Constructive LMI Approaches for Anti-Windup Compensator Design for Systems Subject to Saturation

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Part VII - Solution via dynamic anti-windup

Outline of part VII

1. System description
2. Problem statement
3. Preliminary results
4. Full-order controller
5. Fixed-order controller
6. Illustrative examples
7. Conclusion and prospectives

1. System description (1)

- ☞ We want to build an anti-windup dynamic compensator of order n_{aw}.
- ☞ The output of the dynamic compensator will allow to modify
  - ▷ the controller dynamics,
  - ▷ the output of the controller

1. System description (2)

☞ Consider the system (with the same hypothesis as in the previous chapters):

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & Ax(t) + Bv(t) \\ v(t) & = & sat_{u_0}(u(t)) \\ y(t) & = & Cx(t) \end{array} \right. \quad (1)$$

☞ As previously done, we suppose that a dynamic controller of order n_c has been designed (without taking into account the saturation):

$$\begin{array}{rcl} \dot{\eta}(t) & = & A_c\eta(t) + B_c y(t) + B_r r(t) \\ y_c(t) & = & C_c\eta(t) + D_c y(t) + D_r r(t) \end{array} \quad (2)$$

where $\eta(t) \in \Re^{n_c}$ is the controller state, $y(t)$ and $r(t)$ are the controller inputs (r being the reference signal) and $y_c(t)$ is the controller output. Matrices A_c, B_c, C_c, D_c, B_r, D_r are of appropriate dimensions.

1. System description (3)

☞ Consider that the measured variables available to implement anti-windup scheme are:

$$v = sat_{u_0}(y_c) \text{ (system input) and } y_c \text{ (controller output)} \quad (3)$$

☞ To alleviate the windup effects due to the saturation, we build a dynamic anti-windup controller

$$\begin{cases} \dot{x}_{aw}(t) = A_{aw}x_{aw}(t) + B_{aw}(sat_{u_0}(y_c(t)) - y_c(t)) \\ y_{aw}(t) = C_{aw}x_{aw}(t) + D_{aw}(sat_{u_0}(y_c(t)) - y_c(t)) \end{cases} \quad (4)$$

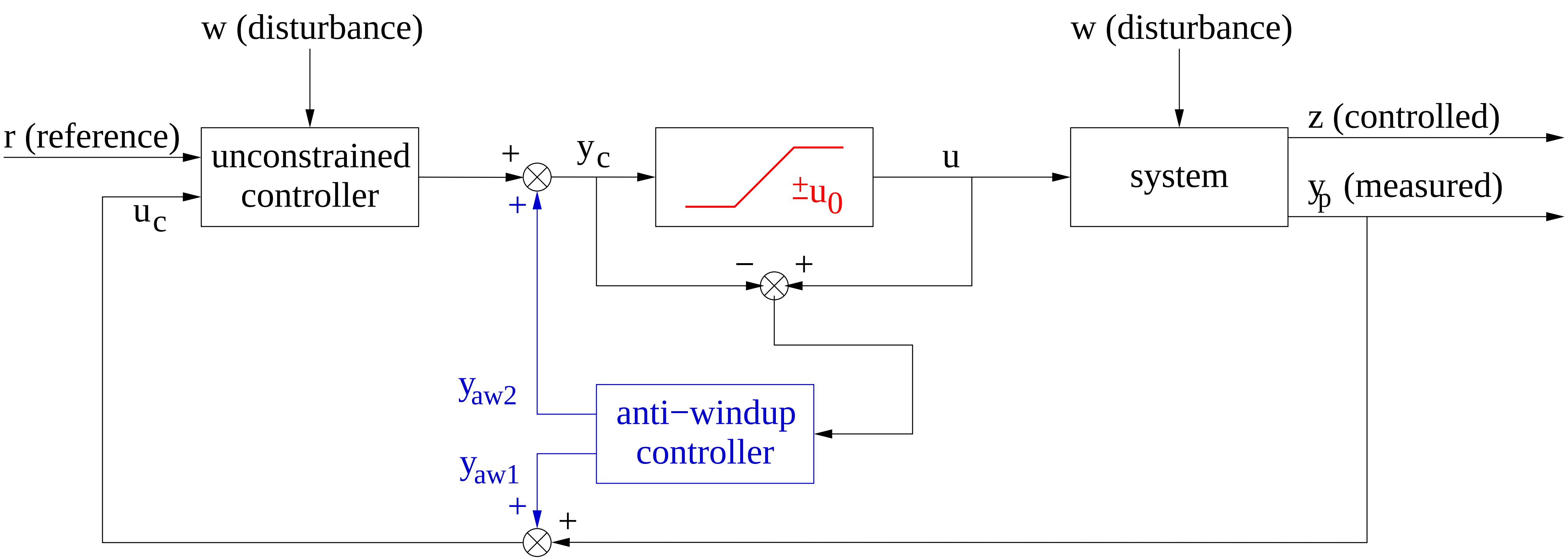
▷ The vectors $x_{aw} \in \Re^{n_{aw}}$ and $y_{aw} \in \Re^{n_c+m}$ are the state and the output of the dynamic anti-windup controller.

▷ The matrices $A_{aw}, B_{aw}, C_{aw}, D_{aw}$ are of appropriate dimensions and have to be determined.

1. System description (4)

☞ The output of the anti-windup controller (4) is inserted in the dynamic controller (2) as follows:

$$
\begin{aligned}
\dot{\eta}(t) &= A_c\eta(t) + B_c y(t) + B_r r(t) + y_{aw1}(t) \\
y_c(t) &= C_c\eta(t) + D_c y(t) + D_r r(t) + y_{aw2}(t)
\end{aligned}
$$



1. System description (5)

☞ The full closed-loop system (process + dynamic controller + anti-windup dynamic loop) then writes:

$$\begin{array}{lcl} \dot{x}(t) & = & Ax(t) + Bsat_{u_0}(y_c(t)) \\ \dot{\eta}(t) & = & A_c\eta(t) + B_c y(t) + B_r r(t) + y_{aw1}(t) \\ \dot{x}_{aw}(t) & = & A_{aw}x_{aw}(t) + B_{aw}(sat_{u_0}(y_c(t)) - y_c(t)) \\ y_{aw}(t) & = & C_{aw}x_{aw}(t) + D_{aw}(sat_{u_0}(y_c(t)) - y_c(t)) \\ y(t) & = & Cx(t) \\ y_c(t) & = & C_c x_c(t) + D_c y(t) + D_r r(t) + y_{aw2}(t) \end{array} \quad (5)$$

2. Problem statement (1)

☞ Design objective: stability and performance

- ▷ The nonlinear closed-loop plant remains stable even for large reference inputs (r). This is ensured by maximizing the size of a stability domain in a given direction;
- ▷ The behavior of the nonlinear system remains as close as possible to the nominal linear plant. This is ensured by minimizing the energy of the error signal $z_p(t) = y(t) - y_{lin}(t)$, where $y(t)$ is the output of the closed-loop system (5) and y_{lin} is the output signal of the closed-loop system without saturation (that is also without anti-windup loop).

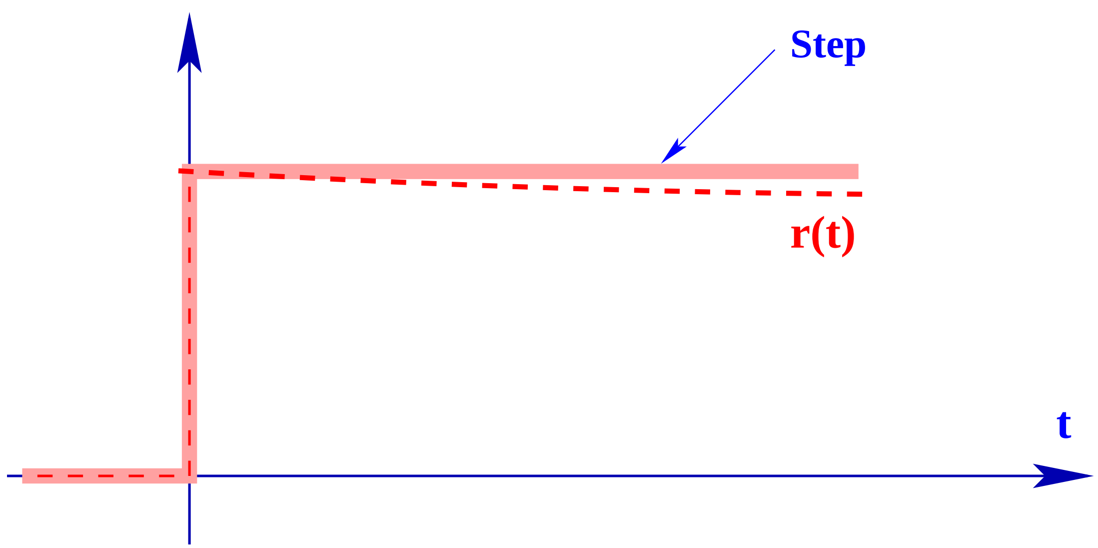
2. Problem statement (2)

☞ The reference inputs $r(t)$ are supposed to be bounded and generated with a step-like profile as follows:

$$
\begin{aligned}
\dot{r} &= -\epsilon r(t) \\
r(0) &= r_0 \\
\| r(0) \| &\leq \rho
\end{aligned}
\qquad (6)
$$

▷ Reference slowly exponentially decreasing

▷ The reference input signal $r(t)$ is viewed as a part of the state-vector.



☞ Problem DAW. Determine the matrices of the anti-windup compensator A_{aw}, B_{aw}, C_{aw} and D_{aw}, such that the stability and performance objectives are satisfied.

☞ The potential interests in using a dynamic anti-windup compensator are:

- ▷ Stability: to increase the degrees of freedom in order to obtain a larger stability domain.
- ▷ Performance: to increase the degrees of freedom in order to degrade less the nominal performance (linear system).

