{max(i)} \\ u_{(i)} & \text{if} \quad -u_{min(i)} \leq u_{(i)} \leq u_{max(i)} \\ -u_{min(i)} & \text{if} \quad u_{(i)} < -u_{min(i)} \end{cases}$$

☞ The symmetry can be recovered by adding a constant input with a sign − before the saturation block and with a sign + after the saturation block:

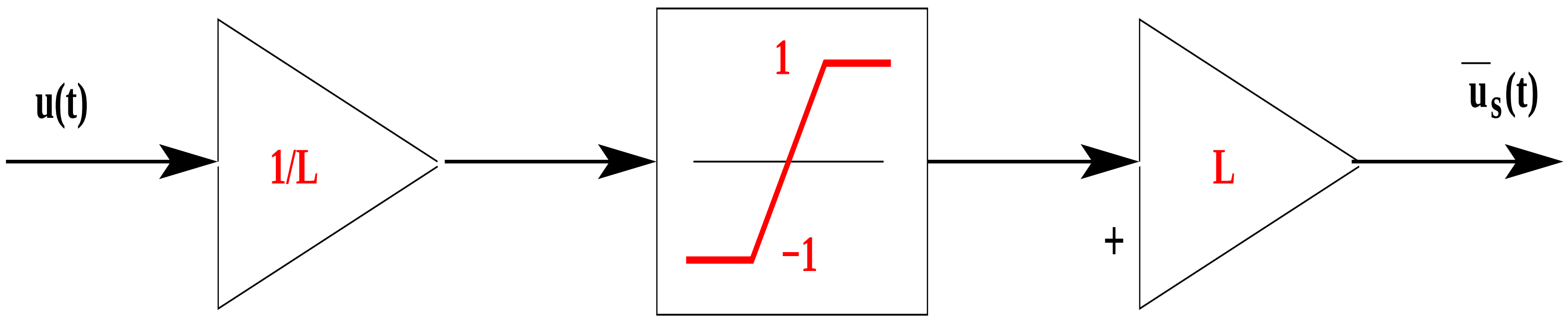


Remark 1 *In general, we will consider, without loss of generality,* $u_{max} = u_{min} = u_0$.

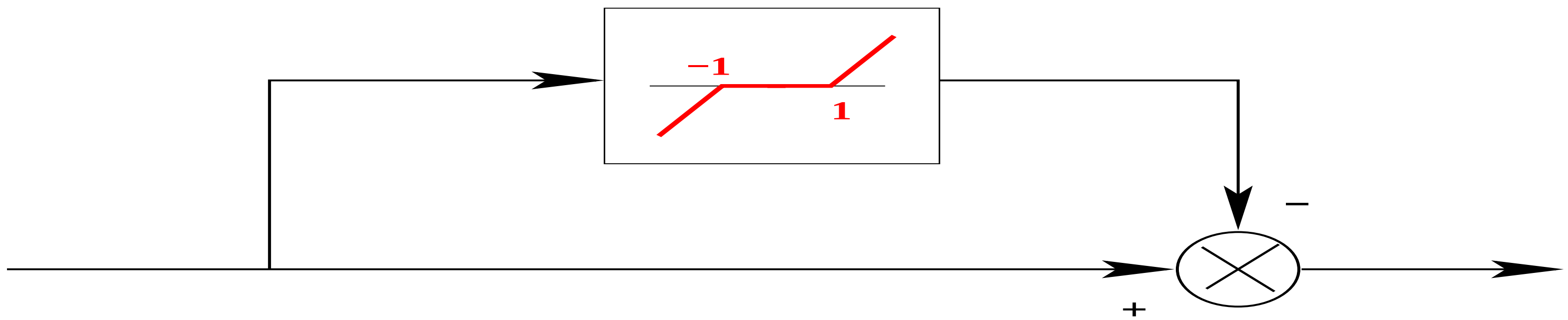
3. Actuator saturation (4)

Normalized saturation in amplitude (position)

☞ The symmetrical saturation can be normalized:



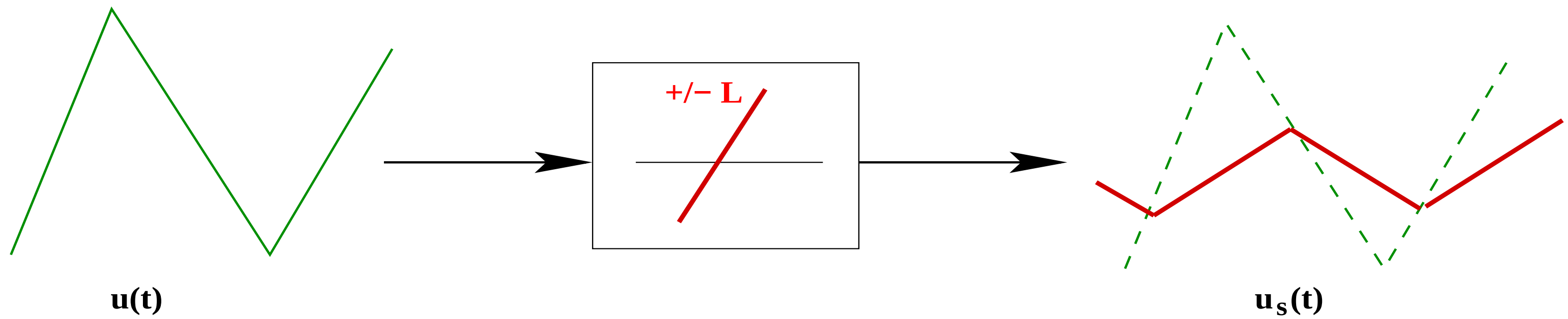
☞ The symmetrical saturation can be replaced by a dead-zone



