Constructive LMI Approaches for Anti-Windup Compensator Design for Systems Subject to Saturation

Sophie Tarbouriech
LAAS-CNRS, Toulouse, France

Part IV - Introduction on anti-windup strategy

Outline of part IV

1. Introduction
2. A bit of history
3. Problem statement
4. Necessary tools

1. Introduction (1)

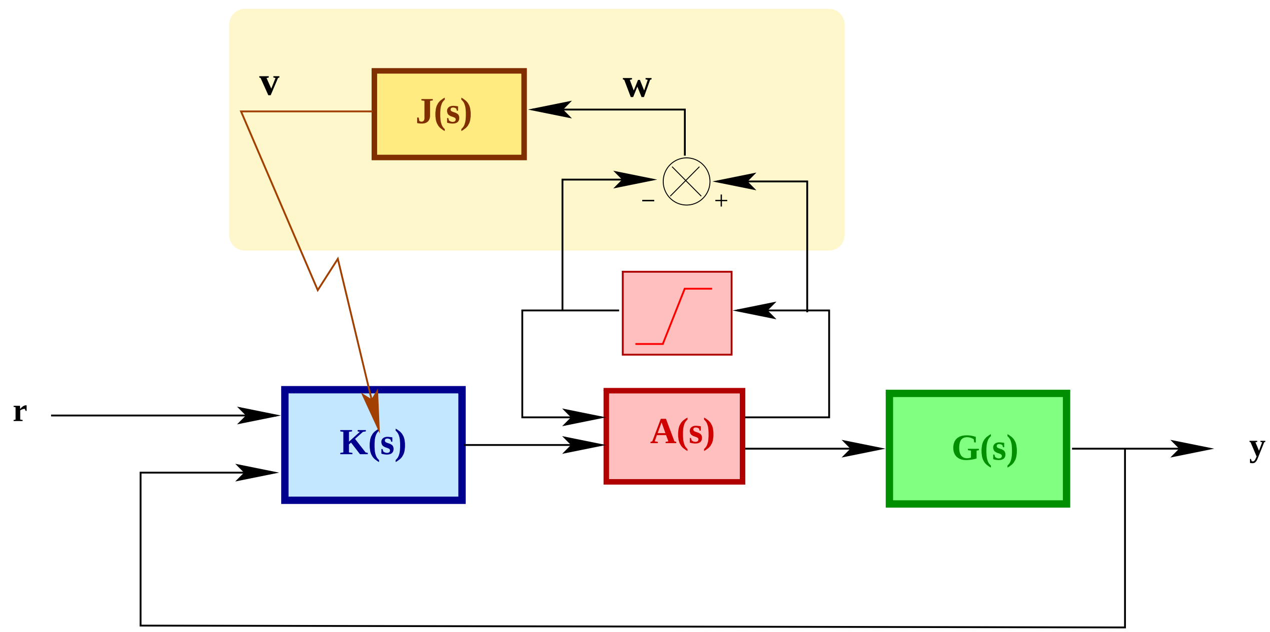
☞ The anti-windup techniques represent an alternative way to analysis and synthesis methods

☞ The anti-windup techniques consist in considering the effects of the nonlinearities like saturations a posteriori

- ▷ Idea: The basic idea underlying anti-windup designs for linear systems with saturating actuators is to introduce control modifications in order to recover, as much as possible, the performance induced by a previous design carried out on the basis of the unsaturated system.

