# Price Competition when Product Quality is Uncertain*

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## 1 Introduction

We model price competition in a market where consumers face uncertainty about the reliability, or more generally, the quality of the products. When making their consumption decisions, consumers take explicit account of the risk associated with buying such products.

Many industries are characterized by the coexistence of products exhibiting different qualities. The mechanisms governing price competition and the choice of products attributes in such markets have been widely investigated since the pioneering contributions of Gabszewicz and Thisse (1979) and Shaked and Sutton (1982). A common feature of these models of "vertical differentiation" is that the quality of the products is certain in the sense that it enters the utility of the consumers as a deterministic variable. Yet, in many instances, the good can be viewed as falling into the category of experience goods, whose true "qualities" are revealed only after they have been consumed. More generally, it often happens that the quality as ascertained by a consumer before he buys differs from what he benefits from upon consumption. For instance, the market for professional services is characterized by the coexistence of "experts" and "non-experts" whose main difference

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lies in the probability that they will provide the adequate service[1]. Recently, Krishna and Winston (1998) offered an interesting reinterpretation of the quality issue. In their paper, the quality of a product depends on the probability that it will "deliver", i.e. yield to the consumer the satisfaction he expected to enjoy. Central to the analysis is that at the time of buying, the consumer faces an intrisic uncertainty. Krishna and Winston (1998) study the impact of such a quality definition on competition through a repeat purchase mechanism. We consider instead that consumers buy a single unit of the good and focus on how the risk involved in the buying decision affects products' valuation and thereby competition.

Second-hand products are another instance of goods of uncertain qualities. In several industries, and specifically those of consumers' durables goods, technological progress is more and more rapidly embedded into new product lines. In consumers' markets such as Personnal Computer, Camera, Hi-Fi and Home-video for instance, technological endowment might even be viewed as *the* key feature distinguishing a new product from an old one. As a consequence of this high-speed technological progress, products become more and more quickly obsolete, even though they still exhibit a high degree of reliability. In such cases, the emergence of second-hand dealers seems to be a natural phenomemon. For instance, at least for PC and Hi-fi goods, specialized magazines attest of the development of such second hand markets, first by publishing "referenced prices" for older product lines as well as by advertising for second-hand dealers, exactly as it has been the case for long in the car market. It is traditional to view second hand product as cheaper alternatives (exhibiting less quality and less reliability) to primary products. Note however that for home computers or non-professional cameras, it is often the case that second-hand dealers are positioned "in between" top quality primary products and the bottom of the range. In other words, they can be viewed as competing with the top of the range by offering fair levels of quality and reliability but for a lower price but they are also offering higher quality (though less reliability) than primary product at the bottom of the range and could thus be view as competing "from above"[2].

What second-hand markets have in common with the markets for services or experience goods is the uncertainty involved in the transactions. Moreover, since in most cases the good can be viewed as a durable one, minimizing risk through diversification is not feasible. In the present paper we explore the implications of this uncertainty on price competition when the risk associated with the decision is explicitely taken into account. To

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1 Bouckaert and Degryse (1998) analyze price competition in such a market but their focus is quite different from ours.

2 Note that the coexistence of second-hand markets and primary ones has been questioned by economic scholars from different point of views. Hendel and Lizzeri (1999) offer a general model in which most of the issues relating a monopolist's strategy and the presence of a second hand market can be dealt with. Anderson and Ginsburgh (1994) deal with price discrimination in second-hand markets within a framework of vertical differentiation. Our analysis departs from theirs' in that we are studying the possibility of competition in local dealers' markets while they focus on the problem of a monopolist producer facing a potential second hand competitive fringe.

this end, we develop a model where products exhibit uncertain qualities and consumers differ in risk aversion. The heterogeneity of consumers' attitude towards risk significantly affects price competition. For instance, it may explain why reliable products would accomodate the presence of less reliable ones instead of preempting the market. We will show in details how the combination of products' attributes combines with the distribution of risk aversion in the population to determine market outcomes. We show that more risky products can be viable in equilibrium but also that choosing to be risky can be an equilibrium strategy whose rationale is to be found in the traditional "market segmentation effect" associated with product differentiation models[3].

Our model can be viewed as an additional application of the vertical differentiation model of Gabszewicz and Thisse (1979). As such it provides a new domain of research where address-models of differentiation prove to be a useful modelling vehicule. The main difference with their approach is that in our setting products are differentiated along two dimensions instead of one. As will be shown in the next section, although each of these two dimensions satisfy the standard vertical differentiation property, their combination often results into horizontal differentiation. Our framework is thus somewhat hybrid: depending on the constellations of products' and population's attributes, either horizontal or vertical differentiation emerges. In this respect, the present analysis can also be viewed as improving our understanding of the nature of price competition in differentiated markets.

The paper is organized as follows. We develop the basic model in section 2 and characterize the Nash equilibrium in prices in section 3. In section 4, we study the relation between equilibrium outcomes and the characteristics of the population. This is done first by performing comparative statics on the Nash price equilibrium and second by extending the model to encompass a stage where products' attributes, specifically products' variances are chosen. This allows us to compare our results with the remaining literature on Vertical Differentiation. Section 5 concludes.

## 2 The Model

Consider an industry where two firms compete in prices by selling differentiated products. Marginal costs are constant, and therefore normalized to zero. A product $i$ ($i = 1, 2$), defined by its random quality $\tilde{q}_i$, is characterized by its expected quality, $\overline{q_i}$ and its variance (risk), $\sigma_i^2$. Consumers are assumed to be risk averse. More precisely, each consumer's preferences are described by a von Neumann-Morgenstern utility function which displays constant absolute risk aversion. Formally, a consumer's utility function can

[3] Note that in doing so, we totally abstract from asymmetric information issues.

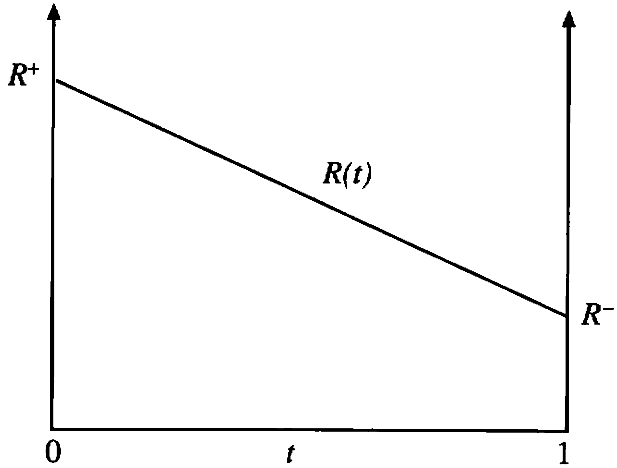


**Figure 1**

be written as:

$$U = -e^{-R(Y-P+q)}$$

where $Y$ stands for income, $P$ for price, $q$ for the level of quality and $R$ represents the absolute risk aversion.

The certainty equivalent, for this consumer, of a product $\tilde{q}_i$ when $Y$ and $p_i$ are assumed to be certain can be approximated by:

$$CE = \overline{q_i} - \frac{1}{2}R\sigma_i^2 + Y - P_i$$

Therefore, we can express the reservation price of the consumer for product $\tilde{q}_i$ as:

$$RP_i = \overline{q_i} - \frac{1}{2}R\sigma_i^2 \tag{2.1}$$

As appears clearly from the inspection of equation (2.1), the type specific characteristic, $R$, affects the reservation price through the variance component only. In other words, in our model, products are characterized by two attributes, expected quality and variance. We assume however that consumers' preferences are heterogeneous with respect to only one of them (the variance).

