Constructive LMI Approaches for Anti-Windup Compensator Design for Systems Subject to Saturation

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Part VI - Solution via static anti-windup (2 loops)

Outline of part IV

1. System description
2. Problem statement
3. Preliminary results
4. Theoretical results
6. Illustrative examples
8. Conclusion and prospectives

1. Introduction (1)

- ☞ The idea is to build several anti-windup loops to modify
  - ▷ the controller dynamics (similar to what has been done in the previous chapter),
  - ▷ the output of the controller
- ☞ Such a type of anti-windup loop allows to deal with nominal static controller (Example: HAVELIMITS model in combat aircraft or control attitude of satellite)

2. System description (1)

☞ Consider the system (with the same hypothesis as in the previous chapter):

$$\left\{\begin{array}{rcl} \dot{x}(t) & = & Ax(t) + Bv(t) \\ v(t) & = & sat_{u_0}(u(t)) \\ y(t) & = & Cx(t) \end{array}\right. \quad (1)$$

☞ As previously done, we suppose that a dynamic controller of order n_c has been designed (without taking into account the saturation):

$$\begin{array}{rcl} \dot{\eta} & = & A_c\eta + B_c u_c \\ y_c & = & C_c\eta + D_c u_c \end{array} \quad (2)$$

where $\eta(t) \in \Re^{n_c}$ is the controller state, $u_c = y(t)$ is the controller input and $y_c(t)$ is the controller output. Matrices A_c, B_c, C_c, D_c are of appropriate dimensions.

2. System description (2)

- As previously, the measured variables available to implement anti-windup scheme are:

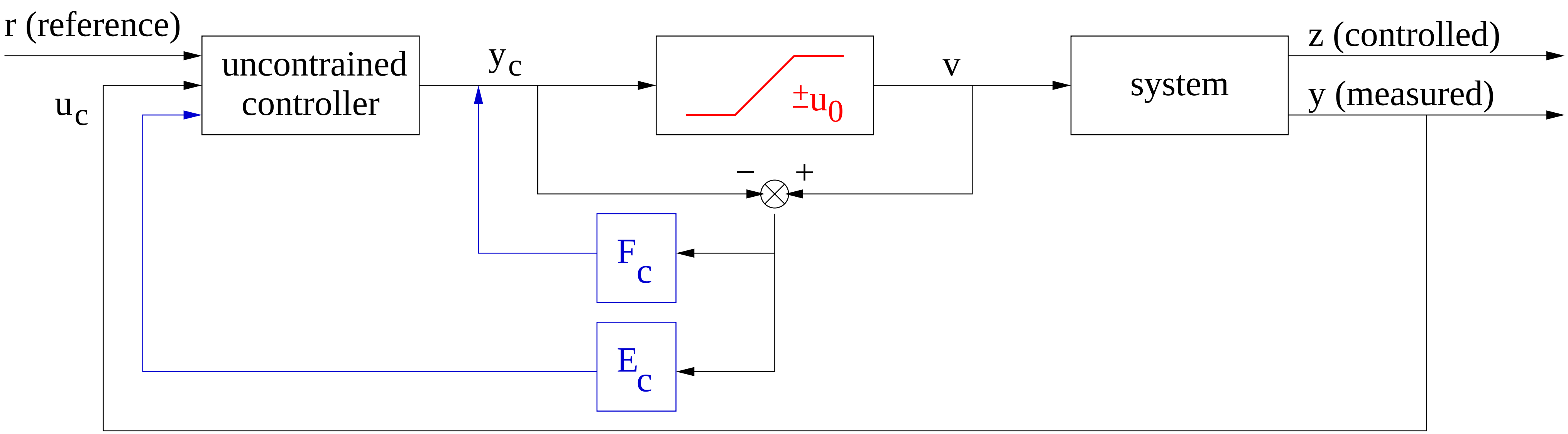
$$v = sat_{u_0}(y_c) \text{ (system input) and } y_c \text{ (controller output)} \tag{3}$$

- To mitigate the windup effects, anti-windup terms are added:
  - ▷ $E_c(sat(y_c(t)) - y_c(t))$ on the controller dynamics,
  - ▷ $F_c(sat(y_c(t)) - y_c(t))$ on the output of the controller

2. System description (3)

☞ The closed-loop system then writes:

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + Bsat(y_c(t)) \\
\dot{\eta}(t) &= A_c\eta(t) + B_cCx(t) + E_c(sat(y_c(t)) - y_c(t)) \\
y_c(t) &= C_c\eta(t) + D_cCx(t) + F_c(sat(y_c(t)) - y_c(t))
\end{aligned}
\tag{4}
$$



2. System description (4)

☞ As previously, one considers the decentralized dead-zone nonlinearity

$$\psi(y_c(t)) = sat(y_c(t)) - y_c(t)$$

☞ The closed-loop system writes:

$$\begin{array}{rcl} \dot{\xi}(t) & = & \mathbb{A}\xi(t) + (\mathbb{B} + \mathbb{B}F_c + \mathbb{R}E_c)\psi(y_c(t)) \\ y_c(t) & = & \mathbb{K}\xi + F_c\psi(y_c(t)) \end{array} \qquad (5)$$

with

$$\mathbb{A} = \begin{bmatrix} A + BD_cC & BC_c \\ B_cC & A_c \end{bmatrix} \; ; \; \mathbb{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

$$\mathbb{R} = \begin{bmatrix} 0 \\ I_{n_c} \end{bmatrix} \; ; \; \mathbb{K} = \begin{bmatrix} D_cC & C_c \end{bmatrix}$$

☞ Problem SAW2. Determine the anti-windup gains E_c and F_c, and an asymptotic stability region for the saturated closed-loop system with anti-windup (5).

☞ This problem to be solved here is similar to the one solved in the chapter V. The complexity comes from the presence of an implicit function:

$$y_c(t) = C_c\eta(t) + D_cCx(t) + F_c(sat(y_c(t)) - y_c(t))$$

▷ To solve this problem, we use Lemma 4 of the chapter III.

4. Preliminary results (1)

☞ Let us consider a generic nonlinearity $\psi(v) = sat_{v_0}(v) - v$, $\psi(v) \in \Re^m$ and the following set:

$$S(v_0) = \{v \in \Re^m, w \in \Re^m; -v_0 \preceq v - w \preceq v_0\} \tag{6}$$

Lemma 1 (Tarbouriech et al, IEEE-TAC2006) *If the vectors v and w belong to the set $S(v_0)$ then the nonlinearity $\psi(v)$ satisfy the following inequality:*

$$\psi(v)'T(\psi(v) + w) \leq 0 \tag{7}$$

for any diagonal positive definite matrix $T \in \Re^{m\times m}$.

☞ In the case of the nonlinearity $\psi(y_c)$, this lemma is applied by considering the following:

$$v = \mathbb{K}\xi + F_c\psi$$

$$w = G_1\xi + G_2\psi$$

$$v_0 = u_0$$

▷ Do we have to choose a particular structure for G_1 and G_2?

4. Preliminary results (2)

☞ Moreover, the nonlinearity $\psi(y_c)$, satisfies the following lemma, directly derived from previous lemma.

