Constructive LMI Approaches for Anti-Windup Compensator Design for Systems Subject to Saturation

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Part V - Solution via static anti-windup (1 loop)

Outline of part IV

1. System description
2. Problem statement
3. Preliminary results
4. Quadratic Lyapunov function
5. Lure type Lyapunov function
6. Numerical procedure
7. Illustrative examples
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1. System description (1)

☞ Consider the system

$$\begin{cases} \dot{x}(t) & = & Ax(t) + Bv(t) \\ v(t) & = & sat_{u_0}(u(t)) \\ y(t) & = & Cx(t) \end{cases} \quad (1)$$

- ▷ $x \in \Re^n$, $v \in \Re^m$ and $y \in \Re^p$ are the state, the input and the measured output.
- ▷ A, B and C are real matrices of appropriate dimensions.
- ▷ Pairs (A, B) and (C, A) are respectively controllable and observable.
- ▷ The vector u_0 is the input saturation limit.

1. System description (2)

☞ We suppose that a dynamic controller of order n_c has been designed (without taking into account the saturation):

$$\begin{array}{rcl} \dot{\eta}(t) & = & A_c\eta(t) + B_c u_c(t) \\ y_c(t) & = & C_c\eta(t) + D_c u_c(t) \end{array} \quad (2)$$

▷ $\eta(t) \in \Re^{n_c}$ is the controller state, $u_c = y(t)$ is the controller input and $y_c(t)$ is the controller output.

▷ Matrices A_c, B_c, C_c, D_c are of appropriate dimensions.

1. System description (3)

- This controller is such that under the interconnection

$$v = u = y_c \ ; \ u_c = Cx \tag{3}$$

with the original system, the closed-loop system is asymptotically stable.

- Due to the presence of input saturation, the real interconnection can be expressed as:

$$u = y_c \ ; \ v = sat_{u_0}(y_c) \ ; \ u_c = Cx \tag{4}$$

- The full closed-loop system is then given by:

$$\begin{aligned} \dot{x} &= Ax + Bsat_{u_0}(C_c\eta + D_cCx) \\ \dot{\eta} &= A_c\eta + B_cCx \end{aligned} \tag{5}$$

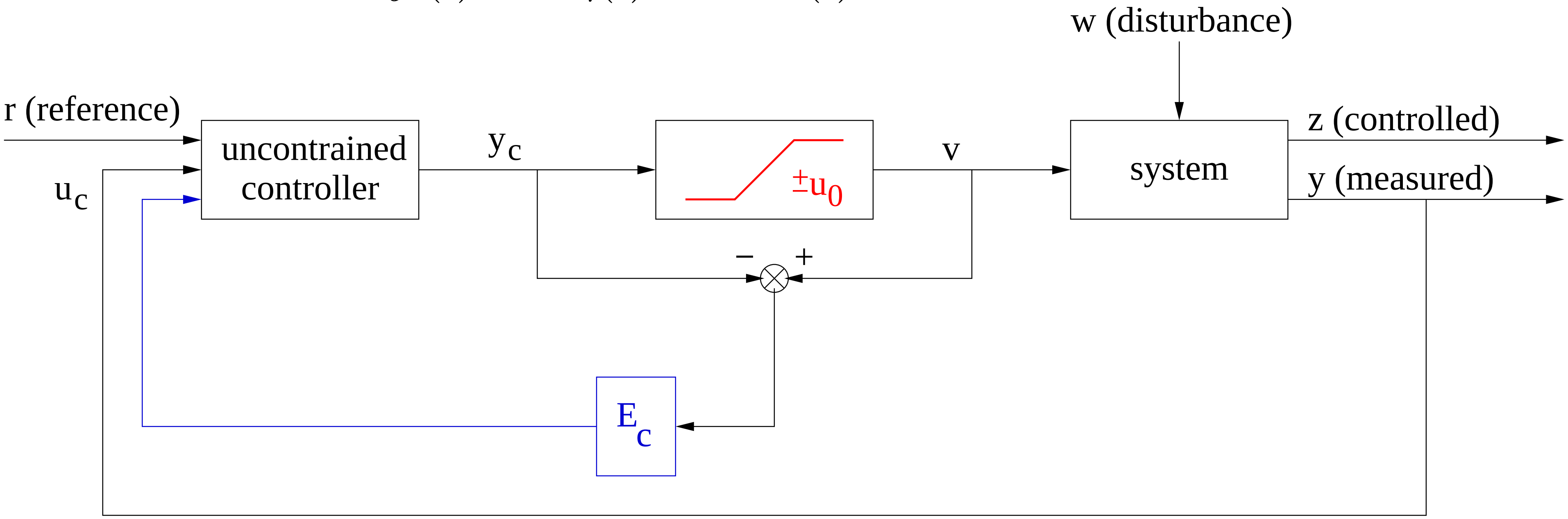
2. Problem statement (1)

☞ The measured variables available to implement anti-windup scheme are:

$$v = sat_{u_0}(y_c) \text{ (system input) and } y_c \text{ (controller output)} \tag{6}$$

