Do workers really benefit from their social networks? *

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1 Introduction

Since the lack of social capital of some groups induces inequalities in employment opportunities, some active labor market policies try to encourage the establishment or the improvement of personal networks (McClure, 2000, OECD, 2001). For example, the *Australians Working Together* program aims at providing people the incentives to stay involved with their communities even if they are economically disadvantaged (OECD, 2003). The McClure Report (2000) claims that "by building their social capital (through stronger networks, trust and shared values), communities can offer individuals more opportunities for economic and social participation. A key part of community capacity building is connecting individuals in ways that enable people to support each other".

Indeed, a large proportion of people (about 50% on average) hear about or obtain jobs through friends and relatives (see: Rees, 1966, Granovetter, 1995, Holzer, 1988, Montgomery, 1991, Topa, 2000, for the U.S., Gregg and Wadsworth, 1996, for the U.K. and Addison and Portugal, 2001, for Portugal). Moreover, to a large extent, employers also use social networks. For example, Holzer (1987) reports that 36 percent of firms interviewed filled their last opening with referred applicants. Campbell and Marsden (1990) find that about half of a sample of 52 Indiana establishments make regular use of referred applicants. Accordingly, this intensive use of networks means that disadvantaged people who do not have access to contact networks have fewer employment opportunities than others. It therefore seems logical that policy makers try to increase the "social capital" of workers. However, do workers always benefit from more and better job contacts? If not, then it is

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important to show under which circumstances an increase in social contacs can be maleficial.

To answer this question we first provide a simple matching model in which unemployed workers and employers in large firms can be matched together through social networks or through more "formal" methods of search. We assume that the labor force is divided between social groups which differ in the efficiency of their social networks. Each employee can forward job offers from his firm to the unemployed workers of his group. Firms can also use formal methods which are more expansive but could generate applicants at an higher rate.

We first identify how the firms subsitute between the methods of search. Indeed, the employer takes the networks into account when it decides the optimal number of advertised vacancies. Moreover, we point out the congestions effects between the groups. While they are competing for the advertised vacancies, they also transmit privately a part of the job offers. Then, we emphasize how the subsitution between the different job search methods have strong implications with respect to the economic policy. Especially, we investigate the impact of new active labor market policies aiming at increasing individuals' social capital. We compare two different policies. The first enlarges, for some "disadvantaged" workers, the access to effective networks. The second improves the efficiency of these workers' existing social contacts.

In both case, we show that this can increase the congestion externalities and induce firms to substitute employees referrals for job advertising. Indeed, if there is an improvement of the informal channel, the firms are more likely to rely on employees' referrals, and decrease their efforts on the costly formal job market. If this substitution is more than proportional, the unemployment can increase and workers' output and welfare decrease.

In economics, some recent contributions investigate the way transmission of job information through contact networks influences the job-worker matching process and its consequences on economic inequalities. To our knowledge, Boorman (1975) was the first to provide a formal network model which described the information structure of finding a job. In Boorman's model, networks are endogenous: contacts are developed by individuals who maximize their probability of getting a new job in the event that they lose their present job. Boorman only focuses on the supply side of the labor market, whereas we take into account both sides of the market. Accordingly, for simplicity's sake, we assume that the network structure is exogenous. Calvó-Armengol and Zenou (2005) use Boorman's framework, but without an endogenous network formation, to provide a matching model in discrete time with contact networks and an endogenous arrival rate of job offers driven by free entry. Cahuc and Fontaine (2002) provide a matching model in continuous time in which individuals (workers and employers) can explicitly choose between different search methods (with or without contact networks). In this framework, they look at the way conditional unemployment benefits can improve welfare and decrease unemployment by coordinating the individuals on the efficient method. Our model is simpler and take into account that there is heterogeneity in the efficiency of networks, inducing wages and job finding inequalities. Moreover, by using a one worker per firm framework, previous papers providing a matching model with contact networks (Calvó-Armengol and Zenou, 2005, and Cahuc and Fontaine, 2002) have bypassed the fact that employees produce applicant for the firm where they work. In this case, the employer has an incentive to take account of the *social* productivity of its employees.

Calvó-Armengol and Jackson (2004) also look at the way job contact networks entail inequalities, but in a different context, in which the job arrival rate is exogenous. In comparison with this contribution, we do not model social networks explicitly but endogenize the job arrival rate and the number of advertised jobs. Indeed, networks can sometimes substitute for market and damage workers excluded from the most efficient networks. In a recent contribution, Bentolila et al. (2003) argue that in the presence of imperfect informations on jobs and workers' characteristics, networks can induce a mismatch of talents. In our model, we show that, even in an homogenous framework without information asymmetries or heterogeneity in workers' productivities, networks can decrease workers' welfare. Besides, larger networks can entail a higher unemployment rate.

The model and its properties are presented in Section 2. We derive the decentralized equilibrium and then consider its efficiency. In Section 3, we investigate the effect of new labor market policies. We show that an economy which increases the "social capital" of the disadvantaged workers does not necessary leads to a lower unemployment rate or higher workers' welfare.

2 A simple matching model with social networks

We provide a matching model where employers take account of the fact that their employees produce applicants. In order to study the economic inequalities due to the use of the informal hiring channel, we assume that there is two types of workers which differ with respect to the efficiency of their social contacts. We derive the decentralized equilibrium and then study the efficiency of the labor market.

2.1 The framework

2.1.1 The matching function

Sociological studies promote the idea that the economic theory of job search bypasses the specific role of social networks in recruitment. Particularly, employees in a firm generate applicants for jobs that are not advertised by that firm. For instance Waldinger (1997) claims that "social networks produce applicants for employers who don't yet have vacancies to fill". In the same way, Granovetter, in his survey of the literature (1995), argues that "if employers do not advertise vacancies, this may be in part because they know they can be filled by friends and relatives of existing employees". If job advertising is used by employers to produce matches, the existing employees can also find applicants. Accordingly, we distinguish in our model vacancies publicly advertised by employers (denoted by V) and offers transmitted by employees (the number of which depends upon the number of employees).

In the same time, empirical evidences suggests that different outcomes in job search could hinge on the inequality of social networks. For instance, Petersen et al. (2000) remark that recruitments "need not be discriminatory in intent or design, but women and ethnic minorities may have lower access to social networks having higher rates of success in hiring". In our model social contacts can differ in efficiency. Moreover, homophily, that is the tendency of socially similar people to band together, implies that an employee of a social group mostly forwards job offers to unemployed workers belonging to the same group (McPherson et al., 2001). In this paper, for sake of simplicity, we assume that employees forward job offers to unemployed workers of their group only. This is probably an extreme view of the homophily principle. A more general model would allow some offers to be forwarded to the other group.

Consider a large number of firms producing a numeraire output and a large labor force whose size is denoted by N. Let us assume that the labor force is divided between two groups j with $j = \{1,2\}$ [1]. We denote by γ the proportion of individuals in the labor force from group 1. Accordingly, we have $\gamma N = N_1$ and $(1-\gamma)N = N_2$ with N_1 (respectively N_2) denoting the size of the labor force of the first group (respectively of the second group). In our framework, workers do not differ with respect to their productivity since we only want to study the effects of social networks which depend on inequality with respect to the efficiency of the networks. A firm i produces $F(L_i) = yL_i$, where L_i is the number of employees in firm $i (L_i = L_{1i} + L_{2i})$ and y the efficiency of labor. L denotes the total number of employees in the economy, $L = \Sigma_i L_i$.