It remains now to introduce the differentiation in the population. We assume that consumers' risk aversion is continuously and uniformly distributed in the domain $R = [R^-, R^+]$ with $R^+ > R^- > 0$. Each type of consumer is thus identified by a specific degree of risk aversion.

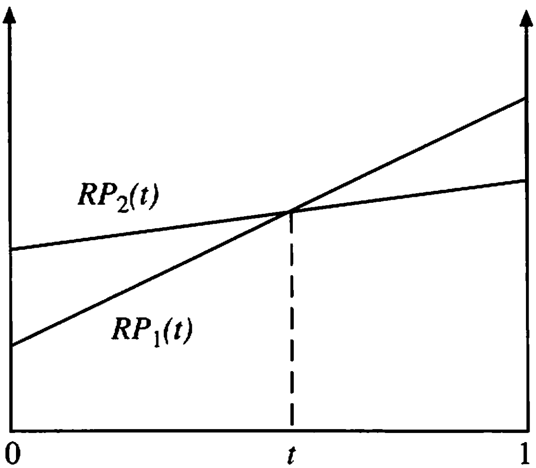


**Figure 2**

In this model it is the attitude towards risk that differentiates consumers. Thus in the absence of uncertainty in the realization of $\tilde{q}_i$ all consumers are identical in their willingness to pay for a product of certain quality $\overline{q_i}$[4].

For convenience, we define a function $R(t)$ with $t \in [0,1]$ where $t$ expresses the fraction of the population whose risk aversion is larger than $R(t)$. $R(t)$ is thus defined as:

$$R(t) = R^+ - (R^+ - R^-)t \quad t \in [0,1] \tag{2.2}$$

Figure 1 provides a typical representation of $R(t)$, given a risk aversion domain. It shows that we have rearranged the population in $[0,1]$ by decreasing order of risk aversion.

Expressions (2.1) and (2.2) allow us to write the reservation price distributions for each of the two products in the market as a function of $t$

$$RP_1(t) = \overline{q_1} - \frac{1}{2}R^+\sigma_1^2 + \frac{1}{2}\sigma_1^2(R^+ - R^-)t \tag{2.3}$$

$$RP_2(t) = \overline{q_2} - \frac{1}{2}R^+\sigma_2^2 + \frac{1}{2}\sigma_2^2(R^+ - R^-)t \tag{2.4}$$

Equations (2.3) and(2.4) provide the explicit expressions for $RP_i(t)$ in the $[0,1]$ interval and lead to a very tractable structure for graphical exposition, as is shown in Figure 2.

Without loss of generality we will use the convention hereafter that $\sigma_1^2 > \sigma_2^2$. This guarantees that the slope of $RP_1(t)$ is steeper than the slope

[4] This model could accomodate for differences in consumers' willingness to pay in the absence of uncertainty by assuming the utility function to be $U^k = -e^{-R(Y-P+\alpha q)}$ with $\alpha$ being a parameter specific to each consumer.

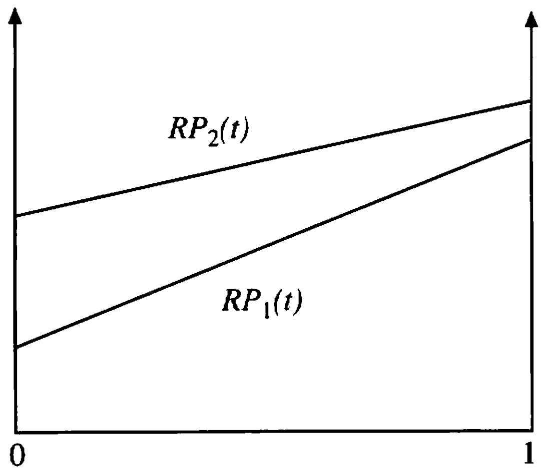


**Figure 3**

of $RP_2(t)$, but imposes no restriction on the relative levels of reservation prices.

Consider Figure 2 which was drawn assuming $\overline{q_1} > \overline{q_2}$. We define natural markets as the share captured by each firm when both prices are equal to the marginal cost (which in this model is assumed to be zero for both firms), i.e. natural market are equivalent to competitive market shares. It is clear that firm 2 has a natural market of size $t^*$ while firm 1 has a natural market of size $1 - t^*$. The argument is intuitive: since product 2 is less risky, it is most attractive for consumers with relatively high risk aversion (types near $t = 0$). As risk aversion decreases, the comparative attractivness of firm 2 becomes less important and eventually firm 1 is preferred by consumers exhibiting a low risk aversion[5]. This argument is the keypoint of the competition process. *Market outcomes, given a risk aversion domain, will always reflect the trade-off between risk differential and expected quality differential.*

Figure 2 depicts only one of the typical situations that may occur. Under the convention that $\sigma_1^2 > \sigma_2^2$ three configurations can be identified.

A) $RP_2(t) > RP_1(t) \quad \forall t \in [0, 1]$

Since, by convention, the slope of $RP_1(t)$ is steeper than the slope of $R_2(t)$ case A occurs whenever $RP_2(1) > RP_1(1)$; this condition can be written as:

$$\overline{q_1} - \overline{q_2} < \frac{1}{2} R^-(\sigma_1^2 - \sigma_2^2) \tag{2.5}$$

In this configuration, firm 2 has a potential for excluding firm 1 from the market by setting a limit price $RP_2(1) - RP_1(1)$. In other

[5] recall that Figure 2 assumes that product 1 presents a higher expected quality than product 2.

words we observe a configuration of vertical differentiation : all consumers would buy the same product if both were sold at the same price.

Clearly, $\overline{q_2} > \overline{q_1}$ is sufficient to obtain case A. Product 2 presents a higher expected quality and a lower risk; this allows firm 2 to exclude firm 1 from the market. However, as shown by (2.5), $\overline{q_2} > \overline{q_1}$ is not a necessary condition for firm 2 to dominate the market[6].

B) $RP_2(t) < RP_1(t) \quad \forall t \in [0,1]$

This situation is the strict reverse of A and will prevail provided $RP_2(0) < RP_1(0)$, that is if and only if :

$$\overline{q_1} - \overline{q_2} > \frac{1}{2} R^+[\sigma_1^2 - \sigma_2^2] \tag{2.6}$$

Under this situation firm 1 has a potential to exclude firm 2 from the market by setting a limit price of $RP_1(0) - RP_2(0)$[7].

Clearly, since firm 1 has the disadvantage of a more risky product, the dominance of firm 1 requires that its expected quality be greater than firm 2's. More precisely, the expected quality differential has to be in favour of firm 1 and sufficiently important to overcome the disadvantageous risk differential, for the most risk averse type ($t = 0$).

C) $RP_2(t) > RP_1(t) \quad \forall t < t^*$ and $RP_2(t) < RP_1(t) \quad \forall t > t^* \quad t^* \in ]0,1[$

This happens when both condition (2.5) and (2.6) are violated, that is, if and only if :

$$\frac{1}{2} R^-[\sigma_1^2 - \sigma_2^2] \leqslant \overline{q_1} - \overline{q_2} \leqslant \frac{1}{2} R^+[\sigma_1^2 - \sigma_2^2] \tag{2.7}$$

In this configuration, as depicted in figure 2, none of the firms has a definitive advantage over the other : both firms will necessarily survive with strictly positive market shares at equilibrium. When (2.7) is satisfied the model describes then a situation of horizontal differentiation. Condition (2.7) defines also the parameters' constellations for which the competitive, and thus efficient, outcome calls for the presence of two active firms.