☞ To mitigate the windup effects, an anti-windup term $E_c(sat(y_c(t)) - y_c(t))$ is added to the controller dynamic equation:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bsat(y_c(t)) \\ \dot{\eta}(t) &= A_c\eta(t) + B_cCx(t) + E_c(sat(y_c(t)) - y_c(t)) \\ y_c(t) &= C_c\eta(t) + D_cCx(t) \end{aligned} \tag{7}$$

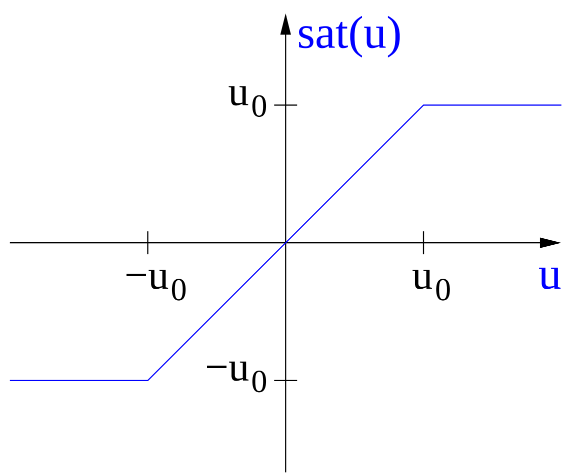


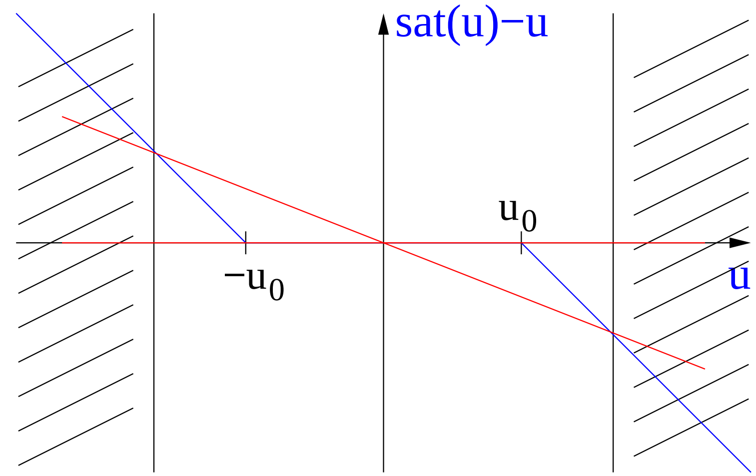
2. Problem statement (2)

☞ Define the augmented state $\xi(t) = \begin{bmatrix} x(t) \\ \eta(t) \end{bmatrix} \in \Re^{n+n_c}$ and

$$\psi(\mathbb{K}\xi(t)) = sat(y_c(t)) - y_c(t) \qquad (8)$$

☞ The nonlinearity $\psi(\mathbb{K}\xi(t))$ is a decentralized dead-zone nonlinearity.





☞ When saturation is inactive, we have $\psi(\mathbb{K}\xi(t)) = 0$. In that case, the closed-loop system is linear and the desired properties are obtained (stability and performances).

2. Problem statement (3)

☞ The closed-loop system writes:

$$\dot{\xi}(t) = \mathbb{A}\xi(t) + (\mathbb{B} + \mathbb{R}E_c)\psi(\mathbb{K}\xi(t)) \quad (9)$$

with

$$\mathbb{A} = \begin{bmatrix} A + BD_cC & BC_c \\ B_cC & A_c \end{bmatrix}; \mathbb{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}; \mathbb{R} = \begin{bmatrix} 0 \\ I_{n_c} \end{bmatrix}; \mathbb{K} = \begin{bmatrix} D_cC & C_c \end{bmatrix}$$

Remarks

- The matrix $\mathbb{A}$ is asymptotically stable by construction. Then when control is not limited, the closed-loop system should be globally stable.
- When the open loop system (matrix A) is unstable, only local stabilization is possible.
- In the context of local stability: necessity of characterizing attraction domain.
- Because the exact determination of the attraction domain is impossible in practice, an interesting problem is to determine an estimate of this domain (as best as possible).

2. Problem statement (4)

☞ Problem SAW1. Find a gain E_c and an asymptotic stability region for the saturated closed-loop system with anti-windup (9).

☞ An implicit objective is to maximize the size of the estimation of the stability region over an appropriate choice of gain E_c.

☞ Two types of Lyapunov function are used

▷ quadratic: $V(\xi(t)) = \xi(t)' P \xi(t), \; P = P' > 0$

▷ of Lure type: $V(\xi(t)) = \xi(t)' P \xi(t) - 2 \sum_{i=1}^{m} \int_0^{\mathbb{K}_{(i)}\xi} \psi(\sigma_{(i)}) N_{(i,i)} d\sigma_{(i)}$, where $P = P' > 0$ and N is a positive definite diagonal matrix.

☞ First recall an important property concerning the nonlinearity $\psi(\mathbb{K}\xi(t))$.

