Positional effects, product quality and regulation in duopoly

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1 Introduction

A wide literature on vertical differentiation investigates the interaction between market competition (typically, in prices) and the choice of product quality. The main focus of these contributions consists in stressing that vertical differentiation softens price competition. This ultimately entails that profit-seeking firms may supply excess differentiation as compared to the social optimum.[1] The role of consumption externalities, such as network and/or positional effects in a vertically differentiated oligopoly has been largely neglected so far.[2] In particular, the consumption of different qualities may well entail status or positional effects. The fact that the purchase of a high-quality variety of any given good can be driven by social motivation rather than intrinsic characteristics is well known from Veblen (1899), Leibenstein (1950) and Hirsch (1976).

To achieve social status, consumers tend to buy particular varieties of goods which, in view of their quality and price (or because they are supplied in limited quantities), are not accessible to everybody. However, note that

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1 See, e.g., Gabszewicz and Thisse (1979) and Shaked and Sutton (1982).

2 Grilo, Shy and Thisse (2001) analyse price competition in a spatial duopoly with both types of externality, under full market coverage. However, they do not investigate the choice of location, which are exogenously given.

'positional' is not necessarily equivalent to 'elitarian' : if a good is elitarian, this means that it is not accessible to all consumers alike; if instead it is positional, this entails that there are several varieties of the same good, out of which only a few or just one may confer status to purchasers, while other varieties only meet functional needs. This is the case, for example, of sunglasses : everybody owns a pair of sunglasses, but only a few brands (e.g., Armani, Ray Ban and Dolce & Gabbana) convey positional effects, while others simply do not.

To grasp the essential features of this situation, we model a duopoly, with one firm selling a positional good and the other one selling a lower quality non-positional substitute, which is bought only for its intrinsic characteristics. The positional effect is such that the high-quality good becomes more desirable as the number of those consumers who purchase it shrinks.[3] We assume full market coverage, so that consumers have to choose among a high-quality (positional) good, and a low-quality (non-positional) good. Single-product duopolists supply vertically differentiated goods whose production entails a variable cost, which is assumed to be increasing in the quality level. The existence of a pure-strategy subgame perfect equilibrium in qualities and prices is proved. We show that the profits of the firm supplying the low-quality (non-positional) good are non-monotone (i.e., first increasing and then decreasing) in the amount of the positional externality. This is due to the fact that unit profits of the low-quality firm are non-monotone while its market share is monotonically decreasing in the positional externality. The same non-monotonicity obtains in the surplus of consumers purchasing the positional good. On the contrary, the surplus of customers of the non-positional good is always decreasing. With regard to quality distortion and social welfare, we obtain two main results. First, duopoly equilibrium qualities are negatively affected by the externality, while the socially efficient ones are independent of the externality. Second, both in the duopoly equilibrium and under social planning, welfare is increasing in the extent of the externality. However, the scope for regulation widens as the externality becomes more important, because beyond a critical threshold of the external effect, the welfare loss due to profit maximisation is also increasing in the amount of positional concern. Accordingly, we illustrate the possibility of regulating such a market through a minimum quality standard.

The remainder of the paper is organised as follows. Section 2 describes the model. The profit-maximising duopoly is analysed in section 3, while the behaviour of the social planner is presented in section 4. Section 5 compares the two regimes and investigates regulation. Concluding remarks are in section 6.

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[3] The behaviour of a monopolist in a market where buyers's satisfaction increases in the number of consumers excluded from purchase is analysed in Basu (1987), who suggests that the positional concern may be traced back to either quality signalling by the firm or status seeking by consumers. The interplay between product quality and positional concerns in monopoly is in Lambertini and Orsini (2002).

2 The setup

Consider a duopoly consisting in two single-product firms offering qualities $q_H \geqslant q_L \geqslant 0$, following Cremer and Thisse (1994).[4] In particular, we focus upon the case where the positional externality is associated with q_H, and all consumers who do not purchase the positional high-quality good buy an alternative low-quality good which does not confer any *status*. Hence, we confine our attention to the full market coverage setting. The population of buyers is uniformly distributed with density equal to one over $\left[\overline{\theta}-1,\overline{\theta}\right]$, with $\overline{\theta}>1$. Parameter $\theta \in \left[\overline{\theta}-1,\overline{\theta}\right]$ measures each consumer's marginal willingness to pay for quality. As an illustration, suppose we are describing the car market. The high-quality positional good could be a flagship sportcar like Porsche Boxster or Ferrari Testarossa, conferring social distinction. Such a car is accessible to rich consumers only, while others may buy anything like Fiat Punto or VW Polo, which can be assumed to be non-positional. All consumers in $\left[0,\overline{\theta}-1\right]$ use either buses or the tube, whose net utility is normalised to zero.

