Altruistic bequests and non-negative savings

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1 Introduction

One of the prominent way in which economists have modelled links between households is based on the concept of altruism. People are said to have altruistic preferences when they care about their relatives and, in particular about their children. This leads them to maximize a combination of their lifetime well-being and their children's well-being. Altruism motivates transfers like inter-vivos gifts and bequests.

Becker (1974) studied the rational game-type interaction which arises between an altruistic parent and his selfish child. The latter's decision consists in choosing actions which may increase his income at the expense of his parent's income. The parent chooses the level of the transfer. Becker's (1974) "Rotten Kid Theorem" states that the altruistic parent can induce his selfish child to behave in the interest of the family. This is achieved when the child takes for granted that his parent is sufficiently altruistic to make a positive transfer. Then the child's best response is to choose his actions in order to maximize the family income, i.e. the sum of his parent's and his income. To obtain this, the parent does not have to engage in strategic behavior to provide the selfish beneficiary with incentives. Instead, if the child expects no transfer from his parent, his best response is to maximize his sole

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income. This result has received a substantial amount of attention (Becker (1976), Hirshleifer (1977), Bernheim *et al.* (1985), Bergstrom (1987)).

However the structure of the child's best response function, i.e. whether a positive or a zero transfer is expected, is not the only crucial feature at work. Temporal aspects also matter. Becker's (1974) setting is a single-period one. He suggested that his result extends to a multi-period setting if the parent is able to follow a retaliatory strategy. Hirshleifer (1977) stressed that, even in a single-period setting, the mere timing of the game decisions is important. If the altruistic father makes a positive transfer before the kid has chosen his action, the theorem does not hold. Indeed, the benefactor must have the "last word" of the game.

Several authors developed these temporal aspects. They adopted a two-period setting to introduce savings in the picture (Bernheim and Stark (1988), Laitner (1988), Lindbeck and Weibull (1988), Bruce and Waldman (1990)). In this framework, a "last word" transfer admittedly produces the child's virtuous behavior but, at the same time, has a significant impact on savings. The recipient clearly anticipates that the gift he is to receive later will be higher the more deprived he presents himself to the benefactor in the second period. This induces strategically low savings and a large second-period transfer. This type of inefficiency is close to what is referred to as the "Samaritan's Dilemma", in the literature on welfare programs (Buchanan (1975), Coate (1995)). In our context a game between an altruistic parent and a sefish child, the inefficiency means the following. The parent's willingness to help his child if he falls on hard times drives the child to decisions which leave him actually deprived. Thus the child's savings are "too low" with respect to the level saved when the parent precommits on the level of the transfer. A first-period transfer, preceding the child's choice of actions, certainly acts as a precommitment but cancels the altruist's benefit of having the last word. Thus the Rotten Kid Theorem does not hold anymore (Bruce and Waldman (1990)).

Problems related to this dilemma are not limited to intertemporal decisions. It also applies to models with uncertainty in which the beneficiary has to make risky decisions, or in models with labor supply in which he has to decide on the level of effort to be devoted to a given activity. In these cases, distorsions of the Samaritan's dilemma type induce respectively more risky choices and a lower level of effort, compared to the prevailing choices and effort when benefactor's precommitment is assumed.

The model of this paper is based on the interaction between a selfish son and an altruistic father and it assumes that the father has the last word, like in Hirshleifer (1977). But it also contains savings decisions like in Bruce and Waldman's (1990) contribution. The transfer from the father to the son is a bequest. But unlike in most bequest models, bequest is invested and transferred only after the father's death (a quite natural feature for bequests). Importantly, we assume that bequest is not a valid collateral for bank loans. As a consequence, there is a credit constraint bearing on the son's consumption possibilities: savings can be positive or zero but not negative.

Laitner (1988) considered the social security system as a means to force savings, thereby limiting the recipient's strategy space. Inefficiencies would then be attenuated. The non-negativity restriction on savings in this model is likely to produce the same outcome.

Nevertheless, as we shall see, the credit constraint impinges on the son's choice of the optimal level of action through the truncation of his budget set. Indeed some levels of action, which we interpret in monetary terms, become undesirable when credit constraints are at work, while they would not be so if savings were free to positive or negative. This leads to a re-examination of the Rotten Kid Theorem under the hypothesis of non-negative savings.

The results can be summarized as follows. Firstly, when the altruistic benefactor's last word produces the offsprings' virtuous behavior, the credit constraint limits the extent of their strategic behavior of the Samaritan's dilemma type; this is an illustration that Rotten Kids, when behaving optimally from the family point of view, are not necessarily able to draw large transfers from their parents. Secondly, the non-negativity constraint on savings can be, in some cases, responsible for the collapse of Rotten Kids' virtuous behavior. This is a contradiction to Becker's (1974) happy result and Hirshleifer's (1977) point. The analysis studies in detail these cases of failure in the son's behavior. They appear when sons and parents have very unequal resources.

The paper is structured as follows. Section 2 sets up the model. We start looking at equilibria in section 3. Then, respectively in sections 4 and 5, we examine two polar cases and characterize conditions under which the Rotten Kid Theorem is invalidated because of the credit constraint. Section 6 comments the two results. Finally, the last section concludes.

2 The model

Since we want to demonstrate the possibility of a contradiction to the Rotten Kid Theorem and the Samaritan's dilemma when credit constraints are at work, we set up a simple partial equilibrium model serving this purpose. Bequest is the only transfer and is carried out after the parent's death. Therefore parents have the last word. Children make intertemporal choices. A more realistic model would of course be needed if we were interested in the responses of aggregate consumption and capital accumulation to the bearing of credit constraints on agents' decision-making. But this is not our purpose.

Time is discrete. We consider the interaction between an altruistic benefactor and a selfish recipient in an economy which just lasts *two periods.*

This economy is populated with altruistic parents, whom we name the P's and who live only in the first period and selfish grown children, the C's, who live two periods. The basic structure is simple: P shares his wealth between consumption and bequest and, simultaneously, C smoothes his two per-period consumptions through savings[1].

At the very beginning of the first period, each C and each P in each family line is endowed, respectively, with $w > 0$[2] and $W > 0$ and with a joint home production technology. The game between P and C has two steps: production choices, followed by consumption choices. These *two stages* of the game between P and C all take place at the beginning of the first period. Indeed, no decisions are made in the second period since P is dead and C only consumes the proceeds of his savings.