1 Let us remark that it is straightforward to extend this framework to n groups of workers. However, for sake of simplicity, and because it does not affect our results, we assume that there are only two types.

Hiring a worker and searching for a job are costly activities. Employers and unemployed workers – the only job seekers, by assumption – are brought together in pairs through an imperfect matching process. However, we take into account the fact that the firms use different search methods. One is a high cost search method where each offer is sent and advertised at a cost h per unit of time. For example, the firm puts advertisements in newspapers and uses public or private agencies. Moreover, we assume that at each period, each employee can try to contact a friend who is seeking for a job.

The matching function presented here is derived from an urn-ball process (cf. Appendix A) in which firms send their vacancies *randomly* to the market, while the employees only contacts unemployed workers from *their* network. In this latter case, they are also supposed to send a ball into the urns. We note λ_j the probability that an employee, belonging to the group j, tries to find an applicant for his firm [2]. Moreover, we suppose that the groups do not have the same effectiveness (denoted by λ_1 and λ_2) in generating employment opportunities. Accordingly, at each period, the number of offers transmitted through the j's social contacts amounts to $\lambda_j L_j$.

We denote by u_j the unemployment rate defined by U_j/N_j, with U_j the number of unemployed workers in group j, and $v = V/N$. We define p as the share of unemployed workers of the first group in the total number of unemployed workers: $p = U_1/(U_1 + U_2)$. Remark that p is also the probability that an offer sent randomly is received by an unemployed worker of the first group. We denote by θ_j the ratio of the number of job offers reaching the unemployed workers of the jth group over the number of unemployed in this group:

$$\theta_1 = (pV + \lambda_1 L_1)/U_1$$

$$\theta_2 = ((1-p)V + \lambda_2 L_2)/U_2$$

We show in Appendix A that the contact rate, denoted $MG(.)$, between the $(\lambda_1 L_2 + \lambda_1 L_2 + V)$ job offers and the $U_1 + U_2$ job seekers is the sum of two homogenous matching functions. These functions represents the contact rate within each of the two groups. The agregate contact rate $MG(.)$ reads

$$MG(\lambda_1 L_1 + pV, \lambda_2 L_2 + (1-p)V, U_1, U_2) = M(\lambda_1 L_1 + pV, U_1) + M(\lambda_2 L_2 + (1-p)V, U_2)$$

Finally, the job filling rate for an offer sent through network 1 reads $m(\theta_1) = M(\lambda_1 L_1 + pV, U_1)/(\lambda_1 L_1 + pV)$ and
$m(\theta_2) = M(\lambda_2 L_2 + (1-p)V, U_2)/(M(\lambda_2 L_2 + (1-p)V, U_2))$ (see the Appendix).

Remark that we assumed that λ_j is an exogenous variable. Although it would be interesting to understand why the employees are willing to help other workers to find a job, this is beyond the scope of this article. On the

[2] We thus assume, for sake of simplicity, that an employee never fowards an offer toward another employee.

one hand, the expected payoff of cooperation could related to future job search (for example in case of a job loss). On the other hand, it can also be related to something outside the labor market. For example, it could be affected by social conventions, social group's traditions or social prestige. λ_j could also be directly linked with the employment rate of the group. For example, the effectiveness of referring is likely to improve of with lower unemployment. Remark that our framework already exhibits such feature but in a different manner. Indeed, the lower the unemployment rate, the lower the congestion effect faced by the offer transmitted through networks. Moreover, the number of offers is increasing with the number of employees and thus the exit rate from unemployment.

Finally, notice that employers are not able in this framework to target their advertised vacancies to one of the groups. However, in real life, different groups or neighborhoods have sometimes different media that an employer can advertise in. Our framework is thus only an extreme case of a model where employers could use a (possibly imperfect) targeting device. The other extreme case would be a model where employers can *perfectly* target their vacancies.

2.1.2 Expected utilities and profits

Let us denote by r the exogenous discount rate, q the job destruction rate and w^j the wage for workers of the group j[3]. The firm chooses the number of vacancies publicly advertised (V_i) and takes into account that its employees also produce applicants. Moreover, when firms decide their optimal employment policies, they have to distinguish between the different types of employees. Thus, the value function for the problem of the firm solves the Bellman equation:

$$r\Pi(L_1^i, L_2^i) = \underset{V_i}{\text{Max}}\,(y(L_1^i + L_2^i) - w^1 L_1^i - w^2 L_2^i - h\,V_i + \dot{\Pi}(L_1^i, L_2^i)) \tag{1}$$

subject to

$$\dot{L}_1^i = (\lambda_1 L_1^i + p\,V_i)m(\theta_1) - qL_1^i \tag{2}$$

$$\dot{L}_2^i = (\lambda_2 L_2^i + (1-p)\,V_i)m(\theta_2) - qL_2^i \tag{3}$$

with p denoting the probability that an offer, sent randomly through the formal channel, is received by an unemployed worker who belongs to the first group. The firm chooses its optimal number of employees, knowing that the positions can be filled using two methods of search. On the one hand, jobs can be advertised by the firm. On the other hand, social networks of employees produce applicants at a rate $\lambda_j L_{ji} m(\theta_j)$ for the group j. Since, at each period, the firm chooses the number of advertised vacancies

[3] Remark that the wage could depend on firm's size. We show in Appendix A.2 that it is not the case.

(V_i), she controls the increase in the number of employees and, consequently, the subsequent number of applicants the social network will produce. Moreover, wages are the subject of bargaining and a firm can refuse to hire an applicant [4]. The fact that the two types of workers are perfect substitute in the production function is a strong assumption. However, it avoids firms to "manipulate" wages and employment strategically [5] (Stole and Zwiebel, 1996a,b) and helps to provide a tractable model without driving our results.

Using (1) and the Kuhn-Tucker conditions, ones gets, at the steady state:

$$h = \frac{pm(\theta_1)(y - w^1)}{r + q - \lambda_1 m(\theta_1)} + \frac{(1-p)m(\theta_2)(y - w^2)}{r + q - \lambda_1 m(\theta_2)} \Leftrightarrow V_i > 0 \tag{4}$$

$$h > \frac{pm(\theta_1)(y - w^1)}{r + q - \lambda_1 m(\theta_1)} + \frac{(1-p)m(\theta_2)(y - w^2)}{r + q - \lambda_1 m(\theta_2)} \Leftrightarrow V_i = 0 \tag{5}$$

(4) defines the optimal employment level when the optimal number of advertised vacancies is positive. In the second case (equation (5)) the expected cost of a job advertisement is higher than the expected profit. Consequently, the optimal number of advertised vacancies amounts to zero. For sake of simplicity, we focus in this paper on the non degenerate case where the formal number of vacancies is positive [6].