The net surplus of a consumer who buys the high quality good is

$$U_H = \theta q_H + \alpha(1-x_H) - p_H \ \forall q_H > q_L \tag{1}$$
$$U_H = \theta q_H - p_H \text{ iff } q_H = q_L$$

where $1 - x_H = x_L$, θq_H is the hedonic component, p_H is the price of the high-quality good and $\alpha(1-x_H) = \alpha x_L$ is the positional component, with parameter $\alpha > 0$ measuring the intensity of the positional externality, while x_H and x_L are market demands for the two varieties. If products are homogeneous, then clearly positional effects disappear, but this would also entail zero profits for firms competing in prices with identical qualities, so such event can be expected to be off the equilibrium path.

The assumption of additive separability between hedonic preferences and the positional effect amounts to saying that the search for status is in fact independent of the taste for intrinsic quality. This is the case, for instance, if one considers that the social status attached to wearing a Hermes sweater may well be the same for everybody, irrespective of anyone's ability to appreciate the quality of the make itself.

The net utility of a consumer buying the low-quality good is $U_L = \theta q_L - p_L$ for all $q_L \in (0, q_H]$. Along the support $\left[\overline{\theta}-1,\overline{\theta}\right]$, define as θ' the location of the consumer who is indifferent between q_H and q_L at generic prices $\{p_H, p_L\}$. Therefore, $1 - x_H = x_L = 1 - \overline{\theta} + \theta'$ and θ' obtains from the solution of $U_H = U_L$:

$$\theta' = \frac{\alpha(\overline{\theta}-1) + p_H - p_L}{\alpha + q_H - q_L} \ . \tag{2}$$

[4] In an alternative setting, Bagwell and Bernheim (1996) describe a market for *conspicuous goods* where products are homogeneous, but a *griffe* adds a status signal to the intrinsic use of a product. This entails pricing above marginal cost by the firm producing the conspicuous good.

Observe that

Lemma 1 *If* $\alpha > (\overline{\theta} - 1)(q_H - q_L) / (q_H - q_L + 1)$ *and*

$$p_H - p_L \in \left((\overline{\theta} - 1)(q_H - q_L), \alpha(q_H - q_L + 1)\right),$$

then $\theta' \in \left(\overline{\theta} - 1, \overline{\theta}\right)$.

Proof. See the Appendix. □

If Lemma 1 is satisfied, the indifferent consumer at θ' is internal to the support of consumer tastes and both firms enjoy positive demand. If so, consumer surplus in the two market segments is, respectively:

$$CS_H = \int_{\theta'}^{\overline{\theta}} U_H d\theta \; ; \; CS_L = \int_{\overline{\theta}-1}^{\theta'} U_L d\theta \, . \tag{3}$$

Firm i's cost function is $C_i = q_i^2 x_i$ and profits are $\pi_i = (p_i - q_i^2)x_i$. Social welfare amounts to

$$SW = CS_H + CS_L + \pi_H + \pi_L \, . \tag{4}$$

3 Profit-maximising duopoly

Strategic interaction between firms takes place in two stages, with firms moving simultaneously in both stages. In the first, firms choose qualities; in the second, they choose prices. As usual, we solve the game by backward induction, the solution concept being subgame perfection. To ease the exposition, we introduce the following:

Definition 1 $\theta'' = 1 + \dfrac{\sqrt{3(4096\alpha^3 + 7680\alpha^2 + 4032\alpha + 675)}}{12(8\alpha + 3)}$.

The solution of the duopoly game is summarised by:

Lemma 2 *Provided that full market coverage obtains, i.e.,* $\overline{\theta} > \theta''$, *the subgame perfect equilibrium prices and qualities are:*

$$p_H^* = \frac{27\left(8\overline{\theta} + 25\right) + 16\left[36\alpha\overline{\theta} + 2\alpha\left(153 + 8\alpha\left(32\alpha + 45\right)\right) + \Psi\right]}{192\left(8\alpha + 3\right)^2};$$

$$p_L^* = \frac{27\left(49 - 40\overline{\theta}\right) + 16\left[8\alpha(8\alpha + 21)\left(4\alpha + 3\right) - 36\alpha\overline{\theta}\left(16\alpha + 11\right) + \Psi\right]}{192\left(8\alpha + 3\right)^2},$$

$\Psi \equiv 3\overline{\theta}^2\left(8\alpha + 3\right)^2;$

$$q_H^* = \frac{12\overline{\theta} + 32\alpha\overline{\theta} + 3}{8(8\alpha + 3)} \ ; \ q_L^* = \frac{12\overline{\theta} + 32\alpha\overline{\theta} - 48\alpha - 15}{8(8\alpha + 3)} \ .$$

Proof. See the Appendix. □

On the basis of the above lemma, we can observe what follows:

Corollary 1 *The absolute degree of differentiation is independent of the extent of the positional externality* α, *in that* $q_H^* - q_L^* = 3/4$.