☞ Consider a matrix $G \in \Re^{m\times(n+n_c)}$ and define the polyhedral set:

$$\mathcal{S}(u_0) = \{\xi \in \Re^{n+n_c}\ ;\ -u_{0(i)} \leq (\mathbb{K}_{(i)} - G_{(i)})\xi \leq u_{0(i)},\ i = 1, ..., m\} \quad (10)$$

Lemma 1 *Consider the function $\psi(v)$ defined by (8). If $\xi \in \mathcal{S}(u_0)$ then the equation*

$$\psi(\mathbb{K}\xi)'T[\psi(\mathbb{K}\xi) + G\xi] \leq 0 \quad (11)$$

is satisfied for all positive definite diagonal matrix $T \in \Re^{m\times m}$.

Local stability

Proposition 1 *If there exist a positive definite symmetric matrix $W \in \Re^{(n+n_c)\times(n+n_c)}$, a positive definite diagonal matrix $S \in \Re^{m\times m}$, a matrix $Z \in \Re^{n_c\times m}$ and a matrix $Y \in \Re^{m\times(n+n_c)}$ such that:*

$$\begin{bmatrix} W\mathbb{A}' + \mathbb{A}W & \mathbb{B}S + \mathbb{R}Z - Y' \\ S\mathbb{B}' + Z'\mathbb{R}' - Y & -2S \end{bmatrix} < 0 \qquad (12)$$

$$\begin{bmatrix} W & W\mathbb{K}'_{(i)} - Y'_{(i)} \\ \mathbb{K}_{(i)}W - Y_{(i)} & u^2_{0(i)} \end{bmatrix} \geq 0, \; i = 1, ..., m \qquad (13)$$

then

- *the anti-windup gain* $E_c = ZS^{-1}$
- *and the set* $\mathcal{E}(P,1) = \{\xi \in \Re^{n+n_c}; \xi' W^{-1}\xi \leq 1\}$, *with* $P = W^{-1}$,

are solutions of Problem SAW1.

4. Solution via quadratic Lyapunov function (2)

☞ Proof.

▷ We use the modified sector condition of lemma 1 with $G = YW^{-1}$.

▷ The satisfaction of equation (13) guarantees that the set $\mathcal{E}(P,1) = \{\xi \in \Re^{n+n_c}; \xi' W^{-1} \xi \leq 1\}$, with $P = W^{-1}$, is included in the domain $\mathcal{S}(u_0)$ defined by (10).

▷ Consider the quadratic Lyapunov function $V(\xi) = \xi' P \xi$ with $P = P' > 0$. Its time derivative expression along the closed-loop system trajectories for all $\xi \in \mathcal{E}(P,1)$ verifies:

$$\dot{V}(\xi) \leq \dot{V}(\xi) - 2\psi(\mathbb{K}\xi)' T(\psi(\mathbb{K}\xi) + G\xi)$$

▷ The right-hand term of this inequality can be written as:

$$\begin{bmatrix} \xi' P & \psi' T \end{bmatrix} M \begin{bmatrix} P\xi \\ T\psi \end{bmatrix} \text{ with } M = \begin{bmatrix} W\mathbb{A}' + \mathbb{A}W & \mathbb{B}S + \mathbb{R}Z - Y' \\ S\mathbb{B}' + Z'\mathbb{R}' - Y & -2S \end{bmatrix}$$

where $W = P^{-1}$, $S = T^{-1}$, $Z = E_c S$ and $Y = GW$.

▷ To satisfy $\dot{V}(\xi) < 0$, we have to satisfy $M < 0$. The satisfaction of (12) ensures that.

Global stability

☞ The previous result gives a condition for the local stability of system (9) in an ellipsoidal region. In the global stability context, the following corollary can be deduced.

Corollary 1 *If there exist a positive definite symmetric matrix* $W \in \Re^{(n+n_c)\times(n+n_c)}$*, a positive definite diagonal matrix* $S \in \Re^{m\times m}$ *and a matrix* $Z \in \Re^{n_c\times m}$ *satisfying:*

$$\begin{bmatrix} W\mathbb{A}' + \mathbb{A}W & \mathbb{B}S + \mathbb{R}Z - W\mathbb{K}' \\ S\mathbb{B}' + Z'\mathbb{R}' - \mathbb{K}W & -2S \end{bmatrix} < 0 \qquad (14)$$

then for $E_c = ZS^{-1}$*, the system (9) is globally asymptotically stable.*

☞ Proof.

- ▷ We consider $G = \mathbb{K}$.
- ▷ Applying Lemma 1, we verify that the sector condition (11) is verified for all $\xi \in \Re^{n+n_c}$.
- ▷ The satisfaction of inequality in Corollary 1 ensures that $\dot{V}(\xi) < 0$, $\forall$ $\xi \in \Re^{n+n_c}$.

4. Solution via quadratic Lyapunov function (5)

Remarks

- By taking $G = \Lambda\mathbb{K}$ with Λ a diagonal matrix ($0 < \Lambda_{(i,i)} \leq 1$), we recover the case of the classical sector nonlinearity condition.
- In that case, conditions (12) and (13) become bilinear with respect to decision variables W and Λ.