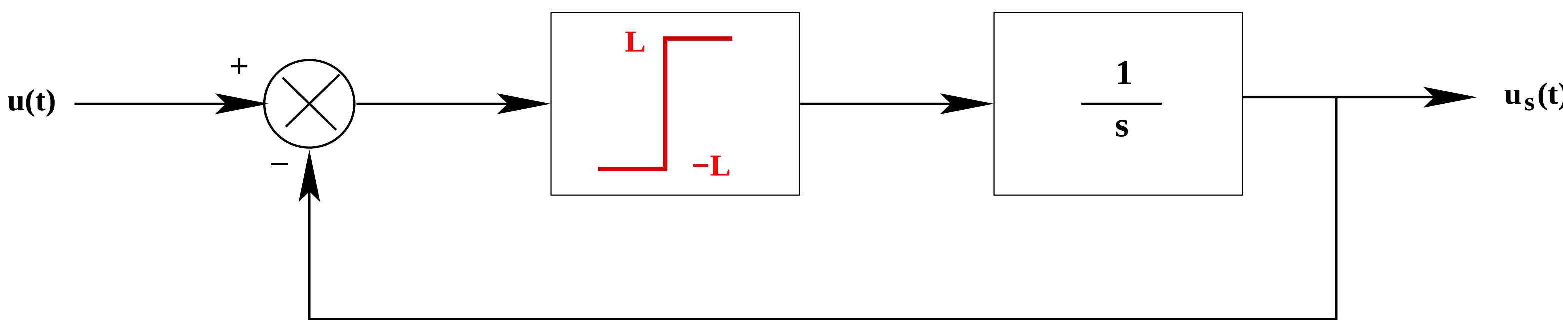
▷ The main advantage of this new nonlinearity is that it is null during its linear behavior $(sat(u) = u)$

Rate saturation

☞ The rate saturation, generally symmetrical, limits the speed of the signal



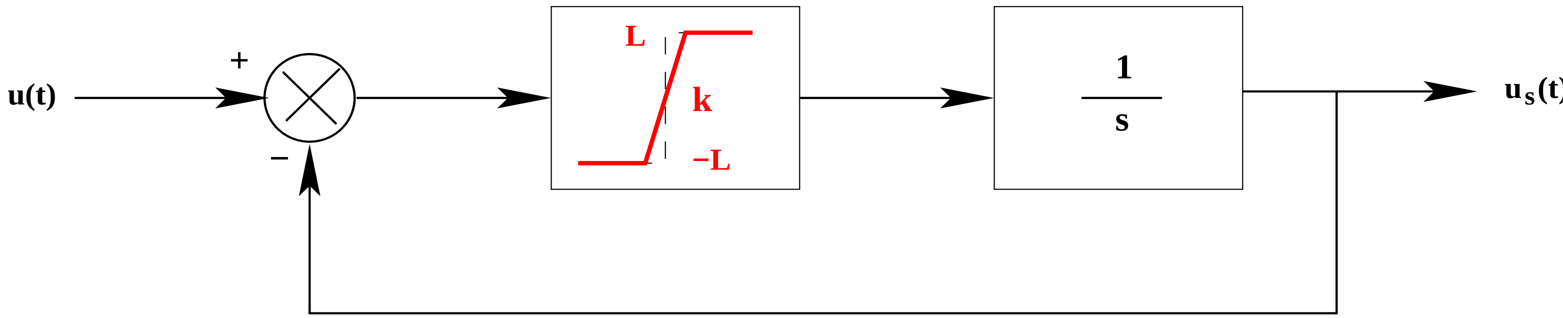
☞ This type saturation can be represented through an integrator closed with a relay:



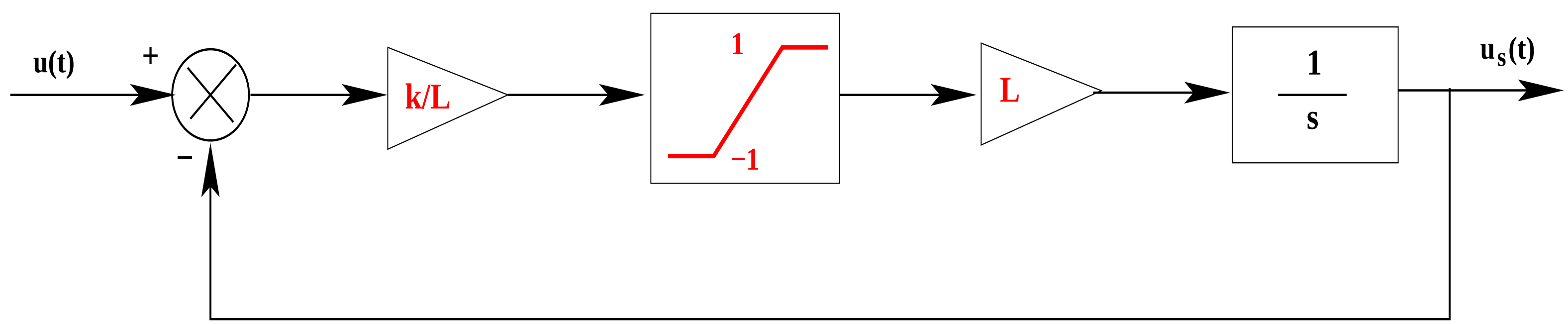
3. Actuator saturation (6)

Rate saturation

☞ Finally, by approximating the relay by a saturation with a high slope, ne can consider the following representation of the rate saturation:



☞ If needed, we can also normalized this saturation as follows:

