

Agenda Control in Coalition Formation^α

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Abstract

Theoretical models of government formation in political science usually assume that the head of state is non-strategic. In this paper, we analyze the power of an agenda setter who chooses the order in which players are recognized to form coalitions in simple games. We characterize those sets of players which can be imposed in the equilibrium coalition and show that the only decisive structures where the agenda setter can impose the presence of any minimal winning coalition are apex games, where a large player forms a winning coalition with any of the small players.

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1 Introduction

Can a head of state choose the composition of a cabinet by selecting the order in which party leaders attempt to form the government? Can the chairman of a board decide on the composition of an executive committee by choosing the order in which board members speak? It is generally believed that the answer to these questions is at least partially positive: under some circumstances, the head of state or the chairman of the board can use their agenda setting power to influence the coalitions formed as the outcome of bargaining between parties or board members. In the political realm, the head of state's ability to influence the formation of government coalitions varies from country to country. While very few constitutions have specific provisions on the order in which party leaders are recognized to form governments, in some countries, the power of the head of state is limited by conventions, which dictate that the largest party should be asked to form the government first. In other countries, like Finland or Italy, the President of the Republic has used his power to choose the order in which parties attempt to form the government (Strom, Budge and Laver, 1994).

In this paper, we develop a formal model of agenda control to analyze the power of the head of state or the chairman of the board in influencing the coalitions formed. In particular, we relate the power of the agenda setter to the underlying decisive structure among players. Is it easier for a head of state to manipulate the outcome of coalitional bargaining when all parties have equal size, or when parties are asymmetric? Are small or large parties more likely to be imposed in the cabinet? How should the head of state design the order of moves in relation to the parties' weights? We provide an answer to these questions in the general context of cooperative simple games, where coalitions of players are divided between losing and winning coalitions.

More specifically, we suppose that the agenda setter selects an ordering

of the players, and that bargaining occurs according to the following simple sequential procedure. When a player is recognized to make a proposal, she offers to form a coalition with other players. If all coalition members agree, the coalition is formed and the game ends. If at least one of the players rejects the offer, the next player in the sequence is recognized to make an offer. If at the end of the sequence, no coalition has been formed, all players obtain their *status quo* outcome.

Our main result shows that the only structure in which the agenda setter can fully manipulate the outcome of coalitional bargaining is the apex structure, with one large player and $n - 1$ small players, where the large player only needs one of the small players to form a winning coalition. In all other structures, the power of the agenda setter is limited: he may be able to force the presence of some of the players in the coalition formed, but cannot manipulate the order to guarantee that any possible minimal winning coalition is formed. If all players are symmetric, the agenda setter's power is even more limited: he can only choose one player to be included in the coalition, and one player to be excluded from the coalition.

Furthermore, for any simple game, we completely characterize the sets of players who can be included in the coalition formed by the agenda setter. We also show that when players can be ordered by desirability, as in parliaments, where parties are more desirable if they have more seats, the agenda setter should choose the least desirable coalition member to be the *first mover*, and the most desirable member outside the coalition to be the *second mover*. In the context of cabinet formation, this implies that the head of state should recognize *first* the smallest party he wants to include in the coalition, and let the largest party he wants to exclude be the *second mover*.

In the political science literature, the agenda setting role of the head of state is usually left unmodeled. For example, in their recent wide ranging analysis of cabinet formation based on policy portfolios, Laver and Shepsle

(1994, pp. 53-54) indicate that "It is at well to be quite clear that [they] assume that the recognition of the actors who can propose new governments is non-strategic. This implies for example that the head of state is not a strategic actor in the government formation process". Similarly, Baron (1991, p. 137) notes that, in his noncooperative model of government formation, "the political bargaining begins with a non-strategic president or monarch who selects a party to be accorded the opportunity to attempt to form a government". Hence, our paper contributes to the noncooperative theory of coalition formation in political science by endogenizing the agenda setting role of the head of state.

Once the head of state has chosen the ordering of proposers, the model of coalitional bargaining we adopt is directly inspired by the noncooperative models of coalition formation initiated by Baron and Ferejdn (1989) and Austen-Smith and Banks (1988). As in Baron and Ferejdn (1989), we assume a purely redistributive framework, and do not incorporate an ideological dimension in party preferences. (See also Baron (1991 and 1993) and Jackson and Moselle (1998) for models of coalition formation where parties have preferences over policy outcomes.) In all these models, the order in which players are recognized is exogenously fixed. For example, Baron and Ferejdn (1989) assume that the probability of being recognized is proportional to the number of seats whereas Austen-Smith and Banks (1988) suppose that parties are given the opportunity to form a government in the order given by their number of seats. In recent empirical work, Merlo (1997) and Diemer and Merlo (1999) investigate the empirical validity of different bargaining procedures. In particular, Diemer and Merlo (1999) note that the data do not support the procedure described by Austen-Smith and Banks (1988), but give some validity to the proportional procedure of Baron and Ferejdn (1989). They also note the incumbent is more likely to be selected to attempt to form the new government.

In the game theoretic literature on coalitional bargaining Chatterjee et al. (1993), Bloch (1996) and Ray and Vohra (1999) have analyzed sequential models where the order of proposers and responders is fixed by an exogenous protocol. These studies emphasize the role played by the protocol in the determination of the equilibrium coalitions and the division of coalitional payoffs. We believe that our work also sheds light on these models of non-cooperative coalitional bargaining by analyzing (in the restricted context of simple games) the relation between the protocol and the coalitions formed in equilibrium.