Local stability

Proposition 2 *If there exist a positive definite symmetric matrix* $W \in \Re^{(n+n_c)\times(n+n_c)}$*, two positive definite diagonal matrices* $S \in \Re^{m\times m}$ *and* $N \in \Re^{m\times m}$*, a matrix* $Y \in \Re^{m\times(n+n_c)}$ *and a matrix* $Z \in \Re^{n_c\times m}$ *such that*

$$\begin{bmatrix} W\mathbb{A}' + \mathbb{A}W & \mathbb{B}S + \mathbb{R}Z - W\mathbb{A}'\mathbb{K}'NS - Y' \\ S\mathbb{B}' + Z'\mathbb{R}' - SN\mathbb{K}\mathbb{A}W - Y & -SN\mathbb{K}(\mathbb{B}S + \mathbb{R}Z) - (\mathbb{B}S + \mathbb{R}Z)'\mathbb{K}'NS - 2S \end{bmatrix} < 0 \tag{15}$$

$$\begin{bmatrix} W & W\mathbb{K}'_{(i)} - Y'_{(i)} \\ \mathbb{K}_{(i)}W - Y_{(i)} & u^2_{0(i)} \end{bmatrix} \geq 0, \ i = 1, ..., m \tag{16}$$

5. Solution via Lure Type Lyapunov Function (2)

then

- the set $\mathcal{D}(V,1) = \{\xi \in \Re^{n+n_c}; V(\xi) \leq 1\}$ with

$$V(\xi) = \xi' W^{-1}\xi - 2\sum_{i=1}^{m}\int_0^{\mathbb{K}_{(i)}\xi}\psi(\sigma_{(i)})N_{(i,i)}d\sigma_{(i)}$$

- and the gain $E_c = ZS^{-1}$

solve Problem SAW1.

5. Solution via Lure Type Lyapunov Function (3)

☞ Proof.

▷ The proof is similar to the one of proposition 1.

▷ We use the modified sector condition of lemma 1: The satisfaction of (16) ensures that the set $\mathcal{E}(P,1) = \{\xi \in \Re^{n+n_c}; \xi' P \xi \leq 1\}$ is included in the domain $\mathcal{S}(u_0)$ defined by (10).

▷ The following inclusion is satisfied (see chapter IV)

$$\mathcal{E}(P + \mathbb{K}' N \mathbb{K}, 1) \subset \mathcal{D}(V,1) \subset \mathcal{E}(P,1) \tag{17}$$

▷ Hence, we conclude that the satisfaction of (16) ensures that the set $\mathcal{D}(V,1) = \{\xi \in \Re^{n+n_c}; V(\xi) \leq 1\}$ is included in the domain $\mathcal{S}(u_0)$.

☞ Proof (continued).

▷ Consider the quadratic Lyapunov function

$$V(\xi) = \xi' P \xi - 2\sum_{i=1}^{m} \int_0^{\mathbb{K}_{(i)}\xi} \psi(\sigma_{(i)}) N_{(i,i)} d\sigma_{(i)}.$$

▷ Its time derivative along the closed-loop system trajectories writes:

$$\dot{V}(\xi) = \dot{\xi}' P \xi + \xi' P \dot{\xi} - 2\psi' N \mathbb{K} \dot{\xi}$$

and for all $\xi \in \mathcal{D}(V, 1)$

$$\dot{V}(\xi) \leq \dot{V}(\xi) - 2\psi(\mathbb{K}\xi)' T (\psi(\mathbb{K}\xi) + G\xi)$$

☞ Proof (continued).

▷ The right-hand term in the previous inequality can be written as:

$$\begin{bmatrix} \xi \\ \psi \end{bmatrix}' \begin{bmatrix} \mathbb{A}'P + P\mathbb{A} & P\mathbb{B} + P\mathbb{R}E_c - \mathbb{A}'\mathbb{K}'N - G'T \\ \mathbb{B}'P + E_c'\mathbb{R}'P - N\mathbb{K}\mathbb{A} - TG & -N\mathbb{K}(\mathbb{B} + \mathbb{R}E_c) - (\mathbb{B} + \mathbb{R}E_c)'\mathbb{K}'N - 2T \end{bmatrix} \begin{bmatrix} \xi \\ \psi \end{bmatrix}$$

▷ By pre and post-multiplying this previous matrix by $\begin{bmatrix} W & 0 \\ 0 & S \end{bmatrix}$, one gets relation (15).

▷ The satisfaction of relation (15) ensures that $\dot{V}(\xi) < 0$.