Once P and C have learned their endowments and technology, the timing of their moves is as follows. In the first stage, production choices are made by C. At the beginning of period 1, he picks up a point on the joint home production frontier. This is what the literature refers to as the choice of action. For the sake of simplicity, we assume that C chooses h, the monetary value of the support he is willing to provide to P, whatever the nature of this support (it can be a transfer in time or in commodities...). Thus, in this first stage, C's choice object is $h \in [0, w)$ and the output which instantaneously accrues to P (or, to be more precise, the monetary value of the benefit for P of C's choice) is $f(h)$, with $f'(h) > 0$ and $f(0) = 0$[3].

In the second stage, the game proceeds with C endowed with $w - h$ and P with $W + f(h)$. Both P and C simultaneously make their consumption and bequest plans.

We assume that bequest is invested and transferred to C after P's death, principal and interest. In the meanwhile, C can only consume up to his first-period income, $w - h$. This is equivalent to assuming that future bequests are not valid collaterals for bank loans. This assumption is crucial.

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1 In this bare-bones model, the two periods we consider can be thought of as cut out from an overlapping generations model. Alternative cut-outs have been used in the literature according to the scope of the authors. The simplest way to think about our structure is the following. The first period is the initial period of an OLG model, period $t = 0$. The first old only live during this period and their income is given; the young agents live during period $t = 0$ and period $t = 1$, which implies savings.

2 We do not model labor participation. Nonetheless one can see this assumption as the child supplying inelastically one unit of time on the labor market.

3 This may seem restrictive only at first sight. Suppose that support is measured in time, say t, spent on old-caring. This kind of support, *home* support, is a substitute for the same kind of services from the market, *market* support. Since we do not model labor participation, in the same course of idea, we do not want to model the choice between leisure and home support. Thus t is not a decision variable in our model. Nevertheless we shall implicitly handle variables like the *monetary values* of home support and market support. The former is the income which C gives up by spending time on caring for P, i.e. w for each unit of time. The latter is the price P would have had to pay if he had not been supported by C, say z for each unit of time. C, by choosing h, can be seen as giving up wt units of income, even if the utility-maximizing choice of t is not modelled as such. Similarly, what P freely gets, $f(h)$, can be valued by what he would pay for similar services on the market, i.e. zt. Then we have $h = wt$ and $f(h) = zt$, which can be rewritten in terms of h as $f(h) = (z/w)h$. The average productivity ratio $f(h)/h = z/w$ is therefore an indicator of how home support services compare to market support services in terms of implicit prices.

If the C's could back on future bequests to consume when young, savings would freely be positive or negative and the timing of bequests would be irrelevant. To keep the analysis simple, we assume that there is no alternative collateral so that savings are constrained to be non-negative. This is not restrictive. Allowing for some other collateral asset would only shift the lower bound downwards but would not cancel it.

No transfers between P and C are made during the first period. In their contribution, Bruce and Waldman (1990) argued that parents may have an incentive to make no transfer during the first period when credit constraints are at work. Indeed such transfers would feed the Samaritan's dilemma problem because children have then an incentive to overconsume any first-period transfer. We shall follow this line of argument. However, Bruce and Waldman (1990) do not model capital market imperfection explicitly. We argue that the non-negativity constraint, bearing on savings, in turn, modifies the domain of validity of the Rotten Kid Theorem.

C's utility and budget constraints are respectively

$$U_c = (1-\beta)\log c + \beta \log d$$

$$\begin{aligned} w - h &= c + s \\ R(s + X) &= d \\ s &\geqslant 0 \end{aligned}$$

where c is his current consumption, R the interest factor and d is C's second-period consumption, after P's death. Taking bequest as given, C maximizes his utility U_c with respect to savings under the above three constraints. This yields the first-order conditions : $(1-\beta)\,(w-h-s)^{-1} \geqslant \beta(s+X)^{-1}$, with equality if $s > 0$[4]. C's behavior is characterized by his saving decision $\hat{s} = \max\{0, \sigma(X,h)\}$ where

$$\sigma(X,h) = \beta(w-h) - (1-\beta)\,X \tag{1}$$

P's utility and budget constraint are

$$U_p = (1-\alpha)\log D + \alpha U_c$$

$$\begin{aligned} W + f(h) &= D + X \\ X &\geqslant 0 \end{aligned}$$

where D is P's consumption and X the bequest to be passed to C after P's death. Taking savings as given, P maximizes his utility U_p under the above two constraints. This yields the first-order conditions $(1-\alpha)(W + f(h) -$

[4] Given prices and incomes, if the non-negativity constraint is not binding, the optimal consumptions are identical to the ones of the free-saving problem, $\left(c^F, d^F\right)$. If, on the contrary, the non-negativity constraint on savings binds, optimal consumptions verify : $c = w - h \leqslant c^F$ and $d = RX \geqslant d^F$.

$X)^{-1} \geqslant \alpha\beta(X+s)^{-1}$, with equality if $X > 0$. P's behavior is described by his bequest decision $\hat{X} = \max\{0, \xi(s,h)\}$ where

$$\xi(s,h) = \frac{1}{1-\alpha(1-\beta)}\left(\alpha\beta\left[W+f(h)\right]-(1-\alpha)s\right) \tag{2}$$

3 Equilibria

As is usual in a two-stage game, we solve it backward. Given h, we are looking for the stage 2 Nash equilibrium (s^*, X^*), i.e. the one which, among all $\left(\hat{s}, \hat{X}\right)$, is a best response for both C and P. A unique Nash equilibrium exists. There are non-negativity constraints on both savings and bequest. Therefore the Nash equilibrium can be one of three alternative disctinct types : either only bequest is positive, either both bequest and saving are positive, or only saving is positive[5].

Let $\omega(h) = (w-h)/\left[W+f(h)\right]$ be the relative endowments after the production decision (choice of h). This variable decreases as h increases. Let $\omega(0) = w/W$ be the *initial relative endowments*. It reflects how C's and P's incomes compare before the choice of support h by C. The wealthier C (respectively, poorer) with respect to P at the beginning of the game, the higher (resp., lower) $\omega(0)$.