Moreover, since $\partial q_H^*/\partial\alpha$ and $\partial q_L^*/\partial\alpha$ are both negative, while $\partial\left(q_H^*/q_L^*\right)/\partial\alpha$ is positive, we can also state:

Corollary 2 *Both equilibrium qualities are everywhere decreasing in the extent of the positional externality and the relative degree of product differentiation, i.e., the ratio* q_H^*/q_L^*, *is everywhere increasing in* α.

Equilibrium output levels and profits are:

$$x_H^* = \frac{32\alpha + 9}{6(8\alpha + 3)} \ ; \ x_L^* = \frac{16\alpha + 9}{6(8\alpha + 3)} \ ; \tag{5}$$

$$\pi_H^* = \frac{(4\alpha + 3)(32\alpha + 9)^2}{144(8\alpha + 3)^2} \ ; \ \pi_L^* = \frac{(4\alpha + 3)(16\alpha + 9)^2}{144(8\alpha + 3)^2} \ . \tag{6}$$

On the basis of (5) and (6), the following results can be quickly derived:

Remark 1 $\partial x_H^*/\partial\alpha > 0$ *and* $\partial x_L^*/\partial\alpha < 0$, *i.e., the market share of the high- (respectively, low-) quality firm increases (resp., decreases) as the positional externality becomes more relevant.* $x_H^* \in [1/2, 2/3)$ *for* $\alpha \in [0, \infty)$.

Note that equilibrium demands satisfy the admissibility condition for duopoly stated in Lemma 1.

Remark 2 $\partial\pi_H^*/\partial\alpha > 0$ *everywhere.* $\partial\pi_L^*/\partial\alpha < 0 \ \forall\alpha \in [0, 3(\sqrt{17} - 3)/32)$; $\partial\pi_L^*/\partial\alpha > 0 \ \forall\alpha > 3\left(\sqrt{17} - 3\right)/32$. *However,* $\partial\left(\pi_H^* + \pi_L^*\right)/\partial\alpha > 0$ *everywhere.*

Remark 1 entails that, positional externalities being absent, the distribution of demand across varieties would be symmetric (as in Cremer and Thisse (1994)). The positional effect exerted by the high-quality good makes it more attractive for consumers. This can be interpreted jointly with corollary 2, as follows. If α is large, firm H may reduce q_H because the associated reduction of the hedonic ability to spend θq_H is compensated by the positional component αx_L. All else equal, this has the desirable property of reducing production costs, which are quadratic in quality. Accordingly, firm H finds it convenient to expand output as α gets larger, because quantity enters the cost function in a linear way. This also has the beneficial effect of increasing revenues. By strategic complementarity, a decrease in q_H^* produces a decrease in q_L^*, while the assumption of full market coverage entails that an increase in x_H^* must yield a decrease in x_L^*.

Remark 2 entails that the behaviour of the low-quality firm's profits is non-monotone in α, due to strategic complementarity in prices. As α increases, the increase in prices and the decrease in qualities jointly determine a larger unit profit. Initially, for low levels of α, this is more than offset by the negative effect on the output level x_L^*. Then, as the positional externality becomes sufficiently large, the opposite happens. Yet, the net effect on industry profits is such that any increase in α improves the profit performance of the whole industry.

Moreover, the behaviour of prices is summarised by the following:

Remark 3 $\partial(p_H^* - p_L^*)/\partial\alpha > 0$ *for all admissible* $\overline{\theta}$.

Together with remark 1, this entails the seemingly counterintuitive result that the market share of the high-quality firm grows larger in α at the same time as the difference in prices increases. This fact can be given the following explanation. For given prices, an increase in α involves an increase in product differentiation as measured by the ratio q_H^*/q_L^* as well as a decrease in both quality levels. This entails that x_H^* must be increasing in α, all else equal. However, prices also change with α. The way prices react to a change in the positional externality significantly differs across firms. Examine first firm H's price: given the price of firm L, an increase in α entails a reduction of q_H^* which, in turn, drives a decrease in p_H. Yet, this is softened by the fact itself that the positional good has become more desirable. This counterbalancing effect is completely absent for the low-quality firm which experiences only a reduction in q_L^*.[5]

At the duopoly equilibrium, social welfare amounts to:

$$SW^* = \frac{144\overline{\theta}(\overline{\theta}-1)(8\alpha+3)^2 + 81 + 1728\alpha + 7680\alpha^2 + 8192\alpha^3}{576(8\alpha+3)^2} . \quad (7)$$