5. Solution via Lure Type Lyapunov Function (6)

Remarks

- Note that by setting $N = 0$ in (15), we retrieve relation (12) of Proposition 1.
- Relation (16) is linear in the decision variables (W and Y).
- Relation (16) is nonlinear in the decision variables, W, N, S and Z, due to the terms $SN\mathbb{K}\mathbb{A}W$ and $SN\mathbb{K}(\mathbb{B}S + \mathbb{R}Z)$ (and their symmetric).
  - ▷ If we fix $N = S^{-1}$, relation (16) becomes an LMI in the decision variables.
  - ▷ Such a solution can be the starting point of an algorithm based on relaxation schemes.
- Another important fact concerns the computation of the region of stability $\mathcal{D}(V, 1)$.
  - ▷ It is practically difficult to compute and therefore to maximize it.
  - ▷ Thus a solution consists in using the inclusion (17). Since $\mathcal{E}(W^{-1} + \mathbb{K}'N\mathbb{K}, 1) \subseteq \mathcal{D}(V, 1)$, one can try to maximize the size of the set $\mathcal{E}(W^{-1} + \mathbb{K}'N\mathbb{K}, 1)$ which will maximize implicitly the size of $\mathcal{D}(V, 1)$.

Global stability

☞ Like in the case of quadratic case (Corollary 1), the global stability can be obtained by taking $G = \mathbb{K}$.

Corollary 2 *If there exist a positive definite symmetric matrix* $W \in \Re^{(n+n_c)\times(n+n_c)}$*, two positive definite diagonal matrices* $S \in \Re^{m\times m}$ *and* $N \in \Re^{m\times m}$*, and a matrix* $Z \in \Re^{n_c\times m}$ *such that*

$$\begin{bmatrix} W\mathbb{A}' + \mathbb{A}W & \mathbb{B}S + \mathbb{R}Z - W\mathbb{A}'\mathbb{K}'NS - W\mathbb{K}' \\ S\mathbb{B}' + Z'\mathbb{R}' - SN\mathbb{K}\mathbb{A}W - \mathbb{K}W & -SN\mathbb{K}(\mathbb{B}S + \mathbb{R}Z) - (\mathbb{B}S + \mathbb{R}Z)'\mathbb{K}'NS - 2S \end{bmatrix} < 0 \quad (18)$$

then with the gain $E_c = ZS^{-1}$*, the system (9) is globally asymptotically stable.*

Optimization problems formulation

- In practice the following two cases are of interest:
  - ▷ Case 1. A set of admissible initial conditions, $\Xi_0 \subset \Re^{n+n_c}$, for which asymptotic stability is desired, is given. Can it really be stabilized by the anti-windup loop?
  - ▷ Case 2. What is the larger set of initial conditions in which the asymptotic stability is guaranteed? The objective is to design an anti-windup gain maximizing the size of the estimation of the associated attraction domain.
- Consider the case of a set Ξ_0 with a given form and a scale factor β. For example, suppose

$$\Xi_0 = Co\{v_1, v_2, \ldots, v_r\}, \; v_r \in \Re^{n+n_c}, \; r = 1, \ldots, n_r$$

6. Numerical procedure (2)

Quadratic approach

- (Proposition 1) We find W, Y, S, Z such $\beta\ \Xi_0 \subset \mathcal{E}(W^{-1}, 1)$.
- Ξ_0 defines the directions of maximization of $\mathcal{E}(W^{-1}, 1)$.
- The maximization of β can be done by solving the following optimization problem:
- **Algorithm 1.**

$$\begin{array}{c} \min\limits_{W,Z,Y,S,\mu} \mu \\ \text{under} \\ \text{relations } (12)-(13), \ \begin{bmatrix} \mu & v_r' \\ v_r & W \end{bmatrix} \geq 0, \ r = 1, \ldots, n_v \end{array} \qquad (19)$$

- One has $\beta = \frac{1}{\sqrt{\mu}}$.

6. Numerical procedure (3)

Lure approach

☞ From the inclusion

$$\mathcal{E}(W^{-1} + \mathbb{K}' N \mathbb{K}, 1) \subset \mathcal{D}(V, 1) \subset \mathcal{E}(W^{-1}, 1)$$

it follows that the inclusion $\beta \Xi_0 \subset \mathcal{D}(V, 1)$ can be obtained if

$$\beta \Xi_0 \subset \mathcal{E}(W^{-1} + \mathbb{K}' N \mathbb{K}, 1)$$

☞ **Algorithm 2.**

Step 1. Fix $N = S^{-1}$. Then the products NS and by the way N disappear from the conditions.

Step 2. Solve the following problem for W, S, Y and Z (N being fixed):

$$\min_{W,S,Y,Z,\mu} \mu$$

$$\text{subject to relations (15)} - \text{(16)}, \begin{bmatrix} \mu & v_r'\mathbb{K}' & v_r' \\ \mathbb{K}v_r & S & 0 \\ v_r & 0 & W \end{bmatrix} \geq 0, \; r = 1, \ldots, n_v$$

Step 3. Keep the values of W, Z and S and solve the following problem for N, Y and μ:

$$\min_{N,Y,\mu} \mu$$

subject to

$$\text{relations (15)} - \text{(16)}, \; \mu - v_r'(W^{-1} + \mathbb{K}'N\mathbb{K})v_r \geq 0, \; r = 1, \ldots, n_v$$

☞ One has $\beta = \frac{1}{\sqrt{\mu}}$.