All in all, we note that our model may exhibit either horizontal or vertical differentiation. More precisely, for horizontal differentiation to prevail we not only need a negative correlation between quality and variance rankings but also to satisfy boundary conditions on the distribution of consumers' characteristics. In other words, products' characteristics alone do not provide enough information to assess the vertical or horizontal nature of differentiation.

6 We say that a firm dominates the market when the distribution of its reservation prices is above the other firm's throughout the market.

7 This is not saying that at equilibrium the firm with the potential to exclude its rival from the market by setting a limit price will choose to do so.

## 3 Market Equilibrium

We are now in a position to characterize the market equilibrium. In computing the price equilibrium, we will retain the "market coverage" assumption : i.e. we concentrate on the domain of parameters for which, at equilibrium, every consumer buys one of the two products[8]. Our equilibrium concept is Nash equilibrium in prices. Straightforward computations yield the following characterization.

**Proposition 1** *Given a compact domain of risk aversion* $[R^-, R^+]$, *the unique Nash equilibrium is given by :*

*1. If,* $\overline{q_1} - \overline{q_2} \in \left[\frac{1}{2}(2R^- - R^+)(\sigma_1^2 - \sigma_2^2), \frac{1}{2}(2R^+ - R^-)(\sigma_1^2 - \sigma_2^2)\right]$.

$$p_1^* = \frac{\overline{q_1} - \overline{q_2} + \frac{1}{2}(R^+ - 2R^-)(\sigma_1^2 - \sigma_2^2)}{3}$$

$$p_2^* = \frac{\overline{q_2} - \overline{q_1} + \frac{1}{2}(2R^+ - R^-)(\sigma_1^2 - \sigma_2^2)}{3}$$

*2. If,* $\overline{q_1} - \overline{q_2} > \frac{1}{2}(2R^+ - R^-)(\sigma_1^2 - \sigma_2^2)$.

$$p_1^* = \overline{q_1} - \overline{q_2} - \frac{1}{2}R^+(\sigma_1^2 - \sigma_2^2) \qquad p_2^* = 0$$

*3. If,* $\overline{q_1} - \overline{q_2} < \frac{1}{2}(2R^- - R^+)(\sigma_1^2 - \sigma_2^2)$.

$$p_1^* = 0 \qquad p_2^* = \overline{q_2} - \overline{q_1} + \frac{1}{2}R^-(\sigma_1^2 - \sigma_2^2)$$

Proposition 1 tells us that in equilibrium, one of the two firm may drive the other out of the market (cases 2 and 3). This is not surprising since we know that our model may exhibit vertical differentiation . For instance we have seen that under condition (2.7) no firm has a definitive advantage over the other and so both firms should be active at equilibrium. In Proposition 1, condition (2.7) ensures that the relevant equilibrium region is the first one. Notice however that the domain of parameters[9] for which the market is shared in equilibrium includes also some of the parameters constellations that describe case A and case B. More precisely, not all values of the parameters that allow firm 2 to exclude firm 1 from the market make it profitable

[8] This restriction amounts to restrict admissible values of the parameters to a domain where equilibrium price of product 2 is less then the lowest reservation price (type $R^+$). Formally, we consider

$$R^+ < R^- \frac{\sigma_1^2 - \sigma_2^2}{2\sigma_1^2 + \sigma_2^2} + 2\frac{2\overline{q_2} + \overline{q_1}}{2\sigma_1^2 + \sigma_2^2}.$$

[9] In this context by parameters we mean not only $R^-$ and $R^+$ but also $\overline{q_1}, \overline{q_2}, \sigma_1^2$ and $\sigma_2^2$.

for firm 2 to do so. The exact same argument can be made regarding firm 1 and case B. In other words, the domain of parameters describing the shared equilibrium in proposition 1 covers situations of horizontal differentiation (case C) as well as of vertical differentiation (cases A and B). Recalling that condition (2.7) defines the constellations where an efficient outcome calls for two active firms in the market, we observe that imperfect competition here is a source of inefficiency because it *allows for equilibrium configurations where two firms are active whereas it would be more efficient that only one of them remains.* Last, recall that by exhibiting a lower variance, product 2 could be viewed as a proxy for a new product. According to configuration 2 in proposition 1, we see that, once consumers' are risk averse, it is not only possible for a risky product to survive, but even to exclude a more reliable one from the market. A necessary condition being of course that a higher expected quality is associated with more risk.

We now briefly comment on relative market power at the shared equilibrium. Simple computations lead to the following conclusion :

$$p_1^* > p_2^* \Longleftrightarrow \overline{q_1} - \overline{q_2} > \frac{\mu_R}{2}(\sigma_1^2 - \sigma_2^2) \tag{3.1}$$

where $\mu_R$ defines the mean risk aversion in the population i.e. :

$$\mu_R = \frac{R^- + R^+}{2}$$

So, firm 1 will set higher prices and make higher profits as long as the expected quality differential exceeds the risk premium differential for the mean type consumer[10]. Moreover, (3.1) can be extended in exactly the same way to demands and profits. *Relative market power directly reflects the trade-off between the characteristic with respect to which the population is homogenous (expected quality) and the type-specific appreciation of the products (risk premium).* Remark that, in the domain of parameters for which, at equilibrium, the market is shared, the dispersion of risk aversion in the population plays no role in determining which firm sets higher prices or makes higher profits. This suggests that as far as the population's distribution is concerned the distribution of market power depends on mean risk aversion whereas its dispersion determines market structure.

In the next section we study in details how population's and products' characteristics concur to determine market outcomes.

10 (3.1) can alternatively be written as: $p_1^* > p_2^* \Longleftrightarrow \overline{q_1} - \frac{\mu_R}{2}\sigma_1^2 > \overline{q_2} - \frac{\mu_R}{2}\sigma_2^2$
which means that firm 1 will set higher prices and make higher profits whenever product 1's reservation price is higher than product 2's for the consumer whose risk aversion is the mean risk aversion in the population.

# 4 Populations characteristics, products' attributes and equilibrium outcomes

The aim of this section is to identify the extent to which population's heterogeneity influences the characterization of market equilibrium outcomes. To this end we perform a comparative static exercice on the impact of the domain of risk aversion on price equilibrium (subsection 4.1). In subsection 4.2 we allow firms to choose the variance of their product before price competition takes place.

## 4.1 Comparative Statics on the Domain of Risk Aversion

Proposition 1 showed that a necessary condition for the possibility of two risky products to coexist in equilibrium is precisely that risk aversion differ across the population. One may then wonder to which extent the distribution of risk aversion in the population determines the nature of equilibrium outcomes.

In order to simplify notation, we represent all the products' characteristics in one variable $K$, defined as :

$$K = \frac{2(\overline{q_2} - \overline{q_1})}{\sigma_1^2 - \sigma_2^2} \tag{4.1}$$

$K$ is thus a ratio of expected quality differential to risk differential. Since by convention $\sigma_1^2 > \sigma_2^2$, the sign of $K$ is that of the numerator.

Whenever $\overline{q_2} > \overline{q_1}$, that is for $K$ positive, firm 2 has an advantage over firm 1, whatever the domain of risk aversion, since it combines lower risk with higher expected quality. This means that for $K > 0$ firm 2 can never be excluded from the market; thus, only two types of equilibrium are compatible with a positive $K$ : quasi-monopoly of firm 2 (case 3) and shared equilibrium (case 1). Moreover, when $K$ is positive and the market is shared at equilibrium, firm 2 will set higher prices and make higher profits than firm 1 (this can be seen using (3.1)).

On the contrary, if $\overline{q_1} > \overline{q_2}$, that is if $K$ is negative, we cannot exclude a priori any particular equilibrium outcome.