Differentiating (7) w.r.t. α, we obtain:

$$\frac{\partial SW^*}{\partial\alpha} = \frac{4096\alpha^3 + 4608\alpha^2 + 2016\alpha + 243}{36(8\alpha+3)^3} > 0 \quad \forall\alpha \geqslant 0; \overline{\theta} \geqslant \theta'' . \quad (8)$$

Therefore, we have proved

Proposition 1 *The level of social welfare associated with the duopoly equilibrium is everywhere increasing in the extent of the positional externality.*

The above result stems from the possibility for the high-quality firm to extract more surplus from her customers as α increases, as we have illustrated above. This, in turn, produces a positive externality on the low-quality firm in the same direction. Indeed, when we consider the behaviour of consumer surplus in the two market segments, it emerges that

[5] Note that the behaviour of both prices is non-monotone in α. However, the slope of the best replies in the price space is always equal to 1/2, ensuring that $p_H^* - p_L^*$ is increasing in α also in the range where both prices get larger when α does.

$\partial CS_i^*/\partial\alpha < 0$, $i = H, L$, for all $\alpha \geqslant 0$. Given that $\partial(\pi_H^* + \pi_L^*)/\partial\alpha > 0$ from Remark 2, the effect of the externality on welfare is due to the fact that $\partial(\pi_H^* + \pi_L^*)/\partial\alpha > -\partial(CS_H^* + CS_L^*)/\partial\alpha$.

4 Social planning

As a benchmark, we briefly illustrate the first best where a benevolent social planner chooses prices and qualities to maximise welfare (4). The price of the low-quality good[6] is $p_L^{SP} \in [q_L^2, (\overline{\theta}-1)q_L]$, while the price of the high-quality good is the solution of the first order condition $\partial SW/\partial p_H = 0$:

$$p_H^{SP} = p_L^{SP} + \frac{\alpha(\alpha + \overline{\theta}q_H + q_H^2 - \overline{\theta}q_L - q_L^2) + (q_H^2 - q_L^2)(q_H - q_L)}{q_H - q_L + 2\alpha}\,. \tag{9}$$

This entails that prices are not linearly independent, and there exist infinitely many price pairs $\{p_H^{SP}, p_L^{SP}\}$ allowing the planner to maximise social welfare.[7] The price of the high-quality good can be rewritten as follows:

$$p_H^{SP} = p_L^{SP} + q_H^2 - q_L^2 + \alpha x_L \tag{10}$$

which obviously coincides, if $\alpha = 0$, with the socially optimal price in absence of the positional externality. For any quality pair, the planner extracts from the status-seeking consumers a tax revenue of the same size as the positional externality.

The system $\{\partial SW/\partial q_H = 0; \partial SW/\partial q_L = 0\}$ of first order conditions w.r.t. qualities has five critical points, out of which

$$q_H^{SP} = \frac{4\overline{\theta} - 1}{8};\ q_L^{SP} = \frac{4\overline{\theta} - 3}{8} \tag{11}$$

is the only pair satisfying second order conditions. This entails:

Remark 4 *The socially optimal quality levels are independent of the positional externality.*

In particular, they coincide with the socially optimal qualities associated with the maximisation of social welfare in a standard model without positional externality (see Lambertini (1997)). Moreover, $q_H^{SP} - q_L^{SP} = 1/4$.

Social welfare amounts to $SW^{SP} = [16(\overline{\theta}^2 - \overline{\theta} + \alpha) + 5]/64$. Equilibrium prices are $p_H^{SP} \in [(16\overline{\theta}^2 - 8\overline{\theta} + 1)/64 + \alpha/2, (4\overline{\theta}^2 - 5\overline{\theta} + 2)/8 + \alpha/2]$ and

[6] We impose that the break even condition on the low-quality good be satisfied. However, in a general equilibrium analysis, the planner could also price the low-quality good below its marginal cost.

[7] Observe that this stems from the assumption of full market coverage, which implies that the size of the social surplus is given (see Lambertini and Mosca (1999)).

$p_L^{SP} \in [(4\overline{\theta} - 3)^2/64, (\overline{\theta} - 1)(4\overline{\theta} - 3)/8]$, while output levels are $x_H^{SP} = x_L^{SP} = 1/2$. Knowing that the first order conditions on prices are not linearly independent, the planner must design the aforementioned price intervals so as to make the median (and average) consumer coincide with the individual who is indifferent between H and L, because any departure from an equal split of total demand across varieties would damage welfare. Accordingly, the lower and upper bounds of the optimal price range for the high-quality good are both shifted upwards by the full amount of the external effect. This is equivalent to imposing a tax on the positional variety, equal to $\alpha(1 - x_H) = \alpha/2$.