☞ Thus, by considering $w = E_1\xi + y_c = (E_1 + \mathbb{K})\xi + F_c\psi(y_c)$, one gets:

Lemma 2 *Consider a matrix $E_1 \in \Re^{m\times n}$. If ξ belongs to $S(u_0)$*

$$S(u_0) = \{\xi \in \Re^{n+n_c}; -u_0 \preceq E_1\xi \preceq u_0\} \tag{8}$$

then the nonlinearity $\psi(y_c)$ satisfies the following inequality:

$$\psi(y_c)'T(\psi(y_c) + (E_1 + \mathbb{K})\xi + F_c\psi(y_c)) \leq 0 \tag{9}$$

or equivalently

$$\psi(y_c)'T(sat(y_c) + E_1\xi) \leq 0 \tag{10}$$

for any diagonal positive definite matrix $T \in \Re^{m\times m}$.

▷ This solution corresponds to choose $G_1 = E_1 + \mathbb{K}$ and $G_2 = F_c$

5. Theoretical results (1)

Local stability

Proposition 1 *If there exist a positive definite symmetric matrix $W \in \Re^{(n+n_c)\times(n+n_c)}$, a positive definite diagonal matrix $S \in \Re^{m\times m}$, two matrices $Z_1 \in \Re^{n_c\times m}$, $Z_2 \in \Re^{n_c\times m}$ and two matrices $Y_1 \in \Re^{m\times(n+n_c)}$, $Y_2 \in \Re^{m\times m}$ such that:*

$$\begin{bmatrix} W\mathbb{A}' + \mathbb{A}W & \mathbb{B}S + \mathbb{R}Z_1 + \mathbb{B}Z_2 - W\mathbb{K}' - Y_1' \\ S\mathbb{B}' + Z_1'\mathbb{R}' + Z_2'\mathbb{B}' - Y_1 - \mathbb{K}W & -2S - Z_2 - Z_2' \end{bmatrix} < 0 \qquad (11)$$

$$\begin{bmatrix} W & Y_{1(i)}' \\ Y_{1(i)} & u_{0(i)}^2 \end{bmatrix} \geq 0, \; i = 1, ..., m \qquad (12)$$

then

- ▷ *the anti-windup gains* $E_c = Z_1 S^{-1}$ *and* $F_c = Z_2 S^{-1}$
- ▷ *and the set* $\mathcal{E}(P,1) = \{\xi \in \Re^{n+n_c}; \xi' W^{-1} \xi \leq 1\}$, *with* $P = W^{-1}$,

solve Problem SAW2.

☞ Proof.

- ▷ It follows the same lines as the proof of Proposition 1 of the previous chapter V.
- ▷ We use the modified generalized sector condition of Lemma 1 with $G_1 = Y_1 W^{-1}$ and $G_2 = Y_2 S^{-1}$, to ensure that the set $\mathcal{E}(P,1) = \{\xi \in \Re^{n+n_c}; \xi' P \xi \leq 1\}$, with $P = W^{-1}$, is included in the domain $S(u_0)$ defined by (8).
- ▷ Hence, one has to verify:

$$\xi' E'_{1(i)} \frac{1}{u^2_{0(i)}} E_{1(i)} \xi \leq 1$$

$$\text{for all } \xi \text{ such that } \xi' P \xi \leq 1$$

- ▷ The satisfaction of relation (12) (using the Schur's complement) guarantees that the set $\mathcal{E}(W^{-1},1)$ is included in the domain $S(u_0)$ defined in (8).

5. Theoretical results (3)

☞ Proof (continued).

▷ Consider the quadratic Lyapunov function $V(\xi) = \xi' P \xi$ with $P = P' > 0$. Its time derivative along the closed-loop system trajectories for all $\xi \in \mathcal{E}(P, 1)$ satisfies:

$$\dot{V}(\xi) \leq \dot{V}(\xi) - 2\psi' T(sat(y_c) + E_1 \xi)$$

▷ The right-hand term in the previous inequality can be written as:

$$\begin{bmatrix} \xi' P & \psi' T \end{bmatrix} M \begin{bmatrix} P\xi \\ T\psi \end{bmatrix} \text{ with}$$

$$M = \begin{bmatrix} W\mathbb{A}' + \mathbb{A}W & \mathbb{B}S + \mathbb{R}Z_1 + \mathbb{B}Z_2 - W\mathbb{K}' - Y_1' \\ S\mathbb{B}' + Z_1'\mathbb{R}' + Z_2'\mathbb{B}' - Y_1 - \mathbb{K}W & -2S - Z_2 - Z_2' \end{bmatrix}$$

where $W = P^{-1}$, $S = T^{-1}$, $Z_1 = E_c S$, $Z_2 = F_c S$, and $Y_1 = E_1 W$.

▷ Hence, to satisfy $\dot{V}(\xi) < 0$, we have to satisfy $M < 0$. The satisfaction of (11) ensures that.

Global stability

☞ The previous result gives a condition for the local stability of system (5) in an ellipsoidal region. In the global stability context, the following corollary can be deduced.

Corollary 1 *If there exist a positive definite symmetric matrix* $W \in \Re^{(n+n_c)\times(n+n_c)}$*, a positive definite diagonal matrix* $S \in \Re^{m\times m}$ *and two matrices* $Z_1 \in \Re^{n_c\times m}$ *and* $Z_1 \in \Re^{m\times m}$ *satisfying:*

$$\begin{bmatrix} W\mathbb{A}' + \mathbb{A}W & \mathbb{B}S + \mathbb{R}Z_1 + \mathbb{B}Z_2 - W\mathbb{K}' \\ S\mathbb{B}' + Z_1'\mathbb{R}' + Z_2'\mathbb{B}' - \mathbb{K}W & -2S - Z_2 - Z_2' \end{bmatrix} < 0 \quad (13)$$

then for $E_c = Z_1 S^{-1}$ *and* $F_c = Z_2 S^{-1}$*, the system (5) is globally asymptotically stable.*

5. Theoretical results (5)

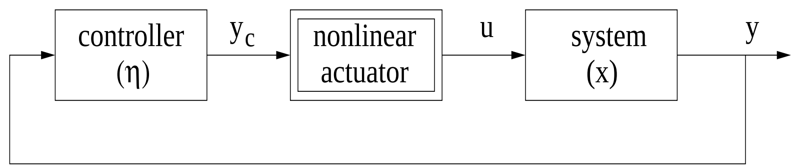
☞ Proof.

- ▷ We consider $E_1 = 0$.
- ▷ Applying Lemma 2 of this chapter, we verify that the sector condition (9) is satisfied for all $\xi \in \Re^{n+n_c}$.
- ▷ The satisfaction of the inequality in Corollary 1 ensures that $\dot{V}(\xi) < 0$, $\forall$ $\xi \in \Re^{n+n_c}$.