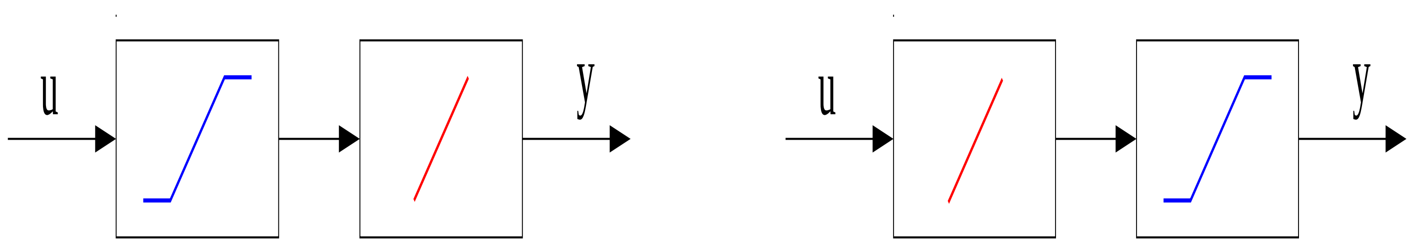


Constraints in amplitude and rate

☞ The control input u and its time-derivative $\dot{u}$ are limited in amplitude:

$$\mathcal{U}_0 = \{u \in \Re^m; -u_{0(i)} \leq u_{(i)} \leq u_{0(i)}, i = 1, ..., m\}$$
$$\mathcal{U}_1 = \{u \in \Re^m; -u_{1(i)} \leq \dot{u}_{(i)} \leq u_{1(i)}, i = 1, ..., m\}$$

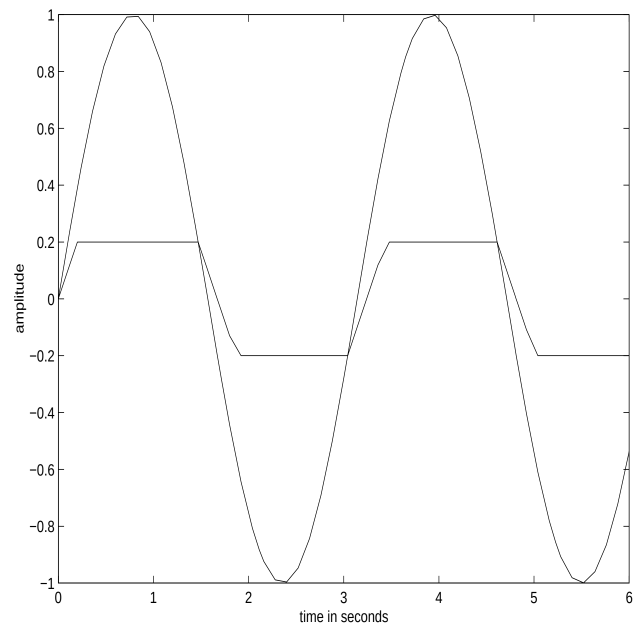
☞ One main point deals with the place of the rate limiter with respect to the position limiter



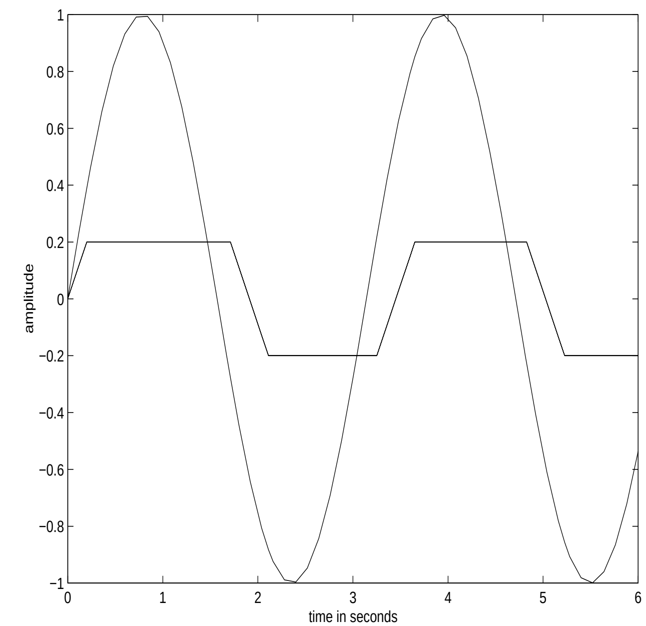
(a) (b)

3. Actuator saturation (8)

▷ Consider a sinusoidal signal with amplitude 1 and frequency $2rd/s$ with $u_0 = 0.2$ and $u_1 = 1$.



(a)



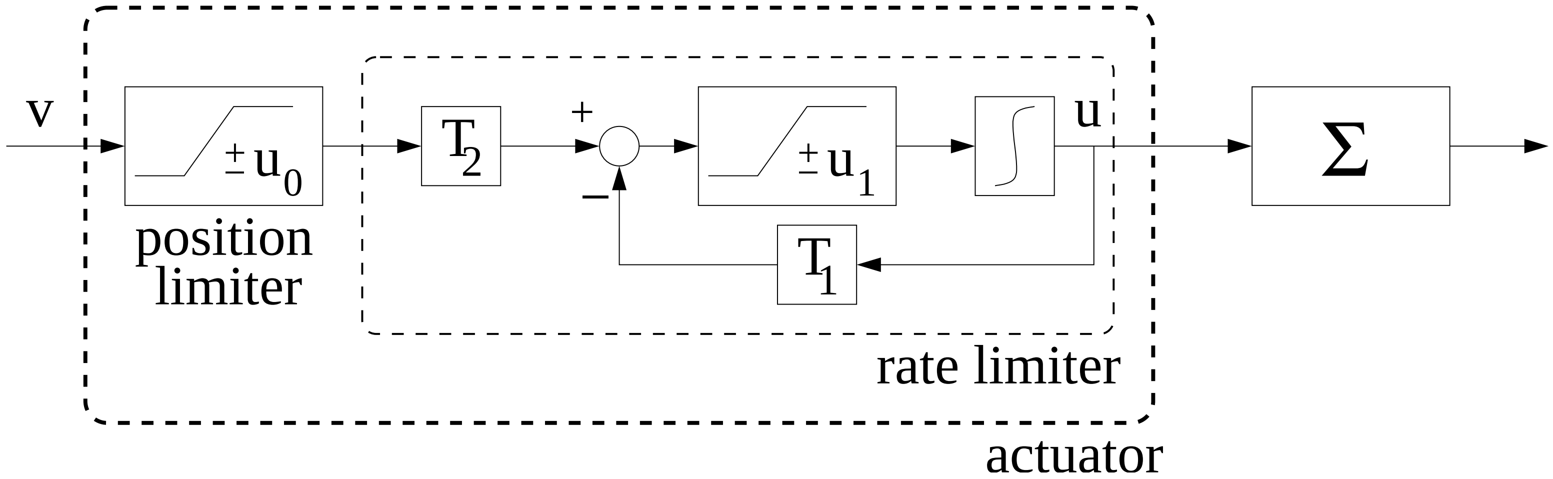
(b)