In order to investigate the exact role of the mean risk aversion and the risk aversion dispersion in the population a graphical representation proves useful. We depict hereafter in the space $(R^-, R^+)$ the domains characterizing the three types of equilibrium. To this end, we define iso-$\mu_R$ as the set of combinations of $R^+$ and $R^-$ which lead to the same mean risk aversion in the population, $\mu_R$. These combinations are described by a straight line of slope $-1$, parameterized by $\mu_R$, in the $(R^-, R^+)$ space. Since by definition $R^+ \geqslant R^-$ only the upper half (above the 45° line) of each iso-$\mu$ is relevant.

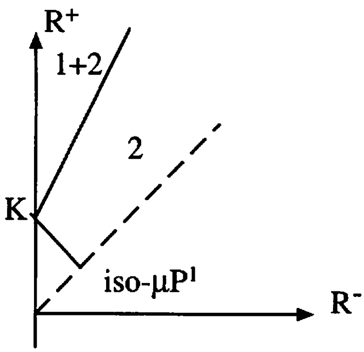


Figure 4 : $K > 0$

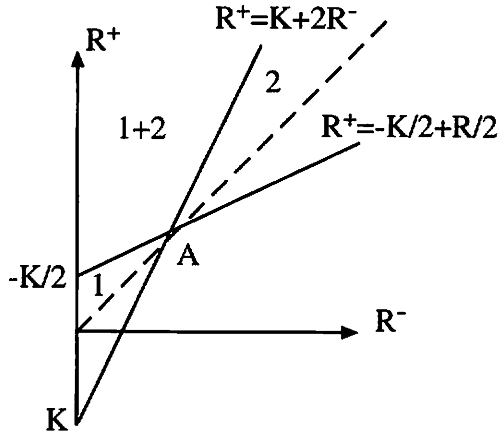


Figure 5 : $K < 0$

Remark that along any particular iso-$\mu_R$ dispersion increases from right to left. If $R^+$ is very close to $R^-$ dispersion is minimized while if $R^- = 0$ dispersion is at its maximum for that particular value of $\mu_R$.

Given a particular value of $K$, we can reexpress the parameters domains separating equilibrium outcomes in Proposition 1. More precisely, market sharing equilibrium occurs if and only if:

$$R^+ > \frac{-K + R^-}{2} \text{ and} \tag{4.2}$$

$$R^+ > K + 2R^- \tag{4.3}$$

Whenever (4.2) does not hold firm 1 is alone in the market, that is, (4.2) separates shared equilibrium from quasi-monopoly of firm 1. Similarly, whenever (4.3) does not hold firm 2 keeps the whole market, so, (4.3) separates shared equilibrium from quasi-monopoly of firm 2.

Clearly, if $K > 0$, (4.2) always holds and quasi-monopoly of firm 1 cannot occur, which means, as we have already mentioned, that a firm selling a product of higher expected quality and presenting less risk cannot be driven out of the market.

Figures 4 and 5 provide typical representations of the conditions stated in (4.2) and (4.3) for $K$ positive and negative, respectively.

Consider figure 4, which was drawn for a particular value of $K > 0$. For points above the line $R^+ = K+2R^-$ a shared equilibrium emerges while for points below, quasi-monopoly of firm 2 arises (recall that with $K > 0$ (4.2) is automatically fullfiled). Figure 4 shows that along any iso-$\mu_R$ below iso-$\mu_R^1$, quasi-monopoly of firm 2 occurs for any level of dispersion. On the contrary, along any iso-$\mu_R$ above iso-$\mu_R^1$, depending on the dispersion, either quasi-monopoly of firm 2 or shared equilibrium may occur. Note that for any given level of $\mu_R > \mu_R^1$ low levels of dispersion lead to quasi-monopoly and there is a critical level of dispersion above which firm 2 *prefers* to share the market.

Consider now figure 5, which was drawn for a particular level of $K < 0$. For points below $R^+ = \frac{-K+R^-}{2}$ and above the 45° line (since by definition $R^+ > R^-$) quasi-monopoly of firm 1 occurs. Similarly, for points below $R^+ = K + 2R^-$ and above the 45° line quasi-monopoly of firm 2 emerges. Finally, in the region where both $R^+ > \frac{-K+R^-}{2}$ and $R^+ > K + 2R^-$ the market is shared at equilibrium. Notice that these two separating lines intersect each other over the 45° line at point A $(R_A^+ = R_A^- = -K)$. Clearly, the iso-$\mu_R$ passing through point A refers to a level of mean risk aversion given by $\mu_R = -K$. A population's distribution corresponding to point A is thus an homogenous population. It is easily shown that for the level of mean risk aversion $\mu_R = -K$, reservation prices are identical. In other words, consumers' valuation of the products are identical, exactly as if products were homogenous. Quality differentials are exactly balanced by variance differentials. At point A, Bertrand equilibrium prevails and when population's heterogeneity increases (we move North-East along the iso$-\mu$), both prices rise symmetrically. When the dispersion in the population increases, price competition is relaxed.

Inspection of figure 5 reveals two interesting features :

a) when $K$ is negative and for any fixed level of mean risk aversion, two types of equilibrium at most, can occur (quasi-monopoly of one of the firms or shared market). Stated otherwise, given products' attributes, only two equilibrium configurations are possible, irrespective of the population's characteristics.

b) along any particular iso-$\mu_R$ one of the firms enjoys an advantage over the other *whatever the dispersion level.* By advantage we mean that either the firm is in a situation of quasi-monopoly or, if it shares the market, its price and profits are higher than the other firm's.

When $K$ is negative no firm is a priori the dominant one. There is a critical level of mean risk aversion below which firm 1 enjoys an advantage over firm 2 and above which the reverse prevails. These observations can be spelled out in more detail in the following way : whenever $\overline{q_1} > \overline{q_2}$, firm 1 keeps the whole market at equilibrium when the mean risk aversion in the population is sufficiently low $(\mu_R < -\frac{K}{4})$. For intermediate levels of mean risk aversion $(-\frac{K}{4} < \mu_R < -K)$ either quasi-monopoly of firm 1 or shared market may occur depending on dispersion (high levels of dispersion being associated with a shared equilibrium), but in any case firm 1 will be in a better position than firm 2. Finally, for high levels of mean risk aversion $(\mu_R > -K)$ it will be firm 2 who enjoys an advantage over firm 1, and either quasi-monopoly of firm 2 (for low levels of dispersion) or shared equilibrium, with firm 2 quoting higher prices and making higher profits, will be observed.

To sum-up, with the help of figures 4 and 5 we were able to identify the following stylized facts:

*Given that firm 1 is more risky,*

- *When firm 2 exhibits a higher expected quality ($K > 0$) firm 1 needs a sufficiently high mean risk aversion ($\mu_R > K$) and a sufficiently high dispersion to benefit from a positive market share.*
- *When firm 2 exhibits a lower expected quality ($K < 0$) there exists a critical level of mean risk aversion ($\mu_R = -K$) above which firm 2 is better off than firm 1 and below which the reverse prevails.*
- *Furthermore, equilibrium outcomes where the market is shared are the more likely the higher is the dispersion, whatever the products' dominance.*

We now give some intuitive explanation of these facts. Consider first the situation where $\overline{q_2} > \overline{q_1}$. In these circumstances firm 2 is in the best competitive position since it sells a product of higher expected quality and lower risk. When mean risk aversion increases, the advantage in expected quality becomes less relevant and firms are competing more on the risk component. When $\mu_R$ is sufficiently high (above $K$), firm 1 captures, at equilibrium, a positive share of the market provided dispersion is sufficiently high. This happens because for this given level of $\mu_R$ an increase in dispersion lowers the limit price and leads firm 2 to prefer the shared outcome to quasi-monopoly with limit price. Exclusion becomes unprofitable. So, when $\overline{q_2} > \overline{q_1}$, firm 1 needs a sufficiently high mean risk aversion to benefit from the population's dispersion effect, which is the only channel through which firm 2 might be led to share the market. Due to an increase in the population risk aversion's heterogeneity, effective product differentiation increases and both firms benefit from relaxed price competition.