The zero-saving and positive-bequest subgame equilibrium is obtained by taking $s=0$ in (2), which yields

$$X^*(h) = \frac{\alpha\beta}{1-\alpha(1-\beta)}\left[W+f(h)\right] \tag{3}$$

This happens when $\omega(h) \leqslant \omega_s$, with $\omega_s = \alpha(1-\beta)/\left[1-\alpha(1-\beta)\right]$. The positive-saving and positive-bequest subgame equilibrium is obtained by solving $\sigma\left(\hat{X}, h\right)$ and $\xi\left(\hat{s}, h\right)$ for s and X, which yields

$$s^*(h) = (1-\alpha(1-\beta))(w-h) - \alpha(1-\beta)\left[W+f(h)\right] \tag{4}$$

$$X^*(h) = \alpha\left[W+f(h)\right] - (1-\alpha)(w-h) \tag{5}$$

This happens when $\omega_s < \omega(h) < \omega_x$, with $\omega_x = \alpha/(1-\alpha)$. Finally, positive-saving and zero-bequest subgame equilibrium is obtained by taking $X=0$ in (1), which yields

$$s^*(h) = \beta(w-h) \tag{6}$$

This happens when $\omega(h) \geqslant \omega_x$. The second-stage unique equilibrium can be any of the these three ones, depending on the value of the relative endowments $\omega(h)$.

5 The case of both zero-saving and zero-bequest cannot be an equilibrium of the second stage game since one of them has to be positive to insure positive consumption d after P's death.

Intuitively, given h, P will leave a bequest only if he is sufficiently wealthy with respect to C. This happens when $\omega(h) = (w-h)/[W+f(h)]$ is low enough, i.e. $\omega(h) < \omega_x$. He will leave no bequest if C is so wealthy that only a negative bequest would be optimal for him. He is then constrained on his bequest decision. The same kind of reasoning applies for C's saving decision. He would like to dissave ($s < 0$) in the case he is so poor with respect to his parent that $\omega(h) \leqslant \omega_s$. This means that he would be willing to consume in period 1 a share of next-period bequest. But since bequest is not a valid collateral, this is not feasible. Thus C only consumes his first-period income, with savings being zero.

We now go one step backward in the game. C then chooses the optimal level of support to his P, $h^* \geqslant 0$. He will depart from the initial relative endowments $\omega(0)$ by choosing a positive h only if this is optimal for him.

Let us first think on what it means, for C, to choose the level of support to be provided to P. The choice of h determines the position of C's intertemporal budget constraint. As a reference, suppose that savings are free to be positive or negative. Then C's only problem would be to choose the level of $h \geqslant 0$ which places his budget constraint at the highest position in order to widen his consumption possibilities. This is equivalent to saying that he would choose

$$h^* = \arg \max_{h \in [0,w)} w - h + X(h)$$

where $w - h + X(h)$ is C's present value lifetime income. Thus, in a free-saving problem, the first stage of the game would consist in choosing the value of $h \geqslant 0$ maximizing the lifetime-income, with $X(h) = \alpha\,[W + f(h)] - (1-\alpha)(w-h)$ or zero. If an optimal positive level of support exists, it means that increasing h away from zero shifts the intertemporal budget constraint upward.

When savings are constrained to be non-negative, instead, there is an additional effect to this upward move of the budget line. To illustrate it, let us set $h = 0$ and assume $X(0) > 0$ and $s(0) > 0$. Any consumptions (c, d) verifying $c > w$ and $d < RX(0)$ is not feasible given the non-negativity constraint on savings. Graphically (figure 1), in the (c, d) diagram, all the pairs of consumptions which are located in the "south-east" area with respect to the "max-c min-d point", $(w, RX(0))$, point B, are not feasible since they would imply negative savings. Let us name this area the "south-east share", BCD. Consumption possibilities are defined by the budget constraint: $w + X(0) = c + d/R$ and by the condition $c \leqslant w$.

Now, suppose the budget constraint moves upward as h increases (arrow heading north-east), i.e. there is room for a positive support from C to P ($X'(0) > 1$). What happens to the south-east share? It gets larger (from BCD to B'C'D'). Indeed, the upper bound on first-period consumption decreases ($c < w - h$) (from D to D') and C has to give up some consumption possibilities which were opened to him when $h = 0$. He simultaneously gains ABB'A' and looses BDD'E.

Therefore, when credit constraints are at work, the choice of the optimal value of support is not merely one of choosing the highest position for the budget line like in the free-saving problem. Indeed, the higher the budget line position, the larger the south-east share is and, thus, the shorter the budget line (from AB to A'B').

If any positive support is to be optimal for C, then not only, the budget line must lie above its initial position like in the free problem but also it should not shrink away too much to the "north-west". Therefore a trade-off should arise between consumption possibilities gained in the north-west side of the (c, d) diagram and those lost in the south-east side.

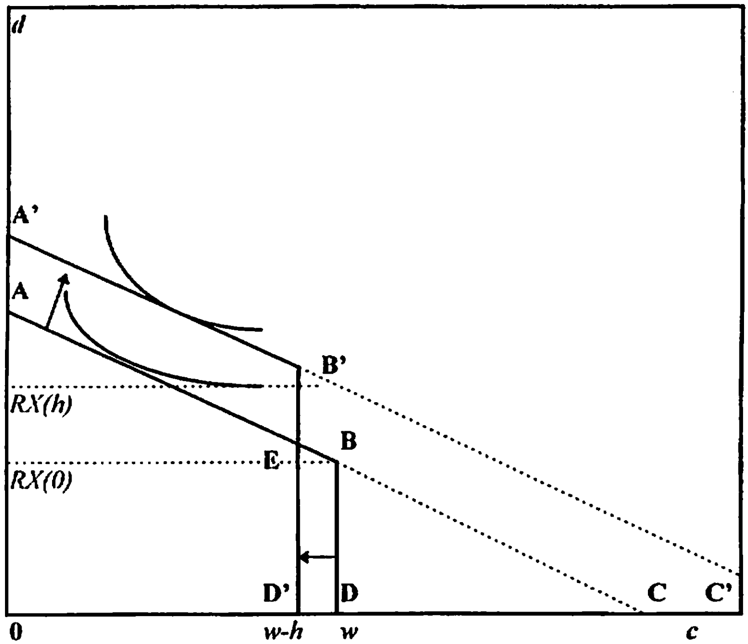

Figure 1

We shall assume a linear home production funtion for simplicity: $f(h) = \theta h$. Furthermore, we assume that $\theta > 1$. Given our interpretation of h and $f(h)$ in monetary terms, the assumption $\theta > 1$ is equivalent to assuming that home support is less costly than similar market support[6].

Since $\omega(h) = (w - h)/[W + f(h)]$ is a decreasing function of h, we necessarily have $\omega(h) \leqslant \omega(0), \forall h \in [0, w)$. By choosing h^*, C determines

[6] When $\theta < 1$, it is easy to prove that the gains from providing a positive level of support are always smaller than the losses. No support to P is then ever provided by C, an outcome in accordance with the fact that market support services are, in that case, relatively less costly than home support services. When $\theta = 1$, the indirect utility function does not depend on h in the region with positive savings and bequest and the analysis gets poorer. We leave aside these two uninteresting cases.

the relative endowments $\omega(h^*)$ for the second stage game and, thus, the associated equilibrium $(s^*(h^*), X^*(h^*))$. Three cases are conceivable.