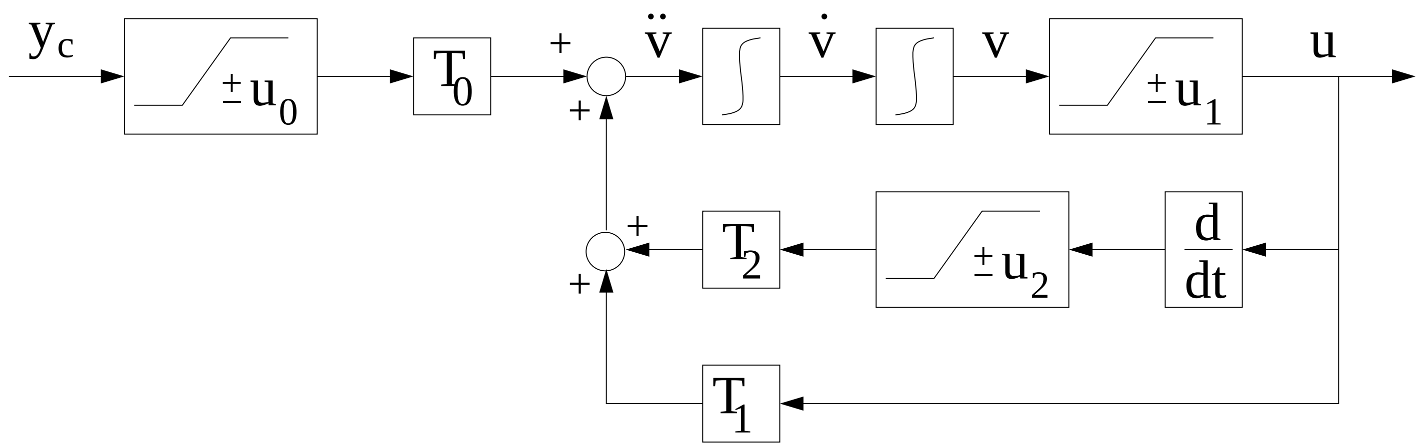
6. Illustrative example (1)

Example 1

☞ Let us consider the following general scheme of a closed-loop system with nonlinear actuator:



☞ Let us consider the actuator described by the following scheme:



▷ Such an actuator corresponds to what may be found in launchers (such as Ariane5). It allows to represent limitations in tuyère angles and their time-derivative, which are of main importance during some phases of the flight, and especially during the atmospheric phase.

6. Illustrative example (2)

☞ The state-space model of the process writes:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned}$$

with

$$A = 0.1 \; ; \; B = 1 \; ; \; C = 1$$

☞ The controller (2) is a PI controller defined by:

$$A_c = 0 \; ; \; B_c = -0.2 \; ; \; C_c = 1 \; ; \; D_c = -2$$

☞ The model of the actuator writes:

$$\begin{aligned} \ddot{v} &= T_0 sat_{u_0}(y_c) + T_1 u + T_2 sat_{u_2}(\dot{u}) \\ u &= sat_{u_1}(v) \end{aligned} \tag{14}$$

with

$$T_0 = 25 \; ; \; T_1 = -25 \; ; \; T_2 = -10 \; ; \; u_0 = 3 \; ; \; u_1 = 2 \; ; \; u_2 = 50$$

6. Illustrative example (3)

☞ To express the state space equation of this actuator,

▷ one defines the following state-space vector: $x_a = \begin{bmatrix} v \\ \dot{v} \end{bmatrix} \in \Re^2$

▷ From relations (14), the following state-space equations of the actuator directly follow:

$$\begin{aligned} \dot{x}_a(t) &= A_a x_a(t) + B_{a0} sat_{u_0}(y_c(t)) + B_{a1} sat_{u_1}(C_a x_a(t)) \\ &\quad + B_{a2} sat_{u_2}(sat_{u_1}(C_a x_a(t))^{(1)}) \\ u(t) &= sat_{u_1}(C_a x_a(t)) \end{aligned} \tag{15}$$

with

$$A_a = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \in \Re^{2\times 2}; B_{a0} = \begin{bmatrix} 0 \\ T_0 \end{bmatrix} \in \Re^{2\times 1}; B_{a1} = \begin{bmatrix} 0 \\ T_1 \end{bmatrix} \in \Re^{2\times 1}$$

$$B_{a2} = \begin{bmatrix} 0 \\ T_2 \end{bmatrix} \in \Re^{2\times 1}; C_a = \begin{bmatrix} 1 & 0 \end{bmatrix} \in \Re^{1\times 2}$$

▷ In (15), $sat_{u_1}(C_a x_a(t))^{(1)}$ corresponds to the time derivative of $sat_{u_1}(C_a x_a(t))$.

6. Illustrative example (4)

☞ Let us consider the augmented vector $\xi = \begin{bmatrix} x \\ x_a \\ \eta \end{bmatrix} \in \Re^4$. The closed-loop system writes:

$$\dot{\xi}(t) = \mathbb{A}_0\xi(t) + \mathbb{B}_0 sat_{u_0}(\mathbb{K}\xi(t)) + \mathbb{B}_1 sat_{u_1}(\mathbb{C}\xi(t)) + \mathbb{B}_2 sat_{u_2}(sat_{u_2}(\mathbb{C}\xi(t))^{(1)}) \quad (16)$$

with

$$\mathbb{A}_0 = \begin{bmatrix} A & 0 & 0 \\ 0 & A_a & 0 \\ B_cC & 0 & A_c \end{bmatrix} \in \Re^{4\times4}; \quad \mathbb{B}_0 = \begin{bmatrix} 0 \\ B_{a0} \\ 0 \end{bmatrix} \in \Re^{4\times1}; \quad \mathbb{B}_1 = \begin{bmatrix} B \\ B_{a1} \\ 0 \end{bmatrix} \in \Re^{4\times1}$$

$$\mathbb{B}_2 = \begin{bmatrix} 0 \\ B_{a2} \\ 0 \end{bmatrix} \in \Re^{4\times1}; \quad \mathbb{C} = \begin{bmatrix} 0 & C_a & 0 \end{bmatrix} \in \Re^{1\times4}; \quad \mathbb{K} = \begin{bmatrix} D_cC & 0 & C_c \end{bmatrix} \in \Re^{1\times4}$$

6. Illustrative example (5)

- Let us assume that the variables $v = C_a x_a(t)$, $u = sat_{u_1}(C_a x_a(t))$ (variables of the actuator), y_c and $sat_{u_0}(y_c)$ (input of the actuator) are measured.

- Then, we want to add the following terms:

$$
\begin{aligned}
\psi_0 &= sat_{u_0}(y_c(t)) - y_c(t) \\
\psi_1 &= sat_{u_1}(C_a x_a(t)) - C_a x_a(t) \\
\psi_1^{(1)} &= (sat_{u_1}(C_a x_a(t)) - C_a x_a(t))^{(1)}
\end{aligned}
$$

to the dynamic controller and, by the way, to the system, via the adequate gains.

- This modified controller then becomes:

$$
\begin{aligned}
\dot{\eta}(t) &= A_c \eta(t) + B_c C x(t) \\
&\quad + G_c(sat_{u_0}(y_c(t)) - y_c(t)) + E_c(sat_{u_1}(C_a x_a(t)) - C_a x_a(t)) \\
y_c(t) &= C_c \eta(t) + D_c C x(t) \\
&\quad + F_c(sat_{u_1}(C_a x_a(t)) - C_a x_a(t)) + H_{c1}(sat_{u_1}(C_a x_a(t)) - C_a x_a(t))^{(1)}
\end{aligned} \tag{17}
$$

▷ E_c, F_c, G_c and H_{c1} are the anti-windup gains to determine.

☞ Let us also consider the nonlinearity:

$$\psi_2 = sat_{u_2}(sat_{u_1}(\mathbb{C}\xi(t))^{(1)}) - sat_{u_1}(\mathbb{C}\xi(t))^{(1)}$$

☞ One can check that, according to the structure of $\mathbb{C}$ and $\mathbb{B}_i$, all products $\mathbb{C}\mathbb{B}_i = 0$.