Finally, we derive the workers' value functions. It is worth recalling that offers forwarded by an employee only reach unemployed within his contact network, i.e., by assumption, within his group. Hence, $\lambda_1 L_1 + pV$ offers reach the job seekers of the first group and are filled at a rate $m(\theta_1) = M(\lambda_1 L_1 + pV, U_1)/(\lambda_1 L_1 + pV)$. Accordingly, the rate at which an unemployed individual belonging to group 1 gets an offer amounts to

$$\underbrace{(\lambda_1 L_1 + pV)m(\theta_1)}_{\text{Contact rate within the first group}} \cdot \underbrace{\frac{1}{U_1}}_{\text{Probability that an unemployed worker } j \text{ gets the offer}}$$

$$= \theta_1 m(\theta_1) \tag{6}$$

In the same way, the exit rate from unemployment for a worker of the second group is simply $\theta_2 m(\theta_2)$. We assume that an unemployed worker

4 However, the Nash bargaining rule ensures that it is always profitable for a firm to hire an applicant (see below).

5 If workers are complementary in the production function, the wage of the first group is going to depend on the employment level of the second group.

6 The alternative case has been studied in a previous version of the paper (see Fontaine (2004)).

benefits from an income flow z. Since all firms are assumed identical, the value functions of the workers read:

$$rS_j = z + \theta_j m(\theta_j)(E_j - S_j) \quad (7)$$

$$rE_j = w^j + q(S_j - E_j) \quad (8)$$

with S_j and E_j the value functions for an unemployed worker and an employee of the group j.

2.1.3 Wage bargaining

"The employers view workers' social connections as resources in which they can *invest*, and which might yield economic returns in form of better hiring outcomes" claim Fernandez et al. (2000). In our model, social connections are valuable since they increase the matching rate of the firm. This changes the value of jobs and influences wages through bargaining [7].

In our model, wages are subject to bargaining between the firm and the worker and can be renegotiated each period at no cost. The surplus of each match is shared according to the Nash solution of the bargaining problem [8]

$$\beta \frac{\partial \Pi}{\partial L_j} = (1 - \beta)(E_j - S_j) \quad (9)$$

with $\beta \in [0,1]$ the share that accrues to the worker (identical for all workers). Remark that we assume that the employer observes the type of the worker. Of course, it is possible when the type is defined by an observable characteristics or when this information can be deduce from the employment history of the individual. However, the result would probably hold in a more general model. Indeed, game theoretical literature shows that, with incomplete information during bargaining and heterogeneity between players, strategic delays between offers and counteroffers are used by agents to communicate their types [9] (Cramton, 1992). Hence, even with asymmetric information, the wage can thus hinge on workers' type.

Using the firm's Bellman equation, we derive, at the steady state, the value of a marginal job

$$\frac{\partial \Pi}{\partial L_j} = \frac{y - w^j}{r + q - \lambda_j m(\theta_j)}$$

7 We present in a previous version of the paper (Fontaine, 2004) another equilibrium where employers give a bonus when an employee finds an unemployed worker to fill the position. We show that this equilibrium is equivalent as regards the employment level of the economy and the number of job vacancies sent by the firms.

8 For a analysis of the wage bargaining in a large firms framework see, for example, Cahuc et al. (2005).

9 Formally, individuals with a high outside option (a high job arrival rate, here the workers belonging to the group with the highest λ) does not accept the first firm's offer and delays counteroffer long enough to separate themselves from low types workers.

and thus

$$w^j = (1-\beta)\left[\frac{r+q-\lambda_j m(\theta_j)}{r+q-(1-\beta)\lambda_j m(\theta_j)}\right] rS_j + \beta\frac{r+q}{r+q-(1-\beta)\lambda_j m(\theta_j)} y \quad (10)$$

While, the wage does not hinge on the number of employee in the firm, the firm takes into account the *social* productivity of its employees in the discount rate of the value of a marginal job $(r+q-\lambda_j m(\theta_j))$. Taken the market tightness θ_j as given, the higher the efficiency of the networks, the higher the wage.

2.2 The decentralized equilibrium

To make the main mechanisms of our model clear, we begin by presenting the case where there is only one type of worker. Especially, we point out the substitution effect between the informal job offers and the number of job offers advertised by the firm. We then study an economy with two groups of workers.

2.2.1 The homogeneous case

Consider the case where $\lambda_1 = \lambda_2 = \lambda$. Using (13) and (10), the wage reads

$$w = z + \beta\frac{r+q+\theta m(\theta)}{r+q+\beta\theta m(\theta)-(1-\beta)\lambda m(\theta)}(y-z) \quad (11)$$

Equation (11) is very close to that of standard matching models (see for example Pissarides (2000)). Nevertheless, in our framework, wages are increasing functions of the efficiency of networks λ because firms directly take into account during wage bargaining that their employees produce applicants (represented by $-(1-\beta)\lambda m(\theta)$ in (11)). An higher efficiency induces an higher job filling rate for the firm.

It is worth noting that empirical evidence suggests that networks have an ambiguous effect on wages (e.g. Simon and Warner, 1992, Granovetter, 1995, Marmaros and Sacerdote, 2002, Kugler 2003, Bentolila et al., 2003). In our simple model with homogenous firms and workers, the individuals have the same wage regardless of the channel by which they have found their jobs. However, the existence of networks has an effect on wages by defining workers' outside opportunities and by reducing firms' search costs. Remember that we focus on the impact of networks on the job arrival rate and search costs, and not on the allocation of workers accross occupation.

We now consider the equilibrium value of the labor market tightness. (4) together with (11) imply that labor market tightness satisfies the following condition:

$$\frac{h}{m(\theta)} = \frac{(1-\beta)(y-z)}{r+q+\beta\theta m(\theta)-(1-\beta)\lambda m(\theta)} \quad (12)$$

In equilibrium, the expected cost of an advertised vacancy, represented by the left–hand side, equates to the expected profit of an advertised vacancy, represented by the right-hand side. It can easily be checked that (12) defines a unique equilibrium value of labor market tightness. The higher the number of employees, the higher the number of job offers transmitted by networks. However, this induces a substitution effect and the number of advertised jobs decreases. This substitution effect ensures the uniqueness of equilibrium (displayed on Figure 2).

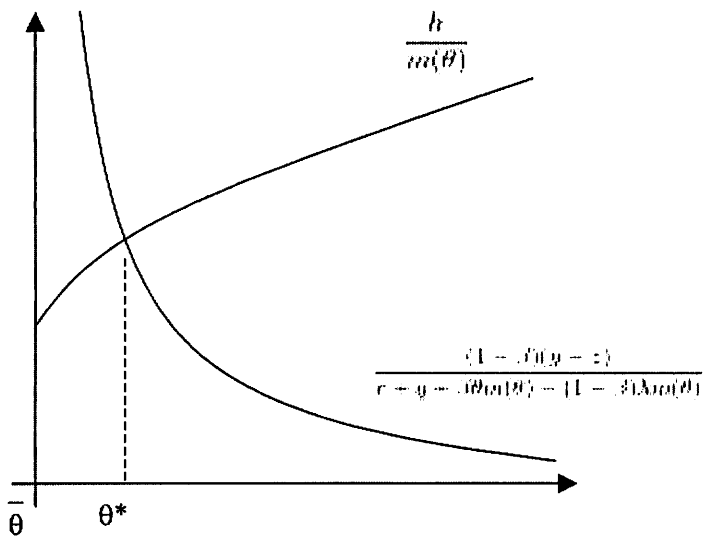

Figure 1: *The decentralized equilibrium with both methods of search*

It is worth noting that the right hand side of (12) is only defined when the value of a marginal job is positive, that is if $r+q+\beta\theta m(\theta)-(1-\beta)\lambda m(\theta)>0$. We denote by $\bar{\theta}$, the value of the labor market tightness such that $r+q+\beta\bar{\theta}m(\bar{\theta}) = (1-\beta)\lambda m(\bar{\theta})$. For $\theta \in \left]\bar{\theta}, +\infty\right[$, the value of a marginal job is decreasing with θ, whereas the expected cost of a advertised vacancy increases with the labor market tightness. Consequently, the equilibrium value of θ is always greater that $\bar{\theta}$ and unique.