1. Introduction (2)

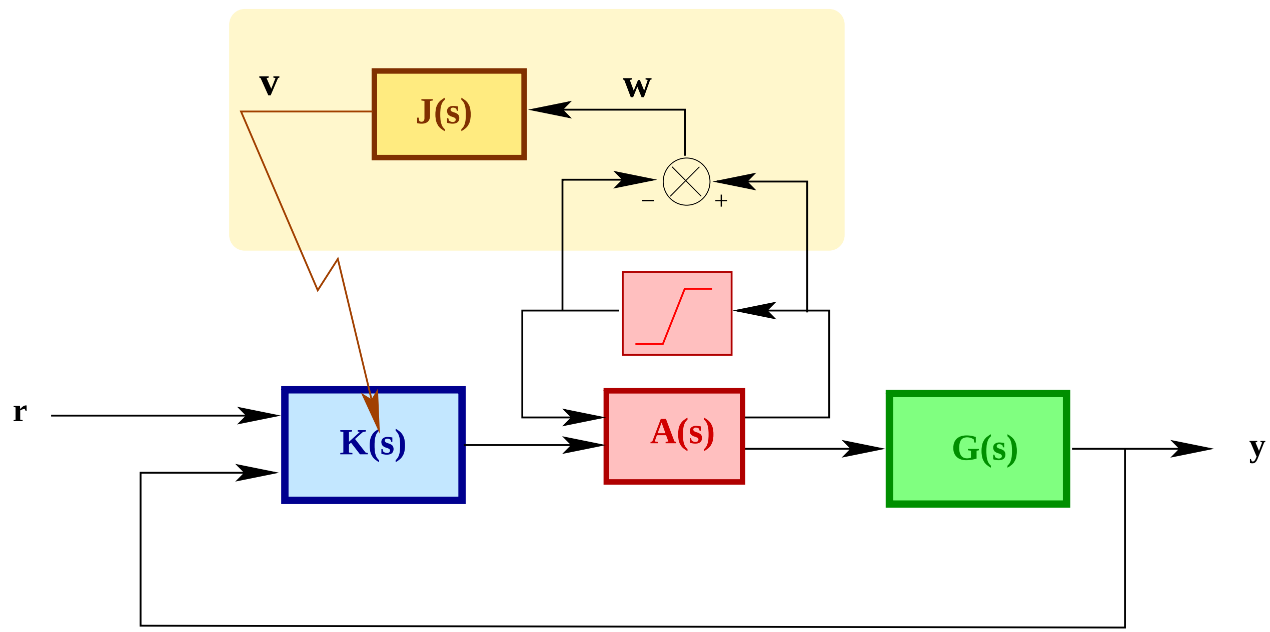
☞ Thus, the general principle of the anti-windup scheme can be depicted in the following figure.



▷ In this Figure, the (unconstrained) signal produced by the controller is compared to that which is actually fed into the plant (the constrained signal).

▷ This difference is then used to adjust the control strategy in a manner conducive to stability and performance preservation.

1. Introduction (3)



☞ In the absence of saturation ($w = 0$) the anti-windup loop is not more active in a progressive way or instantaneously depending on the dynamics of $J(s)$