Finally, our model bears a more distant relationship to the study of agenda control in binary voting surveyed in Rosenthal (1990) and Miller (1995). The description of the role of the agenda setter in those models is quite different from ours. In binary voting the agenda setter has the ability to choose the proposal to be pitted against the status quo (as in the seminal work of Riker and Rosenthal (1978)) or the complete sequence of binary votes (as in the tournament context of Miller (1980) and Banks (1985)). Once a pair of alternatives has been chosen, agents vote between the two alternatives. As in our work, the main objective of the literature is to characterize the set of outcomes which can be obtained for any agenda and study the strategic role of the agenda setter. However, differences in the models preclude a direct comparison between the findings of the literature on binary voting and our results.

The rest of the paper is organized as follows. In Section 2, we discuss fully the power of the head of state in the formation of a cabinet with n parties or less. In Section 3, we introduce our general framework and derive, for any fixed ordering of proposers, the subgame perfect equilibrium of the noncooperative model of coalition formation. In Section 4, we analyze the power of the agenda setter and prove our main theorem, characterizing the decisive structures under which the agenda setter can fully manipulate the

outcome of the procedure of coalition formation. The last Section contains our conclusions, and a discussion of the limitations of our analysis.

2 Cabinet Formation with 3, 4 and 5 Parties

To illustrate the problem of the agenda setter in coalition formation, we analyze in this section the role of the head of state in the formation of a cabinet with three, four and ...ve parties. At the beginning of the game, the head of state chooses the sequence in which the different party leaders attempt to form a government.¹ We suppose that each party leader has an opportunity to form the government, and that no party can be called twice in the sequence.² When a party leader is recognized to form a government, she makes an offer to the other parties to join in a government. If all prospective members accept the offer, the government is formed and the game ends. If at least one of the coalition members rejects the offer, the game continues, and the next party in the sequence is recognized to form the government. If all offers have been rejected at the end of the sequence, all parties obtain their status quo outcome.

We suppose, as in Baron and Ferejohn (1989), that parties' preferences only depend on their share in the government benefits, and not on the policy

¹We assume that the head of state commits to the entire sequence of moves at the beginning of the game. Alternatively, one could consider that the head of state only chooses the identity of the next proposer after an attempt to form the government has failed. To model this situation, the head of state has to be treated as a player in the sequential game, who can condition his choice on the past history. This would lead to a different model, which is more complex to analyze, and does not allow for a simple description of the relation between the order of moves and the outcome of bargaining.

²As long as the sequence of moves is finite, the assumption that no party moves twice in the game is made without loss of generality. (See the discussion in the last Section of the paper.)

chosen by the government.³ Hence, a proposal to form the government only entails a division of the government benefits among the parties. To simplify the analysis, we assume that the government benefits are equal to one for all the possible coalitional governments, and that the status quo outcome is identically equal to zero for all the parties.

The only source of heterogeneity among parties in our model thus stems from the distribution of seats in parliament. Letting w_i denote the number of seats of party i in parliament, a coalitional government C can be formed if and only if $\sum_{i \in C} w_i \geq \frac{P}{2}$.⁴ Disregarding the uninteresting case where one party obtains a majority of seats in parliament, or when some parties are "dummy players", who do not play a significant role in parliament, we can easily characterize the decisive structures with three, four and ...ve players. Von Neumann and Morgenstern (1953, pp. 457-462) were the first to note that, while there are of course a large number of possible distributions of seats in parliament, there are only a few prototypical decisive structures which represent the distribution of power among the parties. To characterize these decisive structures, we recall the definition of a weighted quota game. A weighted quota game is defined by a list $[q; w_1; w_2; \dots; w_n]$ where q denotes the quota, w_i the weight of player i and a coalition C is winning if and only if $\sum_{i \in C} w_i \geq q$. Von Neumann and Morgenstern (1953, pp. 457-462) show that there are only six weighted quota games representing the decisive structures for less than ...ve players.

For $n = 3$, the structure $[2; 1; 1; 1]$

For $n = 4$, the structure $[3; 2; 1; 1; 1]$

For $n = 5$, the structures $[3; 1; 1; 1; 1; 1]; [4; 2; 2; 1; 1; 1]; [4; 3; 1; 1; 1; 1];$

$[5; 3; 2; 2; 1; 1];$

³We discuss in the conclusion the extension of our model to a situation where parties' preferences depend on policy outcomes.

With three players, the only decisive structure is the simple majority case, where two players out of three are needed to form the government. The structures $[3; 2; 1; 1; 1]$ and $[4; 3; 1; 1; 1; 1]$ are examples of apex games where there exists a single large player, who can form a winning coalition with any of the small players. The structure $[3; 1; 1; 1; 1; 1]$ is a symmetric majority game, where all players are identical and a majority is formed by more than a half of the players. The other two structures, $[4; 2; 2; 1; 1; 1]$ and $[5; 3; 2; 2; 1; 1]$ distinguish between large, middle and small players, and represent more complex distributions of power among the parties.

As a first step in the analysis, we need to determine, for every possible ordering of the parties, the coalitions formed in equilibrium. We will illustrate the basic principles with the case of three parties. Clearly, the last party in the sequence can extract the entire surplus by paying the other parties their status quo outcome of 0. The second party in the sequence thus should never propose to form a coalition with the last party, but should instead propose to form a coalition with the first party, paying the first party its continuation payoff of 0, and keeping a surplus of 1. Hence, at the beginning of the game, the first party never proposes to form a coalition with the second party, but forms instead a coalition with the last party.⁴

The same reasoning can be applied to compute the coalitions formed in equilibrium with four parties. Reorder the parties, so that party 1 is the large party and 2, 3 and 4 are the small parties. Barring symmetries between the small parties, there are four distinct orderings to consider. Consider first the ordering 1; 2; 3; 4. At the last stage of the game, party 4 obtains the entire surplus. Hence party 3 should form a coalition excluding the fourth party,

⁴This result has also been obtained by Austen-Smith and Banks (1988) when parties' preferences are defined over government benefits and policy outcomes. In their model, the coalition is formed by the largest party (the first player in the game) and the smallest party (the last player in the game).