☞ The full system finally writes:

$$\dot{\xi}(t) = \mathbb{A}\xi(t) + (\mathbb{B}_0 + \mathbb{R}G_c)\psi_0 + (\mathbb{B}_1 + \mathbb{R}E_c + \mathbb{B}_0F_c)\psi_1 + \mathbb{B}_2\psi_2 + (\mathbb{B}_2 + \mathbb{B}_0H_{c1})\psi_1^{(1)}$$

with

$$\mathbb{A} = \mathbb{A}_0 + \mathbb{B}_0\mathbb{K} + \mathbb{B}_1\mathbb{C} + \mathbb{B}_2\mathbb{C}\mathbb{A}_0 = \begin{bmatrix} A & BC_a & 0 \\ B_{a0}D_cC & A_a + \begin{bmatrix} B_{a1} & B_{a2} \end{bmatrix} & B_{a0}C_c \\ B_cC & 0 & A_c \end{bmatrix} ; \ \mathbb{R} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

☞ At this stage, one can directly modify Proposition 1 to solve this "slightly" modified problem.

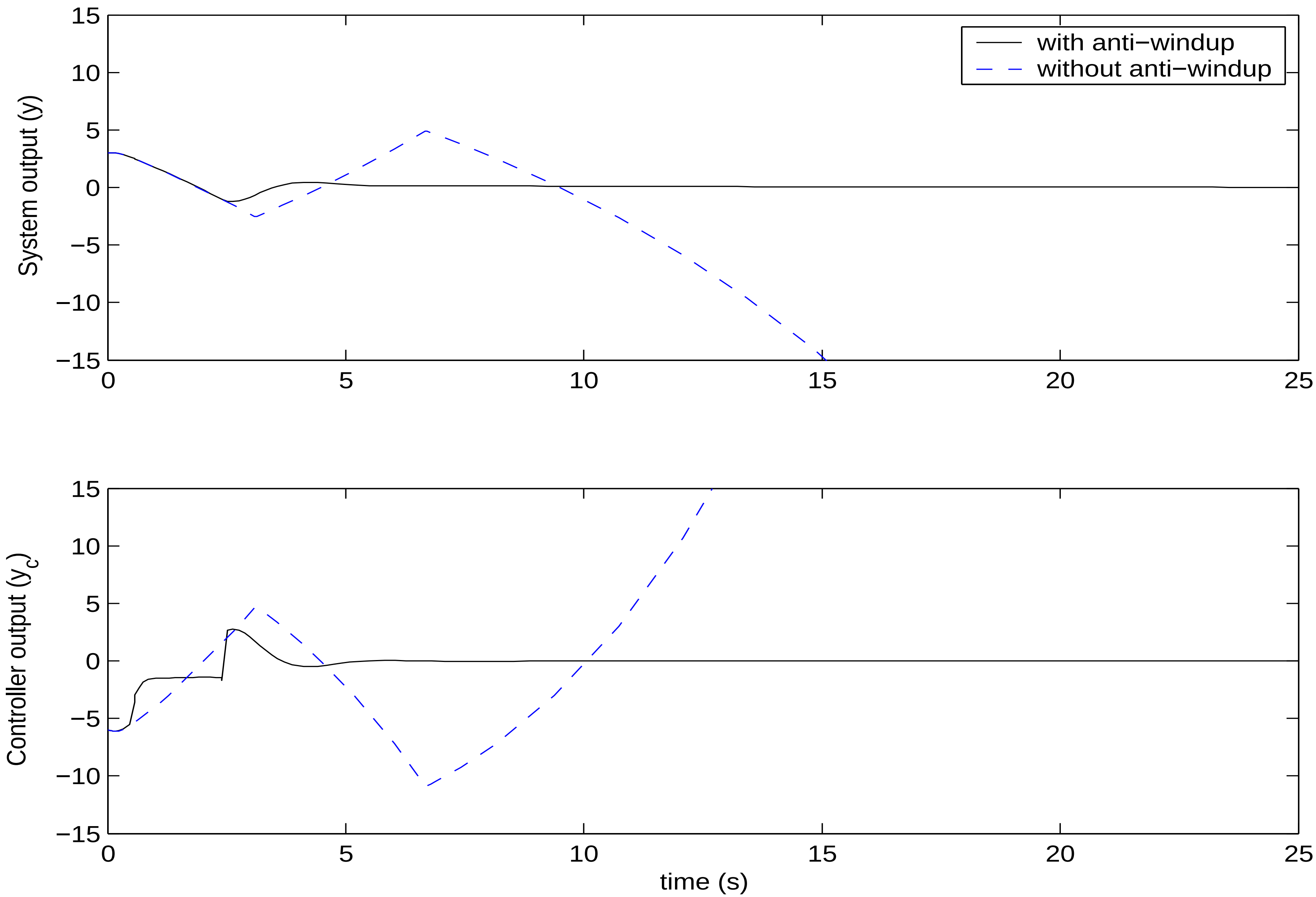
☞ The following anti-windup gains are obtained:
$E_c = 0.1417$, $F_c = 3.2280$, $G_c = 0.0711$, $H_{c1} = 0.4000$

6. Illustrative example (7)

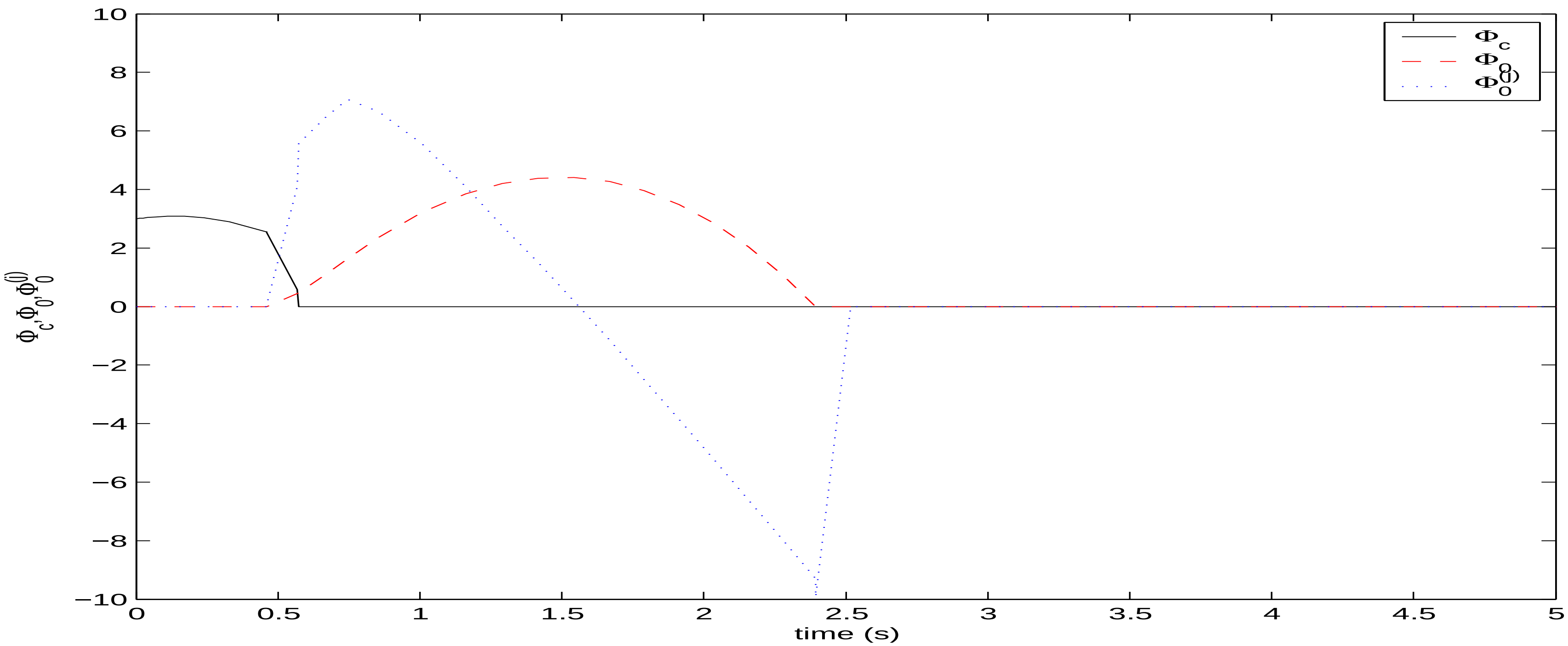
☞ The closed-loop system is simulated, with and without an anti-windup strategy, for the initial condition $\xi(0) = \begin{bmatrix} 3 & 0 & 0 & 0 \end{bmatrix}'$.