Hence, we have proved the following:

Proposition 2 *The socially optimal quality spectrum and allocation of consumers across qualities coincide with those which would obtain the externality being absent. To attain the same allocation notwithstanding the externality, the planner must impose a tax on the positional good, whose size is the same as the positional externality itself.*

It goes without saying that implementing this type of taxation would impose too much of a requirement on the policy maker in terms of both informational and transaction costs. However, a milder and feasible alternative consists in introducing higher tax rates on status goods. E.g., in Italy, the presence of a swimming pool in the garden of a house qualifies the house itself as a status good, subject to a higher tax rate than an equally large house without a pool.

5 Duopoly vs social planning

First of all, we compare the quality levels supplied in equilibrium under the two alternative regimes:

$$q_L^* < q_L^{SP}\,;\, q_H^{SP} < q_H^* \;\; \forall \alpha \geqslant 0 \tag{12}$$

which entails the well known excess differentiation outcome typically associated with profit-seeking behaviour. From Remarks 2 and 6, we also know that, in the private duopoly, both equilibrium qualities downgrade as α increases, while socially efficient qualities are independent of α. Therefore, the presence of a positional effect implies a decrease in the average quality level available at the duopoly equilibrium.

The foregoing discussion enlightens two alternative directions along which a policy maker taking care of social welfare should regulate the profit-maximising duopoly. Consider first that $\partial q_L^*/\partial \alpha < 0$ and $\partial x_L^*/\partial \alpha < 0$ for all $\alpha > 0$. This entails that the downward quality distortion induced by the positional externality is accompanied by an increasing distortion in the allocation of consumers across qualities as α increases. The following result can be easily proved:

Proposition 3 *The incentive for a benevolent policy maker to regulate the behaviour of the profit-maximising duopolists is non monotone in the extent of the positional externality.*

This can be shown by observing that

$$\Delta SW = SW^{SP} - SW^{*} \tag{13}$$

$$= \frac{81 + 16\alpha\,[27 + \alpha(33 + 16\alpha)]}{144(8\alpha + 3)^2}$$

with

$$\frac{\partial \Delta SW}{\partial \alpha} = \frac{2\alpha\,[8\alpha\,(9 + 8\alpha) - 9]}{9(8\alpha + 3)^3} > 0 \;\forall \alpha > \frac{3\left(\sqrt{13} - 3\right)}{16} \simeq 0.11 \tag{14}$$

The intuitive source of this result lies in the increasing distortion in the allocation of demand across goods under profit-maximising duopoly compared to social planning, as the positional externality increases. While the planner sets $x_i^{SP} = 1/2$ for all α, from Remark 3 we know that $\partial x_H^*/\partial\alpha > 0$. This relates to the fact that p_H^* does not internalise the full extent of the externality, which is instead a characteristic of p_H^{SP}. Therefore, as the positional effect becomes more relevant, the distortion in the allocation of demand due to the price policy of the high-quality firm produces an increasing welfare loss w.r.t. social optimum if the external effect is large enough.

This distortion can be corrected by either taxing the high-quality good,[8] or increasing the low-quality level through the adoption of a minimum quality standard. Obviously, a combination of both measures could also be adopted. However, here, we focus on the adoption of a minimum quality standard (MQS). As it is usually done in the existing literature (Ronnen (1991); Crampes and Hollander (1995); Ecchia and Lambertini (1997)), we suppose that the regulator introduces the MQS given the price behaviour of duopolists, and mimicking to control the low quality level in a simultaneous game against the high-quality firm.

The desirability of MQS regulation can be quickly assessed by evaluating the sign of $\partial SW/\partial q_L$ at (q_H^*, q_L^*):

$$\left.\frac{\partial SW}{\partial q_L}\right|_{q_H^*, q_L^*} = \frac{(16\alpha + 9)\,[27 + 2\alpha\,(63 + 80\alpha)]}{36\,(4\alpha + 3)\,(8\alpha + 3)^2} > 0 \tag{15}$$

[8] The Pareto-improving nature of a tax applied to positional goods has been pointed out by Ireland (1994), where purchasing a positional good is used as a wealth signal. The effect of *ad valorem* taxation on quality and welfare levels in duopoly is illustrated by Cremer and Thisse (1994), who do not consider externalities of any kind.