▷ In general, this is the first case which is considered in physical applications (aircrafts).

▷ However, the second case may also appear in some special cases (see example on combat aircraft in chapter VI)

3. Actuator saturation (9)

☞ For example:



☞ The system may be described by:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ \dot{u}(t) &= sat_{u_1}(T_1 u(t) + T_2 sat_{u_0}(v(t))) \end{aligned}$$

▷ The saturation $sat_{u_0}(v(t))$ corresponds to the position saturation

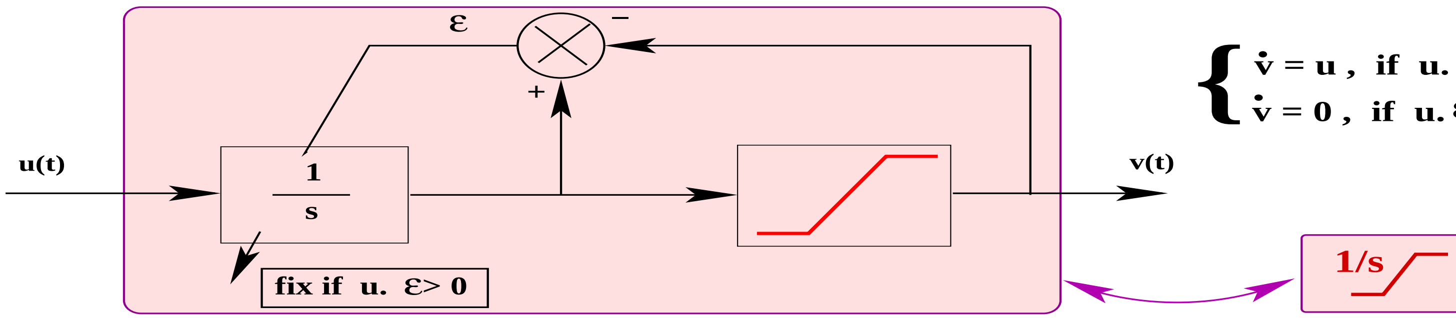
▷ The saturation $sat_{u_1}(T_1 u(t) + T_2 sat_{u_0}(v(t)))$ corresponds to the rate saturation

▷ Example: fighting aircraft

3. Actuator saturation (10)

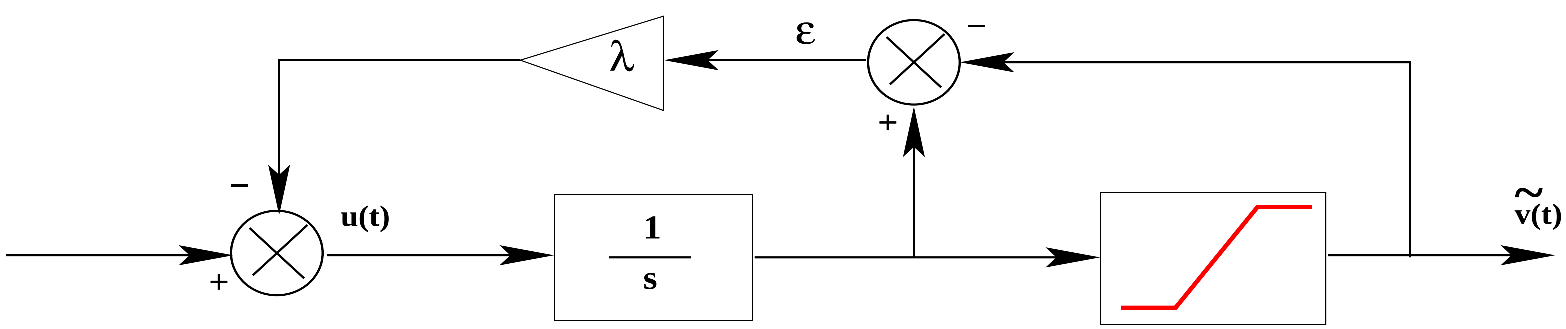
Limited integrator

☞ A limited integrator is a nonlinear operator, that can be defined as:



$$\begin{cases} \dot{v} = u\ , & \text{if } u.\varepsilon <= 0 \\ \dot{v} = 0\ , & \text{if } u.\varepsilon > 0 \end{cases}$$

☞ Such an operator can be approximated as follows:



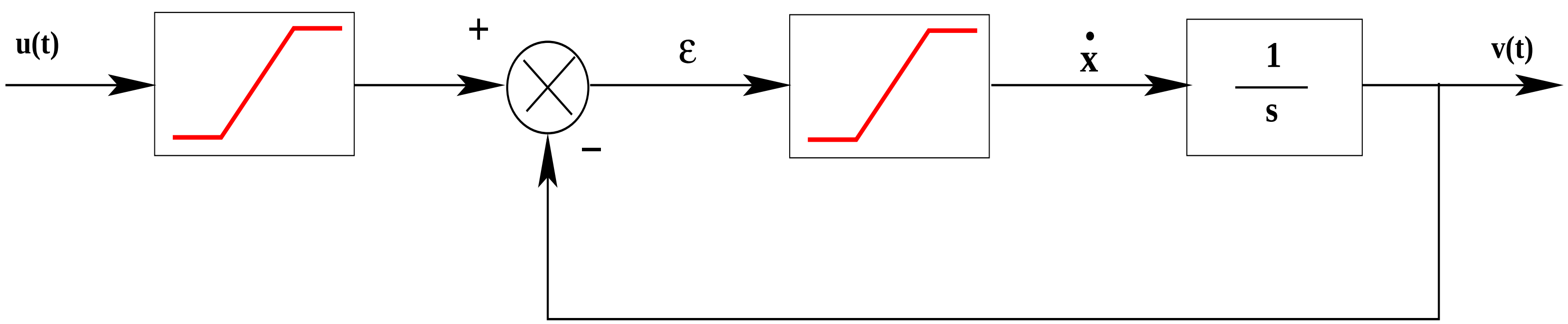
▷ Indeed, one gets: $\lim_{\lambda \to \infty} |v - \tilde{v}| = 0$

3. Actuator saturation (11)

First order saturated actuator

☞ Let us consider now different types of saturated actuator of the first order.

☞ The first model can be depicted as follows:



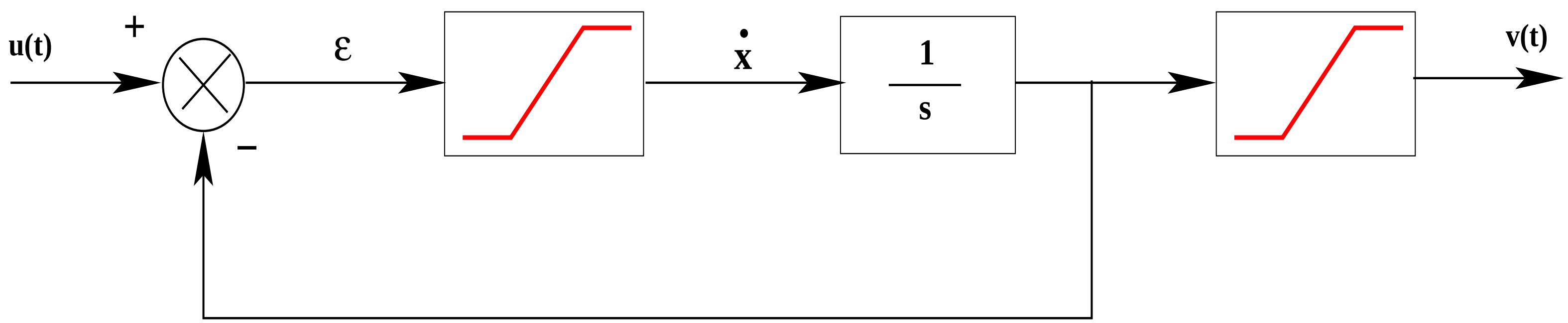
☞ The equations of this actuator are described by:

$$\begin{aligned} \dot{x} &= sat(sat(u) - x) \\ v &= x \end{aligned}$$

▷ Presence of nested saturations.

3. Actuator saturation (12)

☞ The second model of interest can be depicted as follows:

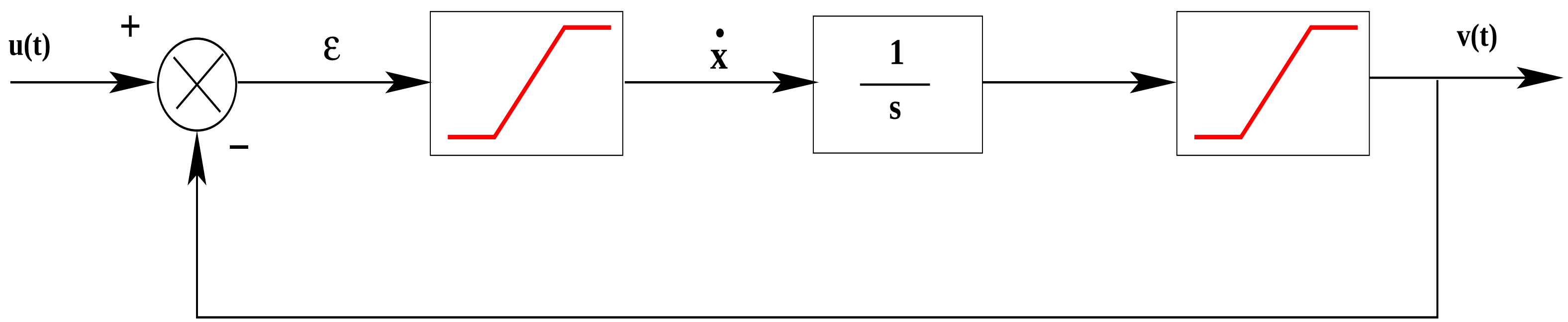


☞ The equations of this actuator are described by:

$$\begin{aligned} \dot{x} &= sat(u - x) \\ v &= sat(x) \end{aligned}$$

▷ No nested saturations.