Firstly, C's initial income w might be so low with respect to P's initial income W that the relative initial endowments w/W lies in the zero-saving region: $\omega(0) \in (0, \omega_s]$. In this case, whether optimal support h^* is strictly positive or zero, the ensuing second stage equilibrium is always one with zero savings and positive bequests as given by (3). In that case, note that C worsens the constraint bearing on his first-period consumption if he chooses to provide a positive level of support. Indeed maximum constrained consumption goes from $c = w$ to $c = w - h$.

In the zero-saving region, $X^*(h) = \alpha\beta[W + \theta h] / [1 - \alpha(1 - \beta)]$. It is an increasing linear function of h. There can be no gains from positive support if the following condition is not met[7] : $X^{*'}(0) > 1$. Given the expression of bequests in the saving-constrained region, this condition is equivalent to the following condition on θ :

$$\theta > 1 + \frac{1-\alpha}{\alpha\beta} \tag{7}$$

This condition is necessary because otherwise the budget line always go down as h increases. This can only lead to lower utility. We shall refer to this condition as the high productivity condition for support (alternatively, home support much less costly than market support).

The second case is when the initial relative endowments verify : $\omega(0) \in (\omega_s, \omega_x)$. Depending on the optimal value of h, the selected second stage equilibrium is either one with both positive savings and bequests ((4) and (5)), or one with zero savings and positive bequests. Let us denote by $h_s > 0$ the threshold value of h such that $\omega(h_s) = \omega_s$. Thus if $h^* < h_s$, the equilibrium is with positive savings and bequests. If $h_s \leqslant h^*$, equilibrium bequests are positive while savings are zero, i.e. C is constrained on savings. In this second case, bequests are given by $X^*(h) = \alpha[W + \theta h] - (1-\alpha)(w - h)$. It is easy to check that $X^{*'}(0) > 1$ is always satisfied $\forall \theta > 1$. Thus, increasing h on $(0, h_s)$ will always result in a higher position of the budget line. Beyond h_s, the budget line goes on upward if condition (7) is satisfied.

The last case obtains when C is relatively wealthy with respect to P : $\omega(0) \in [\omega_x, +\infty)$. Define $h_x > 0$ as the value of support verifying : $\omega(h_x) = \omega_x$. Suppose $h^* \leqslant h_x$. Then bequests are constrained in equilibrium and savings unconstrained : C finds it optimal to give up the possibility of a future bequest from P. If $h_x < h^* < h_s$, bequests are then positive as well as savings. Finally, for $h_s \leqslant h^*$, savings are constrained and bequests unconstrained. In this last case, the budget line goes down as h increases from 0 to h_x. Indeed, for these values of support, C would lower his first-period income but would get nothing anyway in the second period since bequests are constrained. On (h_x, h_s), the budget line always shifts upwards

[7] This condition is obtained by requiring the derivative of $w - h + X(h)$ to be strictly positive at $h = 0$.

$\forall\theta > 1$ (see second case). Finally, beyond h_s, the same holds as in the first case.

It is easy to prove that optimal support is always positive when the initial relative endowments, $\omega(0) = w/W$, lie in the intermediate region with positive savings and bequests (the second case in the discussion above). We want to investigate the effects of credit constraints on the two polar cases. Can poor C's ($\omega(0) \leqslant \omega_s$) optimal support be positive in equilibrium or is it always equal to zero because of the credit constraint ? On the opposite, is it always the case that wealthy C's ($\omega_x \leqslant \omega(0)$) behave optimally from the family point of view, i.e. provide positive support to their P or can they fail in doing so ?

Using $c = w - h - s$ and $d = R(s + X)$ and plugging $s^*(h)$ and $X^*(h)$ into C's utility function, we obtain C's objective function in stage 1, namely his indirect utility function, stemming from stage 2

$$U_c(h) = (1-\beta)\log\left[w - h - s^*(h)\right] + \beta\log\left\{R\left[s^*(h) + X^*(h)\right]\right\} \tag{8}$$

Let us examine successively the case of poor C's, i.e. saving-contrained C's ($\omega(0) \leqslant \omega_s$) and then the case of wealthy C's, i.e. C's who are initially bequest-constrained ($\omega_x \leqslant \omega(0)$).

4 Saving-constrained C's

If C is characterized by $\omega(0) \leqslant \omega_s$, he will always reach a saving-constrained equilibrium in stage 2 : $s^*(h) = 0$. His indirect utility thus writes : $U_c(h) = (1-\beta)\log(w-h) + \beta\log\left[RX^*(h)\right]$, with $X^*(h)$ given by (3), with $f(h) = \theta h$ and $\theta > 1$. At first sight, it seems that poor C's should never be willing to provide support to their P because of the bearing of the credit constraint. We derive conditions under which they nevertheless provide positive support, thereby worsening the constraint on their first-period consumption.

Proposition 1 *Suppose that (i) C is constrained on savings ($\omega(0) \leqslant \omega_s$) and that (ii) productivity of support is high ($\theta > 1 + (1-\alpha)/\alpha\beta$). Then C's optimal support to P is given by $\hat{h} = \max\{0, \beta w - (1-\beta)W/\beta\}$. It is positive when the initial relative endowments $\omega(0) = w/W$ lies in the following interval :*

$$\frac{1-\beta}{\beta\theta} < \frac{w}{W} \leqslant \frac{\alpha(1-\beta)}{1-\alpha(1-\beta)}$$

If $w/W \leqslant (1-\beta)/\beta\theta \leqslant \alpha(1-\beta)\left[1-\alpha(1-\beta)\right]^{-1}$, he chooses to provide no support to P. (Proof in appendix)

Therefore, being saving-constrained at the beginning of the game ($\omega(0) \leqslant \omega_s$) does not mean that optimal support is zero. When productivity of support θ is sufficiently high and initial relative endowments $\omega(0)$ not too low, it might actually be optimal for the selfish recipient C to strengthen even more the credit constraint to reach a higher utility. Let us illustrate this graphically.