At the steady sate, the law of motion for jobs is

$$u = \frac{q}{q+\theta m(\theta)} \tag{13}$$

Accordingly, (12), together with (13), defines an unique value of the unemployment rate. Notice that it decreases with λ. Indeed, an increase in the effectiveness of social networks leads to an improvement in the matching function. On the one hand, it entails a rise in the wage (see (10)) since the expected exit rate from unemployment increases. However, the expected cost of a publicly advertised vacancy decreases and offsets the first effect. Eventually, the number of employees increases.

2.2.2 An equilibrium with two social groups

We now present the decentralized equilibrium with two social groups. Simple replication of previous reasoning implies that wages amount to:

$$w^j = z + \beta\left(\frac{r + q + \theta_j m(\theta_j)}{r + q + \beta\theta_j m(\theta_j) - (1-\beta)\lambda_j m(\theta_j)}\right)(y - z) \tag{14}$$

Moreover, using the first order condition of equation (1), the envelope condition for an optimal choice of v and the wage equations, labor market tightness must satisfy the following condition:

$$\begin{aligned} h = {} & pm(\theta_1)\frac{(1-\beta)(y-z)}{r + q + \beta\theta_1 m(\theta_1) - (1-\beta)\lambda_1 m(\theta_1)} \\ & + (1-p)m(\theta_2)\frac{(1-\beta)(y-z)}{r + q + \beta\theta_2 m(\theta_2) - (1-\beta)\lambda_2 m(\theta_2)} \end{aligned} \tag{15}$$

As usual, in equilibrium, the expected cost to fill a vacancy equates to the expected profit of a filled position. Here, the expected profit of a marginal job is a weighted average of the profit if the advertised job offer is received by an unemployed worker of the first group which occurs with a probability p, and of the profit if the vacancy is received by a unemployed worker of the second group, which occurs with a probability $(1-p)$. Remember, that p is endogenous ($p = U_1/(U_1 + U_2)$). Hence, the weights of the expected profit depend on the relative unemployment rates. Moreover, the equlibrium condition takes into account that the job filling rate is different between the networks.

In order to define the symmetric decentralized equilibrium at the steady state, we use the flow equations (2) and (3) for $\dot{L}_1 = \dot{L}_2 = \dot{L} = 0$. One gets:

$$q\theta_1 - \lambda_1\theta_1 m(\theta_1) = q\theta_2 - \lambda_2\theta_2 m(\theta_2) \tag{16}$$

and

$$u_j = \frac{q}{q + \theta_j m(\theta_j)} \quad \text{for } j = \{1,2\}$$

First, remark that $(q\theta_j - \lambda_j\theta_j m(\theta_j))$ is increasing in θ_j[10]. Hence, if $\lambda_1 > \lambda_2$, then $\theta_1 > \theta_2$, $u_1 < u_2$ and $w_1 > w_2$. More efficient networks lead to a better job offers-job seekers ratio (θ_j), a lower group's unemployment

[10] $[q\theta - \lambda\theta m(\theta)]'_\theta = q - \lambda m(\theta)(1-\eta(\theta))$, with $\eta(\theta) > 0$ the elasticitty of the matching function with respect to the unemployment rate. Since $q > \lambda m(\theta)$, $q - \lambda m(\theta)(1-\eta(\theta)) > 0$.

rate and thus higher wages. However, either (15), nor (16) define an unambiguous relation between θ_1 and θ_2. This could entail multiple equilibria for some parameters values. However, it is very difficult to find simple restrictions on the parameter space which guarantee the uniqueness of the equilibrium. Consequently we check in the next section whether multiple equilibria appear during our numerical simulations for realistic parameters values. They do not.

2.3 Efficiency

This last subsection is devoted to the analysis of the efficient allocation and its comparison with the decentralized equilibrium. We begin by defining the efficient allocation. The social planner chooses the number of vacancies publicly advertised (i.e. V) that maximizes the discounted value of the stream of production. Consequently, the value function of the social planner, denoted by W satisfies:

$$rW = \underset{V}{\mathrm{Max}}\ \left(y(L_1 + L_2) + (N - L_1 - L_2)z - hV + \dot{W}\right)$$

$$\text{subject to } \dot{L}_1 = M(\lambda_1 L_1 + pV,\ N_1 - L_1) - qL_1$$

$$\dot{L}_2 = M(\lambda_2 L_2 + (1-p)V,\ N_2 - L_2) - qL_2$$

The first order and envelope conditions imply that labor market tightnesses (θ_1^* , θ_2^*) satisfy:

$$h = pm(\theta_1^*)\frac{(1-\eta(\theta_1^*))(y-z)}{r + q + \eta(\theta_1^*)\theta_1^* m(\theta_1^*) - (1-\eta(\theta_1^*))\lambda_1 m(\theta_1^*)}$$
$$+(1-p)m(\theta_2^*)\frac{(1-\eta(\theta_2^*))(y-z)}{r + q + \eta(\theta_2^*)\theta_2^* m(\theta_2^*) - (1-\eta(\theta_2^*))\lambda_2 m(\theta_2^*)} \quad (17)$$

with $\eta(\theta_j)$ the elasticity of the group j's matching function with respect to the unemployment rate of this group and given

$$q\theta_1^* - \lambda_1\theta_1^* m(\theta_1^*) = q\theta_2^* - \lambda_2\theta_2^* m(\theta_2^*)$$

Remark that, in case where there is only one social group ($\lambda_1 = \lambda_2 = \lambda$), the comparison of the conditions that define the decentralized equilibrium [11] (equation (12), with the efficient allocation shows that the decentralized economy is efficient if and only if the share of surplus that accrues to the worker, β, is equal to the elasticity of the matching function with respect to the number of unemployed workers. This condition is identical to the well-known efficiency condition of Hosios (1990) and Pissarides (2000).

Our homogenous matching technology is very close to the standard one, so it is not suprising that we get the standard Hosios-Pissarides condition.

[11] Again, we only consider the case where $V > 0$.

Nevertheless this result makes clear that, if we only consider their effects on the job arrival rate, social networks do not change necessarily the *nature* of the matching externalities on the labor market but participate to the standard congestion externalities. In the literature, similar results are often "hidden" by the non homogenous matching framework used. For example, Calvó-Armengol and Zenou (2005) or Cahuc and Fontaine (2002) found a different condition for the efficiency of the decentralized equilibrium. However, it is different from the standard Hosios condition only because the matching function is not an homogenous function. In both papers, networks affect the congestion externality but do not create a *new* externality.