☞ The third model of interest can be depicted as follows:



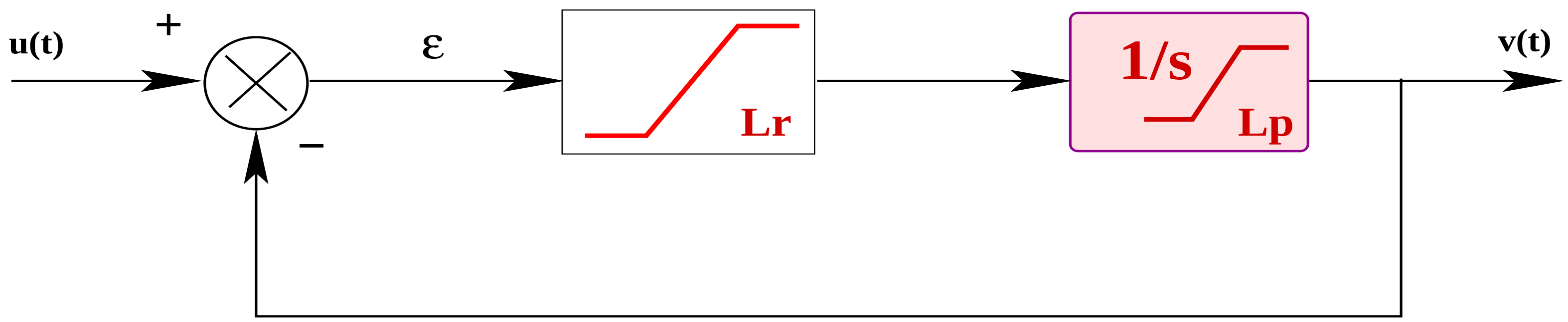
☞ The equations of this actuator are described by:

$$\begin{aligned} \dot{x} &= sat(u - sat(x)) \\ v &= x \end{aligned}$$

▷ Presence of nested saturations.

▷ Here, the activity of the rate saturation is decreasing.

☞ A more physical model for the actuator of the first order can be depicted as follows:

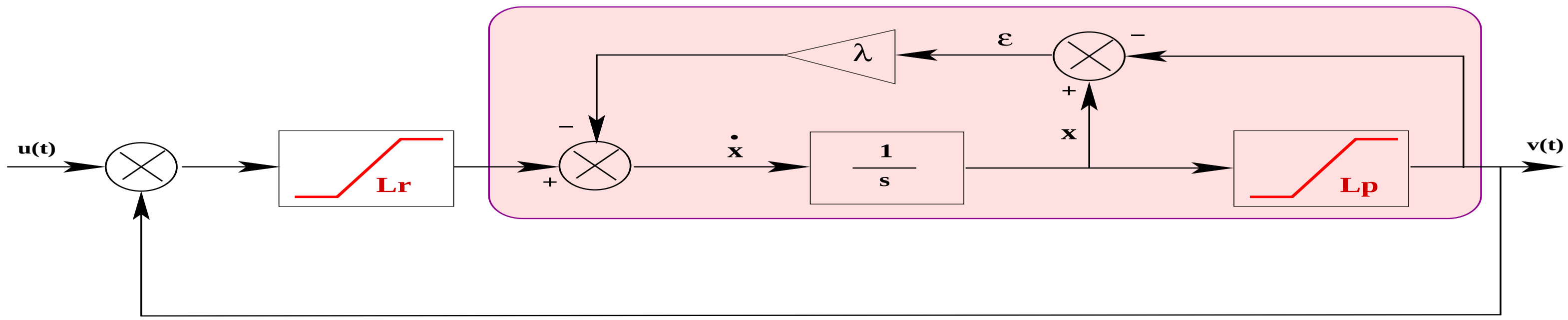


☞ The equations of this actuator are described by:

$$\begin{aligned} \dot{v} &= sat_{L_r}(u - v) \quad \text{if} \quad |x| \leq L_p \\ \dot{v} &= 0 \quad \text{if} \quad sat_{L_r}(u - v)(v - sat_{L_p}(v)) > 0 \\ v &= x \end{aligned}$$

3. Actuator saturation (15)

☞ The previous actuator including a limited integrator can be approximated as follows:



☞ The equations of this actuator are described by:

$$\dot{x} = sat_{L_r}(u - sat_{L_p}(x)) - \lambda(x - sat_{L_p}(x))$$
$$v = sat_{L_p}(x)$$

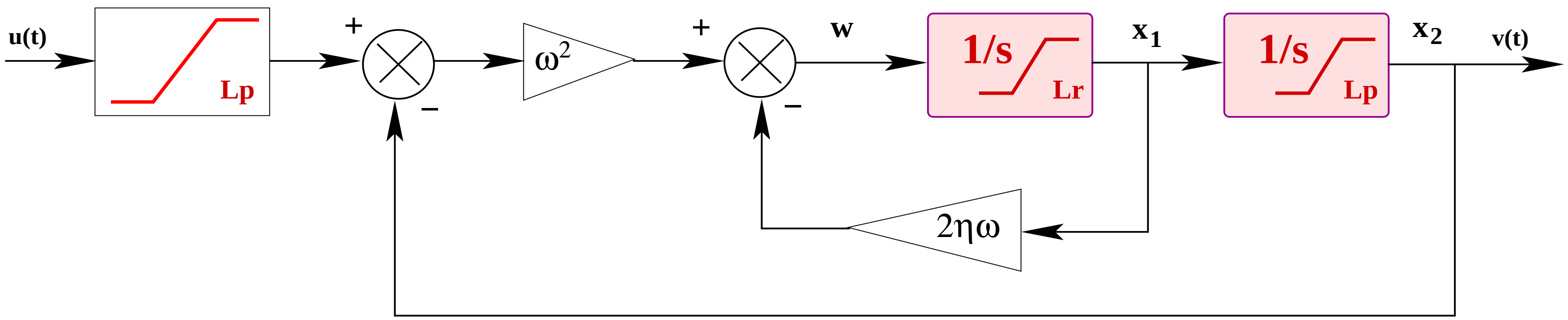
▷ Presence of classical saturations.