▷ Therefore the closed-loop system retrieves a linear (nominal) behavior

Remarks

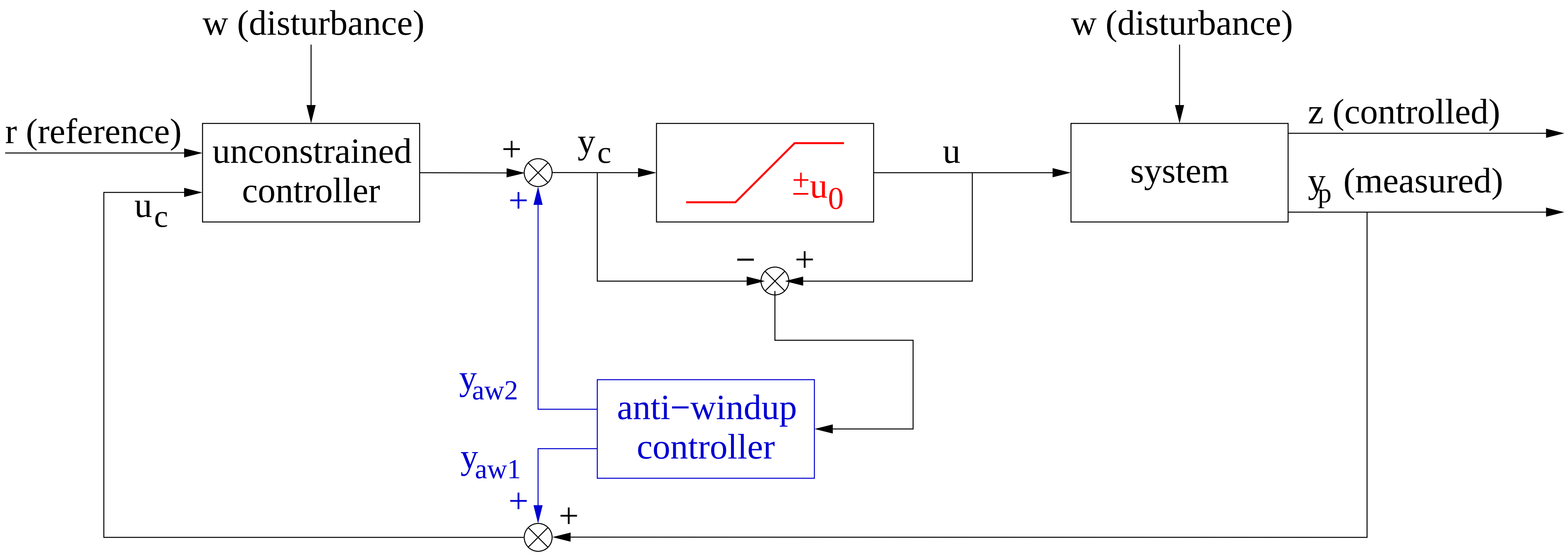
☞ The knowledge of both signals (i.e., the output produced by the linear controller and the saturated version of this) are assumed.

- ▷ In some case, it is not realistic because only the saturated version of the signal is known.
- ▷ This may be problematic for the two-stage (anti-windup) approach, and hence an observer can be used to overcome this difficulty (Tarbouriech et al., 2007).

1. Introduction (5)

☞ The anti-windup compensator itself emits two signals, one which is fed directly into the constrained control signal and one which may be used to drive the controller state equation directly.

☞ Virtually all AW compensators which are present in the literature can be represented in this form and thus the AW compensators discussed hereon will be assumed to have this form.



1. Introduction (6)

☞ The anti-windup techniques are used to mituigate the windup effects:

▷ Successfully used in PID controllers

▷ Stability analysis [Teel, Morari & co-authors]

- ∗ external stability
- ∗ global stability results ☞ application only to open-loop stable systems

1. Introduction (7)

Experience with application of anti-windup strategies

☞ Aeronautical domains: GARTEUR projects (AG15 and AG17),

- ▷ Combat aircraft: PIO avoidance (University of Leicester, SAAB, DLR, DASSAULT, NLR, FOI, ...)
- ▷ Civil aircraft: Robust landing (ONERA Toulouse, Airbus)
- ▷ Civil aircraft: still in progress with Airbus on the pumping effects (rate saturation)

☞ Spatial domain:

- ▷ PIROLA project: robust flying launchers type Ariane 5+ (ONERA Toulouse, CNES, ESA)
- ▷ Control of satellite attitude/closed-loop control of formation flighying of satellites (in progress) (CNES, Thales)

☞ Mechanical domain: flexible beam with piezo-electrical cells (actuator and sensor) (ENS Cachan, France and University of Campinas, Brazil)

1. Introduction (8)

Potential spin-off

☞ Due to several discussions with practical engineers:

- ▷ Reduction of validation costs of control laws
- ▷ Better use of actuator (and/or sensors) capacity
- ▷ Possibility to interact with the engineer at the beginning:
  - for example to choose the actuators and/or sensors (allows to reduce their consummation, size, mass...)

2. A bit of history (1)

☞ The study of anti-windup probably began in industry where practitioners noticed performance degradation in systems where saturation occurred.