Of course, when there is two groups, the decentralized equilibrium can only be efficient if we allow for different bargaining power between the groups and if $\beta_j = \eta(\theta_j)$ for $j = 1,2$. In the other case, and especially if β is the same for both groups, the decentralized equilibrium is not efficient: an unique instrument cannot help to internalize the congestion effects of the two matching sub-functions (namely $M(\lambda_1 L_1 + pV, U_1)$ and $M(\lambda_2 L_2 + (1-p)V, U_2)$).

3 Is it always efficient to increase the social capital of individuals?

For some disadvantaged workers, matching is generally poor because their information about job offers is not sufficient. These workers, unable to use the informal channel, have a lower exit rate from unemployment (Hansen and Pratt, 1991, Petersen et al., 2000). Accordingly, some active labor market policies try to encourage the establishment or the improvement of personal networks (McClure, 2000, OECD, 2001). Sociologists have already discussed various schemes for getting disfavored groups back to work (see Granovetter, 1995, and the references therein). These policies rely on pre-existing social network or construct referral networks artificially. We investigate their effect on unemployment and welfare.

In this section, we show that, surprisingly, an economy with a larger access to the social networks or even with more efficient social contacts does not necessary leads to a lower unemployment rate and to a higher level of welfare for the workers. Indeed, it induces a non-neutral substitution between network and market.

3.1 Parameterization

We consider an economy where social networks *only* differ with respect to the efficiency parameter λ_j. For sake of simplicity, assume that initially the second group of workers has no job contact networks: $\lambda_2 = 0$ and $\lambda_1 = \lambda > 0$. This assumption represents the fact that some individuals are excluded from some social activities, or, more precisely, from networks which have relevant informations about job offers [12]. We parametrize our model to match the French labor market in the late 90's. This example is only illustrative and the results we get are robust to a large range of values. We take the period to be one year and therefore set the discount rate to $r = 0.05$. We assume that the matching functions are Cobb-Douglas [13]. For sake of simplicity, we assume that their elasticities are the same. $M(\lambda_1 L_1 + pV, U_1) = A.(\lambda_1 L_1 + pV)^{\alpha}(U_1)^{1-\alpha}$ and $M(\lambda_2 L_2 + (1-p)V, U_2) = A.(\lambda_2 L_2 + (1-p)V)^{\alpha}(U_2)^{1-\alpha}$. A represent the *ex-ante* efficiency of the matching technology and is calibrated to get a unemployment consistent with the observed one (about 9%). This parameter is supposed not to vary between the groups.

The productivity of a new job is normalized at unity and, as it is usually do in the matching literature, α and β are set to satisfy the Hosios condition for social efficiency and the estimations of the elasticity of the matching function (see Mortensen and Pissarides, 1999).

Cahuc et al. (2006) have estimated on french data that the yearly destruction rate ranges between .03 and .1. To be consistent with these findings, we set the destruction rate to the typical value of .06. Besides, we calibrate the search cost of a firm for an advertised job to represent 30% of the yearly productivity y of an employee to be consistent with survey results reported by Hamermesh (1993). Eventually, the value of leisure amounts to 0.45. Interpreted as unemployment benefit, and given that the simulated average wage is 0.9, this induces a replacement ratio of 50%, consistent with the french replacement ratio.

Descriptive statistics on the french labor force survey [14] (*Enquête emploi*) show that about 80% of the individuals *use* their friends and relatives to find a job whereas one third *find* their job through friends and relative. Accordingly, we set λ in order to get, when $\gamma = .8$ [15],

$$\frac{\lambda_1 L_1}{\lambda_1 L_1 + V} \approx \frac{1}{3}$$

Notice that we control for the possibility of multiple equilibria. In parameters values we adopt for the calibration the equilibrium is always unique. Our calibration is reported in Table 1 .

[12] Moreover, we get exactly the same results in a framework where $0 < \lambda_2 < \lambda_1$.

[13] Most empirical applications of the matching theory assume a Cobb-Douglas form with constant returns to scale (for a survey of the matching functions see Petrongolo and Pissarides (2001)).

[14] We compute these statistics for 1998. Let us remark that they do not really depend on the level of education of the individuals.

[15] It is worth noting that in our model there is no difference between the share of workers embedded in social networks and the share of individuals using their social networks. Indeed, there is no endogenous choice of the search method. See Cahuc and Fontaine (2002) for a model where workers and firms can choose between formal and informal method of search.

Parameters	y	A	h	z	β	α	r	q	λ
Values	1	.5	.3	.45	.5	.5	.05	.06	.07

Tab. 1: *Parameters values (french economy)*

We consider two different kinds of policy. Both aim at reducing the inequalities coming from the use of the informal channel in the hiring process. The first one try to integrate a part of the disadvantaged workers to the job contact networks, that is to the first group [16].

In our model, a policy which extends the proportion of individuals that have a contact network could correspond to a policy inducing a rise in γ. The second policy improves the efficiency of the social contacts of the second group. The benefits of such policy are thus distributed among *all* members of the group. Formally, it corresponds in our model to an increase in λ_2 from 0 to λ_2.

3.2 An enlargement of the access to job contact networks

3.2.1 The unexpected effects of an increase in social capital

In our framework, the parameter γ denotes the share of the labor force who have access to contact networks. At first glance, an increase in the number of workers embedded in a social network may improve the labor market by extending a better matching technology to more workers. However, as illustrated by Figures 2 and 3, an higher number of individuals embedded in networks can have negative impacts on workers, whatever their type.

First, since $u_1 < u_2$, an increase in the proportion of workers with social networks (γ) induces mechanically a fall in the global unemployment rate ($u = \gamma u_1 + (1-\gamma)u_2$), leading, taken all parameters as given, to an increase in the expected cost of a publicly advertised vacancy (*congestion effect*). Taking the optimal employment level as given, the rise in the number of the workers embedded in networks increases the matching rate and consequently decreases the number of vacancies that the firm has to advertise to find applicants (*substitution effect*).

[16] Since we only have very limited evidence on the dynamics of networks over time (McPherson et al., 2001), it is difficult to now how to design such a policy. However, we already have some insights about possible tools. If one considers networks initially based on ethnicity, urban policies could be used enlarge in the long run the existing networks. Indeed, there is empirical evidences that urban areas, with their greater diversity, produce networks with higher levels of racial and ethnic heterogeneity (see McPherson et al., 2001). Hence, there is a relationship between the distribution of characteristics accross space and their distribution within networks. Urban policies could thus be used to increase this heterogeneity and break the initial bounds. Promoting mixed schools could also be a powerful tool.