namely coalition $f1;3g$ or coalition $f1;2;3g$. Party 2 then should choose to form a coalition with party 1 or parties 1 and 4. At the beginning of the game, party 1 thus either forms the coalition $f1;3g$ or $f1;4g$ or $f1;3;4g$. Next consider the ordering $2;1;3;4$. Party 4 again obtains the entire surplus, so that party 3 should form either the coalition $f1;3g$ or $f1;2;3g$. Party 1 thus selects to form coalition $f1;2g$, coalition $f1;4g$ or coalition $f1;2;4g$. Party 2 then forms a coalition excluding party 1, namely the coalition of small parties $f2;3;4g$. If now the ordering is $2;3;1;4$, at the last stage of the game, party 4 still captures the entire surplus. Hence party 1 forms any of the three coalitions $f1;2g$, $f1;3g$ or $f1;2;3g$. Party 3 then chooses to form the coalition $f2;3;4g$, and party 2 forms a coalition excluding party 3, namely any of the coalitions $f1;2g$ or $f1;2;4g$. Finally, if the ordering is $2;3;4;1$, party 1 captures the entire surplus at the last stage of the game. Hence party 4 forms $f2;3;4g$. Party 3 then forms either $f1;3g$ or $f1;2;3g$ and party 2 forms either $f1;2g$ or $f1;2;4g$.

The analysis of the coalitions formed at equilibrium for $n = 3$ and $n = 4$ shows that at each stage of the game, the party which is recognized can capture the entire surplus, by forming a winning coalition which excludes the next party in the sequence. Hence, in equilibrium, the ...rst party forms any winning coalition excluding the second party. This result shows that the only relevant part of the sequence of moves chosen by the head of state are the identities of the ...rst and second parties. Hence, when computing the equilibrium coalition, we can restrict our attention to the identities of the ...rst two parties in the sequence.

We now consider the coalitions formed with 5 parties in the different decisive structures. In the symmetric structure $[3;1;1;1;1]$, all orderings are equivalent. If parties 1 and 2 are the ...rst two parties, the equilibrium minimal winning coalitions are $f1;3;4g$, $f1;3;5g$ and $f1;4;5g$ and $f1;3;4;5g$. Consider now the structure with two large parties and three small parties,

[4; 2; 2; 1; 1; 1]. Let 1 and 2 denote the large parties and 3, 4 and 5 the small parties. Ignoring symmetries, there are four orderings of the...rst two parties to consider: either the large parties are placed on top, or one large and one small party, or one small and one large party, or two small parties. The following table summarizes the coalitions formed in the four cases.

order	equilibrium coalition
12	134; 135; 145; 1345
13	12; 145; 124; 125; 1245
31	234; 235; 2345
34	123; 135; 235; 1235

In the apex game, with party 1 denoting the large party, there are three orderings of the...rst two parties to consider: either party 1 is...rst, or a small party is...rst and party 1 second, or the...rst two parties are small parties.

order	equilibrium coalition
12	13; 14; 15; 134; 135; 1345
21	2345
23	12; 124; 125; 1245

Finally, in the structure [5; 3; 2; 2; 1; 1], we let 1 denote the large party, 2 and 3 the middle parties and 4 and 5 the small parties. There are now 8 different orders of the...rst two parties to consider.

order	equilibrium coalition
12	13; 145; 1345
14	12; 13; 125; 135
21	234; 235; 2345
23	12; 124; 125; 1245
24	12; 123; 125; 1235
41	234; 2345
42	145; 134; 1345
45	124; 134; 1234; 234

These tables can now be used to analyze the power of the agenda setter in the different decisive structures. It is obvious that the head of state can always choose one party to be included in the coalition (the...rst in the order) and one party to be excluded in the coalition (the second in the order). Can the head of state impose the presence of other parties in the government? The answer to this question depends crucially on the decisive structure of the game. For example, consider the symmetric structure with ...ve parties, $[3; 1; 1; 1; 1]$. In that symmetric majority game, the head of state can never guarantee the presence of more than one party in the government. If party 1 is recognized ...rst, she may form a coalition with either parties 3 and 4, 4 and 5 or 3 and 5. There is no way to force the presence of either of those three parties in the government. By contrast, consider the structure with one large party and four small parties, $[4; 3; 1; 1; 1]$. This game has ...ve minimal winning coalitions: $f1; 2g$; $f1; 3g$; $f1; 4g$; $f1; 5g$ and $f2; 3; 4; 5g$. We claim that the head of state can guarantee the presence of any minimal winning coalition in the government. In order to guarantee a coalition with two parties, $f1; 2g$, the head of state can choose any order placing 2 ...rst and 1 in third, fourth or ...fth position. In order to guarantee the formation of

the coalition of small parties, the head of state should place the large party second. In the structure with two large parties and three small parties, the head of state can guarantee the presence of one small party and one large party in the coalition, say parties 2 and 3, by placing party 3 ...rst and party 1 second, but cannot guarantee the presence of any other group of parties in the equilibrium coalition. In the structure with one large party, two middle parties and two small parties, $[5; 3; 2; 2; 1; 1]$, the head of state can guarantee the presence of one large and one middle party, say parties 1 and 2, by placing party 2 ...rst and party 3 second. He can also impose the presence of the minimal winning coalition consisting of the two middle parties 2 and 3 and one of the small parties, player 4, by placing party 4 ...rst and party 1 second. However, the head of state cannot force the presence of the minimal winning coalition $\{1; 4; 5\}$ in the government: there exists no ordering of the parties which guarantees that $\{1; 4; 5\}$ always belongs to the equilibrium coalition.

The study of government formation with ...ve parties or less thus shows that the agenda setter's ability to impose the presence of a player in the equilibrium coalition is related to the underlying decisive structure of the game. In the next sections of the paper, we consider a general model of coalition formation in simple games, and investigate fully the relation between the power of the agenda setter and the underlying simple games.