☞ Take care: when the saturation is no more effective, the dynamic anti-windup compensator goes on acting during a transient period.

- ▷ Importance of the dynamics of this compensator (A_{aw}) and of the transient period to converge towards 0.

2. Problem statement (4)

Remarks

☞ To recover the static case (1 or 2 loops): $A_{aw} = 0$, $B_{aw} = 0$ and $C_{aw} = 0$. The static anti-windup gains are issued from D_{aw}.

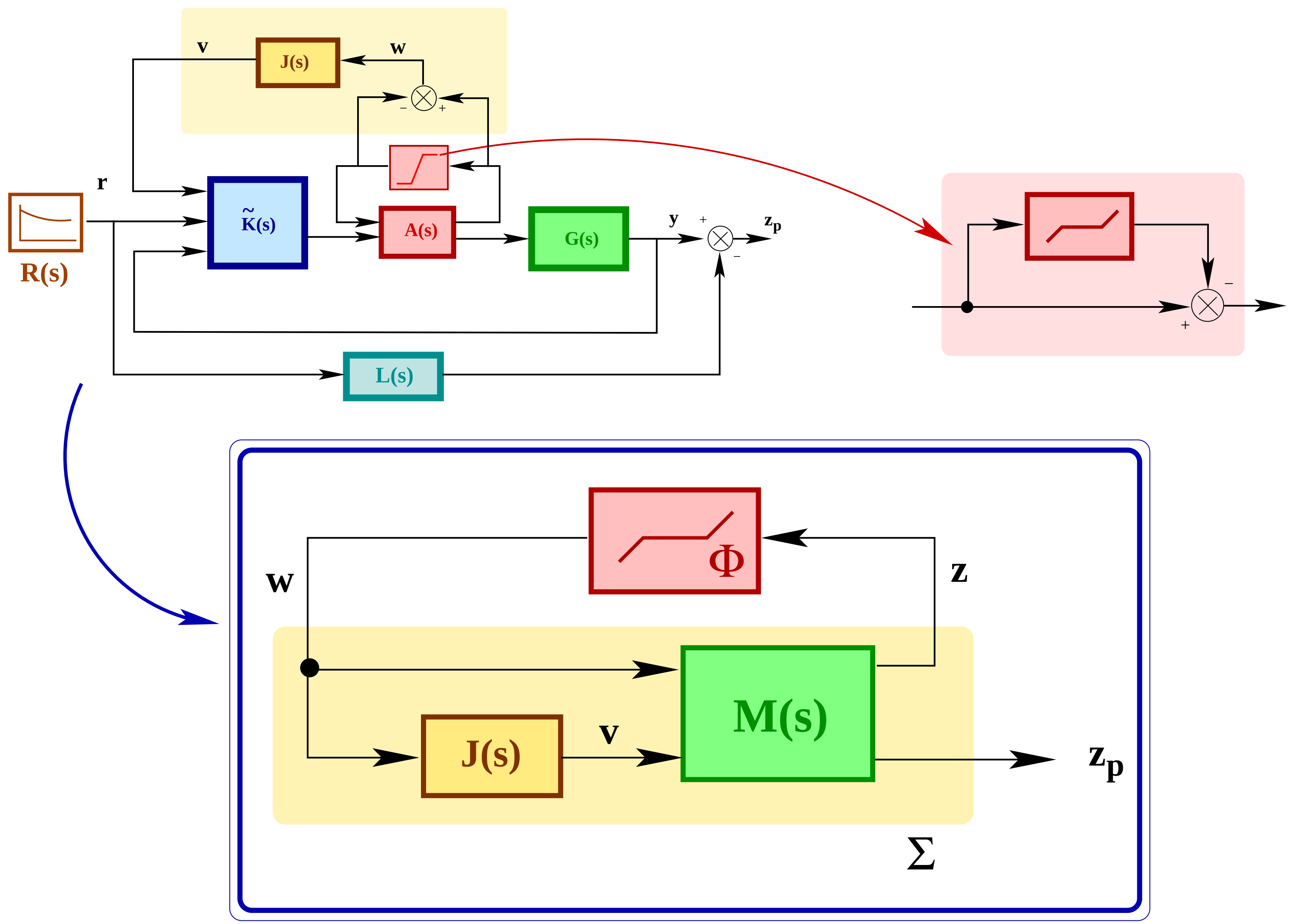
☞ The study of problem DAW corresponds to be able to simultaneously solve:

- ▷ a static output feedback problem (determination of matrices A_{aw} and C_{as})
- ▷ and a state feedback problem (determination of matrices B_{aw} and D_{as})

☞ Problem DAW will be considered into two case:

- ▷ the full-order case
- ▷ the fixed-order case.

☞ Let us define the closed-loop system in a compact form:



☞ Consider the augmented state vector

$$\xi = \begin{bmatrix} r & \rightarrow & \text{state of the reference signal} \\ x_L & \rightarrow & \text{state of the nominal linear system (without saturation)} \\ x & \rightarrow & \text{state of the plant} \\ \eta & \rightarrow & \text{state of the dynamic controller} \\ x_{aw} & \rightarrow & \text{state of the anti-windup controller} \end{bmatrix} \in \Re^{n_M + n_{aw}}$$

☞ Assumption 1. No nested saturation: that implies that no direct transmission in the definition of the complex closed-loop system.

☞ The closed-loop system then reads:

$$\begin{array}{rcl} \dot{\xi} & = & \begin{bmatrix} A & B_v C_{aw} \\ 0 & A_{aw} \end{bmatrix} \xi + \begin{bmatrix} B_\phi + B_v D_{aw} \\ B_{aw} \end{bmatrix} \phi(z) \\ z & = & \begin{bmatrix} C_\phi & 0 \end{bmatrix} \xi \\ z_p & = & \begin{bmatrix} C_p & 0 \end{bmatrix} \xi \end{array} \qquad (7)$$

with $\xi \in \Re^{n_M + n_{aw}}$, $z \in \Re^m$ and $z_p \in \Re^p$.

▷ Set $n = n_M + n_{aw}$

▷ The level of the saturation and therefore of the associate dead-zone nonlinearity is supposed to be normalized.

☞ Let us also define: $\xi = \begin{bmatrix} r \\ \zeta \end{bmatrix}$.

3. Preliminary results (4)

Remarks

☞ The augmented model (denoted $M(s)$) is asymptotically stable by assumption and construction when $\phi(z) = 0$.

☞ Regarding the absence of nested saturation. When the actuator $A(s)$ is strictly proper, such a property is often preserved for the complete closed-loop system $M(s)$.

- ▷ Nevertheless in presence of multiple saturations like position/rate, non-null elements may appear in the direct transmission of $M(s)$.
- ▷ These elements imply the presence of nested saturations. A specific approach can be used or sometimes nested saturation can be eliminate by adequate filtering element.

☞ Suppose matrices A_{aw}, B_{aw}, C_{aw} and D_{aw} are given.

☞ Proposition 1. If there exist matrices $Q \in \Re^{n\times n}$, $S = diag(s_1, \ldots, s_m)$, $Z \in \Re^{m\times n}$, 2 positive scalars γ and $\overline{\rho}$ such that the following conditions hold:

$$\begin{pmatrix} Q & \star \\ \begin{bmatrix} \overline{\rho}\, I_p & \mathbf{0} \end{bmatrix} & I_p \end{pmatrix} > 0 \tag{8}$$

$$(\star) + \begin{pmatrix} \begin{bmatrix} A & B_v \mathbf{C_{aw}} \\ 0 & \mathbf{A_{aw}} \end{bmatrix} Q & \begin{bmatrix} B_\phi \\ \mathbf{B_{aw}} \end{bmatrix} S & 0 \\ S\mathbf{D_{aw}}' \begin{bmatrix} B_v' & 0 \end{bmatrix} - Z & -S & 0 \\ \begin{bmatrix} C_p & 0 \end{bmatrix} Q & 0 & -\frac{\gamma}{2} I_p \end{pmatrix} < 0 \tag{9}$$

$$\begin{pmatrix} Q & \star \\ Z_{(i)} + \begin{bmatrix} C_{\phi_{(i)}} & 0 \end{bmatrix} Q & 1 \end{pmatrix} > 0\,, \quad i = 1, \ldots, m \tag{10}$$

Then for all $\rho \leq \overline{\rho}$, and any reference signal $r(t) \in \mathcal{W}_\tau^p(\rho)$,

▷ the closed-loop nonlinear system is asymptotically stable for any initial condition ζ_0 belonging to the domain of performance $\mathcal{E}(\rho)$ defined by:

$$\mathcal{E}(\rho) = \left\{ \zeta \in \Re^{n-p}; \forall r \in \mathcal{W}_\tau^p(\rho), \begin{bmatrix} r \\ \zeta \end{bmatrix}' P \begin{bmatrix} r \\ \zeta \end{bmatrix} \leq 1 \right\} \quad (11)$$

with $P = Q^{-1}$.

▷ the energy of the error signal z_p satisfies:

$$\int_0^\infty z_p(t)^T z_p(t)\, dt \leq \gamma \quad (12)$$

☞ Proof.

- ▷ It is quite similar to that ones of the previous chapters.
- ▷ First we use the modified sector condition with respect to $\phi(z)$. Hence, the satisfaction of relation (10) guarantee the inclusion of the ellipsoid $\mathcal{E}(\rho)$ in the polyhedral set $S = \{\xi \in \Re^{n_M}; -1 \leq (\begin{bmatrix} C_{\phi_{(i)}} & 0 \end{bmatrix} + Z_{(i)}Q^{-1})\xi \leq 1\}$.
- ▷ Consider the quadratic Lyapunov function $V(\xi) = \xi' P \xi$ with $P = P' > 0$ and $P = Q^{-1}$. Its time derivative expression along the closed-loop system trajectories for all $\xi \in \mathcal{E}(\rho)$ verifies:

$$\dot{V}(\xi) + \frac{1}{\gamma} z_p' z_p \leq \dot{V}(\xi) + \frac{1}{\gamma} z_p' z_p - 2\phi(z)S^{-1}(\phi(z) + ZQ^{-1}\xi) < 0$$

for $\xi \in \mathcal{E}(\rho)$.