☞ Many authors note that the term windup was a phenomenon associated with saturation in control systems with integral controllers and alluded to the build up of charge on the integrator's capacitor during saturation.

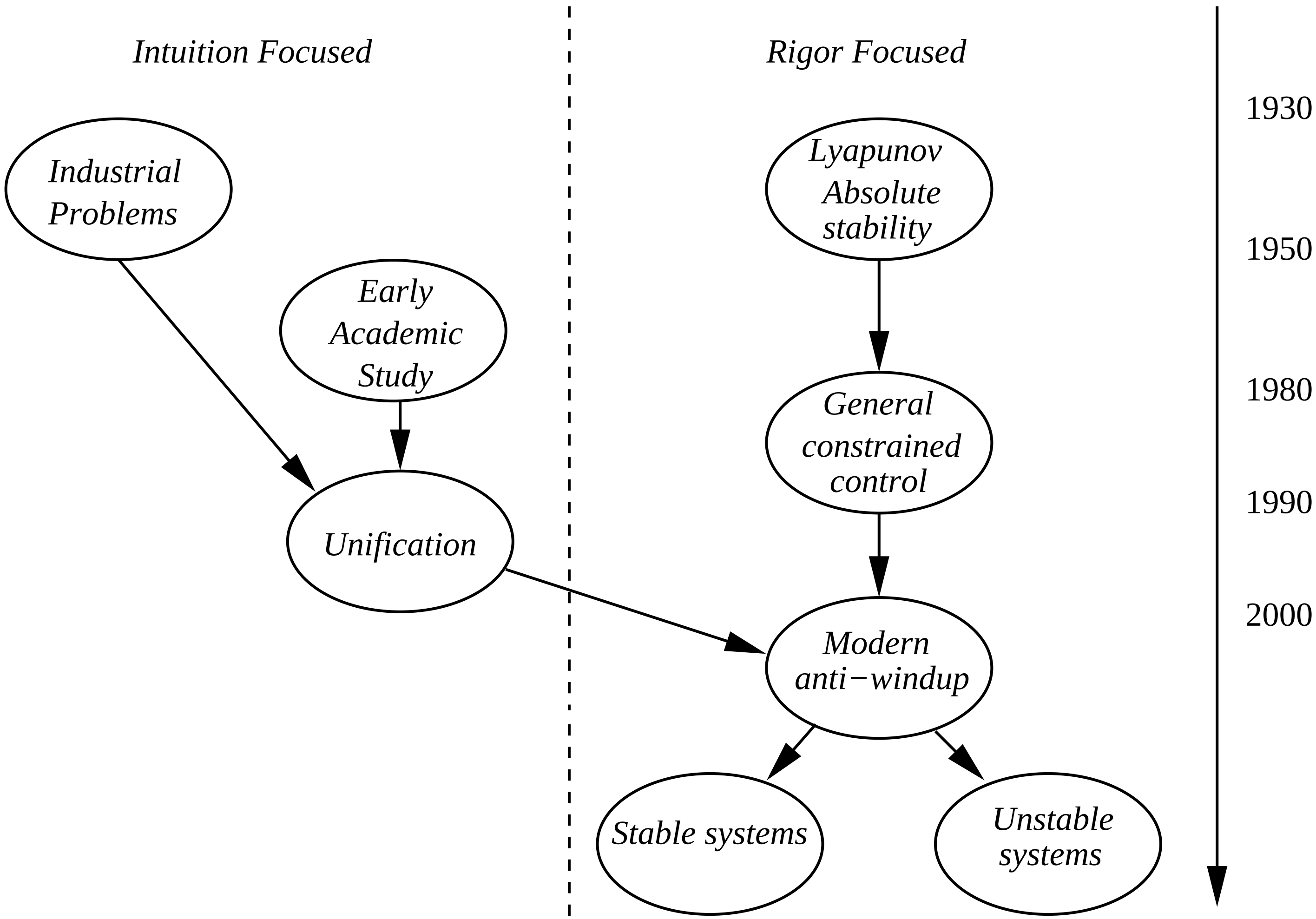
- ▷ The subsequent dissipation of this charge would then cause long settling times and excessive overshoot, thereby degrading the system's performance.
- ▷ Modifications to the controller which avoided this charge build-up were often termed anti-windup modifications and hence the term anti-windup was born. Since then however, the term "anti-windup" has evolved and it now means the generic two-step procedure for controller design which was described earlier.