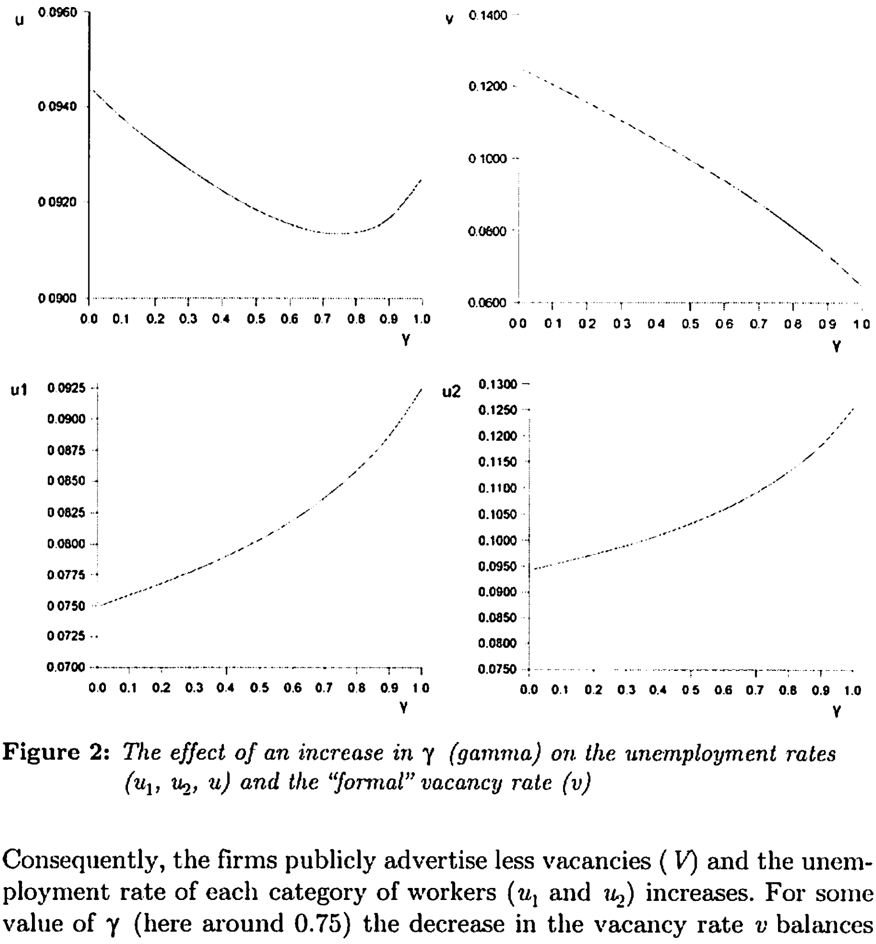

Figure 2: *The effect of an increase in γ (gamma) on the unemployment rates (u_1, u_2, u) and the "formal" vacancy rate (v)*

Consequently, the firms publicly advertise less vacancies (V) and the unemployment rate of each category of workers (u_1 and u_2) increases. For some value of γ (here around 0.75) the decrease in the vacancy rate v balances the rise in the number of workers with a contact network and the unemployment rate eventually increases. Hence an increase in the social capital of disadvantaged unemployed can entail a higher unemployment rate.

3.2.2 Who benefit really from social networks?

Eventually, Figure 3 shows that a larger access to networks leads to a fall in the value of both employment and unemployment. On the one hand, the expected values of a job for both types of workers (rE_1, rE_2) decrease since the wages decrease. Moreover, since the rise in the unemployment rates u_1 and u_2 induces a decrease in the probability of finding a job, the expected value of unemployed workers (rS_1, rS_2) decreases.

As regards total welfare W, it increases with the share of workers who belong to a social network (see Figure 4). First, firms advertise less vacancies and, accordingly, the total cost of the vacancies hv decreases and offsets

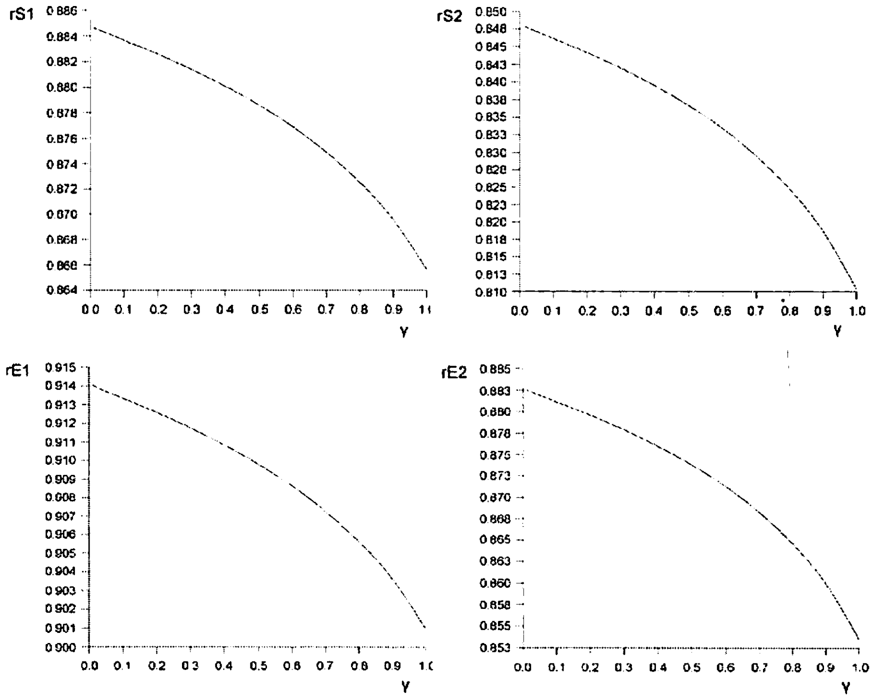

Figure 3: *The effect of an increase in* γ *(gamma) on the intertemporal utilities*

the negative effect of γ on welfare. Moreover, workers who didn't previously belong to a network and who have now access to this search channel experience an utility increase. Nevertheless the increase in γ harms the workers who stay in the same group as before (with or without network, see on Figure 4).

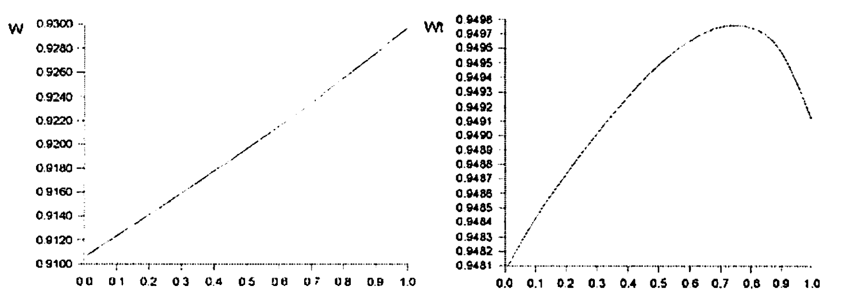

Figure 4: *The effect of an increase in* γ *on total welfare W and workers' output Wt*

Let us denote by W total welfare and Wt workers' output. The comparison between W and Wt help to distinguish between the effect on the output and the effect on the advertising cost (hv).

$$\begin{cases} W = y \times (1-\gamma u_1 - (1-\gamma)u_2) + z(\gamma \mu_1 + (1-\gamma)u_2) - hv \\ Wt = y \times (1-\gamma u_1 - (1-\gamma)u_2) + z(\gamma u_1 + (1-\gamma)u_2) \end{cases}$$

If we only take into account total welfare, the increase in the number of individuals in social networks has an unambiguous effect. Indeed, this increase improves the matching technology. However, for some values on γ (beyond the point $\gamma = 0.75$ in Figure 4) it is mostly the firms which benefit from the rise. Beyond some threshold, workers' output Wt decreases and the rise in total welfare is mostly due to the reduction of advertising cost: the firms advertise less vacancies, hv falls.