When $\overline{q_2} < \overline{q_1}$ and the mean risk aversion is low we can follow the same argument where $\mu_R$ is sufficiently low for the expected quality differential, now favourable to firm 1, to be the preponderant element in the consumers' decision. In this case, firm 1 gains from low dispersion. When $\mu_R$ reaches a critical level the argument is reversed and since the risk differential becomes the most important factor both firms benefit from increases in the risk aversion dispersion.

In conclusion, our results emphasize the permanent trade-off between that part of the product's valuation which is independent of the population's heterogeneity (expected quality differential) and those depending on it (variance differential). The mean risk aversion decides then which element is the most relevant and dispersion in the population, given $\mu_R$, determines the market structure. In any case, low levels of dispersion are required for quasi-monopoly outcomes to arise.

Let us illustrate the case of competition in a local retail market between two dealers: one sells a primary product and the other a second-hand one. This situation typically involves $K > 0$ (i.e. higher expected quality of the primary product) with product 2 as the primary product (lower variance product). Our analysis shows that *the presence of the second-hand dealer is accomodated if and only if both the mean risk aversion and the dispersion are high enough.* In this case indeed, killing off the second-hand market would involve a too large price cut. Note finally that $K < 0$ covers a situation where the primary product is competing with a second-hand product of a higher expected quality. Such cases might illustrate the situation that prevails in markets for rapidly evolving high-tech products. Products become quickly obsolete according to the Top-Range standards of the industry (typically professional users), although exhibiting adequate performances for the low range standards (typically non-professionals). In such industries there is room for a second-hand market where products exhibit high performance while being less reliable, due to their second-hand nature. Our analysis suggests that the emergence of a second hand market for these products is more likely to occur not only if the level of mean risk aversion is lower but also if dispersion is high.

## 4.2 Endogenous Choice of Risk

In the preceeding sections we have treated the product characteristics of each firm as given. This section deals with the endogenous determination of the levels of risk associated with each product. We will not address the question of endogenous choice of expected quality in this paper. There are several reasons to this. First, the fact that the population is homogeneous with respect to the expected quality attribute makes the analysis rather trivial if we assume that expected quality is not costly[11]. Second, and more importantly, we limit our aim to illustrate with the help of a simple example the possibility that firms would *choose* to sell risky products. This strategy seems indeed rather unwise at first sight but we show hereafter that it can be perfectly rational in specific circumstances. In particular, a product which already exhibits a lower expected quality may be better off not compensating this disadvantage through a lower risk. In order to show this, we assume that there exists a given expected quality differential but firms choose the level of the variance at no cost. This assumption is of course not realistic but as will become clear from the analysis that follows, our results would not be affected qualitatively by the introduction of a cost to variance reduction. Neglecting the cost issue allows us to focus exclusively on these firms' incentives that are purely related to differentiation arguments.

We thus allow both firms to simultaneously choose their level of risk, at no cost, before the price game is played. We assume that each firm chooses a level of variance in the interval $[0, \overline{\sigma}^2]$[12].

**Proposition 2** *When the choice of variance is made endogenous :*

[11] If on the contrary we assumed the possibility of a costly change in expected quality the results would depend so crucially on the specification of this cost that any result might be obtained.

[12] For many distributions, the approximation used in (2.1) is valid only for relatively small values of variance.

- *There is no room for two firms in a subgame perfect equilibrium when consumers' heterogeneity is low ($R^+ - 2R^- < 0$). The firm with the higher expected quality preempts the market at any subgame perfect equilibrium,*
- *If $R^+ - 2R^- > 0$, The only subgame perfect equilibria that allow for two active firms are equilibria entailing maximal risk differentiation.*

The detailed proof of Proposition 2 has been relegated to the appendix. It is mainly a matter of tedious computations whose underlying argument can be summarized as follows.

Two cases have to be distinguished, according to the sign of $R^+ - 2R^-$. Whenever $R^+ - 2R^- < 0$, a firm with higher variance and lower expected quality has no place in the market. This result was expected since a parameter restriction on the bounds of the distribution of consumers' preferences such as $R^+ - 2R^- < 0$ is well known to imply that the market is a "natural monopoly" (see Shaked and Sutton(1983)) in a model of vertical differentiation and heterogeneous consumers. The firm with the lower expected quality could in principle keep the whole market provided its variance is small enough. But since $R^+ - 2R^- < 0$, our comparative statics analysis revealed that the firm with higher expected quality necessarily dominates the market. The variance game is thus unimportant in the sense that the market structure in the price game is market preemption.

Whenever $R^+ - 2R^- > 0$, a firm with higher variance and lower expected quality can survive price competition. Moreover, in order to preempt the whole market a firm has to present both a lower variance and a higher expected quality. In other words, consumer preferences satisfying $R^+ - 2R^- > 0$ are less penalizing to the "disadvantaged" firm, i.e. the firm exhibiting the larger variance, than the ones in the previous case. This is materialized by the fact that even against a zero variance, a product of lower expected quality could secure positive profits by choosing a large enough variance. On the other hand, against a large variance, the lower expected quality firm is better off replying with a zero variance. In other words, the firm which is dominated along the expected quality dimension gains from maximizing product differentiation along the variance dimension. Summing up, Proposition 2 reveals that whenever the distribution of risk aversion allows for two active firm in equilibrium, maximal differentiation takes place.

## 4.3 Comments

Let us illustrate our results by considering a market where a primary product dealer and a second-hand one are competing. Choosing a zero variance amounts to selling a brand new product. Then, we have shown that the presence of a second-hand seller, offering a product of lower expected quality and higher variance, can be sustained in equilibrium if and only if there is enough dispersion in consumers' risk aversion. In this case indeed, the second-hand dealer will be accomodated because it would be too costly to drive him out of the market. We have also shown that there is room for selling a second-hand product exhibiting higher expected quality and higher variance, i.e. a product that competes from above with a new one. Proposition 2 may help us understand why firms would voluntarily choose to sell a risky product, even when reducing the risk is not costly. Once the nature of price competition in our model is understood, the intuition is straightforward: *the firm indirectly segments the market by focusing on that part of the population which tends to attach more value to expected quality than to reliability. The reliable product may then prefer to accommodate the presence of this competitor rather than kill off the market.* A necessary condition for this to happen is a high dispersion in the population since this makes market preemption less attractive. Selling a "non-reliable" product appears to be the most efficient way of segmenting the market. This is a survival condition for the second hand product. A similar argument can be developed for the more general case of services or products exhibiting different degrees of reliability.

It is fair to argue that our maximal differentiation results obtains as a corner solution because costs, and singularily the cost of reducing variance, are nil.[13] It is clear however that the qualitative nature of our results does not depend on our zero cost assumption. Relaxing this assumption basically rules out the cases where a continuum of equilibria prevails and modifies the critical levels of variance which support the equilibrium where the lower expected quality product is also the more risky shares the market. However, two equilibria exhibiting an "optimal" degree of differentiation will still prevail. At a more general level, our analysis shows that products whose quality is uncertain, and more precisely products performing the same task but exhibiting notoriously different degree of reliability may coexist while being direct competitors. It may even be rational to specialize on risky products in order to segment the market.

