

Output Monitoring in Teams*

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Abstract

This paper investigates the role of output quality control in a multi agent setting with moral hazard. The principal is in charge of a team of agents who produce the output. The marketing of this output can be either a success or entail huge losses. At the time of marketing the product, the principal is uncertain about its quality and can only observe an imperfect signal of it. This creates an ex post inefficiency (a successful project may not be undertaken) and a room for monitoring output's quality. In the paper, we describe when the principal will pay for this costly monitoring and its effect on agents' incentives to exert effort. We show that there are distortions ex ante in the contract offered by the principal and ex post in the continuation decision. The monitoring can only ensure ex post efficiency. The ex ante efficiency requires effort observability.

Résumé

Dans ce papier, nous étudions le rôle du contrôle de la qualité d'un produit dans le cadre d'un modèle d'aléa moral avec plusieurs agents. Le principal est responsable d'une équipe d'agents qui produisent l'output. La commercialisation du produit est soit un succès soit elle entraîne de fortes pertes pour le principal. Au moment de commercialiser le produit, le principal ne connaît pas la qualité du produit et ne peut observer qu'un signal imparfait de celle-ci. Cela crée une inefficacité ex post (un projet profitable peut ne pas être commercialisé) et une place pour le contrôle de la qualité du produit. Dans ce papier, nous décrivons quand le principal va payer pour ce contrôle coûteux de la qualité et son effet sur les incitations des agents à fournir de l'effort. Nous montrons que dans ce cadre, il y a des distortions à la fois ex ante dans les contrats offerts et ex post dans la décision de continuation. Le monitoring peut supprimer l'inefficacité ex post. L'efficacité ex ante requiert l'observabilité des efforts.

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1 Introduction

This paper investigates the role of output quality control in a multi agent setting with moral hazard. The principal is in charge of a team of agents¹ who produce an output for her. The marketing of this output can be either a success or entail huge losses. At the time of selling the output, the principal is uncertain about its quality and can only observe an imperfect signal of it. This creates an ex post inefficiency (a successful project may not be undertaken) and a room for monitoring output's quality. In the paper, we describe when the principal will pay for this (costly) monitoring and its effect on agents' incentives to exert effort. We add to the problem of team production an interaction between the productive units (agents) and the principal. The principal doesn't participate in the production process but influences it by choosing the level of effort he puts in monitoring and by choosing when she sells the product. We study how these decisions affect the moral hazard problem in the team. The main question raised by this paper is the relation between information gathered after the design of the contract and the incentives provided to the agents.

The relations we have in mind are for example two divisions of a firm who have to cooperate for the development of a new product. The marketing of this product may be successful or not. If the divisions cannot be interested in firm's profit and losses, the firm will pay the division contingent on the decision of marketing. In this paper, we investigate when the firm will exert a control on output. And what is the effect of such a control on divisions incentive to work. An other examples is teamwork, where the effort of all team members creates the value of output. In team, it is impossible to distinguish individual contributions. Team members are paid contingents on a commonly observable variable (ex. joint output). We investigate if the team owner will invest in control activities to check the (at that time uncertain) value of the output and how the control activity affects team members incentives.

Recent works in incentive theory have study the role of information acquisition (monitoring) in principal agent relationships. The papers of Khalil and Lawarée [1995, 1998] study the role of input and output monitoring in the adverse selection case. Crémer [1994, 1995] study the role of monitoring in a moral hazard setting.

Crémer [1995] shows that it is not always optimal for the principal to acquire new information about the circumstances under which the agent has performed his task. He shows an investigation of the reasons of a bad result may increase the cost of agent's incentives. It is sometimes better to not use the monitoring, even if it is costless and keep the agent at "arm's length".

Crémer [1994] studies the incentives to monitor of one (or two) principals who delegate productive investment to an agent. His model is a model of vertical integration in which the the monitoring decision is taken either by one or two principals. The monitoring decision depends on the cost and benefits of associated to this decision which includes the change in incentives' cost. Our framework is more or less the same. We extend his analysis to one principal multi-agent case and we modify the role of monitoring. Monitoring is in our model output quality control while in Crémer, monitoring is an investigation of the

¹We will refer as 'she' for the principal and 'he' for the agents

reasons of success or failure like in his 95's paper. In his paper, monitoring increases the cost of agent's incentives while in our model it is the opposite. Our and Crémer's models describes the incentives to monitor and the effect of monitoring on agents incentives. Including team production in Crémer framework allows us to a richer interaction between the agents and the principal.

The literature on team has focused mainly on the question of implementing an efficient production scheme (Alchian and Demsetz [1972], Holmström [1982], McAfee and McMillian [1991]). The difficulty comes from the fact that efforts of team members are unobservable and cannot be inferred from the observed joint output. Our point is different: we assume that the joint output is not observable and we focus on the design of agents' contracts by the principal and the induced effort and monitoring behaviour.