C uses h to shift his budget constraint upward (see the upward arrow in figure 2). He can do so because condition

$$(1-\beta)/\beta\theta \leqslant \alpha(1-\beta)\left[1-\alpha(1-\beta)\right]^{-1}$$

is equivalent to condition $\theta > 1 + (1-a)/\alpha\beta$ which guarantees that the budget line can go upward as h increases in the zero-saving region. An optimum with positive support is like point B' with coordinates : $c^*\left(\hat{h}\right) = w - \hat{h}$, $d^*\left(\hat{h}\right) = RX^*\left(\hat{h}\right)$. It is the max-$c$ min-d point with positive support $\hat{h}$. It is located to the north-west with respect to B, the max-c min-d point with zero support $(c^*(0) = w, d^*(0) = RX^*(0))$ because of the growing truncation of the budget set as h increases (the arrow heading to the left). Importantly, it lies on a higher indifference curve than the one passing through B.

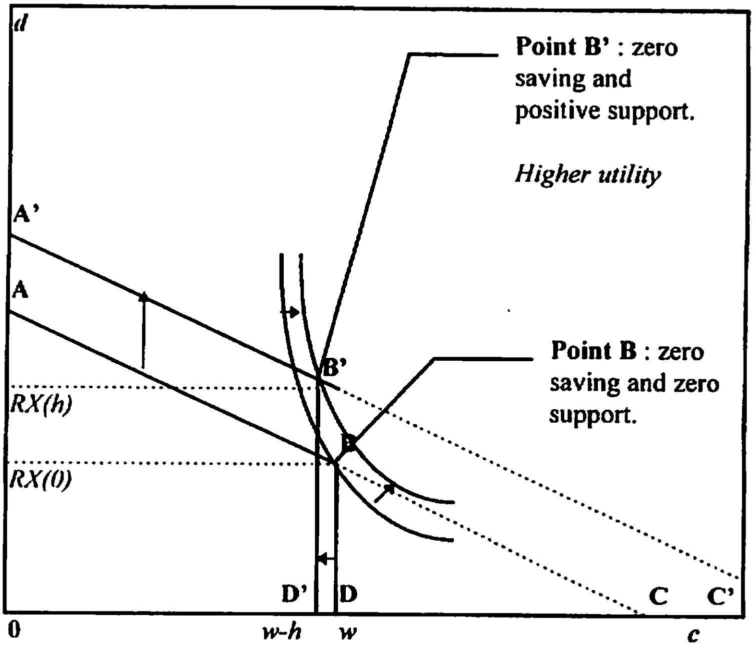

Figure 2

On the contrary, if any positive support shifts point B onto a lower utility contour, like in figure 3, no support is provided. Indeed, even for high θ, there exists a threshold, $\hat{\omega}$, of the relative initial endowments $\omega(0) = w/W$ below which the optimal level of support to P is zero : $w/W \leqslant (1-$

$\beta)/\beta\theta \leqslant \alpha(1-\beta)\,[1-\alpha(1-\beta)]^{-1}$. What this case of very low-wage earners ($\omega(0) < \hat{\omega}$) tells us is that, when C is sufficiently poor with respect to P, any positive level of support brings more losses than gains. When this happens, the desired level of support is actually negative, i.e. C wants support from P.

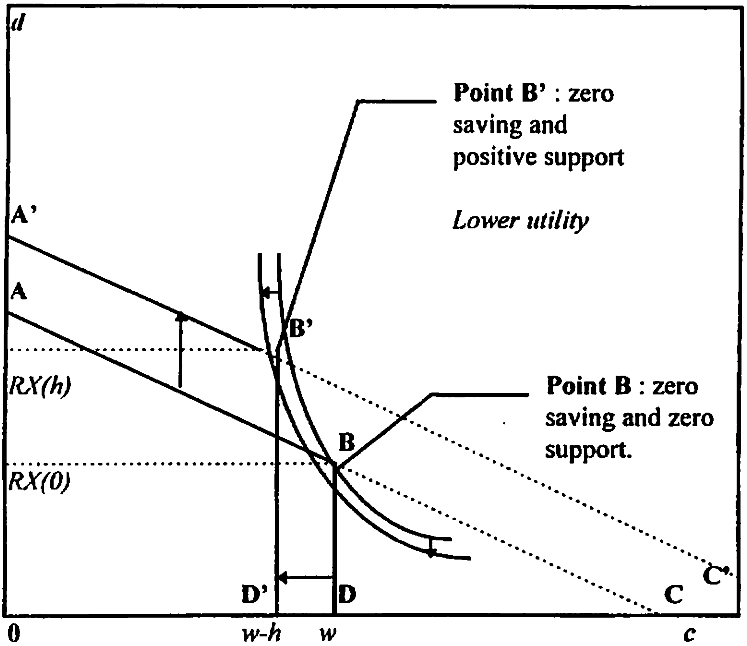

Figure 3

Since C is saving-constrained, as figure 3 shows, the indifference curve associated with zero support passes through the max-c min-d point B. As h increases, bequest rises and the budget constraint goes upward but the upper bound on first-period consumption, $w-h$, simultaneously goes down (shift from D to D'). If, for any positive level of support, this cost of giving up consumption possibilities in the south-east area (from BCD to B'C'D') is so important and/or indifference contours are so steepy that the new truncated budget constraint lies completely on the left side of the indifference curve passing through B, then C will abstain from supporting P.

When C is free to dissave (his budget constraint then includes the south-east share B'C'D'), he does not incur such a cost so that the upward move of the budget constraint (from ABC to A'B'C') is enough to convince him to support P. This means that, in this case, C would be better off if, in some way, bequest could be a valid collateral for bank loans : from his welfare point of view, it would thus be desirable to relax the non-negativity constraint on savings.

The fact that the non-negativity constraint can be responsible for the failure of C to support P illustrates that the simple existence of a P with strongly altruistic preferences (Becker (1974)) and his having the last word (Hirshleifer (1977)) in an intertemporal framework do not necessarily provide C with the automatic incentive to behave optimally from the household point of view. This is true if intertemporal choices are free. The first teaching of the introduction of credit constraints is thus that poor C's may fail to behave optimally from the family point of view because of the non-negativity constraint on savings.

Of course, this does not guarantee that P, if given the opportunity, would be willing to provide support to C or, to be more precise, that he would be willing to make both an inter-vivos gift *and* a bequest in the sense that we assumed. Here we might find again the trade-off between inefficient action (support) and inefficient savings (Bruce and Waldman (1990)).

Let us summarize the saving-constrained case. If "having the last word" means leaving a bequest (and there is no reason for there to be a contradiction), then there is no more the guarantee that the Rotten Kid Theorem holds for the same range of relative endowments $\omega(0)$ as when no constraint bears on savings : the reason is that, when savings are constrained to be non-negative, the exercise of choosing the highest position for the budget constraint is no more a costless exercise. It is not worth the trouble if the price to pay is too high.