Last, it is instructive to relate our present results to some well-known results of the vertical product differentiation literature. We noted in section 2 that our model could not be classified a priori either as vertical or horizontal. And it is easy to show that the Subgame perfect equilibrium exhibiting a "high risk, low expected quality" firm may yield either horizontal or vertical differentiation in the price subgame, depending on the values of the parameters. Horizontal differentiation is not even a necessary condition for the low expected quality firm to be active in a subgame perfect equilibrium. The degree to which the model produces horizontal or vertical differentiation depends in fact on the correlation between the two characteristics across products and the parameters of the distribution describing consumers' tastes. Thus, although each characteristic when taken separetely exhibits vertical differentiation whatever the population's distribution, the nature of differentiation that results from their combination definitely depends on it. Note finally that in our model the population is heterogeneous

[13] It also results from the fact that we restrict attention to fully covered market. Allowing for partial coverage would often invalidate the maximal differentiation result but the incentives to choose a large variance against a low one would remain.

in one dimension only. We have not made the choice of expected quality endogenous but given the fact that consumers do not value it differently, it seems highly reasonable to assume that firms would end up choosing identical expected qualities and therefore differentiate in the risk dimension[14]. This result could be viewed as an illustration of the principles put forward by Irmen and Thisse (1998) in the case of horizontal differentiation. They have shown that when there are several horizontal characteristics, firms are inclined to differentiate along one dimension only. This is also the case in our model where vertical characteristics are present.

# 5 Final Remarks

In this paper, we have shown that products of uncertain and different qualities may coexist in a market where firms compete directly in prices. The critical ingredient to obtain this result is the heterogeneity of consumers' attitude towards risk. Obviously, the model is too stylized but despite of its numerous restrictive assumptions, the intuition it conveys is robust: products which are notoriously less reliable (for instance second-hand products) require a sufficiently high degree of heterogeneity in the population's risk aversion to be viable. We have also shown that even when reducing risk is not costly, it may be an equilibrium strategy to specialize in selling highly risky products. This might in fact be a necessary condition for survival in the market. In particular, we have shown that if the second-hand product exhibits lower expected quality, the seller has no incentive to lower her variance but rather to maximize it.

[14] From the conditions stated in the appendix, we may observe that the presence of an expected quality differential essentially enlarges the set of population's dispersion which supports market preemption. Without this differential indeed, market preemption would require that $R^+ - 2R^- < 0$, which is a standard condition for market preemption prevailing in unidimensional models of vertical differentiation. See Tirole (1988), chap 7.

# A : Appendix : Proof of Proposition 2

In what follows we adopt the following notation :

-Firm 1 designates the firm with the higher variance

-Firm A designates the firm with the higher expected quality

Straightforward computations (using proposition 1) allow us to state that, in the range of parameters corresponding to a shared equilibrium, we have that :

$$sign\frac{d\Pi_1}{d\sigma_1^2} = sign(R^+ - 2R^- + K) \text{ and } \frac{d^2\Pi_1}{d(\sigma_1^2)^2} > 0$$

$$sign\frac{d\Pi_2}{d\sigma_2^2} = sign(K - 2R^+ + R^-) \text{ and } \frac{d^2\Pi_2}{d(\sigma_2^2)^2} > 0$$

## Case A- $R^+ - 2R^- < 0$

**Lemma 1** *When $R^+ - 2R^- < 0$, the firm with the higher expected quality chooses a zero variance level against any variance level of the rival firm. The firm with the lower expected quality chooses a zero variance level against any positive variance level of the rival firm and, against a zero variance level of the rival, is indifferent among any level of variance.*

**Proof of Lemma 1** *Variance choice of firm A*

- For any level of $\sigma_A^2$ such that $\sigma_A^2 \leqslant \sigma_B^2$ firm A keeps the whole market and makes positive profits given by $\Pi_A = \overline{q_A} - \overline{q_B} + \frac{R^-}{2}(\sigma_B^2 - \sigma_A^2)$. So, within this range firm A's profit is maximal for $\sigma_A^2 = 0$.
- For levels of $\sigma_A^2$ such that $\sigma_A^2 \geqslant \sigma_B^2$ firm A will be alone in the market and making a profit given by $\Pi_A = \overline{q_A} - \overline{q_B} - \frac{R^+}{2}(\sigma_A^2 - \sigma_B^2)$ for levels of $\sigma_A^2$ close to $\sigma_B^2$ . For higher levels of $\sigma_A^2$ firm B will enter the market and at this shared equilibrium we have that $sign\frac{d\Pi_A}{d\sigma_A^2} = sign(R^+ - 2R^- + K) < 0$, given that $R^+ - 2R^- < 0$ and $K < 0$ (because $\overline{q_A} > \overline{q_B}$ and $\sigma_A^2 > \sigma_B^2$). For even higher values of $\sigma_A^2$ firm A will be forced out of the market.

In conclusion, we have seen that firm A's profit is decreasing in $\sigma_A^2$ and so firm A's best reply against any $\sigma_B^2$ is to set $\sigma_A^2 = 0$.

*Variance choice of firm B*

Since firm B is by definition the firm with the lower expected quality its profit will be zero for any level of $\sigma_B^2$ such that $\sigma_B^2 \geqslant \sigma_A^2$. For levels of $\sigma_B^2$ such that $\sigma_A^2 > \sigma_B^2$ either $(\sigma_A^2 - \sigma_B^2)$ is large enough so that firm B can keep the whole market (i.e. : $(\sigma_A^2 - \sigma_B^2) > \frac{2(\overline{q_A} - \overline{q_B})}{2R^- - R^+}$), in which case its profit is given by $\Pi_B = \overline{q_B} - \overline{q_A} + \frac{R^-}{2}(\sigma_A^2 - \sigma_B^2)$, decreasing in $\sigma_B^2$ ; or $(\sigma_A^2 - \sigma_B^2)$ is

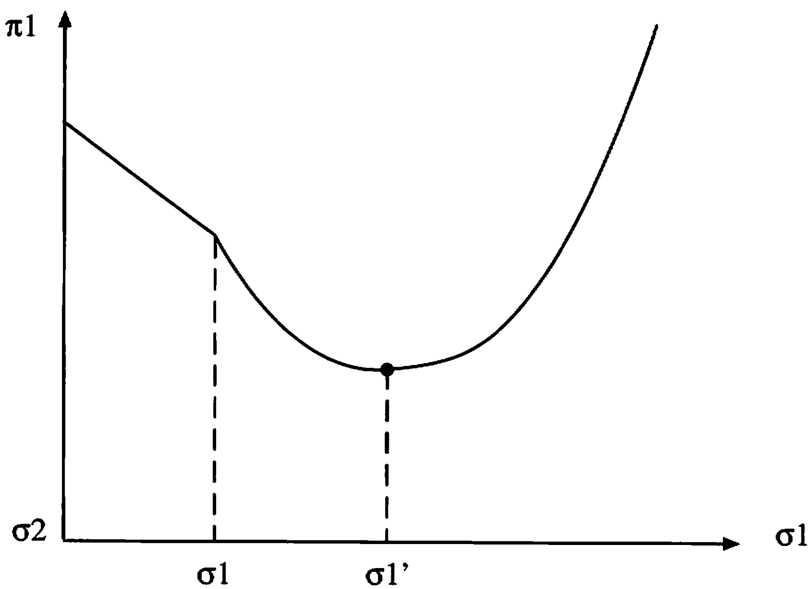


**Figure A.1**

not large enough and a shared equilibrium will emerge. Under this shared equilibrium we know that $sign\frac{d\Pi_B}{d\sigma_B^2} = sign(K - 2R^+ + R^-)$ which is negative since $K$ is negative. So, the profit of firm B is decreasing in $\sigma_B^2$ in the range $\sigma_B^2 < \sigma_A^2$ and is invariantly nill in the range $\sigma_B^2 \geqslant \sigma_A^2$. This means that firm B will reply with $\sigma_B^2 = 0$ against any $\sigma_A^2 > 0$ and, against $\sigma_A^2 = 0$, will be indifferent among any level of $\sigma_B^2$ since its profit will then be zero. □

## Case B- $R^+ - 2R^- > 0$

We start by studying the best reply, in the variance game, of the firm with the highest expected quality.