This proves that a marginal increase in q_L above the non-cooperative duopoly equilibrium increases welfare.[9] This, in turn, implies an increase in the market share of the low-quality (non-positional) supplier :

$$\left.\frac{\partial x_L}{\partial q_L}\right|_{q_H^*, q_L^*} = \frac{12\,(16\alpha+9)}{27+16\alpha\,(8\alpha+7)} > 0 \tag{16}$$

The following second-order derivative :

$$\left.\frac{\partial^2 x_L}{\partial q_L \partial \alpha}\right|_{q_H^*, q_L^*} = -\frac{768\,(4\alpha+3)\,(8\alpha+3)}{[27+16\alpha\,(8\alpha+7)]^2} < 0 \tag{17}$$

entails

Remark 5 *Albeit always positive, the beneficial effect of the MQS on x_L shrinks as the weight attached to the positional externality becomes more relevant.*

This property can be intuitively explained, as follows. As the MQS induces an increase in the demand for the low-quality product, the positional externality enjoyed by those consumers who purchase the high-quality product becomes larger. This partially offsets the beneficial effect of the MQS on x_L.

6 Concluding remarks

We have modeled the role of positional effects in a duopoly for vertically differentiated goods, where the positional externality is associated with the high-quality good, while the low-quality variety is non-positional. Then, we have evaluated the welfare performance of the duopoly against the behaviour of a social planner operating with the same number of varieties.

Due to the fact that the distortion in the allocation of consumers across varieties increases as the externality effect becomes larger, the welfare loss generated by profit-seeking behaviour becomes increasing in the externality, provided the latter is sufficiently high. As to the bearings of positional effects upon quality levels, as the positional concern increases, we observe a decrease in the quality levels of both varieties, irrespective of market affluency, while the quality ratio measuring the degree of differentiation becomes higher.

Concerning the behaviour of a social planner offering two varieties, we have established that the socially efficient allocation of consumers across

9 As qualities are strategic complements (at least in the neighbourhood of the equilibrium), any standard imposing an increase to the low quality would also imply an increase in the high quality (see Crampes and Hollander (1995); Ecchia and Lambertini (1997)).

goods obtains by fully internalising the positional effect through a tax of the same amount. Moreover, the socially efficient quality spectrum is independent of positional concerns, and the planner prices the high-quality good so as to tax positional consumption. Finally, we have assessed the feasibility of a regulation based upon the introduction of a minimum quality standard. As it is usually observed in the available literature, the adoption of the MQS increases the market share of the low-quality firm, albeit this effect is softened by the positional externality.

Appendix

Proof of Lemma 1

To prove this result, we proceed in two steps. First, we have

$$\theta' < \overline{\theta} \Leftrightarrow p_H - p_L < \alpha\,(q_H - q_L + 1)\,. \tag{18}$$

Second, we also have

$$\theta' > \overline{\theta} - 1 \Leftrightarrow p_H - p_L > (\overline{\theta} - 1)\,(q_H - q_L)\,. \tag{19}$$

Then, $\alpha\,(q_H - q_L + 1) > (\overline{\theta} - 1)\,(q_H - q_L)$
if and only if $\alpha > (\overline{\theta} - 1)\,(q_H - q_L)\,/\,(q_H - q_L + 1)$. □

Proof of Lemma 2

The first order conditions w.r.t. prices, given qualities q_H and q_L chosen at the first stage, are:

$$\frac{\partial \pi_H}{\partial p_H} = \frac{p_L - 2p_H + \overline{\theta}(q_H - q_L) + q_H^2 + \alpha}{q_H - q_L + \alpha} = 0\,; \tag{20}$$

$$\frac{\partial \pi_L}{\partial p_L} = \frac{p_H - 2p_L + (1 - \overline{\theta})(q_H - q_L) + q_L^2}{q_H - q_L + \alpha} = 0\,, \tag{21}$$

whose simultaneous solution yields candidate equilibrium prices:

$$p_H^* = \frac{(1 + \overline{\theta})(q_H - q_L) + 2q_H^2 + q_L^2 + 2\alpha}{3}\,; \tag{22}$$

$$p_L^* = \frac{(2 - \overline{\theta})(q_H - q_L) + q_H^2 + 2q_L^2 + \alpha}{3}\,. \tag{23}$$

Such prices satisfy second order conditions. Plugging (22-23) into the profit functions and simplifying, we obtain the relevant expressions for profits at the first stage of the game:

$$\pi_H = \frac{\left[(1+\overline{\theta})(q_H - q_L) - q_H^2 + q_L^2 + 2\alpha\right]^2}{9(q_H - q_L + \alpha)}\,; \tag{24}$$

$$\pi_L = \frac{\left[(2-\overline{\theta})(q_H - q_L) + q_H^2 - q_L^2 + \alpha\right]^2}{9(q_H - q_L + \alpha)}\,. \tag{25}$$