5 Bequest-constrained C's

To the opposite, very rich C's (i.e. those with $\omega(0) \in [\omega_x, \infty])$ will have the opportunity to select a value for $\omega(h)$ in any of the three regions : the zero-bequest region $[\omega_x, \infty]$, the positive-saving positive-bequest region (ω_s, ω_x) or the zero-saving region $[0, \omega_s)$. Given this, the maximum of the objective function : $U_c(h) = (1-\beta)\log[w - h - s^*(h)] + \beta \log\{R[s^*(h) + X^*(h)]\}$ is more difficult to find. Indeed, the derivative of $X^*(h)$ in $h = h_x$ and the one of $s^*(h)$ in $h = h_s$ are not uniquely defined. Remind that these values of h verify $\omega(h_x) = \omega_x$ and $\omega(h_s) = \omega_s$.

Thus we proceed in two steps in order to find $h^* = \arg\max_{[0,w)} U_c(h)$. The first one consists in calculating

$$\tilde{h} = \arg \max_{h \in [0, h_x]} U_c(h)$$

$$\bar{h} = \arg \max_{h \in [h_x, h_s]} U_c(h)$$

$$\hat{h} = \arg \max_{h \in [h_s, w)} U_c(h)$$

and the second step is to select among $\tilde{h}$, $\bar{h}$ and $\hat{h}$, the one which maximizes $U_c(h)$, i.e. $h^* = \arg\max_{\{\tilde{h},\bar{h},\hat{h}\}} U_c(h)$. We show in appendix that $\tilde{h} = 0$, $\bar{h} = h_s$ and that $\hat{h} = h_s$ when productivity of support is low.

Proposition 2 *Suppose that (i) C is bequest-constrained* ($\omega_x < \omega(0)$) *and that (ii) productivity of support is low* ($1 < \theta \leqslant 1 + (1-\alpha)/\alpha\beta$). *Then C's optimal support is positive and is given by* $h^* = h_s$ *when the initial relative endowments* $\omega(0) = w/W$ *lie in the following interval:*

$$\frac{\alpha}{1-\alpha} < \frac{w}{W} < \frac{\alpha}{1-\alpha\left(1+\beta\left(\theta-1\right)\right)}$$

If, instead, $\alpha/(1-\alpha) < \alpha/\left[1-\alpha\left(1+\beta(\theta-1)\right)\right] < w/W$, *C's optimal support is zero. (Proof in appendix)*

The second teaching of this paper is therefore the following. When C is relatively rich with respect to P ($\omega_x < \omega(0)$), there exists a threshold value, $\alpha/\left[1-\alpha\left(1+\beta\left(\theta-1\right)\right)\right]$, for the initial relative endowments $\omega(0)$, beyond which optimal support is zero whereas, below it, a positive level of support would be provided by C.

This is not specific to the model with non-negative savings. Indeed, if savings were free to be positive or negative, there would also be such a threshold. Indeed, when savings are free, the difference $U_c(0) - U_c(w)$ is positive, zero or negative according to

$$\omega(0) \gtreqless \frac{\alpha}{1-\alpha\theta} \tag{9}$$

It is easy to check that the free-problem threshold $\alpha/(1-\alpha\theta)$ is lower than the constrained-problem threshold $\alpha/\left[1-\alpha\left(1+\beta(\theta-1)\right)\right]$[8]. Why is it so? The explanation is again to be found in the particular changes in consumption possibilities arousing when saving must be non-negative.

Indeed, for a given low productivity θ (i.e. for $\theta \in (1, 1+(1-\alpha)/\alpha\beta)$), when C is that rich with respect to P, the ratio $\omega(0) = w/W$ is so far above $\omega\left(h_x\right) = \omega_x$ that it takes a high level of support h_x for $\omega(h)$ to reach the critical value ω_x. Only after h_x has been spent, there starts to be some

[8] Note that the same alternative threshold would obtain if, instead of bequest and free savings, a simple inter-vivos gift is assumed. Of course savings would be different and always positive

$$s(h) = \alpha\beta\left(w - h + W + \theta h\right) > 0$$

But the critical value separating positive support from zero support under the case $\omega(0) > \omega_x$ would also be $\alpha/(1-\alpha\theta)$. It is easy to check that we have the following ordering

$$\omega_x = \frac{\alpha}{1-\alpha} < \frac{\alpha}{1-\alpha(1+\beta(\theta-1))} < \frac{\alpha}{1-\alpha\theta}$$

gains to provide support, these gains being an increasing positive bequest. Indeed, as figures 4 and 5 show, in the range $[0, h_x]$, C's intertemporal budget constraint moves downward a great deal (arrow heading south-west) since bequest remains zero on the whole way. The further $\omega(0)$ from ω_x, the larger the downward move of the budget constraint (from ABC to A'C'). Note that as long as $h < h_x$ consumption possibilities are not truncated. They are all associated with positive savings since there is no second period income when bequest is zero. Beyond h_x, as h goes on increasing, the budget constraint starts to move upward (arrow heading north-east on figures 4 and 5) and truncation appears. But it is not sure that it can go back and over its initial zero-support position ABC, i.e. it is not sure that $X(h_s) > h_x$. If it cannot, then optimal support is zero. This outcome is common to the constrained and the free problem.