3 Coalition Formation in Simple Games

We consider a TU cooperative game $(N; v)$ where N is the set of players and v a coalitional function, assigning to each nonempty subset C of N , a real number $v(C) \geq 0$. Since our aim is to study the power of the agenda setter in the determination of the coalition formed, rather than the division

of payoffs among coalition members, we focus in this paper on simple games. Recall that a cooperative game is simple if and only if $v(C) \in \{0, 1\}$. A coalition C is called winning if $v(C) = 1$ and losing if $v(C) = 0$.

We first recall some well known properties of simple games (see Weber (1994), p.1291). A simple game is monotone if $v(C) = 1 \Rightarrow C \subseteq D \Rightarrow v(D) = 1$. For any monotone simple game, we can define minimal winning coalitions as the smallest winning coalitions of the game. A simple game is proper if $v(C) = 1 \Rightarrow v(\bar{C}) = 0$, where \bar{C} denotes the complement of the set C . A simple game is strong if $v(C) = 0 \Rightarrow v(\bar{C}) = 1$. A simple game is decisive if it is proper and strong. Player i is called a veto player if she belongs to all winning coalitions. Player i is more desirable than player j if and only if, for any coalition C excluding i and j , $v(C \cup \{j\}) = 1 \Rightarrow v(C \cup \{i\}) = 1$. Finally, a player i is a dummy player if she does not belong to any minimal winning coalition.

The procedure of coalition formation that we consider is a simple extensive form noncooperative game inspired by recent models of coalitional bargaining (Chatterjee et al. (1993), Bloch (1996), Ray and Vohra (1999)). Players are ordered according to a fixed protocol π . Player i is called the π - k -th player if $\pi(i) = k$. The π -first player starts the game by proposing a coalition C to which she belongs and a division of the coalitional payoffs among the members of C , namely a vector of payoffs $y_{i \in C}$ such that $y_i \geq 0$ for all $i \in C$ and $\sum_{i \in C} y_i = 1$. All the members of C respond to the offer in the ordering specified by π . If all members accept the offer, the coalition is formed, and players obtain payoffs $x_i = y_i$ if $i \in C$, $x_i = 0$ if $i \notin C$. If at least one of the members rejects the offer, the π -second player makes a proposal. If the proposal of the second player is rejected, the third player makes a proposal. The game continues in that way until the π -last player makes a proposal. If her proposal is in turn rejected, all players obtain their status quo payoff,

$x_i = a_i$ with $a_i \geq 0$ and $\sum_{i=1}^n a_i < 1$.⁵ Since the noncooperative game depends on the underlying simple game v and the protocol π , we denote it by $G(v; \pi)$:

While this game of coalition formation embodies strong assumptions on the bargaining procedure, we have adopted it because it is the simplest and most transparent game where the protocol matters. As the game is finite, and the ordering of moves is deterministic, the payoffs of players are completely determined by the protocol. It is this feature of the game that we emphasize here, because we focus on the relationship between the protocol and the coalitions formed in equilibrium.

A subgame perfect equilibrium of the game of coalition formation is a sequence of proposals and responses such that, at any point in the game, all players act optimally. An equilibrium coalition is a coalition formed at a subgame perfect equilibrium of the game. We are now ready to characterize the equilibrium coalitions for any protocol π and any simple game v without veto players.

Proposition 1 Suppose that the game v is monotone and does not have any veto player. Then any coalition containing the π -first player and excluding the π -second player is an equilibrium coalition of $G(v; \pi)$. Furthermore, the π -first player captures the entire coalitional surplus.

Proof. Let $k = 1; 2; \dots; n$ denote the π -kth player. For any subgame perfect equilibrium, we denote by C^k the coalition proposed by the k th player in equilibrium. We will show by induction that, for any subgame perfect

⁵In the political interpretation, the status quo payoff represents the payoff of the parties in a caretaking government, or when new elections are called. The specification of a vector of status quo payment also allows us to extend the analysis to an infinite horizon game where the infinite sequence of offers is repeated. The status quo payoffs can then be interpreted as the continuation payoffs of the players in a stationary perfect equilibrium of the game, and the analysis of the game remains unchanged.

equilibrium of the game, for any $k = 1, 2, \dots, n-1$, the coalition formed by the k th player excludes player $k+1$, i.e. $(k+1) \notin C^k$.

We first define the continuation payoff that player i obtains after the offer of player k is rejected, u_i^k . If the offer of player n is rejected, all players obtain their status quo payoff, so $u_i^n = a_i$. Consider now the second to last player, player $n-1$ and let S and T be two winning coalitions containing player $n-1$ with $n \notin S$ and $n \notin T$. (It is always possible to find a winning coalition containing $n-1$ and excluding n since N itself is winning and the game is monotone.) The minimal amount that player $n-1$ must give to his coalition partners in coalitions S and T is given by $\sum_{i \in S, i \neq n-1} u_i^{n-1}$ and $\sum_{i \in T, i \neq n-1} u_i^{n-1}$ respectively. Now clearly, $u_i^{n-1} = 0$ if $i \notin C^{n-1}$; $u_i^{n-1} = a_i$ if $i \in C^{n-1}$; $i \neq n$ and $u_n^{n-1} = 1 - \sum_{i \in C^{n-1}, i \neq n} a_i$. We thus compute

$$u_i^{n-1} = \begin{cases} 0 & \text{if } i \notin C^{n-1} \\ a_i & \text{if } i \in C^{n-1}, i \neq n \\ 1 - \sum_{i \in C^{n-1}, i \neq n} a_i & \text{if } i = n \end{cases}$$

Next observe that since T is a winning coalition containing player n , player n could have chosen to form T . Hence

$$\sum_{i \in T, i \neq n} a_i \leq \sum_{i \in C^n, i \neq n} a_i$$

But

$$a_i = \sum_{i \in T, i \neq n} a_i + \sum_{i \in T \cap C^n, i \neq n-1} a_i + \sum_{i \in T \setminus C^n, i \neq n-1} a_i + a_{n-1}$$

So

$$u_i^{n-1} \leq 1 - \sum_{i \in T \cap C^n, i \neq n-1} a_i - a_{n-1}$$

Now

$$\sum_{i \in T \cap C^n, i \neq n-1} a_i + a_{n-1} + \sum_{i \in S \setminus C^n, i \neq n-1} a_i + \sum_{i \in N} a_i < 1$$

so we finally obtain

$$\bigvee_{i \in T; i \neq n-1} u_i^{t+1} > \bigvee_{i \in S; i \neq n-1} u_i^{t+1};$$

showing that, in any subgame perfect equilibrium of the game, player $n-1$ forms a coalition which excludes player n .