2. A bit of history (2)

☞ It is hard to pin-point exactly the origins of anti-windup compensation due to lack of published work on the subject in the early years of control.

- ▷ Teel and co-authors, in their ACC03 Workshop T-1: Modern Anti-Windup Synthesis, trace the discovery back to the 1930s and in particular cites the paper of Lozier as being one of the key early academic papers in identifying the windup problem.
- ▷ Teel then describes the evolution of anti-windup compensation on a time-line spanning the last 80 years or so.

☞ In a similar manner, we view the development of anti-windup as that depicted in the following figure where several stages of anti-windup development are identified as described in the following picture.



2. A bit of history (4)

☞ We can partition the history of the studies on anti-windup technique into four main stages, namely:

- ▷ The first stages in anti-windup development
- ▷ Early academic study
- ▷ Constrained input control
- ▷ Modern anti-windup

2. A bit of history (5)

☞ The first stages in anti-windup development.

- ▷ Around 1940-1950.
- ▷ The practitioners were aware of the problems which saturation caused from the early days of control and they adopted ad hoc solutions to the problem. (Paper in 1940 on absolute stability).
- ▷ This awareness also manifests itself in some of the early work on absolute stability which was published from the 1940's onwards, and where formal treatment of the saturation problem seems to have begun.
- ▷ Note however, there is a separation from the AW problem and the work on absolute stability, which was more general than just anti-windup, but arguably less practical.

2. A bit of history (6)

☞ Early academic study.

- ▷ Some years after this, academics began to study the problem of saturation in control systems. Lozier was able to explain saturation problems in the integrator portion of PI controllers.
- ▷ Fertik and Ross proposed perhaps the first properly documented anti-windup methodology. This was also accompanied by various work on intelligent integrators (Krikelis) and later the celebrated conditioning technique of Hanus et al.
- ▷ Ad hoc techniques which seemed to overcome certain practical problems, but provided no guarantees.
- ▷ Several researchers (Aström, Campo, Kothare) provided a unification of many anti-windup schemes related to modern robust control ideas.
- ▷ This unification was later continued Postlethwaiste where a state-space interpretation was given in "generic" form for many compensators.

2. A bit of history (7)

☞ Constrained input control.

- ▷ While anti-windup was being developed from a very applied perspective, new results were beginning to be developed in the more general constrained input control field, particularly from the mid-1980s onwards.
- ▷ Such work was focused on using Lyapunov's second method to develop *one step* controllers which could guarantee stability of the *nonlinear* closed-loop systems.
- ▷ Although this work was not anti-windup *per se*, this line of work provided important theoretical results which anti-windup would later benefit from. Some useful references on this subject are papers from Lin, Tyan and Bernstein, Saberi, Suarez, ...

2. A bit of history (8)

☞ Modern anti-windup.

- ▷ It is difficult to define exactly what constitutes a modern anti-windup technique but we view it as a systematic methods which can be used to design an AW compensator providing rigorous guarantees of stability and or performance.
- ▷ Most of these techniques were developed from the late 1990s (Miyamoto, Teel, Turner, Gomes da Silva Jr., Tarbouriech, Zaccarian) in the contexts of global stability or local stability and performance.
- ▷ Note that there was also a split in the development of anti-windup controllers. Some researchers chose to investigate the problem of enforcing global stability and performance properties for anti-windup compensators (see the papers from Weston, Grimm, Teel, ...), while others began to look at local stability and performance properties, which was necessary for unstable plants (see the papers from Gomes da Silva Jr., Tarbouriech, Lin).