- ▷ The satisfaction of relation (9) ensures this last inequality. This establishes both the stability and performance properties (by integration).
- ▷ Note that the inequality (8) enforces the condition on r.

3. Preliminary results (8)

Remarks

☞ Let us now focus on the synthesis issue as stated in problem DAW.

☞ In such a case, the analysis variable Q which is introduced in Proposition 1 and the state-space matrices of $J(s)$ have to be optimized simultaneously. As a result, the inequality (9) which is a priori a BMI, is no longer convex.

☞ However, in the full-order case (i.e. $n_{aw} = n_M$), the constraints (8)-(10) exhibit some particular structures which can be exploited to derive a convex characterization.

4. Full-order controller (1)

☞ Define $u(\overline{\rho}) = \begin{bmatrix}\overline{\rho} I_p & \mathbf{0}\end{bmatrix}'$.

☞ Theorem 1. Consider the nonlinear closed-loop plant. There exists an anti-windup controller $J(s)$ such that the conditions of Proposition 1 are satisfied if and only if there exist

- $X = X', Y = Y' \in \Re^{n_M \times n_M}$
- $S = diag(s_1, \ldots, s_m) > \mathbf{0}$ (13)
- $W = \begin{bmatrix}U & V\end{bmatrix} \in \Re^{m \times (n_M + n_M)}$

such that:

$$u(\overline{\rho})' X u(\overline{\rho}) \leq 1 \tag{14}$$

$$\begin{bmatrix} A'X + XA & \star \\ C_z & -\gamma I_p \end{bmatrix} < \mathbf{0} \tag{15}$$

$$\begin{bmatrix} N_v'(AY + YA')N_v & \star & \star \\ (SB_\phi' - V)N_v & -2S & \star \\ C_zYN_v & 0 & -\gamma I_p \end{bmatrix} < \mathbf{0} \quad (16)$$

$$\begin{bmatrix} X & \star & \star \\ I_{n_M} & Y & \star \\ U_i & V_i + C_{\phi_i}Y & 1 \end{bmatrix} > \mathbf{0}\,, \quad i = 1 \ldots m \quad (17)$$

where N_v denotes any basis of the null-space of B_v'.

4. Full-order controller (3)

☞ Proof.

▷ Following a scheme proposed in (Gahinet and Apkarian, 1994), the inequality (9) of Proposition 1 is first rewritten in a compact form by capturing the synthesis variables A_{aw}, B_{aw}, C_{aw} and D_{aw} into a single matrix Ω which then can be eliminated by the projection lemma.

▷ As a result, (9) is equivalent to (15) and (16) where the variables X and Y correspond to the left-upper parts of P and Q respectively.

▷ Next, using the completion lemma and standard matrix manipulations the equivalence between (17) and (10) can be established.

▷ Finally, by definitions the equivalence between (14) and (8) is readily verified which concludes the proof.

Remarks

☞ In order to optimize the size of the stability domain by increasing ρ, the performance constraint (12) can be relaxed by setting $\gamma = \infty$.

▷ In that case, the LMI constraints (15) and (16) become respectively:

$$A'X + XA < \mathbf{0} \tag{18}$$

$$\begin{bmatrix} N_v'(AY + YA')N_v & \star \\ (SB_\phi' - V)N_v & -2S \end{bmatrix} < \mathbf{0} \tag{19}$$

☞ The conditions given in Proposition 1 and Theorem 1 concern the stabilization problem in a local context. If the open-loop matrix is Hurwitz, the global asymptotic stabilization problem can be addressed by considering

▷ $Z = -\begin{bmatrix} C_\phi & \mathbf{0} \end{bmatrix} Q$ in Proposition 1

▷ and $U = \mathbf{0}$, $V = -C_\phi Y$ in Theorem 1.

4. Full-order controller (5)

Algorithm

☞ Using Theorem 1, the existence of a full-order anti-windup compensator $J(s)$ is easily checked by solving a finite set of LMIs.

▷ Then the matrix Q (from Proposition 1) is obtained as:

$$Q = \begin{bmatrix} Y & I_{n_M} \\ N & \mathbf{0} \end{bmatrix} \begin{bmatrix} I_{n_M} & X \\ \mathbf{0} & M \end{bmatrix}^{-1} \tag{20}$$

with $M'N = I_{n_M} - XY$.

▷ The synthesis variables A_{aw}, B_{aw}, C_{aw} and D_{aw} are finally calculated as the solution of (9) which becomes convex as soon as Q is fixed.

▷ Moreover, using a change of variables $\tilde{B} = B_{aw}S$, $\tilde{D} = D_{aw}S$ it can be observed that the matrices S and Z do not have to be fixed. This offers some additional degrees-of-freedom that can be used for example to add constraints on the controller matrix A_{aw}.

4. Full-order controller (6)

☞ Algorithm:

1. Fix ρ,
2. Minimize γ under the LMI constraints (14), (15), (16) and (17) with respect to the variables X, Y, S and W. If the problem is infeasible, decrease ρ and go back to step *(ii)*.
3. Compute Q as the solution of (20),
4. Fix Q in inequality (9) and solve the convex feasibility problem with respect to the variables A_{aw}, B_{aw}, C_{aw} and D_{aw}.

4. Full-order controller (7)

Remarks

☞ As a preliminary step to the above algorithm, it is interesting to compute the largest admissible stability parameter ρ without performance constraint.

▷ This can be done by simplifying the first two steps of the algorithm as follows:

1. Define $\mathcal{W}(\rho)$ according to the design objective,
2. Minimize $1/\rho^2$ under the LMI constraints (14), (18), (19) and (17) with respect to the variables X, Y, S and W.

☞ Let us underline that the notion of full-order anti-windup has different meaning depending on the authors in the literature. For example, in (Grimm et al., 2003), (Wu and Soto, 2004), (Hu et al., 2005), the authors use full order to mean plant order (i.e., $n_{aw} = n_{plant}$). At the contrary, in (Biannic et al., 2007) or in the current talk, the full order means $n_{aw} = n_M$.

5. Fixed-order controller (1)

☞ Let us now focus on a special case where the matrices A_{aw} and C_{aw} of the anti-windup compensator are a priori fixed.

☞ This allows not only to control the order of $J(s)$ but also to reduce the computational efforts as is stated by the following proposition.

▷ Proposition 2. The BMI constraint (9) of Proposition 1 is convex as soon as the matrices A_{aw} and C_{aw} of the anti-windup controller are fixed.

▷ Proof. It immediately follows from a classical change of variables: $\tilde{B} = B_{aw}S$, $\tilde{D} = D_{aw}S$ which is here justified since $S > \mathbf{0}$ and thus non-singular.

5. Fixed-order controller (2)

☞ Based on this result, a simple algorithm can be derived.

Algorithm

1. Choose appropriate A_{aw} and C_{aw},
2. Fix ρ and minimize γ under the LMI problem constraints (8), (9), (10) with respect to the variables Q, S, Z, $\tilde{B}$, $\tilde{D}$.
3. Compute B_{aw} and D_{aw} by inverting the aforementioned change of variables.

☞ The main difficulty in the above algorithm consists in choosing the matrices A_{aw} and C_{aw} correctly.

5. Fixed-order controller (3)

☞ Nevertheless, the choice of matrices A_{aw} and C_{aw} may appear intuitive and natural. Consider the following decomposition of $J(s)$:

$$J(s) = M_0 + \sum_{i=1}^{n_1} \frac{M_{i1}}{s+\lambda_i} + \sum_{i=1}^{n_2} \frac{M_{i2}}{s^2 + 2\eta_i\omega_i + \omega_i^2} \tag{21}$$

▷ $D_{aw} = M_0$ and B_{aw} contains the collections of matrices M_{i1} and M_{i2}.

▷ For this decomposition, the fixed matrices A_{aw} and C_{aw} can be chosen as:

$$\begin{aligned} &\bullet\ A_{aw} = \text{diag}\left(-\lambda_1, \ldots, -\lambda_{n_1}, A_1, \ldots, A_{n_2}\right) \\ &\bullet\ C_{aw_k} = \Big[\underbrace{1 \ldots 1}_{n_1}\underbrace{[1\ 0] \ldots [1\ 0]}_{n_2}\Big],\ k = 1, \ldots, p_J \end{aligned} \tag{22}$$

$$\text{with}\ \ A_i = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\eta_i\omega_i \end{bmatrix},\ i = 1, \ldots, n_2 \tag{23}$$

▷ From this observation, the first step of the algorithm simply boils down to the choice of a list of poles of the anti-windup controller whose matrices A_{aw} and C_{aw} are then immediately deduced from the equations (22) and (23).

Remarks

☞ Note that the order of the controller is now given by $n_{aw} = n_1 + 2n_2$.

- ▷ The two parameters n_1 and n_2 (defined in (21)) should be chosen sufficiently small, so that $n_{aw} < n_M$.

☞ The poles of the anti-windup controller can be chosen by selecting a part of those which were obtained in the full order design case.

- ▷ Typically, the slow and fast dynamics are eliminated.
- ▷ Alternatively, an iterative procedure starting from the static case can be used.
- ▷ The list of poles is then progressively enriched until the gap between the full and reduced order cases becomes small enough.

Anti-windup synthesis with fixed-dynamics

1. Perform a full-order synthesis:
   (a) Fix ρ and minimize γ under the LMI constraints of Theorem 1.
   (b) Compute the Q matrix and the compensator $J(s)$
2. Perform a fixed-order synthesis:
   (a) Select a set of relevant poles from the full-order compensator;
   (b) Build the matrices A_{aw} and C_{aw} of the reduced-order compensator;
   (c) Minimize γ under the LMI constraints of Proposition 1 w.r.t the variables Q, S, Z, $\tilde{B}$ and $\tilde{D}$.
   (d) Compute B_{aw} and D_{aw} by inverting the aforementioned change of variables.