6. Numerical procedure (5)

Constraint on the structure of the gain

☞ A constraint on the structure of the anti-windup gain can be considered in the optimization problem (19).

▷ Because $E_c = ZS^{-1}$ we have $E_{c(i,j)} = Z_{(i,j)}S^{-1}_{(j,j)}$. Thus, if $\begin{bmatrix} S_{(j,j)}\sigma & Z_{(i,j)} \\ Z_{(i,j)} & S_{(j,j)} \end{bmatrix} \geq 0$ then $\sigma - Z_{(i,j)}S^{-1}_{(j,j)}Z_{(i,j)}S^{-1}_{(j,j)} \geq 0$, that is $(E_{c(i,j)})^2 \leq \sigma$.

▷ Other structural constraints could be considered. For example, some elements of E_c can be fixed equal to zero.

Example 1

☞ Consider the following system which is open loop unstable:

$$\dot{x}(t) = 0.1x(t) + u(t)$$
$$y(t) = x(t)$$

☞ It can be stabilized by the following PI controller:

$$\dot{\eta}_c(t) = -0.2y(t)$$
$$v_c(t) = \eta_c(t) - 2y(t)$$

☞ We have:

$$u_0 = 1 \; ; \; \Xi_0 = Co\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} ; \begin{bmatrix} 1 \\ -1 \end{bmatrix} ; \begin{bmatrix} -1 \\ 1 \end{bmatrix} ; \begin{bmatrix} -1 \\ -1 \end{bmatrix} \}$$

☞] The quadratic approach (Algorithm 1) leads to:

$$\beta = 6.5956$$

$$W = \begin{bmatrix} 99.9997 & 32.3392 \\ 32.3392 & 145.3100 \end{bmatrix}$$

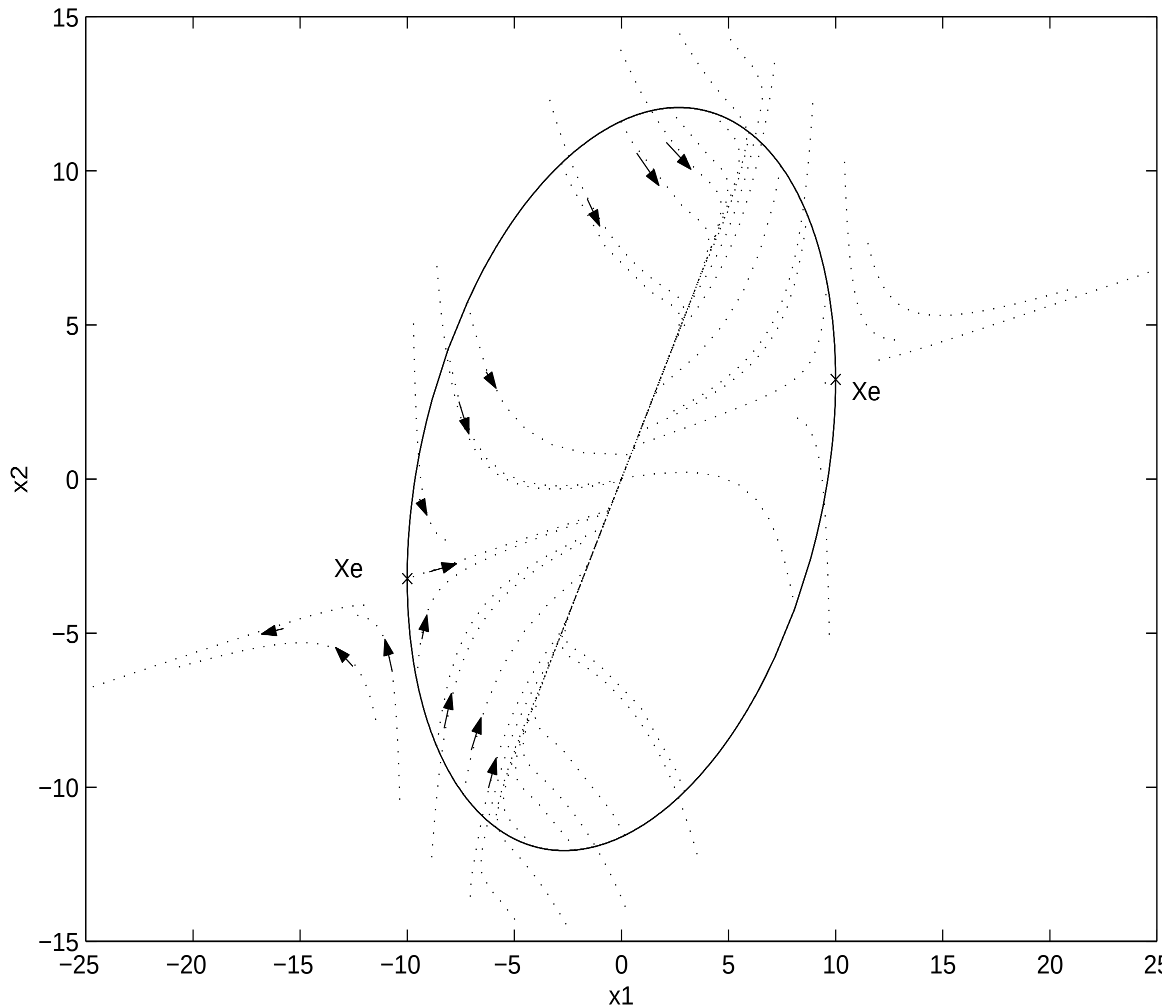
$$E_c = 0.1269$$

☞ All the trajectories initiated in the ellipsoid $\mathcal{E}(W^{-1}, 1)$ converge asymptotically to the origin.