Thus, the extension of the use of the informal channel (an increase in γ) have ambiguous outcomes. On the one hand, it betters the matching technology. On the other it entails congestion and substitution effects which induce a decrease in the number of advertised jobs. The unemployment rates of both types of workers can increase and workers' output decrease. Besides, while workers who now benefit from networks are better off, the utility of others has decreased [17].

3.3 An increase in the efficiency of the social contacts

3.3.1 The substitution effect

In the previous policy, some workers are chosen to be integrated to the first group. We consider now an economic policy which aims at improving the efficiency of the existing social contacts of the disadvantaged workers. In our model , it could be represented by an increase in λ_2 from 0 to λ_1 (= .07). During our simulations, we assume that 80% of the population belongs to the first group to be consistent, when $\lambda_1 = \lambda_2 = .07$, with the observed share of the jobs found through networks.

First, since we increase the efficiency of the informal job search channel of the second group, its unemployment rate decreases (Figure 5). However, as before, the overall improvement of the informal channel generates a substitution effect. The number of vacancies advertised by the firms decreases.

[17] In the extreme case where employers would be able to target *perfectly* their advertised vacancies and reach a given type of workers, the effect of an increase in γ would be neutral for the individuals intially in the first group. Indeed, it would be exactly like an increase in the population size in a Pissarides model (Pissarides, 2000). However, this is only the case if the employer is able to *perfectly* target its vacancy.

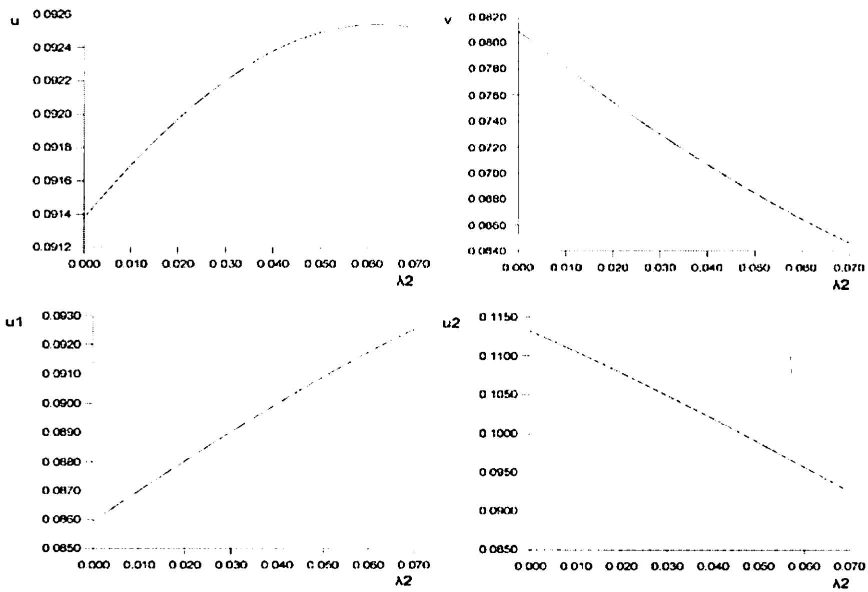

Figure 5: *The effect of an increase in λ_2 on the unemployment rates (u_1, u_2, u) and the "formal" vacancy rate (v)*

The first group's workers suffers from the fall in the number of formal job offers which increases its unemployment rate. Consequently, the overall effect on the unemployment can be negative. In our case, the average unemployment rate initially rises, driven by the unemployment rate of group 1, and only decreases slightly when λ_2 is above .06. It stays above the initial level.

Of course, this policy reduces the income inequalities between employees. The difference between the wages (not displayed here) disappears when both groups have the same efficiency, but the equilibrium wage is lower than the initial wage of the first group. However, notice that this is also due directly to the fact that workers of the first group suffer from the substitution effect. Indeed, the increase in their unemployment rate lowers their bargaining power and thus their wage.

3.3.2 Welfare and output

As regards the total welfare, a rise in λ_2 increases unambiguously the welfare. By improving the matching technology, it helps to save search costs and the fall in the unemployment rate of the second group increases its workers' intertemporal utilities. On the contrary, the welfare of the first group decreases with the rise in its unemployment rate. For this group, the policy lowers the exit rate from unemployment and the wages. More generally, the

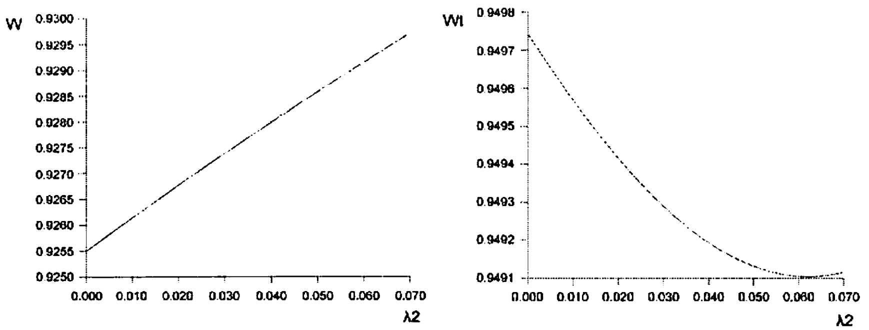

Figure 6: *The effect of an increase in λ_2 on total welfare W and workers' output Wt*

workers' output Wt suffers from the increase in the unemployment rate u, as in the previous case. The results of the policy are mixed, but again the strengthening of the informal search channel has strong adverse effects [18].

It is worth noting that the effects of the two types of policies are not the same. In both cases, the total welfare increases. However, in the first case, the congestion and substitution effects induce, for *both groups*, an increase in the unemployment rates and a decrease in the intertemporal utilities. The workers newly integrated in the first group are the only one to benefit from the policy. On the contrary, in a distributed policy, *all* workers in the second group benefit unambiguously from the improvement of their matching technology. If one compare these two policies through a rawlsian criterion, the distributed policy is the only one to increase the utility of the worst-off worker (S_2). However, since we do not model the cost of the different policies, it is difficult to compare their relative efficiency.

One could wonder why the effect of an improvement of the networks efficiency seems to be monotonic in the homogenous case (one social group) while the effects of these social policies are mixed. It is worth noting that an increase in γ in the homogenous case cannot be directly compared with an enlargement of the access to the networks or an increase of the networks efficiency for *one* workers type. Indeed, in both cases, only some of the workers benefit from the policy. In the first case the workers who join the first group, in the second case the workers of the second group. We point out that for the others such policies could induce strong adverse effects (due to congestion and substitution effects).

[18] Remark that, if employers were able to target their vacancies, the expected profit of a job must be the same for each group at the equilibrium. They were thus linked by an indifference condition. Consequently, an increase in λ_2 would induce a substitution between the vacancies sent to the second and the vacancies sent to the first group. Consequently, we would still observe a welfare loss for the first group.

4 Discussion

In this paper, we provide a simple matching model with large firms in which unemployed workers and employers can be matched together through social networks or through more "formal" methods of search. We emphasize how the subsitution between the different job search methods have strong implications with respect to the economic policy. Especially, we investigate the impact of new active labor market policies relying on social networks. Two types of policies are considered: an increase in the number of workers who access to the (effective) social networks and an improvement in the efficiency of the *existing* social contacts of the disadvantaged workers. Surprisingly, both policies can decrease workers' welfare and employment. They induce firms to desinvest in formal job advertising and this desinvestment can be more than proportionnal. The fact that we observe these effects in both cases interestingly shows that the substitution effect underlined by our model could matter for a large range of policies.