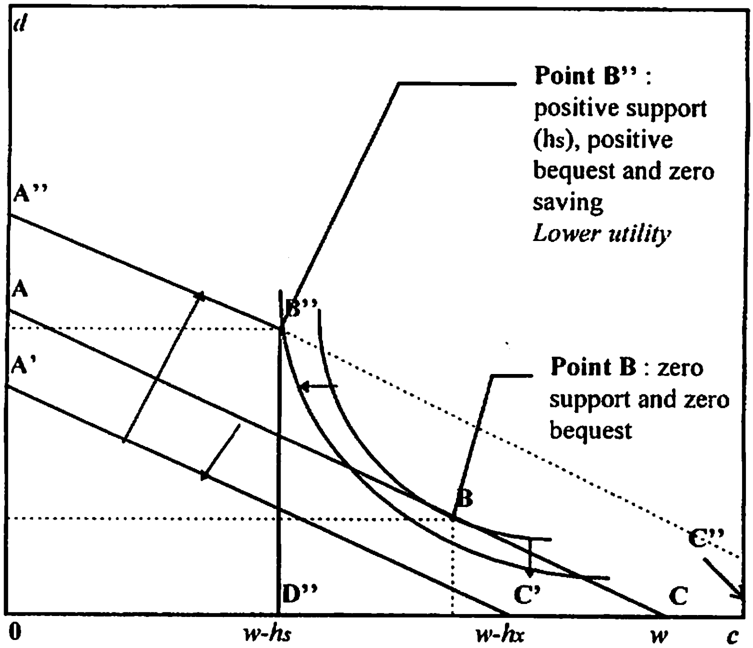

Figure 4

If, instead, the budget line can go back and over its position ABC, then the free problem will always yield a positive support, because the budget line includes the dotted line B"C", but the constrained problem might not yield a positive optimal support, due to the cost in terms of consumption possibilities (the share B"C"D"). Indeed it is not sure that the budget constraint of the constrained problem, once above its initial position ABC, will not be so truncated that it stands all on the left side of the indifference curve passing through B, i.e. in a lower utility region. An example of this subcase is given in figure 4 (the shift from B to B" with

$h = h_s$). Only if the max-c min-d point B" is on the good side (right side) of the indifference curve passing through B does C choose to provide positive support (figure 5).

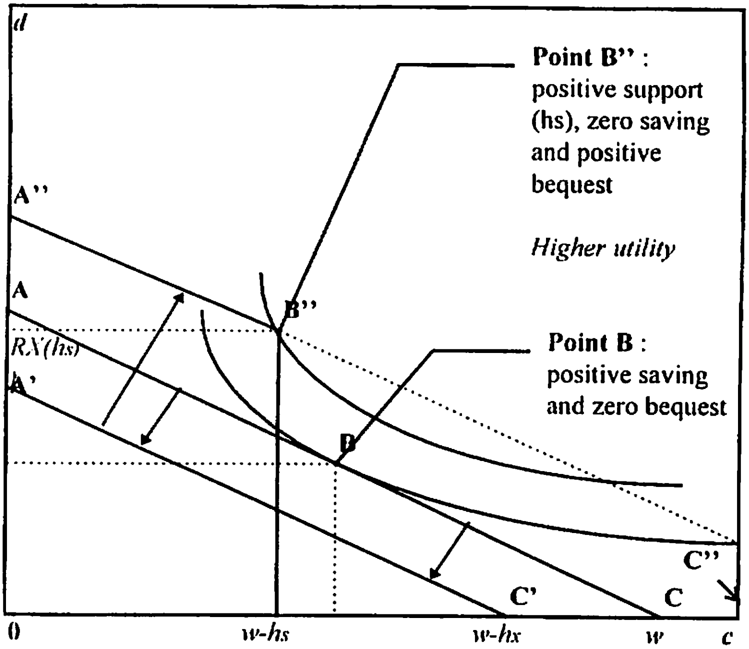

Figure 5

It is easy to illustrate with an example that the validity domain of Becker's (1974) happy result is not as large as it is when savings are free. Let us denote by h_c^* and h_f^*, respectively, the optimal support of the constrained and the free problem. For relative initial endowments $\omega(0)$ in between the above two threshold values: $\alpha/\left[1 - \alpha\left(1 + \beta(\theta - 1)\right)\right] < \alpha/\left(1 - \alpha\theta\right)$, the free-problem optimal support is positive while the constrained-problem optimal support is zero. This is precisely the case in which the intertemporal budget constraint reaches a higher general position $\left(h_f^* > 0\right)$ but the set of points situated north-west of $w - h$ have got the meanest share and constrained C's choose not to support their P's $(h_c^* = 0)$.

Thus, for the same productivity θ, when C can borrow against his future bequest, he provides a positive level of support to P whereas, when he cannot, the optimal support is zero. The non-negativity constraint on saving widens the upper range over which C is too rich to be willing to help P. This is so because it has been so costly, in terms of consumption opportunities, to bring bequest and resources that high, that only lower-utility points are reachable with respect to the zero-support option.

6 Non-negative savings, the Samaritan and the Rotten Kid

Let us summarize the underlying hypotheses of this analysis. In this model, the benefactor P has the last word because he leaves a bequest after the recipient's choice of the level of support. But there is more at work here: bequest is carried out after the recipient's saving decision (bequest as a *post-mortem* transfer). In a sense, it comes out late. So let us refer to the exercise of that type of lastword by the expression "benefactor's last word with lateness". Given credit constraints, future bequest cannot be used to finance current consumption.

In that context, inter-vivos gifts, unmodelled here, have a tendency to be overconsumed by recipients, as Bruce and Waldman (1990) explained. We derived conditions under which this exercise of the lastword with lateness associated with credit constraints could, on the one hand, limits the space over which the selfish recipient can behave strategically but, on the other hand, still guarantees the happy result of the Rotten Kid Theorem. Under the free-saving hypothesis, when the benefactor has the lastword, the recipient under-saves with respect to the level he would save if the benefactor could commit on bequest, and he does so to present himself destitute in the second period and draw a large bequest from his benefactor. In our model, when optimal support is positive, i.e. when the Rotten Kid Theorem holds, C cannot consume more than $w - h$ in his first period of life. The non-negativity constraint on savings therefore places an upper bound on the extent to which C can "free-ride" on P's benevolence. We found that C can sometimes consider optimal to worsen the constraint bearing on him by providing a positive level of support. This is the good side of the problem. But we also showed the other one, namely that the benefactor's "lateness" can undo the benefactor's "last word".

This happens when the recipient fails to support the benefactor while he would choose a positive support in the absence of credit constraints. Indeed, in both cases of poor and rich C's, what is responsible for the failure of C to support P is that growing parts of the the recipient's south-east consumption possibilities are destroyed by support. Those losses might be so important that only lower utility levels can be reached by providing positive support. As a consequence, the range of initial relative endowments $\omega(0)$, over which the Rotten Kid Theorem holds, shrinks when there are credit constraints and when the parent's last word is exercised with "lateness". As a consequence, in those cases, P is also worse off.

7 Conclusions

In this paper, we just slightly amended the altruistic bequest model: our point was that, bequest being what it is, namely a post-mortem transfer, it was all but far-fetched to assume that heirs cannot back on a future bequest to finance their current consumption by bank loans. Surprisingly enough, this assumption has received scarce attention in the literature on altruism. We showed how the induced non-negativity constrained on savings generates a twofold result.