☞ Taking an initial condition outside, but close to the limit, we observe the instability of the system. We can compute the following equilibrium points:

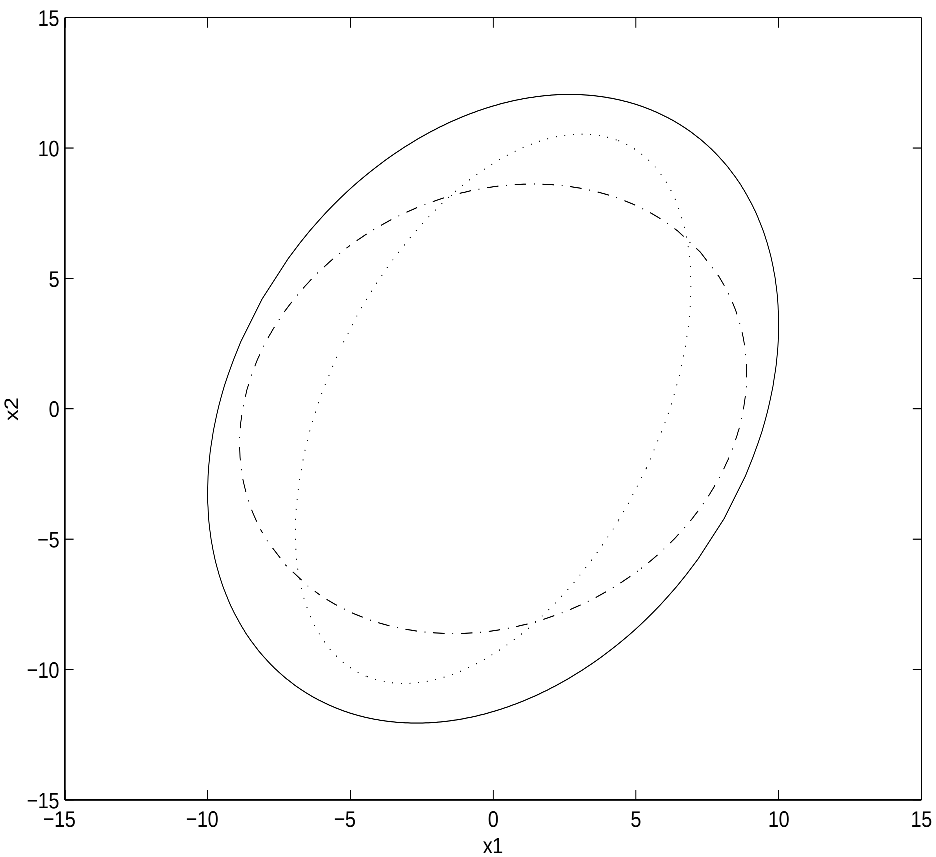
$$X_e = \pm \begin{bmatrix} 10 \\ 3.2339 \end{bmatrix}$$

☞ The obtained stability domain and trajectories are plotted in the following figure.



7. Illustrative examples (4)

- Recall the value of β obtained: $\beta = 6.5956$
- This value is larger than the value obtained in [Gomes da Silva Jr., Tarbouriech, Reginatto/CCA02] using the classical sector condition ($G = \Lambda\mathbb{K}$): $\beta = 5.6872$.
- Considering $E_c = 0$ (that is taking $Z = 0$ in (12)), we obtain $\beta = 4.3514$.
- The stability domains are given:
  – in the presented case (solid),
  – for the approach proposed in [Gomes da Silva Jr., Tarbouriech, Reginatto/CCA02] (dash-dotted),
  – without anti-windup, i.e., for $E_c = 0$ (dotted).



7. Illustrative examples (5)

Exemple 2

☞ Consider the multivariable system [Gomes da Silva Jr., Tarbouriech, Reginatto/CCA02]:

$$A = \begin{bmatrix} 0.1 & -0.1 \\ 0.1 & -3 \end{bmatrix} ; \quad B = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix} ; \quad C = I_2 \; ; \; u_0 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

☞ The dynamic controller is given by

$$A_c = \begin{bmatrix} -171.2 & 27.2 \\ -68 & -626.8 \end{bmatrix} ; B_c = \begin{bmatrix} -598.2 & 5.539 \\ -4.567 & 149.8 \end{bmatrix} ; C_c = \begin{bmatrix} 0.146 & 0.088 \\ -6.821 & -5.67 \end{bmatrix} ; D_c = 0_2$$