Consider now the general step, and suppose that for all $t > k$, in all subgame perfect equilibria of the game, $t+1 \in C^t$. Consider again two winning coalitions containing player k , S and T with $k+1 \in S$ and $k+1 \in T$. The minimal amount that player k must give to his coalition partners in coalitions S and T is given by $\bigvee_{i \in S; i \neq k} u_i^t$ and $\bigvee_{i \in T; i \neq k} u_i^t$ respectively. Notice that, for any $t \geq k$, the continuation payoffs of the players are given by the recursive expressions: $u_i^t = 0$ if $i \in C^{t-1}$, $u_i^t = u_i^{t-1}$ if $i \in C^{t-1}$ and $i \neq t+1$ and $u_{t+1}^t = 1 - \bigvee_{i \in C^{t-1}; i \neq t+1} u_i^{t-1}$. Hence we obtain

$$\begin{aligned} \bigvee_{i \in S; i \neq k} u_i^k &= \bigvee_{i \in S \setminus C^{k+1}; i \neq k} u_i^{k+1} \\ \bigvee_{i \in T; i \neq k} u_i^k &= 1 - \bigvee_{i \in C^{k+1}; i \neq k+1} u_i^{k+1} + \bigvee_{i \in T \setminus C^{k+1}; i \neq k; i \neq k+1} u_i^{k+1} \end{aligned}$$

Since player $k+1$ could have chosen to form the coalition T ,

$$\bigvee_{i \in T; i \neq k+1} u_i^{k+1} \geq \bigvee_{i \in C^{k+1}; i \neq k+1} u_i^{k+1};$$

It now follows that

$$\bigvee_{i \in T; i \neq k+1} u_i^{k+1} = \bigvee_{i \in T \cap C^{k+1}; i \neq k; i \neq k+1} u_i^{k+1} + \bigvee_{i \in T \setminus C^{k+1}; i \neq k; i \neq k+1} u_i^{k+1} + u_k^{k+1}$$

Hence

$$\bigvee_{i \in T; i \neq k} u_i^k \geq 1 - \bigvee_{i \in T \cap C^{k+1}; i \neq k; i \neq k+1} u_i^{k+1} + \bigvee_{i \in T \setminus C^{k+1}; i \neq k; i \neq k+1} u_i^{k+1} + u_k^{k+1}$$

and

$$\sum_{i \in T; i \neq k} x_i^k + \sum_{i \in S; i \neq k} x_i^{k+1} + \sum_{i \in T \cap C^{k+1}; i \neq k; i \neq k+1} x_i^{k+1} + \sum_{i \in S \setminus C^{k+1}; i \neq k} x_i^{k+1}.$$

To finish the proof, two cases must be distinguished. If $k+2 \notin T$, then
 $\sum_{i \in T \cap C^{k+1}; i \neq k; i \neq k+1} x_i^{k+1} = \sum_{i \in T \cap C^{k+1}; i \neq k; i \neq k+1} a_i + x_k^{k+1} = a_k$ and $\sum_{i \in S \setminus C^{k+1}; i \neq k} x_i^{k+1} = \sum_{i \in S \setminus C^{k+1}; i \neq k} a_i$. Hence

$$\sum_{i \in T; i \neq k} x_i^k + \sum_{i \in S; i \neq k} x_i^{k+1} + \sum_{i \in T \cap C^{k+1}; i \neq k; i \neq k+1} x_i^{k+1} + \sum_{i \in S \setminus C^{k+1}; i \neq k} x_i^{k+1} = \sum_{i \in N} a_i > 0.$$

If $k+2 \in T$, player $k+1$ must strictly prefer the coalition C^{k+1} to the coalition T . Hence

$$\sum_{i \in T; i \neq k+1} x_i^{k+1} > \sum_{i \in C^{k+1}; i \neq k+1} x_i^{k+1}$$

and

$$\sum_{i \in T; i \neq k} x_i^k + \sum_{i \in S; i \neq k} x_i^{k+1} + \sum_{i \in T \cap C^{k+1}; i \neq k; i \neq k+1} x_i^{k+1} + \sum_{i \in S \setminus C^{k+1}; i \neq k} x_i^{k+1}.$$

Now

$$\begin{aligned} & \sum_{i \in T \cap C^{k+1}; i \neq k; i \neq k+1} x_i^{k+1} + \sum_{i \in S \setminus C^{k+1}; i \neq k} x_i^{k+1} \\ = & \sum_{i \in T \cap C^{k+1}; i \neq k; i \neq k+1; k+2} x_i^{k+1} + (1 - x_{k+2}^{k+1}) \sum_{i \in C^{k+2}; i \neq k+2} x_i^{k+1} + \sum_{i \in S \setminus C^{k+1}; i \neq k} x_i^{k+1} \\ = & \sum_{i \in C^{k+2}; i \neq k+2} x_i^{k+2} + \sum_{i \in T \cap C^{k+1}; i \neq k; i \neq k+1; k+2} x_i^{k+1} + \sum_{i \in S \setminus C^{k+1}; i \neq k} x_i^{k+1} \end{aligned}$$