On the one hand, credit constraints and the exercise of the last word through bequest result in standing on the selfish beneficiary's way and stopping him drawing large amount of resources out of the benefactor's generosity. On the other hand, under conditions which we derived, a selfish recipient may fail to provide support to his altruistic benefactor due to costs in terms of consumption possibilities. The benefactor himself is harmed by these costs he inflicts on C. This happens in the case of dissimilar parents and children, but, curiously enough, both rich and poor parents and/or children are concerned. It is what we called their lateness associated with credit constraints which causes the failure of recipients to behave optimally.

One question which could be next on the research agenda is: instead of treating altruistic bequest as an inter-vivos gift, i.e. instead of ruling out credit constraints, it should be worthwhile wondering how bequest with "lateness" *and* inter-vivos gifts can be both accounted for by the altruistic model when credit constraints bear on intertemporal choices?

Appendix

A Optimal support from saving-constrained C's

Proof of proposition 1. In the zero-saving region, the utility function is $U_c(h) = (1-\beta)\log(w-h) + \beta \log[RX^*(h)]$, with $X^*(h) = \alpha\beta\,[W+\theta h]\,/\,[1-\alpha(1-\beta)]$. Support h can take values between 0 and w. The first term of $U_c(h)$ decreases with h (derivative: $-(1-\beta)\,/(w-h))$ and the second one increases with h (derivative: $\beta\theta/\,[W+\theta h]$). We must pick up $h^* = \arg\max_{[0,w)} U_c(h)$.

The function $U_c(h)$ is always decreasing on $[0,w)$ if $(1-\beta)\,/w \geqslant \beta\theta/W$. This condition is equivalent to

$$\omega(0) = \frac{w}{W} \leqslant \frac{\beta\theta}{1-\beta}$$

In this case, the maximum is reached for $h = 0$. On the contrary, if $\omega(0) > \beta\theta/\,(1-\beta)$, $U_c(h)$ starts increasing, its slope $U_c'(h)$ is positive for any h satisfying $h < \hat{h} = \beta w - (1-\beta)\,W/\beta$, beyond, $U_c(h)$ decreases.

The first-order conditions are

$$\frac{1-\beta}{w-h} \geqslant \frac{\beta\theta}{W+\theta h} \tag{10}$$

with equality if $h > 0$. The expression of optimal h on $[0,w)$, if positive, is given by:

$$\hat{h} = \beta w - (1-\beta)\,W/\theta$$

It is always inferior to w. The condition $\hat{h} > 0$ is equivalent to

$$\hat{\omega} = \omega\left(\hat{h}\right) < \omega(0) \tag{11}$$

with $\hat{\omega} = (1-\beta)\,/\beta\theta$. Condition $\hat{\omega} < \omega_s$ has to be verified for (11) to hold for a saving-constrained C. When productivity of support is high this is always true because $\hat{\omega} < \omega_s$ is equivalent to condition (7) on $\theta : \theta > 1 + (1-\alpha)/\alpha\beta$, i.e. the necessary condition we identified earlier on to insure that the budget line goes upward as h increases when w/W is in the zero-saving region.. Thus, conditions $\hat{\omega} < \omega(0) \leqslant \omega_s$ yield a positive optimal support $\hat{h}$ of a saving-constrained C.

B Optimal support from bequest-constrained C

Proof of proposition 2. Taking, in (8), $s^*(h)$ or $X^*(h)$ equal to zero when needed, the indirect utility function reveals its local maxima on the three intervals. A quick inspection of $U_c(h)$ shows that the function is always decreasing in h in the zero-bequest region, reaching a maximum at zero: $\bar{h} = 0$. On $[h_x, h_s]$, the function is strictly increasing: $\bar{h} = h_s$. On $[h_s, w)$, things are similar to the case of saving-constrained C's. We know by the analysis of the previous case ($\omega(0) \leqslant \omega_s$), that, solved over the whole interval $[0, w)$, the zero-saving problem has first-order conditions as in (10). Condition $\hat{\omega} \equiv \omega(\hat{h}) = (1-\beta)/\beta\theta < \omega(0)$ guarantees that $U_c(h)$ is not always decreasing so that the solution is interior to the interval $[0, w)$. This interior maximum is actually reached in the interval $[h_s, w)$, and not for values such that $h \leqslant h_s$[9], when $\hat{h}$ also satisfies $\omega(\hat{h}) = (1-\beta)/\beta\theta < \omega_s = \omega(h_s)$, i.e. when productivity of support is high in the sense we defined. If we have either $\omega(0) \leqslant \hat{\omega}$ or $(1-\beta)/\beta\theta \geqslant \omega_s$ (the last condition being "low productivity of support"), optimal support on $[h_s, w)$ is h_s.

Let us now make the low productivity assumption $\omega_s < \hat{\omega}$.

Thus $\hat{h} = \bar{h} = h_s$ and, because $\omega_x < \omega(0)$, the other candidate is $\bar{h} = 0$. We therefore want to compare $U_c(0)$ and $U_c(h_s)$. Substituting for h in (8) and taking $s^*(h_s) = 0$ and $X^*(0) = 0$ when needed, we get:

$$\begin{aligned} U_c(0) &= (1-\beta)\log\left[(1-\beta)w\right] + \beta\log\left[R\beta w\right] \\ U_c(h_s) &= (1-\beta)\log\left[\alpha(1-\beta)\frac{\theta w + W}{1-\alpha(1-\beta)(1-\theta)}\right] \\ &\quad + \beta\log\left[R\alpha\beta\frac{\theta w + W}{1-\alpha(1-\beta)(1-\theta)}\right] \end{aligned}$$

Thus

$$U_c(0) - U_c(h_s) = \log\frac{\left[1-\alpha(1-\beta)(1-\theta)\right]w}{\alpha(\theta w + W)} \gtreqless 0$$

iff

$$\left[1-\alpha\left(1+\beta(\theta-1)\right)\right]w \gtreqless \alpha W$$

One easily checks that $1-\alpha(1+\beta(\theta-1)) \geqslant 0$ iff

$$\theta \leqslant \left[1-\alpha(1-\beta)\right]/\alpha\beta$$

which is exactly the condition defining the low productivity region. Therefore:

1. If $\dfrac{\alpha}{1-\alpha(1+\beta(\theta-1))} < \dfrac{w}{W}$
$$U_c(0) - U_c(h_s) > 0$$
and optimal support is zero.

2. If $\dfrac{\alpha}{1-\alpha} < \dfrac{w}{W} \leqslant \dfrac{\alpha}{1-\alpha(1+\beta(\theta-1))}$
$$U_c(0) - U_c(h_s) \leqslant 0$$
and optimal support is h_s.

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[9] Otherwise the function is concave but decreasing on $[h_s, w)$.

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