▷ Presence of nested saturations.

3. Actuator saturation (16)

Second order saturated actuator

☞ In many applications, the actuator considered is of the following type:



☞ The equations of this actuator are described by:

$$
\begin{aligned}
\dot{x}_1 &= w && \text{if} \quad |x_1| \leq L_r \\
\dot{x}_1 &= 0 && \text{if} \quad w(x_1 - sat_{L_r}(x_1)) > 0 \\
\dot{x}_2 &= x_1 && \text{if} \quad |x_2| \leq L_p \\
w &= \omega^2(sat_{L_p}(u) - x_2) - 2\eta\omega x_1 \\
v &= x_2
\end{aligned}
$$

3. Actuator saturation (17)

Second order saturated actuator

☞ This previous actuator including a limited integrator can be approximated by using the same way as previously.

- ▷ Presence of classical saturations.
- ▷ No nested saturations.

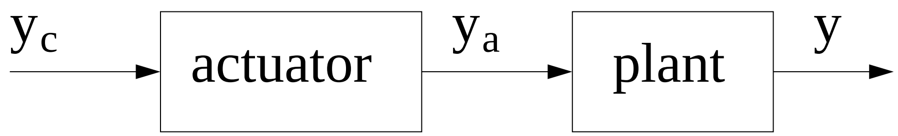
☞ In this model the input saturation may be redundant with the saturation due to the limited integrator in the output.

- ▷ In this case, one can remove it and therefore one can recover a simplified modelling with only two saturations.

3. Actuator saturation (18)

Constraint in higher dynamics

☞ We consider a class of nonlinear systems obtained by cascading linear systems with actuator containing some nonlinearities of saturation type



☞ The actuator reads:

$$\begin{cases} \dot{x}_a(t) = A_a x_a(t) + B_{a0} sat_{u_0}(C_a x_a(t)) + \sum_{j=1}^{q-1} B_{aj} sat_{u_j}\left(sat_{u_0}(C_a x_a(t))^{(j)}\right) + B_{aq} y_c(t) \\ y_a(t) = sat_{u_0}(C_a x_a(t)) \end{cases}$$

with $x_a = \begin{bmatrix} u' & \dot{u}' & \cdots & u^{(q-1)\prime} \end{bmatrix}' \in \Re^{mq}$, $A_a = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \in \Re^{mq \times mq}$,

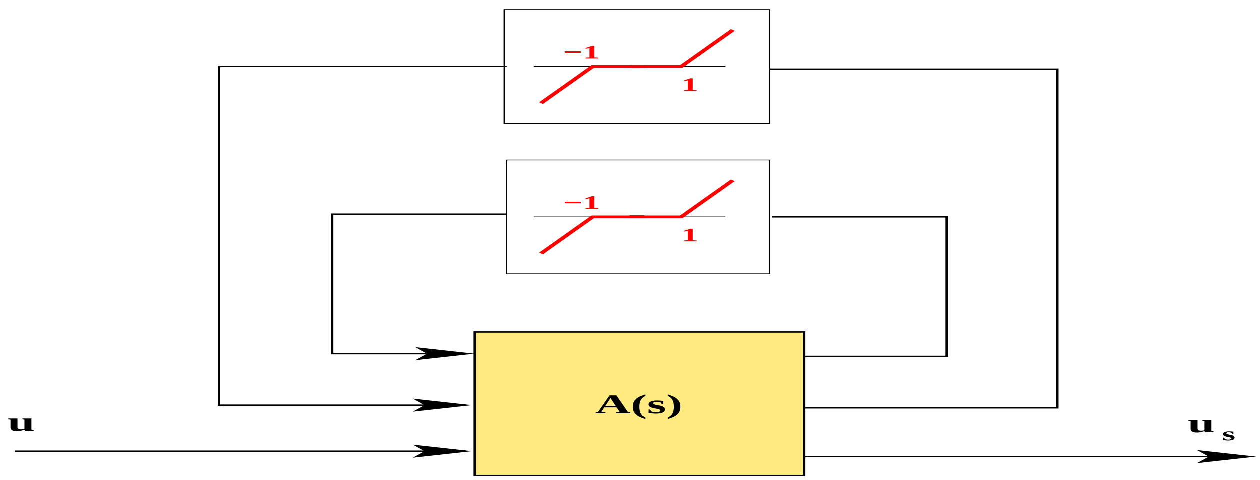
$B_{aj} = \begin{bmatrix} 0 & T_j' \end{bmatrix}' \in \Re^{mq \times m}$, $C_a = \begin{bmatrix} 1 & 0 \end{bmatrix} \in \Re^{m \times mq}$

☞ Example: launcher

Unified representation

☞ From the previous discussion, one can redefine a large class of nonlinear actuator as actuator with amplitude saturations.