☞ The admissible set of initial conditions is given by :

$$\Xi_0 = Co\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} ; \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} ; \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} ; \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \}$$

☞ The quadratic approach leads to:

$$\beta = 250.8677; \quad E_c = 10^4 \begin{bmatrix} 2.4176 & -0.0169 \\ 0.3590 & 0.0011 \end{bmatrix}$$

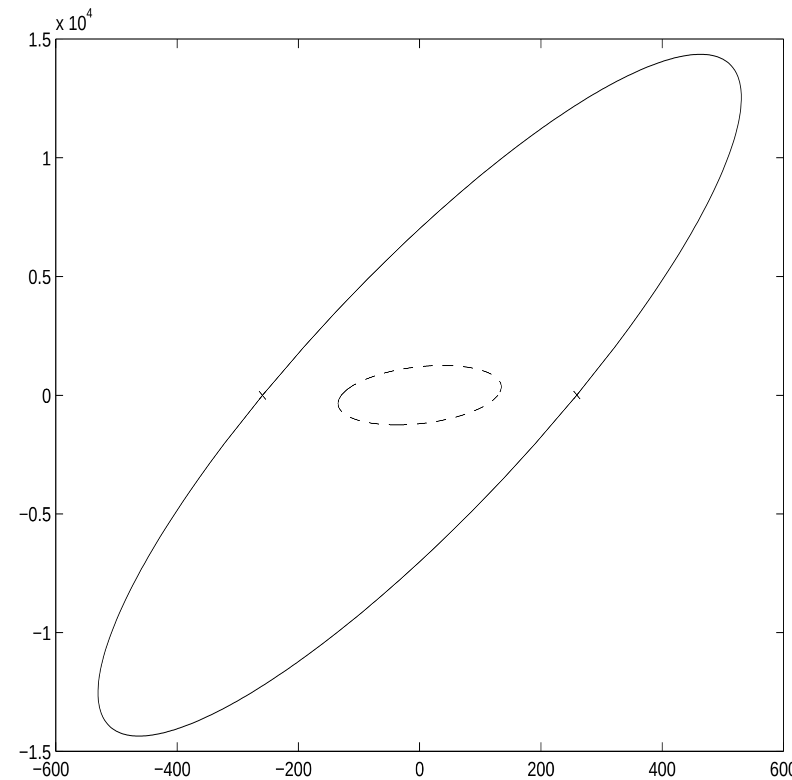
☞ We can compute the following equilibrium points:

$$X_e = \pm \begin{bmatrix} 259.3103 & 9.3103 & -70.3493 & 21.3140 \end{bmatrix}'$$

☞ The part of the stability domain relative to the state of the system (x) is plotted in the following figure in two cases:

– with the presented approach (solid),

– with the classical nonlinearity sector condition (dash-dotted).



Example 3

☞ Consider the system presented in [Cao,Lin,Ward/IEEE02]:

$$A = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix} \; ; \; B = \begin{bmatrix} 1.5 & 4 \\ 1.2 & 3 \end{bmatrix} \; ; \; C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \; ; \; u_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

☞ The dynamic controller is defined as:

$$A_c = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \; ; \; B_c = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \; ; \; C_c = \begin{bmatrix} 0.3333 & 0 \\ 0 & -0.1 \end{bmatrix} D_c = \begin{bmatrix} -3.3333 & 0 \\ 0 & 1 \end{bmatrix}$$

☞ The open loop system is stable.

▷ Corollary 1 can be applied to design an anti-windup gain which leads to the global stability of the closed-loop system.

☞ With the constraint $E_{c(i,j)}^2 \leq 100$, we obtain: $E_c = \begin{bmatrix} 9.6593 & 3.9260 \\ 8.2607 & -0.7235 \end{bmatrix}$.

☞ The gain obtained in [Cao,Lin,Ward/IEEE02] leads only to local stability.

8. Conclusion and prospectives (1)

Conclusion

- A solution to enlarge the stability region has been presented:
  - ▷ via static anti-windup loop acting on the dynamics of the nominal controller
- The results are based on the use of both a modified sector nonlinearity condition (Lemma 1) and a Lyapunov function:
  - ▷ a suitable static anti-windup gain and a set in which the stability of the complete closed-loop system is guarantee are characterized.

- The conditions are formulated
  - through LMI in the quadratic approach;
  - through BMI in the Lure approach.
- An application of the elements presented in this talk was done in collaboration with ONERA (and Dassault) in the context of combat aircraft: (17th IFAC Symposium on Automatic Control in Aerospace (ACA07), Toulouse, France, June 2007)
  - Some elements will be provided in the context of dynamic anti-windup

8. Conclusion and prospectives (3)

Prospectives

- ☞ Take into account the performances: what type of criteria?
- ☞ Multi-loop static anti-windup/Dynamic anti-windup
- ☞ Saturation of sensors
- ☞ Tracking problem: the reference signal is taken into account in the anti-windup loop
- ☞ In the case of systems with several operating points (aircraft), build one compensator or a set of anti-windup compensator (LPV for example)