Now, since $k+2 \geq C^{k+1}$ and $u_i^{k+2} \leq u_i^{k+1}$ for all $i \in k+2$,

$$\sum_{i \in C^{k+2}; i \notin k+2} u_i^{k+2} \leq \left(\sum_{i \in T \cap C^{k+1}; i \notin k; i \notin k+1; k+2} u_i^{k+1} + u_k^{k+1} + \sum_{i \in S \cap C^{k+1}; i \notin k} u_i^{k+1} \right) \leq 0$$

so that

$$\sum_{i \in T; i \notin k} u_i^k - \sum_{i \in S; i \notin k} u_i^k > 0$$

establishing the induction result. To complete the proof, note that player 1 always forms a coalition excluding player 2 and that for all $k \geq 3$, $u_k^1 = 0$. So player 1 is indifferent among all the winning coalitions which exclude player 2 and is able to capture the entire surplus. ■

Proposition 1 completely characterizes the subgame perfect equilibria of the procedure of coalition formation, when the simple game does not have any veto player. All subgame perfect equilibria induce the same distribution of payoffs, with the first player capturing the entire surplus.⁶ The logic underlying the result is that in equilibrium, at any stage of the game, a player offers a coalition which excludes her immediate successor. Running the argument backwards, it appears that at the beginning of the game, player 1 should always exclude player 2. Furthermore, since each player $k \geq 3$ has been excluded from the coalition formed by its immediate predecessor, her continuation payoff in the game is zero and hence the first player captures the entire coalitional surplus.

⁶ Notice that, in any subgame perfect equilibrium of the game, all the players except the first one play weakly dominated strategies. They accept an offer of 0 immediately, precluding the possibility of obtaining a positive payoff at later stages in the game. While these strategies may seem unnatural, they are the only strategies consistent with subgame perfect equilibrium play.

The sequential structure of the game thus yields a very unequal distribution of the surplus among the players. In order to mitigate this problem, we could have assumed that every player has an outside option π_i and must obtain at least her outside option to participate in the government. For a vector of π_i 's small enough, the thrust of the analysis remains unchanged, and in equilibrium, all players must exclude their immediate successor in the coalition they form. With a vector of outside options, the equilibrium coalition formed by the first player becomes determinate: it is the winning coalition excluding player 2 and containing the players with the smallest outside option. Notice in particular, that in this slightly perturbed variant of the game, the equilibrium coalition is always a minimal winning coalition.

The main difficulty in the proof of Proposition 1 stems from the existence of an arbitrary status quo payoff for the players. However, it is striking to note that the equilibrium distribution of the surplus is independent of the status quo and only depends on the ordering of the players. This result is obtained because each player has an opportunity to make an offer, and hence each player is excluded from a coalition at least once in the game, so that her continuation payoff is equal to zero and independent of the status quo. If the protocol did not let all players make an offer, this result would not hold. Some players may never be excluded from the coalition formed, and thus obtain at least their status quo payoff.

The equilibrium characterization of Proposition 1 depends crucially on the assumption that the game v does not have any veto player, so that any player can form a winning coalition excluding her immediate successor. If the simple game v admits veto players, the subgame perfect equilibria of the game are not easily characterized and depend on the status quo payoffs. It appears that since veto players belong to winning coalitions, they must earn at least their status quo payoff in equilibrium. The last veto player in the game can secure for herself a large share of the surplus (one minus

the minimum sum of status quo payoffs of the other players in a winning coalition), and other veto players can similarly obtain a share of the surplus larger than their status quo payoff. Depending on the order of veto and nonveto players and the structure of the game, some nonveto players can also obtain a positive share of the surplus. Overall, the equilibrium coalitions and equilibrium payoffs depend in a complex way on the ordering of players and the status quo outcomes.

The only case where the equilibrium of a game with veto players can easily be characterized is when all status quo payoffs are equal to zero. In that case, the last veto player captures the entire surplus. Let k denote that last veto player. It is easy to see that in equilibrium, the continuation payoffs obtained by the players after the offer of player k has been rejected are 0 for all $i \in k+1$ and 1 for player $k+1$. Since however, by assumption, player $k+1$ is not a veto player, player k can form a winning coalition excluding player $k+1$, and propose 0 to all coalition members. Hence, when it is player k 's turn to make an offer, she can secure for herself a payment of 1. Next, since player k is a veto player, she must belong to any winning coalition, and hence any player j making offers before player k must offer her continuation payoff of 1.

The fact that the last veto player obtains the entire surplus is hardly surprising given the sequential structure of the procedure we analyze. When there are veto players, our game can be construed as a simple extension of an ultimatum game (when all players are veto players, our game is exactly an ultimatum game), and the last player can thus obtain the entire surplus.

4 Agenda Control and Coalitions

In this section, we analyze the power of an agenda setter who chooses the order in which players make offers in the noncooperative game of coalition

formation. We focus here on games without veto players. By Proposition 1, the only relevant part of the protocol π is the choice of the first two players. Proposition 1 also clearly shows that an agenda setter can always impose the presence of one of the players (player 1) and exclude another player (player 2) from the equilibrium coalition. We are thus interested in situations where the agenda setter wants to impose the presence of more than one player in the equilibrium coalition.

Definition 2 The agenda setter can impose the presence of a set S of players in the game v if there exists an ordering π such that the set S belongs to all equilibrium coalitions of the game $G(v; \pi)$.

Using the characterization result of Proposition 1, we can prove the following Proposition.