6. Illustrative examples (1)

Example 1 - Combat aircraft

☞ We consider a detailed application of the proposed algorithms on a real-world example.

☞ The main objective is to improve the performances of a flight control system for a fighter aircraft so that the responses of the nonlinear closed-loop plant remain as close as possible to those of a linear reference model despite actuators saturations.

- ▷ In the context of anti-windup synthesis, this application is very challenging since several saturations (rate and magnitude) can be involved simultaneously and are not directly measurable.
- ▷ The implementation aspects play a key role here and the anti-windup compensator should still work correctly when the anti-windup inputs are delayed.
- ▷ To improve the clarity of our presentation, the application will be mainly focused on the longitudinal axis. However, the proposed technique was successfully applied to the lateral axis as well.

6. Illustrative examples (2)

☞ For a better illustration of the proposed techniques, a critical point (associated to a low dynamic pressure) of the flight envelope is selected: Mach = 0.3, H = 5000 ft.

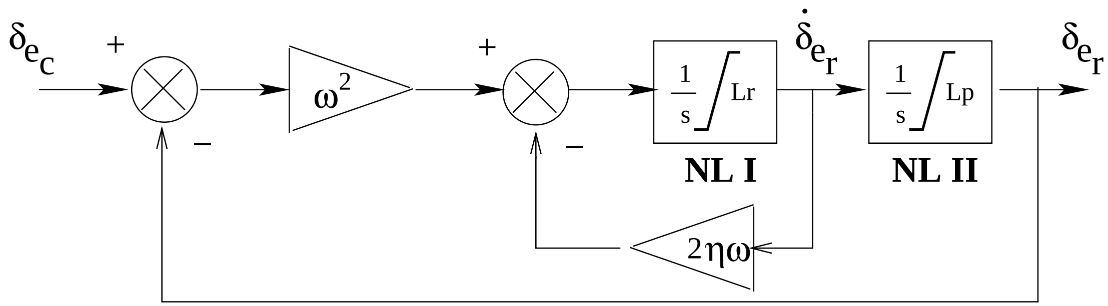
☞ For such a point, the control efficiency is reduced and the plant is unstable.

☞ The linearized short-term dynamics are characterized by the following state-space equations:

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -0.5 & 1 \\ 0.8 & -0.4 \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} -0.2 \\ -5 \end{bmatrix} \delta_{e_r} \qquad (24)$$

where α, q and δ_{e_r} denote the angle-of-attack, the pitch-rate and the realized elevator deflection respectively.

☞ The nonlinear actuator model involves both magnitude and rate limitations.

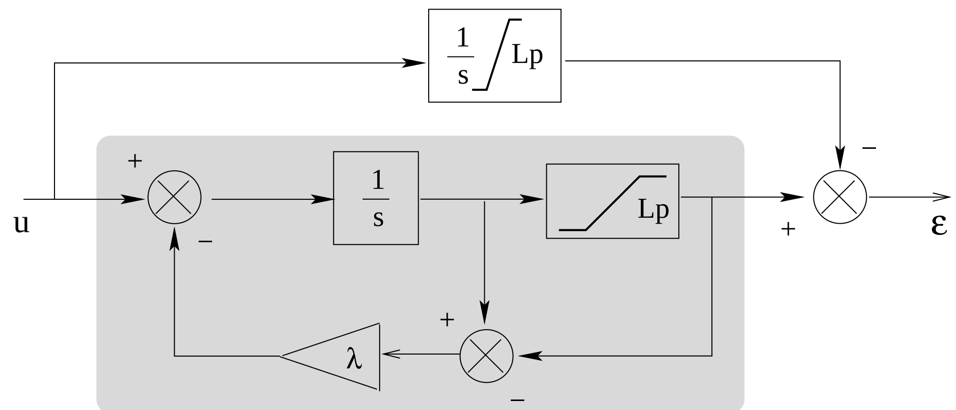


☞ This nonlinear model involves two standard limited integrators which are dynamic nonlinearities.

▷ The first one appears in the internal loop and is clearly associated to the rate limitation $L_r = 80\, deg/s$, while the second one in the external loop, is linked to the magnitude limitation $L_p = 20\, deg$.

6. Illustrative examples (4)

☞ Such a nonlinear model is not directly handled by our approach which is restricted to static nonlinearities. We approximate this nonlinear actuator like in Chapter I.

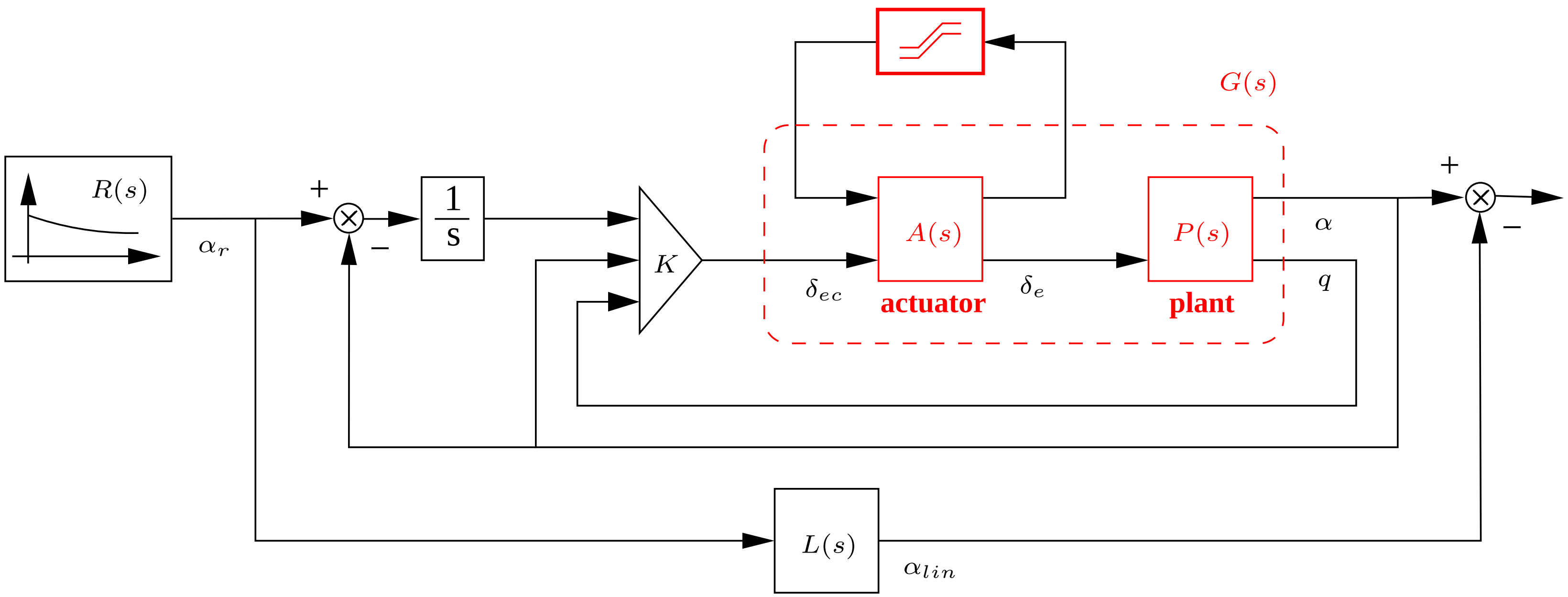


☞ A 4th-order open-loop plant $G(s)$ is obtained as follows:

$$\begin{bmatrix} \dot{\zeta}_1 \\ \dot{\zeta}_2 \\ \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -100 & 0 & 0 & 0 \\ 0 & -100 & 0 & 0 \\ 0 & 0 & -0.5 & 1 \\ 0 & 0 & 0.8 & -0.4 \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \alpha \\ q \end{bmatrix} + \begin{bmatrix} 28 & -675 \\ 5.33 & 100 \\ 0 & -3 \\ 0 & -75 \end{bmatrix} \begin{bmatrix} \psi(\zeta_1) \\ \psi(\zeta_2) \end{bmatrix} + \begin{bmatrix} 45 \\ 0 \\ 0 \\ 0 \end{bmatrix} \delta_{e_c} \tag{25}$$