2. A bit of history (9)

☞ Note that the above description of the development of anti-windup is only a broad approximation and does not include every nuance in the subject's history.

☞ Nevertheless, it does provide a useful overview of how anti-windup developed into its present form.

- ▷ It is important to note that it has developed from the problem-specific ad hoc solutions aimed particularly at PID controllers, to sophisticated synthesis methods which populate today's anti-windup literature.
- ▷ In particular, major improvements in this field have have been achieved in the last decade, as can be observed in papers from Barbu, Kothare, Teel, Miyamoto, ...

2. A bit of history (10)

☞ Many LMI-based approaches now exist to adjust the anti-windup gains in a systematic way: see, for example, (Turner and Zaccarian, 2006) for a quick overview).

▷ Most often, these are based on the optimization of either a stability domain or a nonlinear $\mathcal{L}_2$-induced performance level.

▷ More recently, based on the LFT/LPV framework, extended anti-windup schemes were proposed.

– In these contributions, the saturations are viewed as sector nonlinearities and anti-windup controller design is recast into a convex optimization problem under LMI constraints.

▷ Following a similar path, alternative techniques using less conservative representations of the saturation nonlinearities, yet with sector non-linearities, are proposed in papers due to Teel, Zaccarian, Biannic, Tarbouriech, Gomes da Silva Jr., ...

2. A bit of history (11)

☞ Moreover, during the last phases previously evoked, we can point out several papers dealing with practical experiments with application of anti-windup strategies in various fields like:

- ▷ aeronautical or spatial domains (papers in AST 2006, CCA 2006, ROCOND 2006, ACC 2007, Springer 2007),
- ▷ mechanical domains (papers in CEP 2006, IFAC 2005),
- ▷ open water channels (CEP 2007), nuclear fusion (Automatica 2005),
- ▷ telecommunication networks (CEP 2003) and hard disk drive control (ASME 2004).

☞ The aims:

- ▷ Study of conditions for guaranteing stability (in local and global contexts) of anti-windup schemes
- ▷ To take into account the amplitude limitations on both the input and the output of the system (limited actuator and sensor)
- ▷ In global context:
  - ∗ open-loop properties of stability are required
  - ∗ when the open-loop system is unstable : local stability.
- ▷ In local context: design the anti-windup gain in order to enlarge the basin of attraction of the closed-loop system
- ▷ Propose numerically efficient procedures to compute the anti-windup gains

☞ Different types of anti-windup compensators

3. Problem statement (2)

☞ For developing our approach, we need to consider

- ▷ A state-space description of the system and controller
- ▷ A description of the amplitude saturation on actuator and sensor blocks
- ▷ Some anti-windup techniques
  → What are available measures for building the anti-windup loops?

☞ Tools:

- ▷ A generalized sector condition is used
- ▷ Quadratic and Lure Lyapunov functions
- ▷ Convex optimization

Generalities

☞ Pre and post multiplication. For any symmetric matrix $\mathcal{M}$ and all non-singular matrix N (i.e., $det(N) \neq 0$):

$$\mathcal{M} < 0 \text{ is equivalent to } N'\mathcal{M}N < 0$$

☞ Schur's complement. One gets the following equivalence:

$$\begin{bmatrix} R & N' \\ N & S \end{bmatrix} > 0 \text{ is equivalent to } S > 0,\; R - N'S^{-1}N > 0$$

☞ S-procedure. There exists $x \neq 0$ such that both $x'\mathcal{M}_1 x \leq 0$ and $x'\mathcal{M}_2 x \leq 0$ hold if there exists $\lambda \geq 0$ such that $\mathcal{M}_1 - \lambda\mathcal{M}_2 \leq 0$.

Lure function

☞ Consider a Lure Lyapunov function given by:

$$V(x) = x'Px - 2\sum_{i=1}^{m} \int_0^{K_{(i)}x} \psi(\sigma_{(i)}) N_{(i,i)} d\sigma_{(i)}$$

where $P = P' > 0$ and N is a diagonal positive matrix.