- ▷ The time evolution of the system output $y(t)$ and of the controller output $y_c(t)$ are plotted for the cases with (solid line) and without anti-windup (dashed line).
- ▷ The time evolution of the new controller inputs are also plotted: ψ_0 (solid line), ψ_1 (dashed line) and $\psi_1^{(1)}$ (dotted line).

6. Illustrative example (8)



6. Illustrative example (9)



Example 2

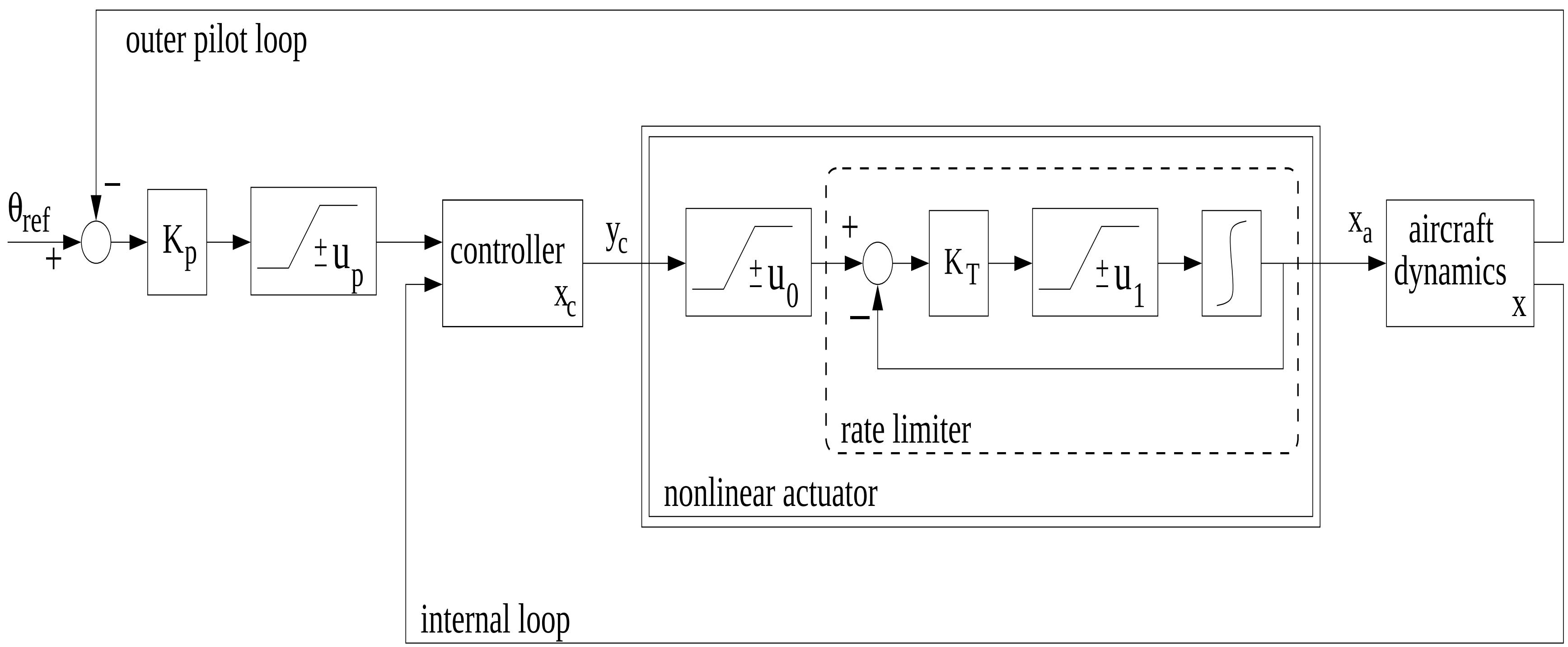
☞ GARTEUR project AG15. The anti-windup technique is applied in the context of two aircraft models: the ADMIRE model (FOI) and the HAVELIMITS model (University of Leicester).

- ▷ Thus, the anti-windup compensator designs are proposed to deal with amplitude and/or rate-limits imposed in those system on control vector.
- ▷ The main objective is twofold and consists in maximizing both the region of stability of the complete closed-loop system (aircraft + actuator + sensor + controller + pilot) and the set of admissible tracking references.
- ▷ The tracking references under consideration are defined in terms of both admissible amplitude and rate limitations.

6. Illustrative example (11)

☞ Let us consider the Admire model (ADMIRE release 3.4c) and in particular pitch control loop, where the actuator is subject to position and rate-limiting constraints.

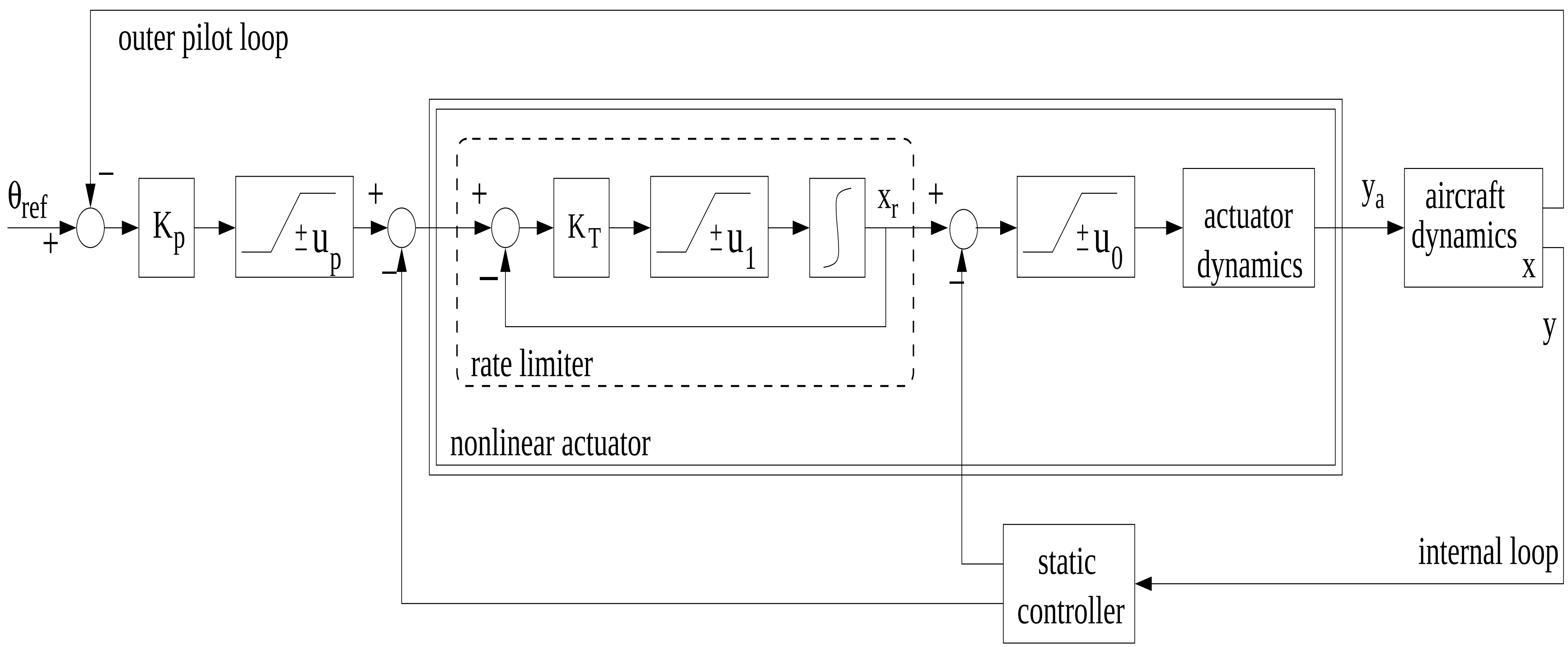
▷ A block diagram of the reduced linear model related to the longitudinal dynamics is described as follows.