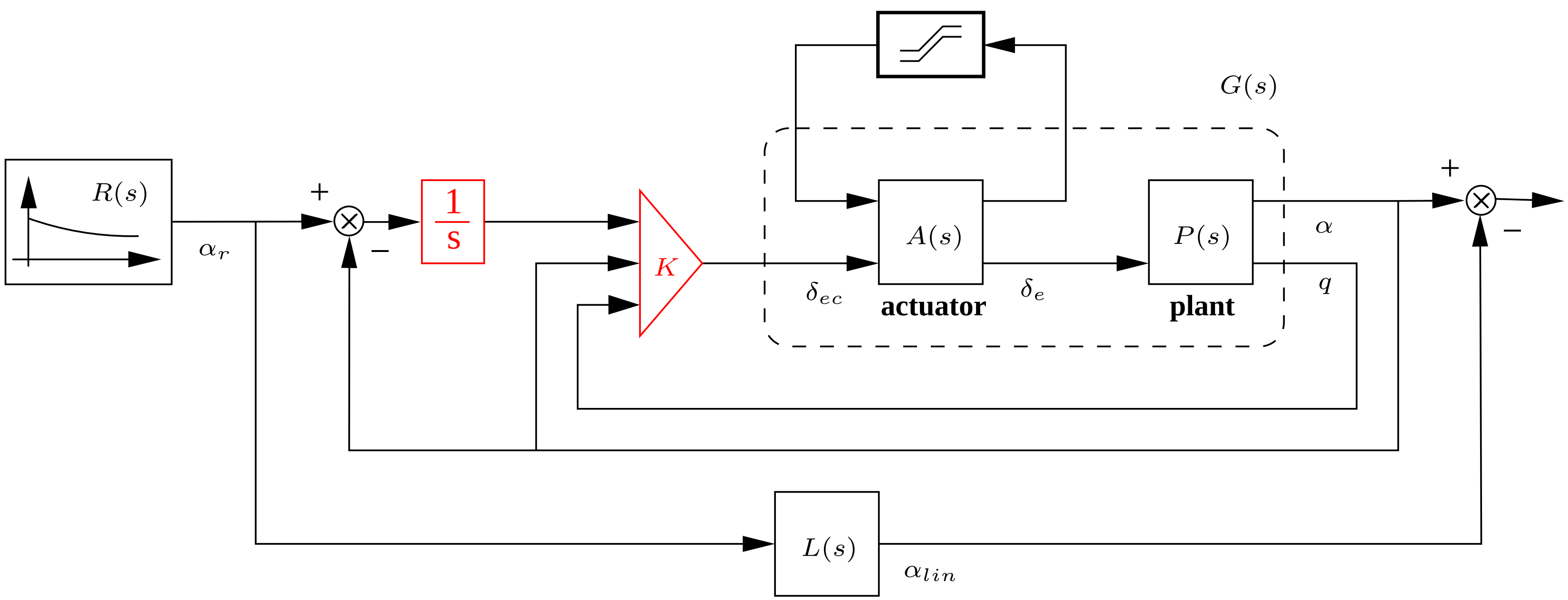
where the two saturation-type static nonlinearities $\psi(.)$ have been normalized and only depend on the components ζ_1 and ζ_2 of the state-vector.