☞ **Fact 1** *The function $V(x)$ above defined is positive definite $\forall x \neq 0$.*

▷ To prove this fact, one considers the definition of $\psi(\sigma_{(i)})$:

$$\psi(\sigma_{(i)}) = \begin{cases} u_{0(i)} - \sigma_{(i)} < 0 & \text{if} \quad \sigma_{(i)} > u_{0(i)} \\ 0 & \text{if} \quad -u_{0(i)} \leq \sigma_{(i)} \leq u_{0(i)} \\ -u_{0(i)} - \sigma_{(i)} > 0 & \text{if} \quad \sigma_{(i)} < -u_{0(i)} \end{cases} \quad , \quad \forall i = 1, \cdots, m$$

▷ Thus, one computes the integral term in each case.

4. Necessary tools (3)

▷ For example, in the first case ($\psi(\sigma_{(i)}) = u_{0(i)} - \sigma_{(i)}$), one gets:

$$
\begin{aligned}
\mathcal{I}_1 &= -2\int_0^{K_{(i)}x} \psi(\sigma_{(i)})N_{(i,i)}d\sigma_{(i)} \\
&= -2\left[\int_0^{u_{0(i)}} \psi(\sigma_{(i)})N_{(i,i)}d\sigma_{(i)} + \int_{u_{0(i)}}^{K_{(i)}x} \psi(\sigma_{(i)})N_{(i,i)}d\sigma_{(i)}\right] \\
&= -2\int_{u_{0(i)}}^{K_{(i)}x} \psi(\sigma_{(i)})N_{(i,i)}d\sigma_{(i)} \\
&= -2\int_{u_{0(i)}}^{K_{(i)}x} (u_{0(i)} - \sigma_{(i)})N_{(i,i)}d\sigma_{(i)} \\
&= -2\left[\left(u_{0(i)}\sigma_{(i)} - \frac{\sigma_{(i)}^2}{2}\right)N_{(i,i)}\right]_{u_{0(i)}}^{K_{(i)}x} \\
&= -2\left[u_{0(i)}K_{(i)}x - \frac{(K_{(i)}x)^2}{2} - u_{0(i)}^2 + \frac{u_{0(i)}^2}{2}\right]N_{(i,i)} \\
&= \left(u_{0(i)} - K_{(i)}x\right)^2 N_{(i,i)} > 0
\end{aligned}
$$

▷ Similar conclusions follow in the other cases.

4. Necessary tools (4)

☞ **Fact 2** *The Lure function $V(x)$ above defined satisfies the following double inclusion:*

$$x'Px \leq V(x) \leq x'(P + K'NK)x \quad (1)$$

▷ To prove this fact, compute as in Fact 1 the integral in each case of $\psi(Kx)$.

▷ Consider the previous $\mathcal{I}_1$.

$$\mathcal{I}_1 = \left(u_{0(i)} - K_{(i)}x\right)' N_{(i,i)} \left(u_{0(i)} - K_{(i)}x\right) \leq \left(K_{(i)}x\right)' N_{(i,i)} \left(K_{(i)}x\right)$$

since $u_{0(i)} - K_{(i)}x < 0$ in this case.

▷ Therefore, for each case, one can verify that:

$$-2\sum_{i=1}^{m} \int_0^{K_{(i)}x} \psi(\sigma_{(i)})N_{(i,i)}d\sigma_{(i)} \leq x'K'NKx$$

▷ Thus, the right inequality follows.

▷ The left inequality is a direct consequence of the definition of $V(x)$.

▷ From (1), it follows that the following double inclusion holds:

$$\mathcal{E}(P + K'NK, \gamma) \subseteq \mathcal{D}(V, \gamma) \subseteq \mathcal{E}(P, \gamma) \quad (2)$$

where $\mathcal{D}(V, \gamma) = \{x \in \Re^n;\ V(x) \leq \gamma^{-1}\}$.