Proposition 3 Let v be a monotone game without veto players. The agenda setter can impose the presence of a set S of players in the game v if and only if there exists a player $i \in S$ and a player $j \notin S$ such that for all $k \in S, k \neq i$, $v(N \setminus \{j, k\}) = 0$. To impose the presence of the set S , the agenda setter chooses $\pi(i) = 1; \pi(j) = 2$:

Proof. Suppose that the condition is satisfied, and consider an ordering where i is placed first and j is placed second. By Proposition 1, any winning coalition containing i and excluding j is an equilibrium coalition. Now if there exists $k \in S, k \neq i$ such that k does not belong to an equilibrium coalition, there must exist a winning coalition excluding j and k , contradicting the assumption.

Suppose now that the condition is violated, and consider any ordering of the players. Let i and j be the first and second players in that ordering. If

$j \notin S$, by Proposition 1, j does not belong to the equilibrium coalition, and hence the agenda setter cannot impose the presence of S . If now $j \in S$, since the condition is violated, there exists $k \in S; k \neq i$ such that $v(N \setminus \{j\}; k) = 1$. Hence, by Proposition 1, $N \setminus \{j\}; k$ is an equilibrium coalition, and the agenda setter cannot impose the presence of S . ■

Proposition 3 provides a simple criterion to check whether a coalition can be imposed by the agenda setter. In the definition of the criterion, two players play a particular role: a member of the coalition i who is placed first by the agenda setter, and a player outside the coalition, player j , who is placed second in the order. It is easy to see that, if players can be ordered by the desirability relation, in order to meet the condition, player i should be selected as the least desirable member of the coalition, and player j as the most desirable player outside the coalition. Formally,

Proposition 4 Let v be a monotone game without veto players. If the agenda setter can impose the presence of a set S of players by choosing $\pi(i) = 1$ and $\pi(j) = 2$, and if $i^0 \in S$ is less desirable than i and $j^0 \notin S$ is more desirable than j , then the agenda setter can impose the presence of the set S by choosing $\pi(i^0) = 1$ and $\pi(j^0) = 2$:

Proof. Since the agenda setter can impose the presence of S by placing i first and j second, $\forall k \in S; k \neq i; v(N \setminus \{j\}; k) = 0$. To prove the proposition, we simply need to show that $\forall k \in S; k \neq i^0; v(N \setminus \{j^0\}; k) = 0$. For all $k \in S; k \neq i; v(\pi^0 [N \setminus \{j\}; j^0; k]) = 0$. Since j^0 is more desirable than j , $v(\pi^0 [N \setminus \{j\}; j^0; k]) = v(N \setminus \{j^0\}; k) = 0$. In particular, $v(N \setminus \{j^0\}; i^0) = v(\pi^0 [N \setminus \{i\}; i^0; j^0]) = 0$. Since i is more desirable than i^0 , $v(\pi^0 [N \setminus \{i\}; i^0; j^0]) = v(N \setminus \{i\}; j^0) = 0$, completing the proof of the Proposition. ■

While the definition of the criterion of Proposition 3 and of the desirability relation may be difficult to interpret in general simple games, they are more

transparent in weighted quota games. Recall that in a weighted quota game, player i is more desirable than player j if and only if $w_i \geq w_j$. We can now combine Proposition 3 and Proposition 4 to characterize the sets of players who can be imposed by an agenda setter in weighted quota games.

Corollary 5 Let v be a weighted quota game. For any coalition C , let a_C and b_C be the smallest and largest players in C . The agenda setter can impose the presence of a set S of players if and only if $\exists i \in S, i \notin a_S; w_i + w_{b_S} > 1 - q$.

To illustrate Corollary 5, consider the present (as of December 1999) composition of the Dutch parliament

party	seats
Social democrats (PvdA)	45
Liberals (VVD)	38
Christian Democrats (CDA)	29
Democracy 66 (D66)	14
Green Left	11
Socialist Party	5
Others	10

Since the Parliament contains 150 seats and a simple majority is needed to form the government, the quota is given by $q = 76$. From Corollary 5, it appears that the head of state can impose the presence of the Social Democrats with any other party but the Liberals, and the presence of the Liberals with any other party but the Social Democrats. These are the only groups of parties that the head of state can impose in the equilibrium coalition.

In the preceding analysis, we have characterized the set of players that the agenda setter can impose in an equilibrium coalition. Irrespective of

the underlying game, the agenda setter can always impose the presence of one agent (the ...rst) in the winning coalition. Proposition 3 shows that, under some restrictions on the underlying simple game, the agenda setter can impose the presence of larger sets of players in the equilibrium coalition. Can the agenda setter impose the presence of all members of a winning coalition? Clearly, if a nonminimal winning coalition is an equilibrium coalition of the game, there exists a minimal winning coalition which is also obtained in equilibrium. Hence, the agenda setter can never impose the presence of all players in a winning coalition which is not minimal. We now characterize the simple games for which the agenda setter can impose the presence of any minimal winning coalition in the equilibrium coalition.

To this end, we restrict our attention to decisive games (games which are both proper and strong) and we exclude dummy players. A decisive game v is called an apex game if there exists a player i such that $v(\{i\}; j) = 1$ for all j in N . We now state our main Theorem.

Theorem 6 Let v be a decisive simple game without veto players nor dummy players. The agenda setter can impose the presence of any minimal winning coalition in the equilibrium coalition if and only if v is an apex game.

Proof. We ...rst show that, if v is an apex game, the agenda setter can impose the presence of any minimal winning coalition. Let i be the large player. Consider ...rst a two player coalition $S = \{i\}; j$. For any $k \notin S$, $v(N \setminus \{i\}; k) = 0$. Now consider the coalition of all the small players, $S = N \setminus \{i\}$. For any $k \notin S$, $v(N \setminus \{i\}; k) = 0$. Hence the condition of Proposition 3 is satisfied for any minimal winning coalition S .