Open-loop saturated plant

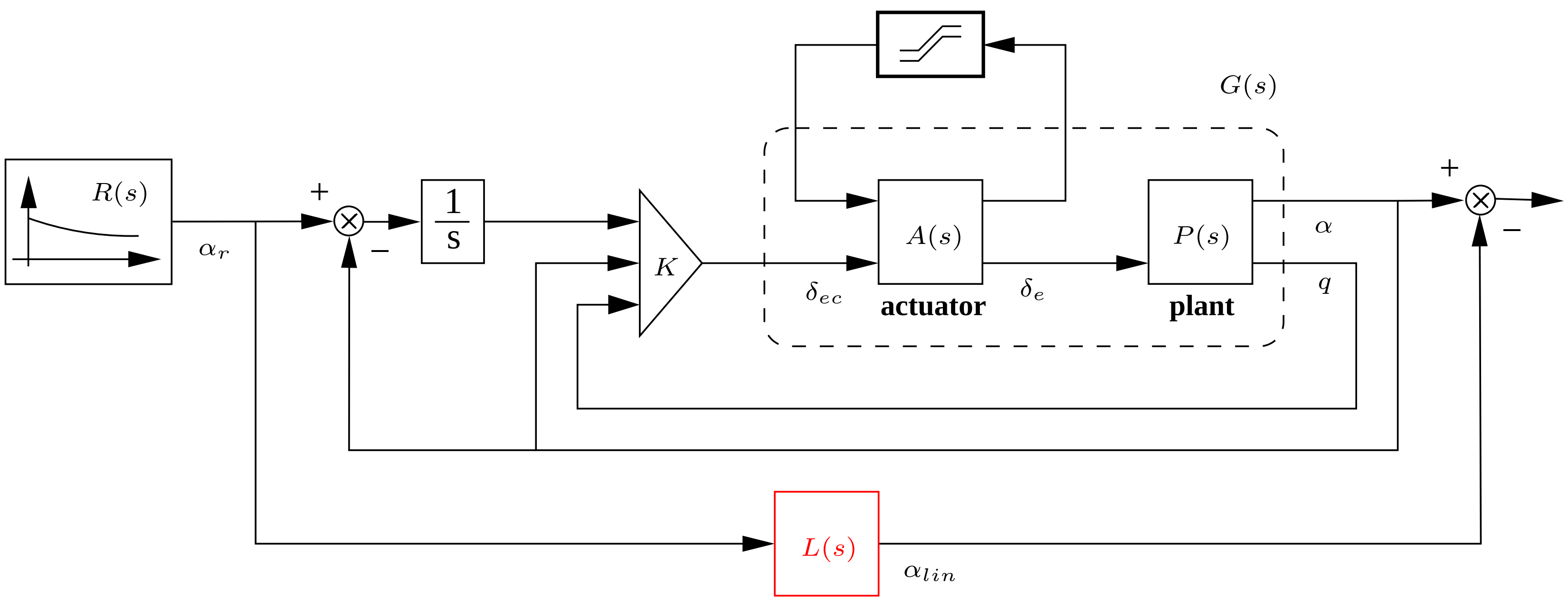


6. Illustrative examples (6)

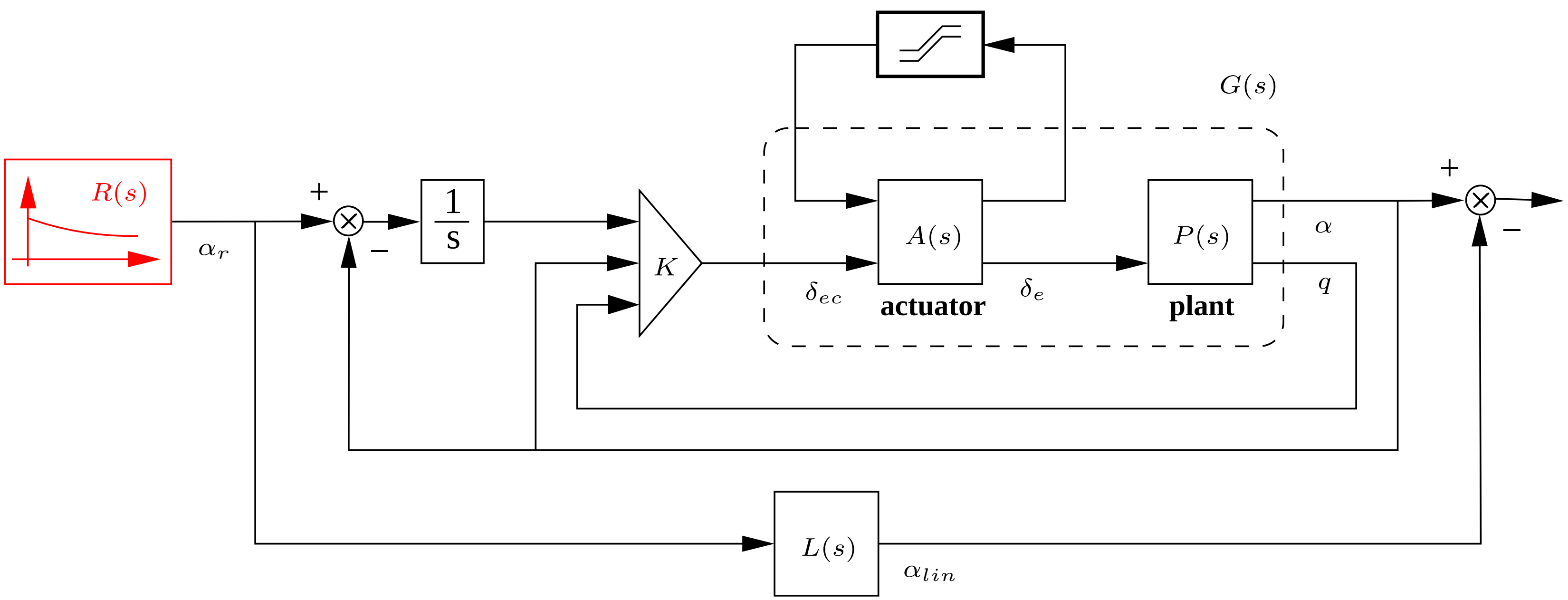
Nominal linear controller



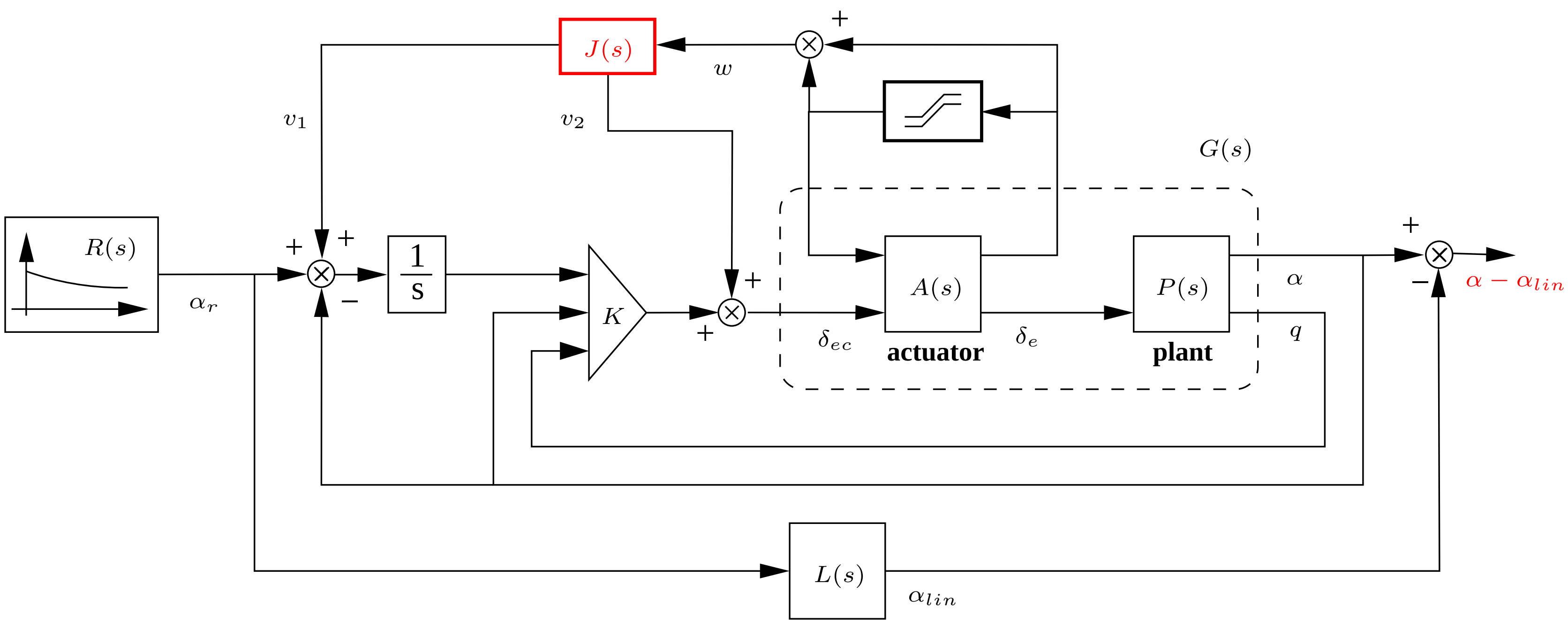
Nominal closed-loop plant



Reference input



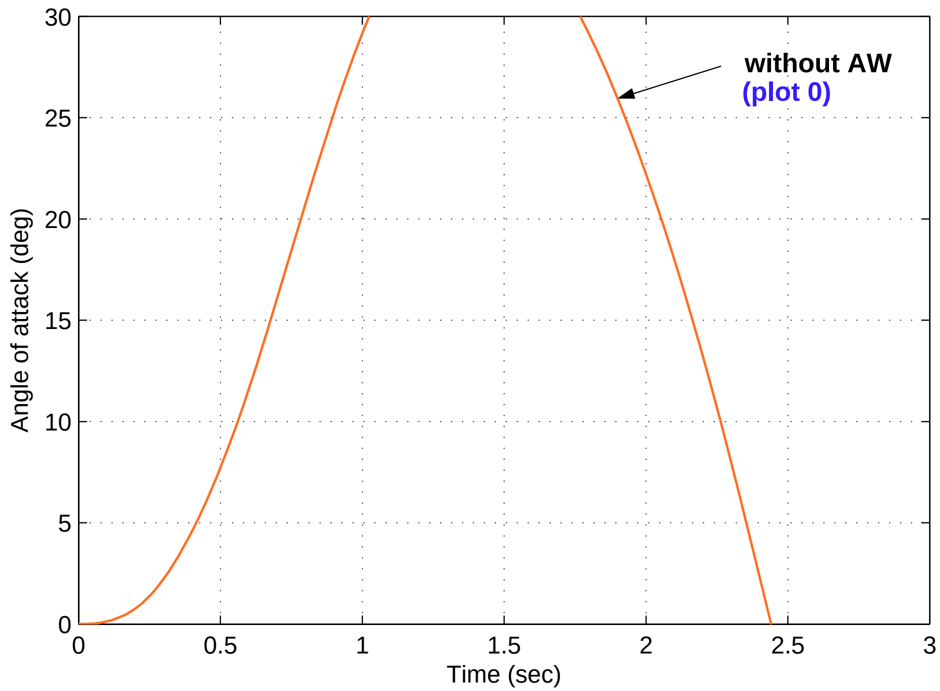
Specifications



☞ Design objective. Design a dynamic anti-windup controller $J(s)$ such that the energy of the tracking error $\alpha - \alpha_{lin}$ is minimized.

6. Illustrative examples (9)

☞ Time-domain responses to a 20 deg step command in angle of attack.



▷ Plot 0 - orange line

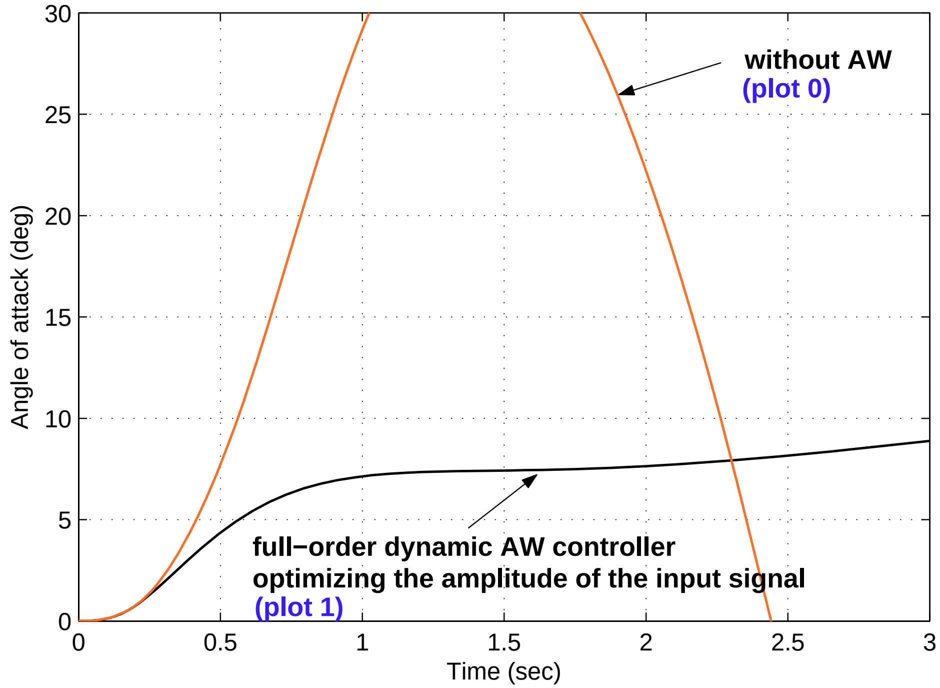
▷ Without anti-windup

▷ Unstable plant

☞ The PID controller performs well as long as the amplitude of α_r does not exceed 7 deg.

☞ Beyond this value, a performance degradation appears and stability is finally lost when the amplitude gets larger than 7.8 deg.

6. Illustrative examples (10)

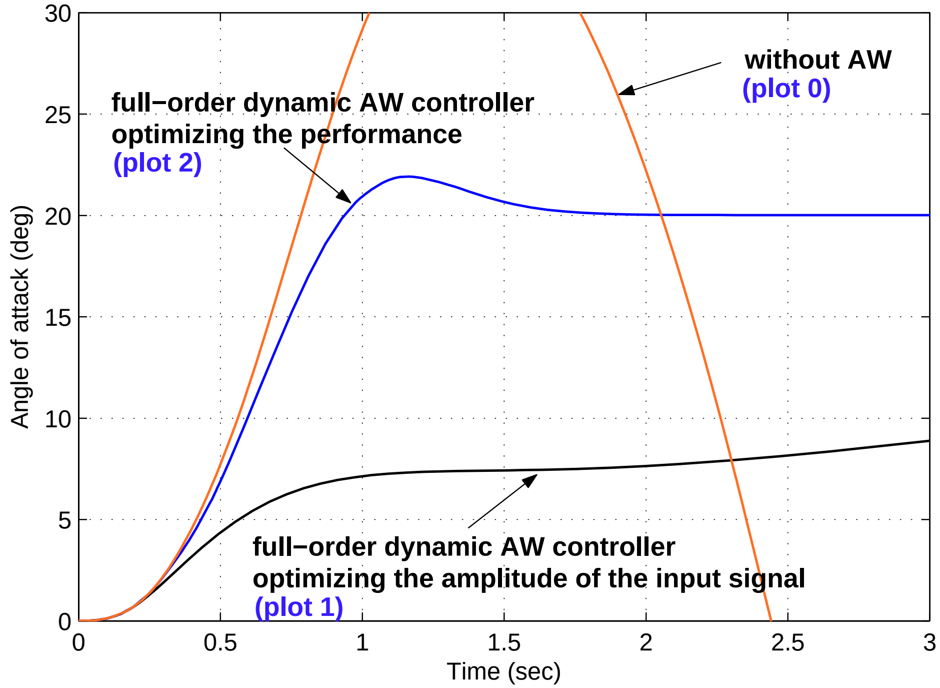


- ▷ Plot 1 - black line
- ▷ Full-order anti-windup
- ▷ Stability
- ▷ But very poor performance level

☞ The maximum value of α_r is computed, for which the plant is guaranteed to be stable.

☞ A very large value is obtained $(29.6\,deg)$, which means that the stability domain is considerably enlarged.

6. Illustrative examples (11)

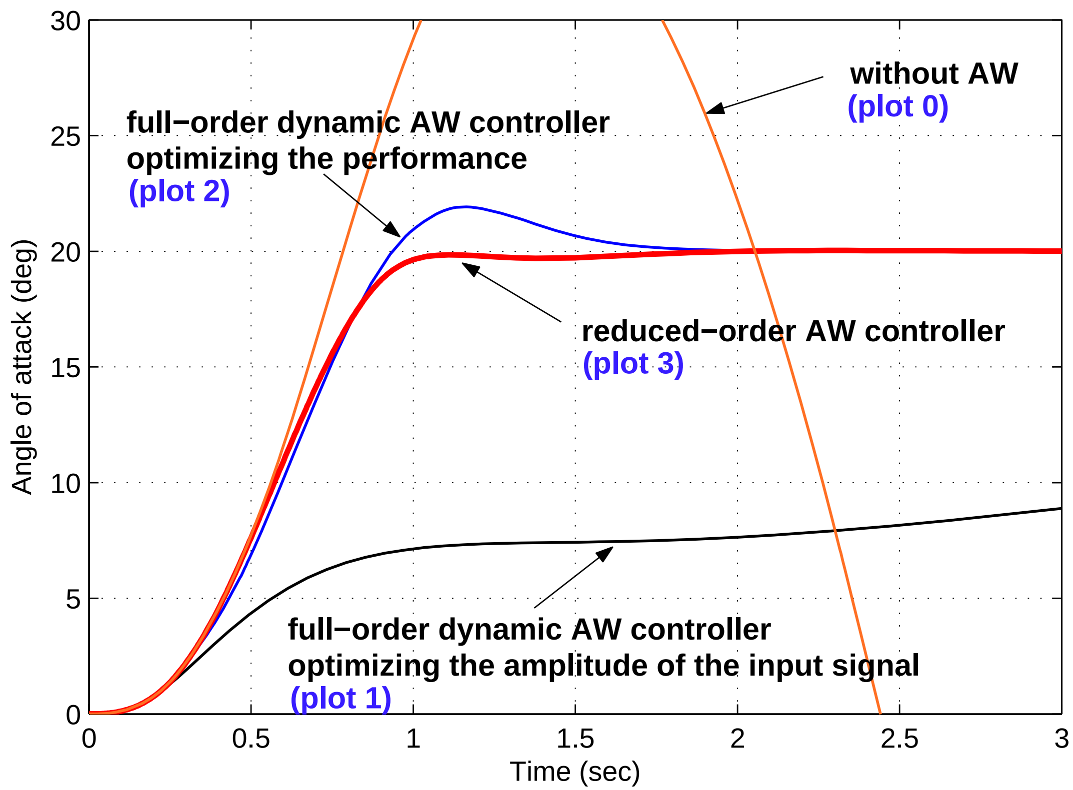


- ▷ Plot 2 - blue line
- ▷ Full-order anti-windup
- ▷ Performance
- ▷ Good step response
- ▷ But small overshoot