6. Illustrative example (12)

☞ Let us turn now to the HAVELIMITS model, which has been generated from a data based issued from flight tests using the USAF Flight Dynamics Laboratory NT-33A aircraft, an extensively modified Lockheed T-33 jet trainer.

▷ A block diagram of the reduced linear model related to the longitudinal dynamics is described as follows.



☞ The main differences with respect to ADMIRE simulator are that:

▷ the controller is a static output feedback controller which acts both at the input of the rate limiter (digital action) and at the input of the saturation element (analogical action);

▷ the rate limiter is placed before the position saturation in the actuator model.

☞ Both ADMIRE and HAVELIMITS models involve a saturation in the pilot loop and a position plus rate limiters in the internal control loop. The rate limiter is modelled by an equivalent model involving a position saturation.

6. Illustrative example (14)

☞ We consider two types of anti-windup scheme:

▷ direct anti-windup loop

▷ use of a fictitious linear actuator.

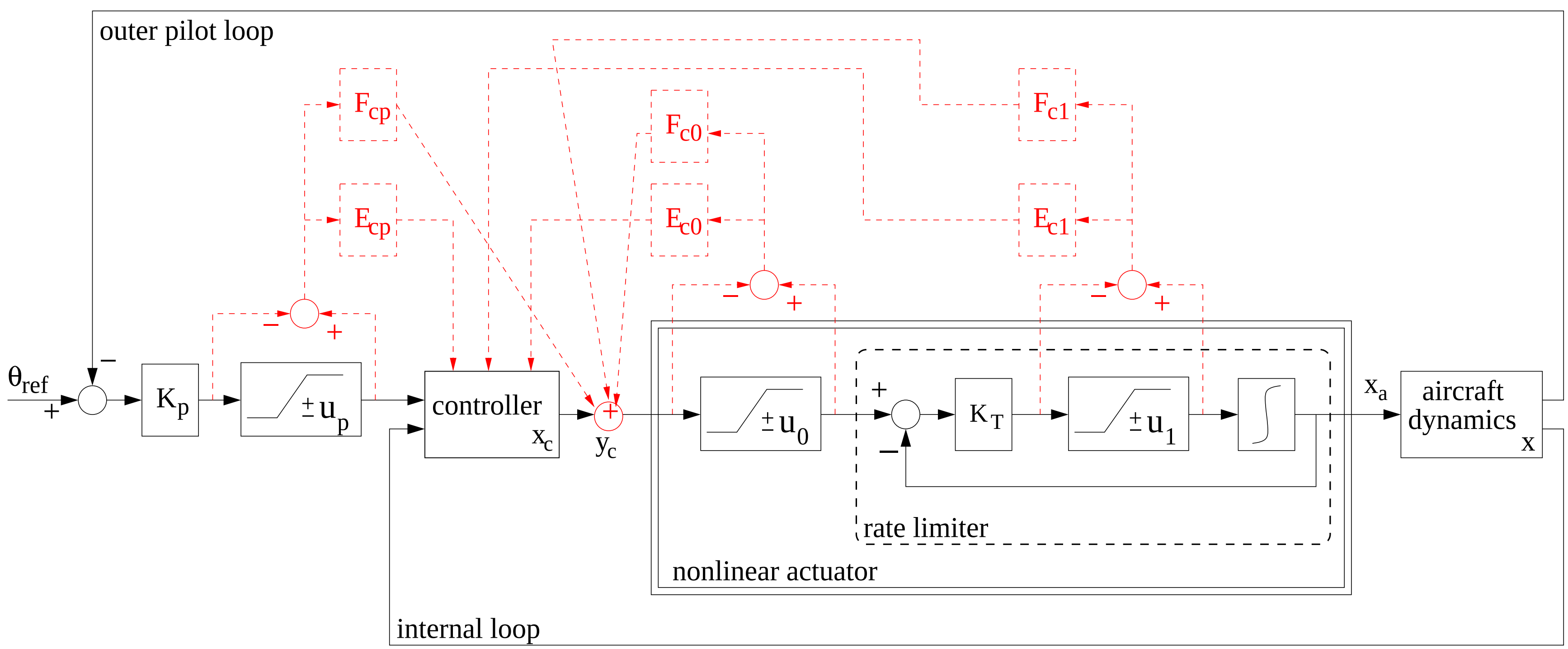
☞ The idea of the direct strategy is to build the anti-windup action by using the direct difference between the input and output of the three saturation elements.

▷ ADMIRE and HAVELIMITS models involving direct anti-windup gains are shown on the following figures

6. Illustrative example (15)

☞ The key point is that the controller dynamics and output are modified with additional terms of the generic form $Ec\psi$ and $Fc\psi$, respectively, that is:

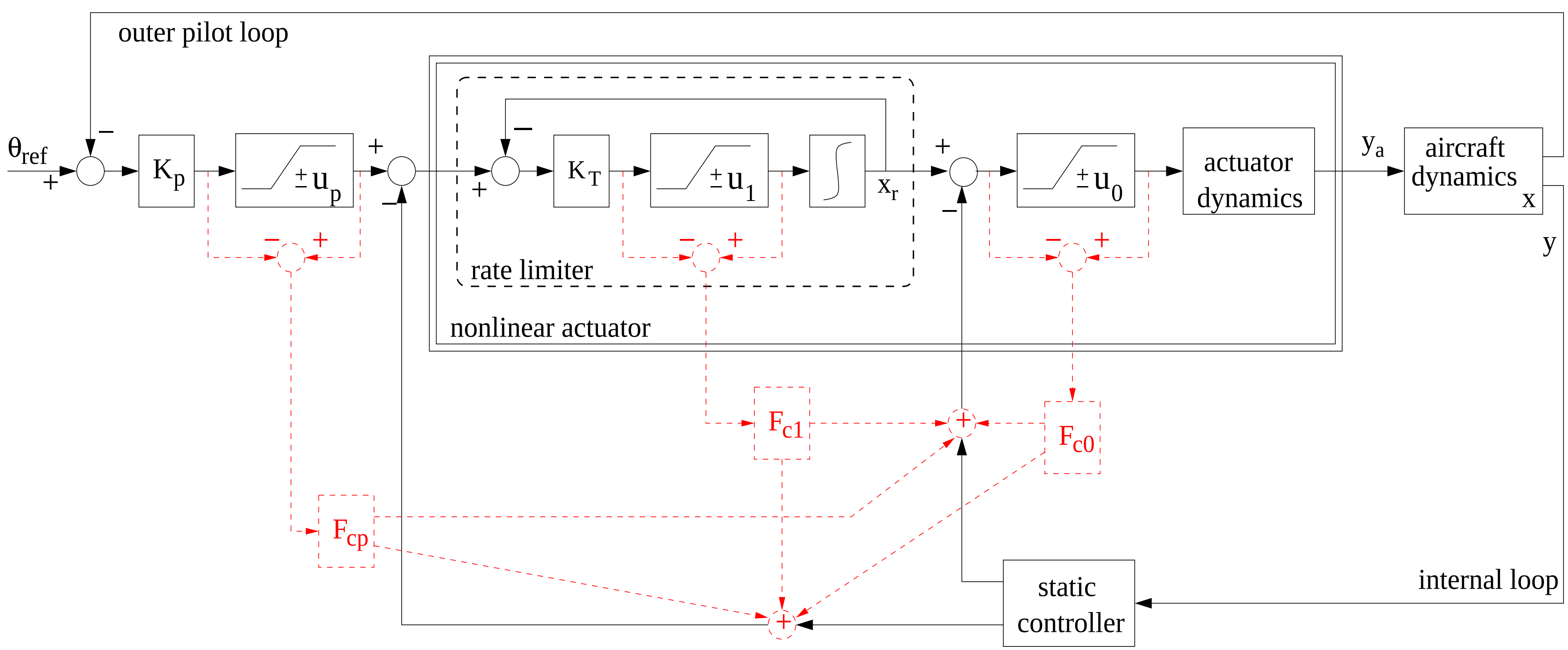
$$\dot{x}_c = \star \rightarrow \dot{x}_c = \star + \begin{bmatrix} E_{cp} & E_{c0} & E_{c1} \end{bmatrix} \psi$$
$$y_c = \star \rightarrow y_c = \star + \begin{bmatrix} F_{cp} & F_{c0} & F_{c1} \end{bmatrix} \psi$$



6. Illustrative example (16)

☞ HAVELIMITS is controlled by a static controller. Therefore, the anti-windup only acts at the output of the controller, that is,

$$y_c = \star \rightarrow y_c = \star + \left[\begin{array}{ccc} F_{cp} & F_{c0} & F_{c1} \end{array} \right] \psi$$



6. Illustrative example (17)

☞ Let us now consider the case where it is assumed that we only have access to the input and output of the full nonlinear dynamic actuator, but not to the internal signals in the actuator.

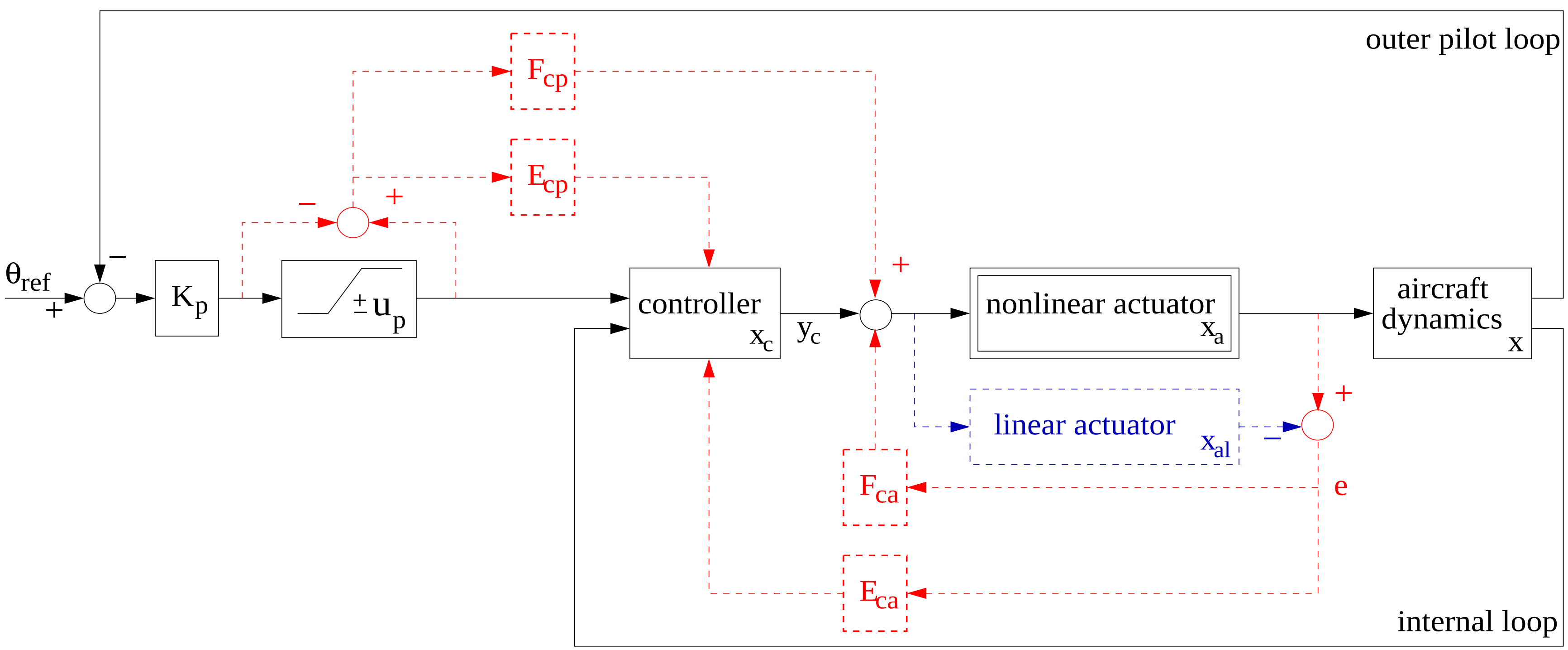
☞ In that case, the output of the nonlinear dynamic actuator is compared with a fictitious linear actuator (same dynamics but without saturations) and the difference e is injected in the dynamic state equation of the controller x_c and in the controller output y_c through static anti-windup gains E_{ca} and F_{ca}, respectively.

☞ The difference between the input and output of the saturation stick limits in the outer pilot loop is, as in the previous direct AW strategy, injected through static gains both in the dynamic state equation and at the output of the predesigned controller.

6. Illustrative example (18)

☞ In the ADMIRE model, the controller dynamics and output are now modified with additional terms as follows:

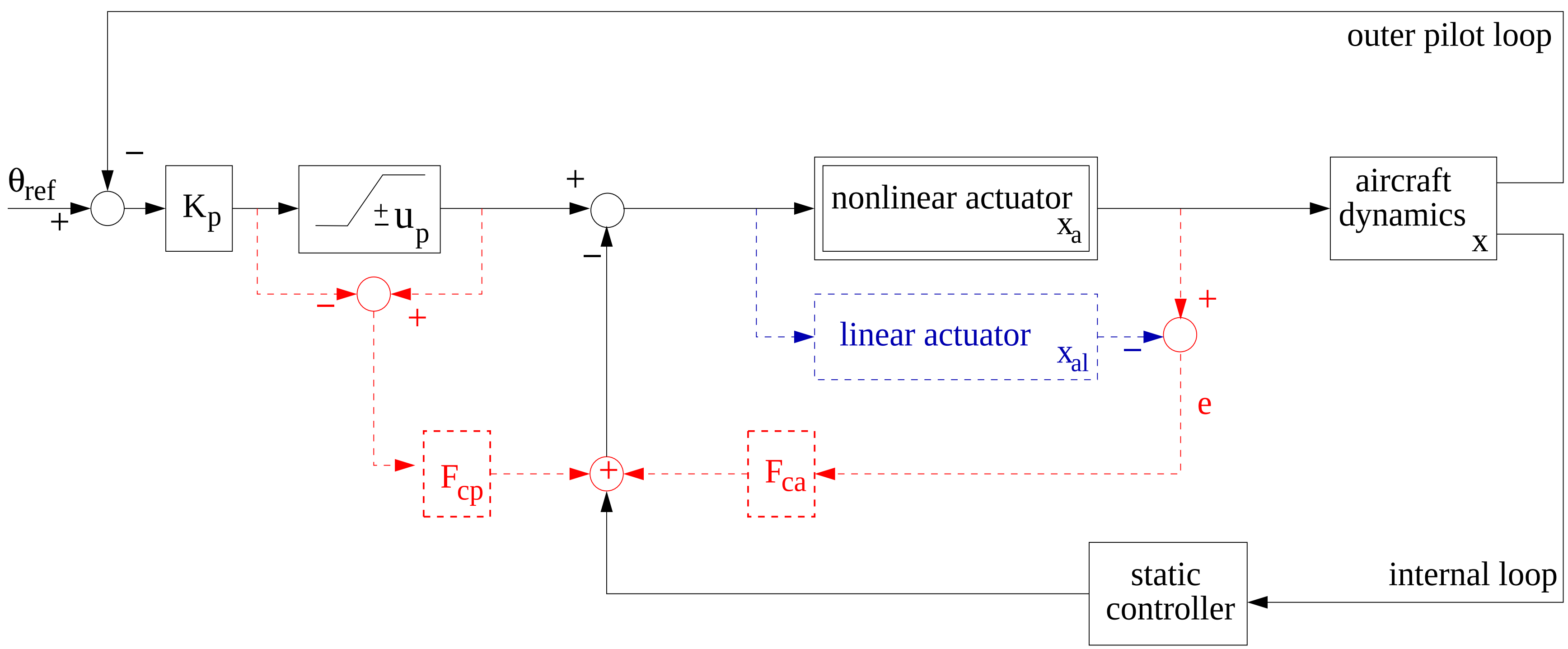
$$\dot{x}_c = \star \rightarrow \dot{x}_c = \star + \begin{bmatrix} E_{cp} & 0 & 0 \end{bmatrix} \psi + E_{ca} e$$
$$y_c = \star \rightarrow y_c = \star + \begin{bmatrix} F_{cp} & 0 & 0 \end{bmatrix} \psi + F_{ca} e$$



6. Illustrative example (19)

☞ In the HAVELIMITS model, the controller output is now modified with additional terms as follows:

$$y_c = \star \rightarrow y_c = \star + \left[\begin{array}{ccc} F_{cp} & 0 & 0 \end{array} \right] \psi + F_{ca} e$$



6. Illustrative example (20)

☞ ADMIRE flight simulator. Let us consider the pitch-axis control (longitudinal system).

- The flight condition used for numerical evaluation are: Mach 0.25, altitude 500m.
- For this flight condition, the pitch-loop pilot gain K_p was chosen as 1378.
- The reduced and transformed longitudinal ADMIRE model is of dimension $n_r = 7$, which corresponds to remove 23 states from the full closed-loop state-space model formed with the control state x_c of dimension $n_c = 4$, the longitudinal state x of dimension $n = 24$ and the actuator state x_a of dimension $m = 2$.
- Actuator limitations are:
  - Canard angles position limits: $\pm$ 25 deg
  - Elevon position limits: $\pm$ 30 deg
  - Canard angles rate limits: $\pm$ 50 deg/s
  - Elevon rate limits: $\pm$ 150 deg/s
  - Pilot stick limits: $\pm$ 80 Newton

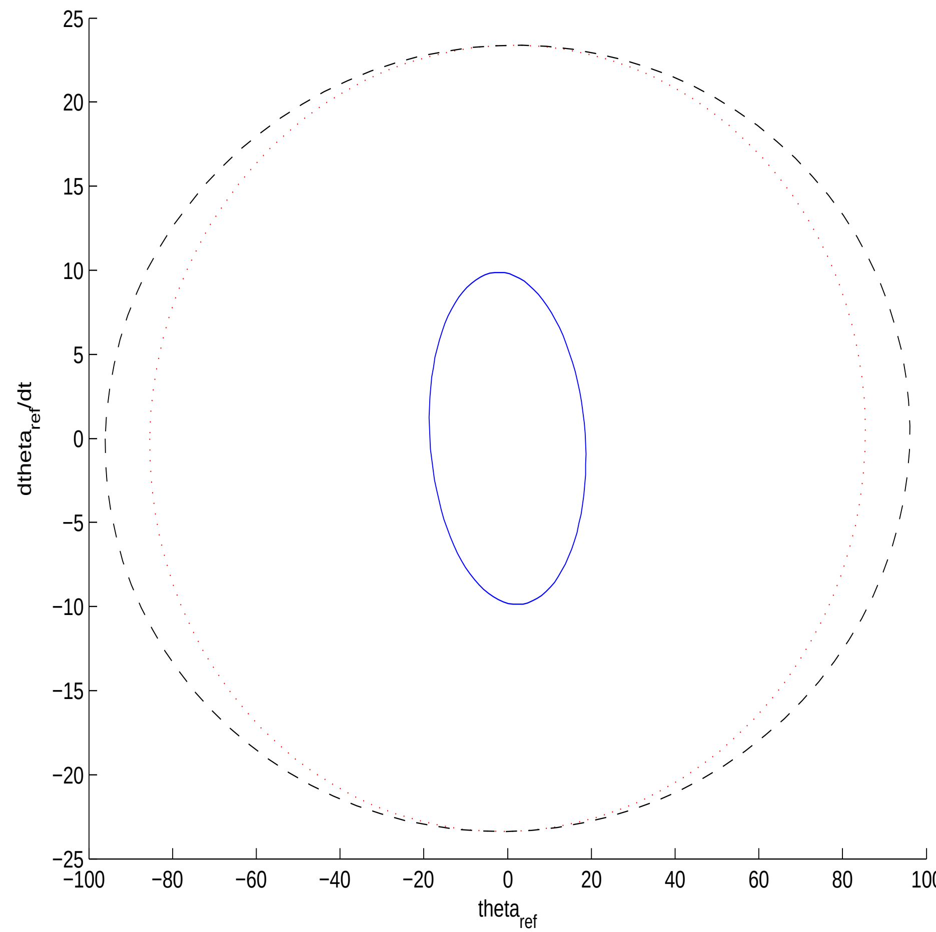
6. Illustrative example (21)

☞ One can modify Proposition 1 to solve the current problem: Ellipsoid of admissible tracking references

☞ Three cases are tested:

- ▷ Analysis $E_c = 0$ and $F_c = 0$: solid line
- ▷ direct anti-windup loop: dashed line
- ▷ use of a fictitious linear actuator: dotted line

☞ The anti-windup strategy using the direct effect of nonlinear elements allows to obtain slightly better results than that one using a fictitious model of the actuator.



7. Conclusion and prospectives

Conclusion

- A solution to enlarge the obtained stability region has been presented.
- The conditions are formulated with LMIs.
- The results have been illustrated on two applications issued from aerospace domain: launcher and combat aircraft.

Prospectives

- Observation of non-measured variables which are necessary to anti-windup loops
  - Partial elements or response by using a fictitious linear actuator.
  - Use of observer to estimate the needed variables.
- Dynamic anti-windup
- Take into account the performances