**Lemma 2** *When $R^+ - 2R^- > 0$, the best reply of firm with the higher expected quality is either a zero level of variance or the highest possible level of variance ($\overline{\sigma}^2$). When the highest level of variance is chosen the market is shared between the two firms.*

**Proof of Lemma 2** *Choice of $\sigma_A^2$ in $[\sigma_B^2, \overline{\sigma}^2]$*

The profit function of firm A in this range is described by $\Pi_1$ which behaves as follows:

For $\sigma_1^2 \in \left[\sigma_2^2, \sigma_2^2 + \frac{2(\overline{q_A} - \overline{q_B})}{2R^+ - R^-}\right]$ the profit of firm A (here firm 1) is given by $\Pi_1 = \overline{q_1} - \overline{q_2} - \frac{R^+}{2}(\sigma_1^2 - \sigma_2^2)$. For levels of $\sigma_1^2$ above $\sigma_2^2 + \frac{2(\overline{q_A} - \overline{q_B})}{2R^+ - R^-}$ firm 2 enters the market and firm 1's profit is decreasing in $\sigma_1^2$ for $\sigma_1^2 < \sigma_2^2 + \frac{2(\overline{q_A} - \overline{q_B})}{R^+ - 2R^-}$ and increasing after that point.

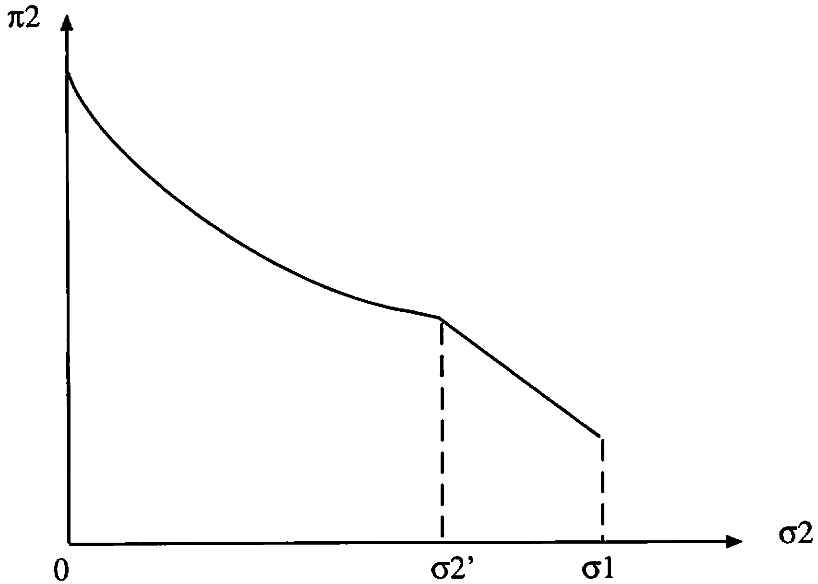

**Figure A.2**

Therefore in the range $[\sigma_B^2, \overline{\sigma}^2]$ firm A chooses either $\sigma_A^2 = \sigma_B^2$ or $\sigma_A^2 = \overline{\sigma}^2$ depending on the value of $\overline{\sigma}^2$.

*Choice of* $\sigma_A^2$ *in* $[0, \sigma_B^2]$

Now the profit function of firm A is described by $\Pi_2$ which behaves as follows:

Firm 2 keeps the whole market if and only if $\sigma_2^2 > \sigma_1^2 - \frac{2(\overline{q_A}-\overline{q_B})}{R^+-2R^-}$ in which case its profit is given by $\Pi_2 = \overline{q_2} - \overline{q_1} + \frac{R^-}{2}(\sigma_1^2 - \sigma_2^2)$.

If on the contrary $\sigma_2^2 < \sigma_1^2 - \frac{2(\overline{q_A}-\overline{q_B})}{R^+-2R^-}$ the market will be shared. At the shared equilibrium $\frac{d\Pi_2}{d\sigma_2^2} = 0 \Leftrightarrow \sigma_2^2 = \sigma_1^2 - \frac{2(\overline{q_A}-\overline{q_B})}{2R^+-R^-}$ which is bigger than $\sigma_1^2 - \frac{2(\overline{q_A}-\overline{q_B})}{R^+-2R^-}$. So, within the shared equilibrium $\frac{d\Pi_2}{d\sigma_2^2} < 0$.

Therefore, in the range $[0, \sigma_B^2]$ firm A chooses $\sigma_A^2 = 0$. Moreover, firm A keeps the whole market if $\sigma_B^2 < \frac{2(\overline{q_A}-\overline{q_B})}{R^+-2R^-}$ and shares the market otherwise.

By continuity of the profit function in the variance level, the choice $\sigma_A^2 = 0$ beats $\sigma_A^2 = \sigma_B^2$.

Thus, in $[0, \overline{\sigma}^2]$ firm A chooses either $\sigma_A^2 = 0$ or $\sigma_A^2 = \overline{\sigma}^2$. Furthermore, the choice $\sigma_A^2 = \overline{\sigma}^2$ implies a shared equilibrium. □

Given Lemma 2 we can study the equilibria of the game by concentrating on the replies of the lower expected quality firm against a zero variance and against the highest variance level.

It can be shown that against a zero variance level the lower expected quality firm can have a share of the market only if the variance differentiation is big enough. Moreover, the lower expected quality firm's profit is increasing in the variance differential. This result can be summarized by the following Lemma.

**Lemma 3** *When $R^+ - 2R^- > 0$, the firm with the lower expected quality (firm B) will reply to a zero variance level by setting the highest variance level ($\overline{\sigma}^2$) and will make positive profits if and only if $\overline{\sigma}^2 > \frac{2(\overline{q_A}-\overline{q_B})}{R^+-2R^-}$ (where $\overline{q_A} > \overline{q_B}$). Otherwise the firm will be indifferent among setting any level of variance and will in any case make zero profits.*

**Proof of Lemma 3**

Given $\sigma_A^2 = 0$ firm B is able to share the market if the variance differential is large enough, that is, if and only if $\sigma_B^2 > \frac{2(\overline{q_A}-\overline{q_B})}{R^+-2R^-}$, in which case its profit is increasing in $\sigma_B^2$ (recall that $sign\frac{d\Pi_1}{d\sigma_1^2} = sign(R^+ - 2R^- + K)$ which is positive in this case). Otherwise firm B is driven out of the market and its profit is zero.