☞ The energy γ of the error signal $\alpha - \alpha_{lin}$ is minimized, where α_{lin} is the step response of the unsaturated closed-loop plant.

☞ A reasonably small value is obtained (0.11)

6. Illustrative examples (12)



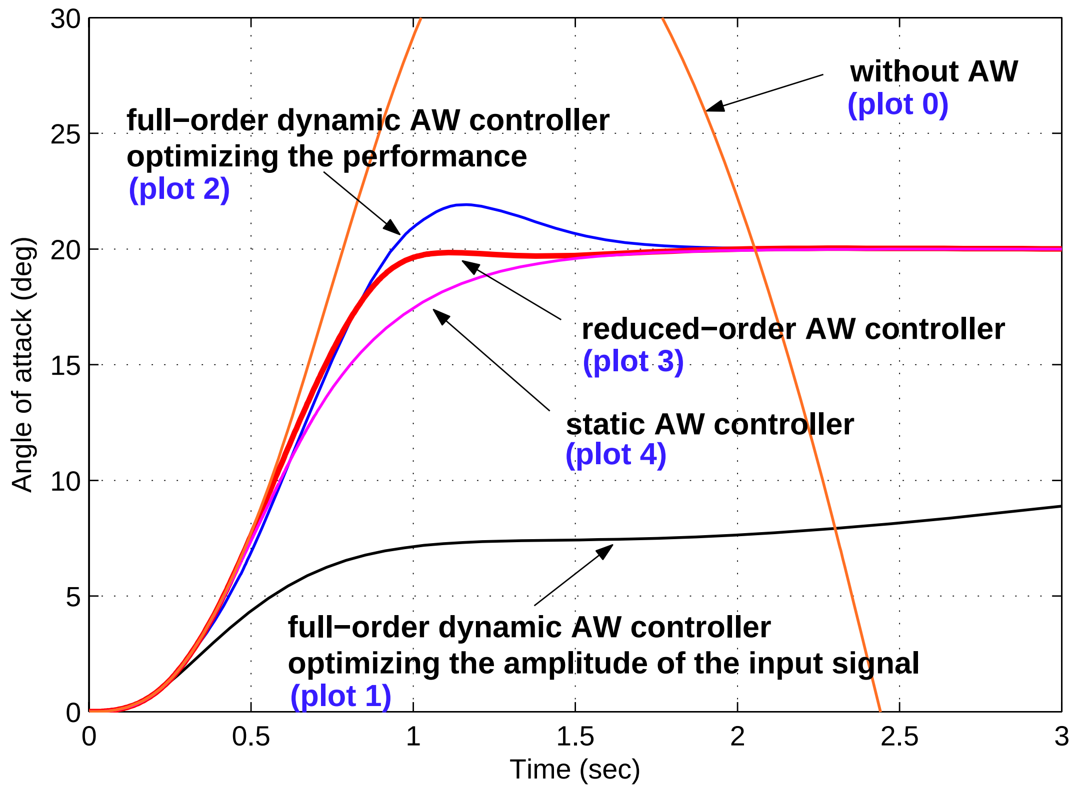
- ▷ Plot 3 - red line
- ▷ Fixed-order anti-windup with fixed poles
- ▷ Fast response with no overshoot

☞ The poles of the full-order anti-windup controller are analyzed.

☞ A selection is made and a reduced-order controller is computed.

$$-0.0013 \quad -0.45 \quad -1.80 \quad -4.21 \quad -5.53 \pm 3.23j \quad -548 \quad -4750$$

6. Illustrative examples (13)



- ▷ Plot 4 - purple line
- ▷ Static anti-windup
- ▷ performance: Good time response
- ▷ But a bit slow

☞ Finally, for the purpose of comparison, a static anti-windup controller is synthesized.

☞ Such a design can be viewed as a special case of the previous one. The controller here reduces to the term D_{aw}.

Example 2 - Civil aircraft

☞ Context. GARTEUR Project AG17 (AIRBUS, SAAB, FOI, NLR, ONERA, Univ. Leicester, Univ. Bristol, Univ. Liverpool, LAAS) 2005-2008: Nonlinear approaches for analysis and synthesis in performance for robust landing and on-ground aircraft control. (Chapter 7 in "Nonlinear Analysis and Synthesis Techniques for Aircraft Control", LNCIS, vol.365, Springer-Verlag, 2007)

6. Illustrative examples (15)

☞ Fly-by-wire systems are now commonly used onboard transport aircraft, allowing for automation of many parts of the flight, including the landing phase.

- ▷ Immediately after touch-down, however, the motion is still controlled manually by the pilot who has to coordinate actions on rudder deflection, engines running speed, wheels brakes and nose-wheel steering system.
- ▷ This piloting task is quite demanding, especially in bad weather conditions: indeed it should be emphasized that the aircraft behavior on ground significantly changes according to the runway state (dry, wet or icy).
- ▷ Moreover, in order to reduce congestion of most big airports, ground phases have to be constantly further optimized. Consequently, there is a real need to develop new control systems improving on-ground aircraft handling qualities.

Need to develop new control systems improving on-ground aircraft handling qualities

6. Illustrative examples (16)

☞ The design objective consists of designing lateral control laws for the on-ground aircraft.

☞ Two types of maneuvers can be distinguished:

- ▷ runway maneuvers mainly deal with lateral wind rejection, to ensure that the aircraft maintains a straight trajectory on the main runway while decelerating,
- ▷ taxiway maneuvers aim at bringing the aircraft from the main runway to the parking area and are performed at lower speeds (below 40kts).

☞ Only the second type of maneuver has been considered.

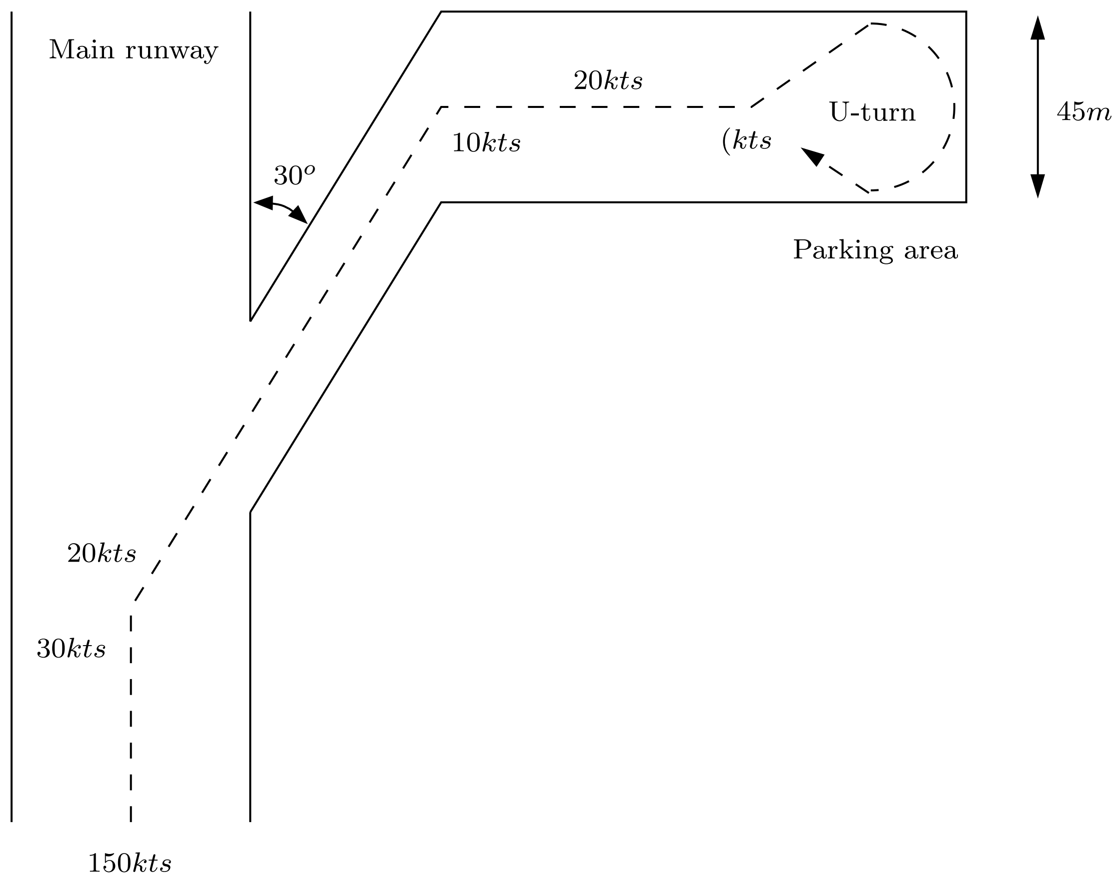
6. Illustrative examples (17)

☞ Particular attention will be paid to the three following sequences:

▷ sequence 1: turn to take a 30^o exit while decelerating from 30kts to $20kts$,

▷ sequence 2: make a 60^o turn at $10kts$,

▷ sequence 3: perform a U-turn at 5kts on a 45m wide runway.