To prove the reverse implication, consider a game v for which all minimal winning coalitions can be imposed. We will show that the game v is an apex game. If $|N| = 3$, the result is immediate. So suppose that $|N| \geq 4$. Since

the game is proper, there must exist a minimal winning coalition S with $|S| \geq 3$. As the head of state can impose the presence of all the players of S in the equilibrium coalition, there exist two players a and b in S and a player $z \in S$ for whom $v(fa; zg) = v(fb; zg) = 1$:

Now define $Z = \{i \in N; i \neq z; v(fi; zg) = 1\}$. The set Z is the set of all players who are winning with z , and by our previous argument, $|Z| \geq 2$. Suppose that there exists a player $i \in N; i \neq z$ such that $i \notin Z$. Since i is not a dummy player, there exists a minimal winning coalition S_i to which she belongs. Now observe that, since the game is proper, for all $a \in Z; v(N \setminus fa; zg) = 0$. This implies that the winning coalition S_i must either contain z or all the players in Z , i.e. either $z \in S_i$ or $Z \subseteq S_i$. We now show that $v(Z) = 0$. Suppose to the contrary that $v(Z) = 1$. By minimality of the winning coalition S_i , we cannot have $Z \subseteq S_i$. Hence $z \in S_i$. Furthermore, as $v(Z) = 1$ and the game is proper, $S_i \setminus Z \neq \emptyset$, implying that there exists $a \in Z$ such that $fa; zg \subseteq S_i$, contradicting the minimality of the winning coalition S_i .

Since $v(Z) = 0$ and the game is strong $v(N \setminus z) = 1$. Hence there exists a minimal winning coalition $S \subseteq N \setminus z$. Clearly, since $Z \subseteq S, z \in S$. We now claim that $|S| = 2$. Suppose by contradiction that $|S| \geq 3$. Since the head of state can impose the presence of all players in S in the minimal winning coalition, there exists $d \in S; d \neq z$ and $y \in S$ such that $v(fd; yg) = 1$: Now recall that $v(fa; zg) = v(fb; zg) = 1$: Since the game is proper, $d \neq z; d \neq a; d \neq b$ and $y \neq z$, we must have $y = a$ and $y = b$, contradicting the fact that a and b are distinct elements of Z . We conclude that there exists a player $d \in Z$ for whom $v(fd; zg) = 1$, contradicting the definition of Z .

Hence, $Z = N \setminus z$, establishing that the game is an apex game. ■

Theorem 6 clarifies the relationship between the agenda setter's control and the underlying structure of power. In a sense, it shows that the agenda

setter's manipulation possibilities are rather restricted: he can only fully choose the minimal winning coalition when the underlying simple game has the special structure of an apex game, with one large player and many small players. In all other situations, the agenda setter's power is very restricted. He may be able to impose some but not all minimal winning coalitions, or even not be able to impose the presence of any more than one player, as would be the case in a symmetric majority game with more than five players, where no two player coalition is winning.

5 Conclusion

In the theoretical models of government formation in political science, it is usually assumed that the head of state has no strategic power. In this paper, we develop a formal model of agenda control to analyze the power of an agenda setter in the formation of coalitions in simple games. We suppose that the agenda setter can choose the order in which players make proposals to form coalitions in a simple, sequential noncooperative procedure of coalitional bargaining. We show that the agenda setter can always impose the presence of one agent in the coalition by placing him first, and exclude another agent by placing him second. The only structure in which the agenda setter can impose the formation of any minimal winning coalition are apex games, where a large player forms a winning coalition with any of the other players. Furthermore, the agenda setter should always place the least desirable member of a coalition first, and the most desirable player outside the coalition second.

While our results provide a first step in the understanding of the relation between the order of proposers and the coalitions formed in a sequential bargaining game, we point out some important limitations of our analysis, which

need to be addressed in future research. First, we focus in this paper on a purely redistributive model of coalition formation. In order to explain the formation of governments in parliamentary democracies, we need to extend the model to accommodate for ideological preferences of the political parties. Austen-Smith and Banks (1988) and Baron (1991 and 1993) have analyzed sequential models of government formation with three parties with preferences over a policy space. In Austen-Smith and Banks (1988), parties' utilities also depend on their share in the government benefits, and government benefits are assumed to be large enough to compensate a coalition member for any loss in utility due to an adverse policy choice. Hence, in equilibrium, the first party chooses its optimal policy outcome, and uses a transfer in government benefits to induce the other coalition member to participate in the government. In essence, the assumption that government benefits are high enough implies that all transfers will be made through a redistribution of government benefits, and the equilibrium is similar to the equilibrium in the purely redistributive game. In Baron (1991 and 1993), parties have purely ideological preferences, and bargain over policy outcomes. In that case, parties' strategies at the government formation stage are much more complex to characterize, and the equilibria may involve different coalitions than the ones obtained in the redistributive case. The characterization of equilibrium coalitions with more than three parties in Baron's models appears to be a formidable task.

Second, we have focussed our attention to simple games, dividing the set of coalitions into only two types, and only allowing for coalitional worths of 0 or 1. As shown in Chatterjee et al. (1993), the study of the relation between the protocol and the equilibrium coalition in general cooperative games with transferable utility is extremely complex. Chatterjee et al. (1993) show that, in order for the grand coalition to be formed for any ordering of the players, the per member worth of the grand coalition must exceed the per

member worth of any other coalition. In all other cases, the coalition formed in equilibrium may not be efficient (i.e. may be different from the grand coalition for strictly superadditive games) and may depend on the protocol. The exact link between the protocol and the coalition formed in equilibrium is however difficult to characterize.

Third, we have considered a specific finite procedure of sequential bargaining where each player is given the opportunity to form a coalition once. As is apparent from the characterization of the subgame perfect equilibrium of the game of coalition formation, allowing players to move more than once in the sequence would not affect our results. However, excluding some players from the order of proposers would clearly change our analysis, since the status quo payoffs would then play a role in the determination of the equilibrium outcome. This implies that the agenda setter could benefit from excluding some players from the order of proposers, and that his power may be enlarged by allowing him to choose incomplete orderings of the players.

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