Differentiating (24) and (25) w.r.t. q_H and q_L, respectively, we obtain the first order conditions (FOCs) at the first stage:

$$\frac{\partial \pi_H}{\partial q_H} = \frac{(1+\overline{\theta})(q_H - q_L) - q_H^2 + q_L^2 + 2\alpha}{9(q_H - q_L + \alpha)} \cdot \Big[2(1+\overline{\theta} - 2q_H) + \\ - \frac{(1+\overline{\theta})(q_H - q_L) - q_H^2 + q_L^2 + 2\alpha}{9(q_H - q_L + \alpha)}\Big] = 0\,; \tag{26}$$

$$\frac{\partial \pi_L}{\partial q_L} = \frac{(2-\overline{\theta})(q_H - q_L) + q_H^2 - q_L^2 + \alpha}{9(q_H - q_L + \alpha)} \cdot \Big[2(2-\overline{\theta} + 2q_L) + \\ + \frac{(2-\overline{\theta})(q_H - q_L) + q_H^2 - q_L^2 + \alpha}{9(q_H - q_L + \alpha)}\Big] = 0\,. \tag{27}$$

FOCs (26-27) are both of degree four, so that in principle one has to expect to find sixteen critical points of the system itself. Indeed, there are only nine, as some solutions coincide:

$$q_{H1} = \frac{2\overline{\theta}+5}{4}; q_{L1} = \frac{2\overline{\theta}-1}{4}, \text{ iff } \alpha = 0 \tag{28}$$

$$q_{H2} = \frac{2\overline{\theta}-5}{4}; q_{L2} = \frac{2\overline{\theta}-1}{4}, \text{ iff } \alpha = \frac{1}{2}$$

$$q_{H3} = \frac{2\overline{\theta}-7}{4}; q_{L3} = \frac{2\overline{\theta}-1}{4}, \text{ iff } \alpha = 0 \tag{29}$$

$$q_{H4} = \frac{2\overline{\theta}+7}{4}; q_{L4} = \frac{2\overline{\theta}-1}{4}, \text{ iff } \alpha = \frac{1}{2}$$

$$q_{H5} = \frac{2\left(\overline{\theta}+1\right) - \sqrt{32\alpha+9}}{4}; q_{L5} = \frac{2\overline{\theta}-1}{4}, \text{ for all } \alpha \neq 0, \frac{1}{2} \tag{30}$$

$$q_{H6} = \frac{2\left(\overline{\theta}+1\right) + \sqrt{32\alpha+9}}{4}; q_{L6} = \frac{2\overline{\theta}-1}{4}, \text{ for all } \alpha \neq 0, \frac{1}{2}$$

$$q_{H7} = \frac{2\left(\overline{\theta}-2\right) - \sqrt{16\alpha+9}}{4}; q_{L7} = \frac{2\overline{\theta}-1}{4}, \text{ for all } \alpha \neq 0, \frac{1}{2} \tag{31}$$

$$q_{H8} = \frac{2\left(\overline{\theta}-2\right) + \sqrt{16\alpha+9}}{4}; q_{L8} = \frac{2\overline{\theta}-1}{4}, \text{ for all } \alpha \neq 0, \frac{1}{2}$$

$$q_{H9} = \frac{12\overline{\theta} + 32\alpha\overline{\theta} + 3}{8(8\alpha + 3)} \; ; \; q_{L9} = \frac{12\overline{\theta} + 32\alpha\overline{\theta} - 48\alpha - 15}{8(8\alpha + 3)} \, . \tag{32}$$

In order to select the Nash equilibrium pair, we proceed as follows. First, there are only two pairs satisfying second order conditions (SOCs),[10] i.e., (q_{H5}, q_{L5}) and (q_{H9}, q_{L9}). In particular, SOCs are always simultaneously met by (q_{H9}, q_{L9}), while they are simultaneously met by (q_{H5}, q_{L5}) only for $\alpha \in [0, 1/2]$. However, in this range of α, (q_{H5}, q_{L5}) can be dismissed as a candidate equilibrium, since it implies $\pi_H = 0$ and $\pi_L \leqslant 0$.