▷ Possibly multiple and nested saturation



3. Actuator saturation (20)

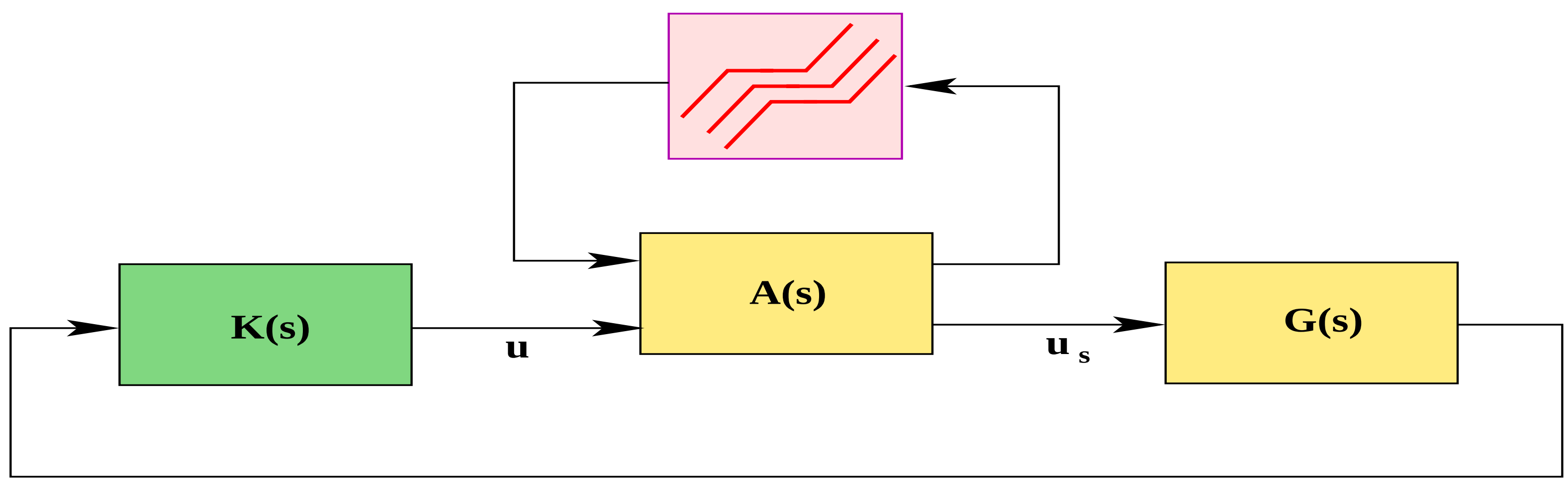
☞ The conversion of the saturation terms into dead-zones guarantees the stability of the linear behavior of the actuator

- ▷ without saturation = nominal behavior= the output of the dead-zone is null

☞ It is possible to take into account the presence of nested saturation

- ▷ Presence of non-zero elements in the direct transmission of the linear system of the actuator.

☞ The closed-loop system is then described as follows:



4. Problem Statement (1)

☞ Consider the closed-loop system (saturation in amplitude):

$$\dot{x}(t) = Ax(t) + Bsat(Kx(t)) \qquad (1)$$

☞ The nonlinear system (1) is locally linear.

▷ Linearity region:

$$S(K, u_0) = \{x \in \Re^n; -u_{0(i)} \leq K_{(i)}x \leq u_{0(i)}, i = 1, ..., m\}$$

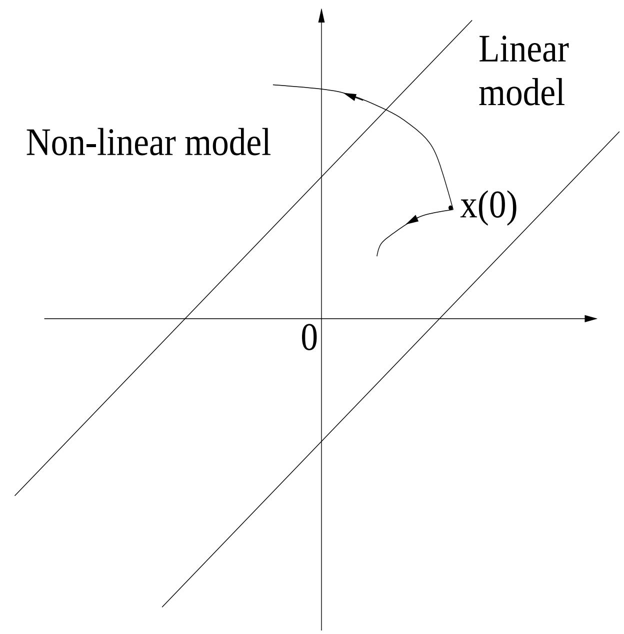
▷ If $x(t) \in S(K, u_0)$:

$$\dot{x}(t) = (A + BK)x(t)$$

4. Problem Statement (2)

☞ Remark: If $x(0) \in S(K, u_0)$ then $x(t)$ does not necessarily belong to $S(K, u_0)$, $\forall t \geq 0$, even if $A + BK$ is Hurwitz.

▷ $\forall\ x(0) \in S(K, u_0)$, $x(k, x(0)) \rightarrow 0$ when $t \rightarrow +\infty$?



▷ It is necessary to take into account the nonlinear behavior of the closed-loop system (1).

4. Problem Statement (3)

Analysis Problem

- Given a stabilizing controller, determine regions of the state space in which the closed-loop system trajectories converge to the origin in presence of saturations.
- Related problems
  - Determination of domain of guaranteed performance
  - Determination of domain of admissible set-points
- Similar problems arise with more complex actuators (saturation in dynamics)

4. Problem Statement (4)

Synthesis Problem

☞ Given a set of admissible initial conditions $\mathcal{I}_0$, design a stabilizing controller ensuring that for all initial condition in $\mathcal{I}_0$, the trajectory converges asymptotically to the origin with a control limited in amplitude.

☞ Related problems

- Impose a particular structure to the controller
- Take into account performance requirements
- Predictive control technique

4. Problem Statement (5)

Anti-windup Problem

☞ Given a stabilizing controller (linearly designed), add some loops active only when saturations effectively occur, i.e. the nominal controller is modified only when saturation occurs. The objectives may be:

- to enlarge the basin of attraction
- to improve the performance (in particular nonlinear performance when saturation occurs).

☞ Related problems

- Impose a set of initial conditions
- Impose a particular structure to the controller
- Improve performance/robustness