Therefore, against $\sigma_A^2 = 0$ firm B will set $\sigma_B^2 = \overline{\sigma}^2$ and make positive profits if $\overline{\sigma}^2 > \frac{2(\overline{q_A}-\overline{q_B})}{R^+-2R^-}$ and will be indifferent among any level of $\sigma_B^2 \in [0, \overline{\sigma}^2]$ if $\overline{\sigma}^2 < \frac{2(\overline{q_A}-\overline{q_B})}{R^+-2R^-}$, in which case its profit will be zero whatever its choice of variance. □

With the help of Lemmas 1,2 and 3 we are now able to identify the incentives of the second-hand firm with respect to the variance. Indeed, it is natural to assume that the primary product exhibits a variance close to zero. Consider first that the second-hand product exhibits a lower expected quality. Then, we know from Proposition 2 that if the population's dispersion is low, the second-hand firm can never survive, so that her variance choice does not matter to her. When the dispersion is high enough, Proposition 4 tells us that the second-hand firm must choose the highest variance level in order to secure positive profits. When on the contrary the second-hand product exhibits the higher expected quality, Proposition 3 tells us that the second-hand firm will either choose the lowest or the highest variance. Moreover it can be shown that the former case applies when $\overline{\sigma}^2$ is large enough. In this case, the market is shared. Therefore, we are led to the following conclusion : whatever the expected quality differential prevailing between the two products, if the secondary market is to coexist with the primary one, then the second-hand seller must choose the highest possible variance.

It remains now to characterize the SPE of the variance game. Lemma 4 considers the reply of the lower expected quality firm against the highest variance level. The only difference with the previous case is that choosing the maximum differentiation means replying with a zero variance level; moreover the size of differentiation needed to make positive profits is higher than in the preceeding case.

**Lemma 4** *When $R^+ - 2R^- > 0$, the firm with the lower expected quality will reply to the highest variance level ($\overline{\sigma}^2$) by setting a zero variance level* *and will make positive profits if and only if* $\overline{\sigma}^2 > \frac{2(\overline{q_A}-\overline{q_B})}{2R^+-R^-}$ *(where* $\overline{q_A} > \overline{q_B}$*). Otherwise the firm will be indifferent among setting any level of variance and will in any case make zero profits.*

The proof of this Lemma can be done along the same lines as the preceeding proof and is thus omited.

We are now in position to determine the equilibria of the variance game.

Proposition 1 and Lemma 3 and 4 tell us that when $\overline{\sigma}^2 > \frac{2(\overline{q_A}-\overline{q_B})}{R^+-2R^-}$ the equilibrium candidates are $(0,\overline{\sigma}^2)$ and $(\overline{\sigma}^2,0)$. Furthermore, given Lemma 3 and 4 we know that these candidates will in fact be equilibria if the higher expected quality firm (firm A) cannot gain from deviating.

It can be shown that :

$$\Pi_A(0,\overline{\sigma}^2) > \Pi_A(\overline{\sigma}^2,\overline{\sigma}^2) \qquad \forall \overline{\sigma}^2$$

So, $(0,\overline{\sigma}^2)$ is a SPE for all values of $\overline{\sigma}^2$ and is a strict SPE if $\overline{\sigma}^2 > \frac{2(\overline{q_A}-\overline{q_B})}{R^+-2R^-}$.

It can also be shown that :

$$\Pi_A(\overline{\sigma}^2,0) > \Pi_A(0,0) \qquad \text{iff:}$$

$$\overline{\sigma}^2 > \frac{(\overline{q_A}-\overline{q_B})(7R^+-5R^-)+3(\overline{q_A}-\overline{q_B})\sqrt{(R^+-R^-)(5R^+-R^-)}}{(R^+-2R^-)^2} \tag{5.1}$$

So, $(\overline{\sigma}^2,0)$ is a (strict) SPE if and only if condition (5.1) is met[15].

It remains to study the existence of non-strict SPE of the form $(0,\sigma_B^2)$ and $(\overline{\sigma}^2,\sigma_B^2)$ in the range $\overline{\sigma}^2 < \frac{2(\overline{q_A}-\overline{q_B})}{R^+-2R^-}$.

When $\overline{\sigma}^2 < \frac{2(\overline{q_A}-\overline{q_B})}{2R^+-R^-}$ firm A, by choosing either $\sigma_A^2 = 0$ or $\sigma_A^2 = \overline{\sigma}^2$ against any value of $\sigma_B^2$ will always keep the whole market, it is then clear that its profit is higher when its variance is lower than that of the rival's firm. Therefore, when $\overline{\sigma}^2 < \frac{2(\overline{q_A}-\overline{q_B})}{2R^+-R^-}$ there is a continuum of equilibria of the form $(0,\sigma_B^2)$ where $\sigma_B^2$ takes any value in $[0,\overline{\sigma}^2]$.

Whenever $\overline{\sigma}^2 \in \left[\frac{2(\overline{q_A}-\overline{q_B})}{2R^+-R^-}\, \frac{2(\overline{q_A}-\overline{q_B})}{R^+-2R^-}\right]$ we have seen that against $\sigma_A^2 = \overline{\sigma}^2$ firm B will choose $\sigma_B^2 = 0$ (proposition 5), we know though that this is not an equilibrium since condition (5.1) is not met. On the other hand we have also seen that against $\sigma_A^2 = 0$ firm B will choose any value of $\sigma_B^2$ (proposition 4). This leads us to determine what are the values of $\sigma_B^2$ for which $\Pi_A(0,\sigma_B^2) \geqslant \Pi_A(\overline{\sigma}^2,\sigma_B^2)$. Straightforward computations allow us to state that[16] :

$$\Pi_A(0,\sigma_B^2) \geqslant \Pi_A(\overline{\sigma}^2,\sigma_B^2) \qquad \text{iff} \qquad \sigma_B^2 > Z$$

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15 Remark that the right-hand side of (A.1) is bigger than $\frac{2(\overline{q_A}-\overline{q_B})}{R^+-2R^-}$.

16 Z is the minus root of $\Delta\Pi = \Pi_A(0,\sigma_B^2) - \Pi_A(\overline{\sigma}^2,\sigma_B^2)$. This value, which is a function of the values of $\overline{\sigma}^2$, $(\overline{q_A}-\overline{q_B})$, $R^+$ and $R^-$, belongs to the interval $]0,\overline{\sigma}^2 - \frac{2(\overline{q_A}-\overline{q_B})}{2R^+-R^-}]$.

Therefore, for $\overline{\sigma}^2 \in \left[\frac{2(\overline{q_A}-\overline{q_B})}{2R^+-R^-}, \frac{2(\overline{q_A}-\overline{q_B})}{R^+-2R^-}\right]$, there is a continuum of equilibria of the form $(0, \sigma_B^2)$ where $\sigma_B^2$ takes any value in $[0, Z]$. Under any of these variance equilibria the price equilibria is such that firm B charges a zero price and firm A charges $P_A = \overline{q_A} - \overline{q_B} + \frac{R^-}{2}\sigma_B^2$.

Our succesive findings can be summarized as follows:

In the domain of $[R^+, R^-]$ for which $R^+ - 2R^- > 0$ we have that:

1. *$(0, \overline{\sigma}^2)$ is an equilibrium for all values of the parameters.*
2. *For given values of $(\overline{q_A} - \overline{q_B})$, $R^+$ and $R^-$ there is a value of $\overline{\sigma}^2$ above which $(\overline{\sigma}^2, 0)$ is also an equilibrium.*
3. *For given values of $(\overline{q_A} - \overline{q_B})$, $R^+$ and $R^-$ there is a value of $\overline{\sigma}^2$ below which there is a continuum of equilibria where firm A sets a zero variance level and firm B sets any variance level.*
4. *For given values of $(\overline{q_A} - \overline{q_B})$, $R^+$ and $R^-$ there is an interval for $\overline{\sigma}^2$ for which there is a continuum of equilibria where firm A sets a zero variance level and firm B sets any variance level in $[0, Z]$ where $Z < \overline{\sigma}^2$.*

The combination of these four claims is summarized in Proposition 2. □

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