Hence, the second step consists in verifying that there are no profitable unilateral deviations from (32). The candidate equilibrium profits in (q_{H9}, q_{L9}) are:

$$\pi_H\left(q_{H9}, q_{L9}\right) = \frac{(4\alpha + 3)(32\alpha + 9)^2}{144(8\alpha + 3)^2} \; ; \; \pi_L\left(q_{H9}, q_{L9}\right) = \frac{(4\alpha + 3)(16\alpha + 9)^2}{144(8\alpha + 3)^2} \, . \tag{33}$$

Now take as given $q_L = q_{L9}$, and verify that $\pi_H\left(q_H, q_{L9}\right)$ takes a maximum in $q_H = q_{H9}$ while it is equal to zero in:

$$\hat{q}_H = \frac{\overline{\theta} + 1}{2} + \frac{\sqrt{(32\alpha + 9)\left(256\alpha^2 + 320\alpha + 81\right)}}{8(8\alpha + 3)} \tag{34}$$

where both $x_H = 0$ and $p_H = \hat{q}_H^2$, i.e., firm H's demand and unit profits are both nil at the same time. Moreover, for all $q_H > \hat{q}_H$, we have $x_H < 0$ and $p_H < \hat{q}_H^2$. To conclude the proof one has to check that

$$\pi_H\left(q_H, q_{L9}\right) \geqslant \pi_H\left(q_{H9}, q_{L9}\right) \tag{35}$$

for all

$$q_H \geqslant \tilde{q}_H = \frac{3\left(4\overline{\theta} + 7\right) + 32\alpha\left(\overline{\theta} + 2\right) + 4\sqrt{(32\alpha + 9)(8\alpha + 3)(4\alpha + 3)}}{8(8\alpha + 3)} \, . \tag{36}$$

Yet, $\tilde{q}_H > \hat{q}_H$, therefore we are in a range where $\left(p_H - q_H^2\right) x_H$ is the product of two negative numbers. This implies that there are no profitable unilateral deviations for firm H. By applying the same procedure, one comes to an analogous conclusion for firm L as well.

Hence, the unique subgame perfect equilibrium qualities are:

$$q_H^* = \frac{12\overline{\theta} + 32\alpha\overline{\theta} + 3}{8(8\alpha + 3)} \; ; \; q_L^* = \frac{12\overline{\theta} + 32\alpha\overline{\theta} - 48\alpha - 15}{8(8\alpha + 3)} \, , \tag{37}$$

which, if $\alpha = 0$, coincide with the equilibrium qualities observed in the well known model without externality (see Cremer and Thisse (1994); Ecchia

10 Second order conditions are omitted for brevity.

and Lambertini (1997), *inter alia*). In addition, $q_L^* > 0$ requires $\overline{\theta} > 3\,(5 + 16\alpha)\,/\,[4(8\alpha + 3)]$.

Qualities (37) can be plugged into (22-23) to obtain the subgame perfect equilibrium prices:

$$p_H^* = \frac{27\left(8\overline{\theta} + 25\right) + 16\left[36\alpha\overline{\theta} + 2\alpha\left(153 + 8\alpha\left(32\alpha + 45\right)\right) + \Psi\right]}{192\left(8\alpha + 3\right)^2}; \tag{38}$$

$$p_L^* = \frac{27\left(49 - 40\overline{\theta}\right) + 16\left[8\alpha\left(8\alpha + 21\right)\left(4\alpha + 3\right) - 36\alpha\overline{\theta}\left(16\alpha + 11\right) + \Psi\right]}{192\left(8\alpha + 3\right)^2}, \tag{39}$$

where $\Psi \equiv 3\overline{\theta}^2\left(8\alpha + 3\right)^2$. Expressions (37-38-39) define the subgame perfect equilibrium of the two-stage game if and only if the market is sufficiently rich to ensure full market coverage. This requires $\overline{\theta} \geqslant \theta''$. In order to calculate the critical level of $\overline{\theta}$, i.e., θ'', above which all consumers are able to buy in equilibrium, consider the condition of the poorest consumer located at $\overline{\theta} - 1$:

$$U_L(\overline{\theta} - 1) = (\overline{\theta} - 1)q_L - p_L = \tag{40}$$

$$= \frac{48(64\alpha^2 + 48\alpha + 9)\overline{\theta}^2 - 4(1536\alpha^2 + 115\alpha + 216)\overline{\theta}}{192(8\alpha + 3)^2} + $$

$$- \frac{4096\alpha^3 + 4608\alpha^2 + 1728\alpha + 243}{192(8\alpha + 3)^2}.$$

Solving $U_L(\overline{\theta} - 1) = 0$ w.r.t. $\overline{\theta}$, we obtain:

$$\overline{\theta} = 1 \pm \frac{\sqrt{3(4096\alpha^3 + 7680\alpha^2 + 4032\alpha + 675)}}{12(8\alpha + 3)}. \tag{41}$$

Obviously, only the larger root $\overline{\theta}_+$ is acceptable, and tends to $9/4$ as α tends to zero (cf. Cremer and Thisse (1994)). Hence, full market coverage is observed in equilibrium for all $\overline{\theta} \geqslant \theta'' \equiv \overline{\theta}_+$. Moreover, $\overline{\theta} \geqslant \theta''$ is also sufficient to ensure that q_L^* be strictly positive. □

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