Finally, notice that this could imply that an economic policy which tries to increase disadvantaged workers' "social capital" should set up transfers in order to lead to a rise in workers' welfare. Firms benefit from the increase since it reduces their search costs, while, in the same time, a large number of workers suffer from congestion effect and from the decrease of the number of advertised vacancies. To some extent, firms could thus contribute to the establishment of the new active policies.

Our results have been obtained with a very simple model. First, it is worth noting that the quantitative impact of these social policies is only illustrative. Especially, the effects could hinge on the homophily degree while we assume perfect segregation between the groups.

Besides, the unemployed workers are passive. They do not choose their search intensity for each method of search. We can expect that if one strengthens the social networks, workers are going to decrease their use of the formal methods of search and save search costs. Does it means that a worker directly benefits, in this case, from the increase in social capital? First, this additional substitution effect induce the firms to decrease again their investment in the formal methods. This decrease can be more than proportional. The same argument can be true on workers' side. Their investment in social contacts could not be sufficient to compensate their desinvestment in the formal methods. In both cases, it is very likely that the unemployment rate and the workers' utilities would fall. Moreover, coordination failures between firms and workers can exist (Cahuc and Fontaine, 2002). There is no reason for the distribution of the search intensity between the methods to be efficient. The improvement of the informal channel could stick workers and firms in an equilibirum where they mostly use social networks although it is inefficient.

More generally, a weakness of our framework is that we treat the network structure as given. In comparison with Calvó-Armengol and Jackson (2004) , our model is particularly well-suited to the study of economic policy and economic inequality. On the contrary, it cannot investigate the dynamics of social isolation on unemployed workers as in Bramoullé and Saint-Paul (2004). A first step could be to let the parameter which drives the effectiveness of the networks (λ) be the outcome of agent's decisions. For example, agents could invest in order to improve it (increase λ). This strategic element could shed some new light on the efficiency of active labor market policies and on the evolution of social inequalities. Besides, in this paper, we investigate how networks can affect labor market outcomes by increasing the job arrival rate of workers. Consequently, we choose to not take into account that networks can generate a mismatch between heterogenous workers and heterogenous occupations. We have now to introduce this dimension. Theses issues are on our research agenda.

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Appendix:

A An urn-ball matching model

We use results from Fontaine (2006). Consider an economy with V advertised vacancies and U unemployed workers. A job offers is represented by a ball which is sent into urns, i.e. the unemployed. Besides, at each period, each employee can contact unemployed friends (send a ball into the urns). We assume that the labor force is divided in N groups indexed by i, $i = \{1, 2, ..., N\}$. Moreover, an employee of a given group forwards job offers only to unemployed workers belonging to the same group. At each period of time, an employee of the group i can contact one of the unemployed worker of the group with a probability λ_i. On the firms' side, we assume that they send their V offers at random to the U urns.

We denote by C_i the probability for an uemployed worker to receive at least one job offer:

$$C_i = 1-\left(1-\frac{1}{U}\right)^V\left(1-\frac{1}{U_i}\right)^{\lambda_i L_i}$$

$$\approx 1-\exp(-(V/U+(\lambda_i L_i)/U_i))$$

Remark that the expected number of offers reaching the members of the ith group reads

$$p_i V+\lambda_i L_i$$

with $p_i = U_i/\sum_{j=1}^{N} U_j$. One gets

$$C_i \approx 1-\exp\left(-\frac{p_i V+\lambda_i L_i}{U_i}\right)$$

We define $\theta_i = (p_i V+\lambda_i L_i)/U_i$. The probability for an offer received by an unemployed worker of the group i to be filled amounts to

$$m(\theta_i) = \frac{1}{\theta_i}(1-\exp(-\theta_i))$$

Consequently, the unconditional probability for an offer send through the formal channel (V) to be filled reads

$$\sum_{j=1}^{N} p_j \frac{1}{\theta_i}(1-\exp(-\theta_i))$$

Eventually, the exit rate from unemployment for a worker of the ith group amounts to

$$\theta_i m(\theta_i) = 1-\exp(-\theta_i)$$

B The wage equation and firm's size

For sake of simplicity, consider the general expression for the value of the firm in the simple case where $\lambda_1 = \lambda_2 = \lambda$ [19]:

$$r\Pi(L_i) = \max_{V_i \geq 0} [yL_i - w(L_i)L_i - hV_i + \dot{\Pi}(L_i)] \tag{18}$$

subject to

$$\dot{L}_i = (\lambda L_i + V_i)m(\theta) - qL_i \tag{19}$$

Using (18), (19) and the Kuhn-Tucker conditions, one gets, at the steady state, the conditions defining the equilibrium with job advertising and the equilibrium without:

$$\frac{h}{m(\theta)} = \frac{y - w(L_i) - w'(L_i)L_i}{r + q - \lambda m(\theta)} \Leftrightarrow V_i > 0 \tag{20}$$

$$\frac{h}{m(\theta)} > \frac{y - w(L_i) - w'(L_i)L_i}{r + q - \lambda m(\theta)} \Leftrightarrow V_i = 0 \tag{21}$$

Let us denote by U the value function of an unemployed worker and by E the value function of an employee in firm i. U and E satisfy at the symmetric steady state:

$$rU = z + \theta m(\theta)(E(L_i) - U) \tag{22}$$

$$rE(L_i) = w(L_i) + q(U - E(L_i)) \tag{23}$$

Wages are subject to bargaining between the firm and the worker and can be renegotiated each period at no cost. Then the surplus gotten by an employee paid wage $w(L_i)$ is

$$E(L_i) - U = \frac{w(L_i) - rU}{r + q} \tag{24}$$

Moreover, the value of a marginal job reads

$$J(L_i) = \frac{y - w(L_i) - w'(L_i)L_i}{r + q - \lambda m(\theta)} \tag{25}$$

The surplus of each match is shared according to the Nash solution of the bargaining problem

$$\beta J(L_i) = (1 - \beta)(E(L_i) - U) \tag{26}$$

with $\beta \in [0, 1]$ the share that accrues to the worker. That leads to the following differential equation

[19] These results hold in the general case.

$$w(L_i) = \underbrace{(1-\beta)\left[\frac{r+q-\lambda m(\theta)}{r+q-(1-\beta)\lambda m(\theta)}\right] r V_u}_{A}$$

$$+ \underbrace{\frac{r+q}{r+q-(1-\beta)\lambda m(\theta)}}_{B}[y - w'(L_i)L_i] \tag{27}$$

The equation is multiplied by $L^{1/B-1}$:

$$L^{1/B-1}w(L_i) + BL^{1/B}w'(L_i) = (A+By)L_i^{1/B-1}$$

$$\Rightarrow \int_0^{L_i}[L^{1/B-1}w(L) + BL^{1/B}w'(L)]\,dL = (A+By)\int_0^{L_i} L^{1/B-1}\,dL$$

We assume that $\lim_{L\to 0} Lw(L) = 0$. One gets,

$$w(L_i) = w = (a+by)$$