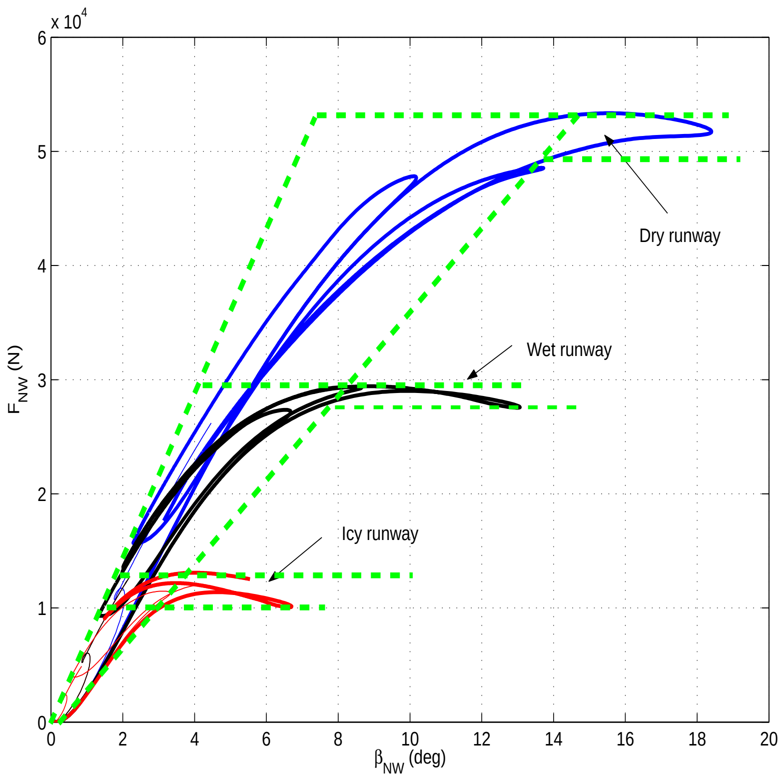


☞ More generally, the design issue can be summarized as follows:

▷ Design challenge.

- Compute a (possibly nonlinear) controller, which ensures a good tracking of the yaw rate r and the heading Ψ:
- with as fast a response as possible,
- without overshoot (especially in heading), whatever the runway state (dry, wet or icy),
- for any aircraft longitudinal velocity between 5 and $40kts$.

6. Illustrative examples (19)

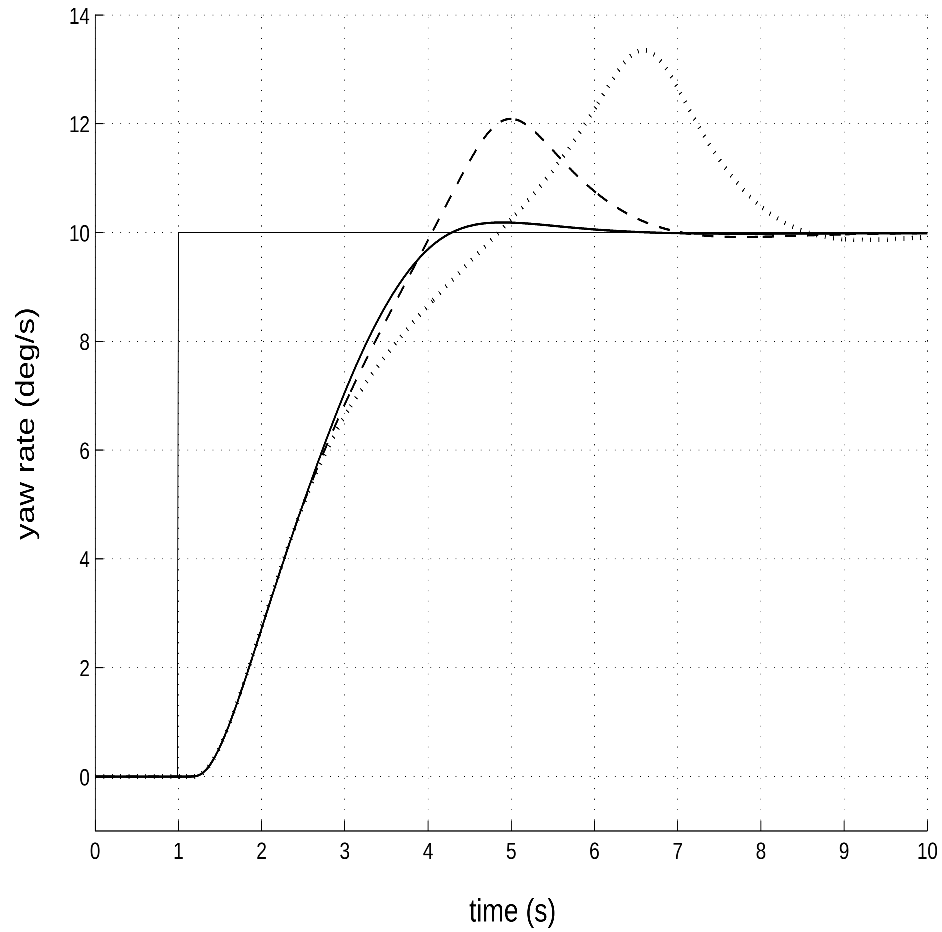


- ▷ The main point consists of an original simplification of the ground forces, which are approximated by saturation-type nonlinearities, where the saturation levels depend on the runway state.
- ▷ It is then shown that these saturation levels can be identified on-line, and that the resulting estimator admits an LFT-based expression.

☞ It is a non-standard application of anti-windup control, which can be considered as an original alternative to dynamic inversion.

☞ Consider the response of the on-ground aircraft to a yaw rate step input on a wet runway for a longitudinal velocity $V_x = 8m/s$.

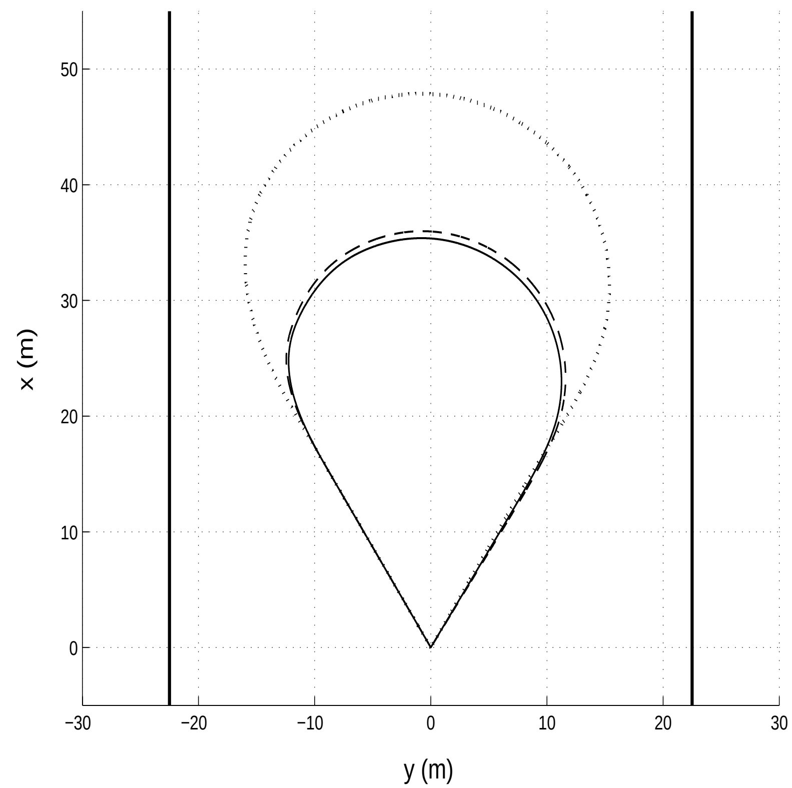
- ▷ Without anti-windup compensation (dotted line): stability but a strong overshoot, 8 seconds to reach the desired value.
- ▷ With anti-windup gain associated with the rate limitation of the nose-wheel steering system is employed (dashed line), the overshoot is reduced.
- ▷ With full anti-windup compensation (solid line), good tracking and a negligible overshoot.



6. Illustrative examples (21)

☞ Sequence 3: perform a U-turn at 5kts on a 45m wide runway.

- ▷ At the beginning of the maneuver, there is a 30^o angle between the aircraft body and the runway axis.
- ▷ The U-turn is then performed, which consists in making a 240^o turn on the right.
- ▷ The aircraft then returns to its initial position, maintaining a 30^o angle with the runway axis.
- ▷ The overall maneuver lasts 34.0s on a dry runway (solid), 34.3s on a wet runway (dashed) and 43.0s on an icy runway (dotted).



7. Conclusion and prospectives (1)

Conclusion

- ☞ Some new results have been presented to compute dynamic or static anti-windup controllers by convex optimization of an original performance criterion.
- ☞ Both the full-order and the reduced-order (with fixed dynamics) cases have been considered.
- ☞ The results have been illustrated on two applications issued from aeronautical domain: combat and civil aircrafts.

Prospectives

☞ When dealing in the context of anti-windup, there are still some open questions.

- ▷ In particular in the context of the fixed order anti-windup compensator, a methodology more systematic to choose the dynamic of the anti-windup needs further developments.
- ▷ A way to fix the order n_{aw} a priori.
- ▷ Robustness properties of the anti-windup compensators so as to facilitate their implementation