Our model is a model of team production in which the agents and the principal invest together to develop a product. Agents are responsible for the production of the good, and the principal invests in control activity. The model is constructed in the following way: at a first stage (contracting stage), the principal offers a wage contract to the agents. If they accept the contract, they exert an effort (production stage). The level of efforts is private information to each agent. The efforts determine (together with a random shock) the output's value. This value remain unknown till the marketing of the product. However, the principal can observe a signal about output's quality. The accuracy of the signal is affected by the principal's monitoring decision. Without monitoring, the signals are noisy. By investing in monitoring, the principal can observe perfectly informative signals. After observing the signal, the principal decides if she sells the output (continuation stage). Finally, the principal collects the surplus and pays wages to the agents. We assume that the monitoring decision and the signal are private information to the principal. Private nature of monitoring and signals implies that agents, when choosing their effort, will form *expectations* about the principal's monitoring decision. And conversely, the principal decides to monitor, evaluating the cost and benefits of this decision according to her beliefs about agents unobservable efforts. Private nature of agents' and principal's decisions form the central point of our paper. It is because these decisions cannot be contracted ex ante that there is an interesting strategic interaction between the principal and the agents.

Our goal in this paper is to describe the optimal contract offered by the principal to the team members. This contract doesn't correspond to the first best contract. Our comparative static shows that the principal extracts less effort compared to a situation in which she can observe the efforts level. This inefficiency has two sources: the lack of proper signal when the principal doesn't monitor and the payment of rents to the agents. These extra wages are necessary to extract the agents' efforts when they work in team.

The paper is organized as follow: in the next section, we present the model. In section 3, we describe the continuation (marketing), monitoring and efforts decisions. In section 4, we compute the contract selected by the principal. We make after some comparative static and comment the results. In section 5, we present an extension of the model. Section 6 concludes.

2 Model

The game played by the principal and the agents is represented by the following sequence of decisions:

Contracting stage (ex ante stage)

- The principal offers a wage contract to the agents.
- Agents accept or reject the contract

Production stage (ad interim stage)

- The agents choose non cooperatively their effort levels.
- The principal decides to monitor or not.
The signal is observed by the principal

Continuation stage (ex post stage)

- The principal decides to continue or to stop the relation. She pays the agents according to the contract and she collects the surplus.

The principal has three decisions to take: first she offers a contract to the agents. This contract specifies the wages she pays to the agents. Second she decides to whether or not control the output (monitoring decision) and finally, she decides if she sells the product (a decision we call continuation decision). After she offers the contract, and before she monitors, agents make an unobservable effort. The complexity (and the richness) of the model comes from this interaction between principal's and agents' decisions. In this subsection, we explain the assumption we make concerning the principal and the agents.

2.1 Productive units (agents)

Agents are risk neutral² and have a separable utility:

$$U_i = W_i - e_i \quad i = 1, 2$$

where W_i is the wage perceived and e_i the disutility of effort. We normalize their ex ante reservation utility to zero. We assume that agents have limited liability³: $W_i \geq 0$. Given the wages offered by the principal, agents choose the level of effort that maximizes their expected utility. For simplicity we take a discrete effort level. They can do either a high or a low effort, noted respectively e_i^h , e_i^l , $i = 1, 2$. A high effort has a disutility of Δe_i^h . Without loss of generality, we normalize the disutility of e_i^l to zero. We assume that the agents' efforts are private information to them. This creates a conflict of interest between the agents and the principal.

²We abstract from risk sharing.

³The limited liability constraint and the non contractibility of the created surplus (see after) implies that it is not possible to make an agent residual claimant of the relation. Then, even with risk neutral agents, the incentive problem is non trivial.

2.2 Principal

2.2.1 Output's value

The value of the output will be either equal to $S > 0$ or $F < 0$. S correspond to a situation where the marketing of the product is a *success*, while F correspond to a *failure*. The principal observes the value of output only at the final stage of the game if she has decided to continue. Otherwise, if she has decided to stop, her payoff is zero. Her gross surplus (before paying wages) is then either S , F or 0.

2.2.2 How efforts generate output?

The value of the output (S or F) is a function of agents' efforts. This relation is stochastic. The efforts affect the probability of having a surplus equals to S . These probabilities are depicted in table 1.

	e_2^h	e_2^l
e_1^h	π	p
e_1^l	p	0

Table 1. Probability of success

The technology used in this model exhibits three characteristics: First, even if the agents may differ with respect to their cost of effort, they have a symmetric impact on the probability of success. In the production process, agents are symmetric. Second, the fact that it is possible to have a positive probability of success with only one agent performing a high effort ($= p$), implies that no agent is indispensable for the production. And, indeed, we will show that there a circumstances in which the principal prefers that only one agent does a high effort.

And third, we assume that the production process exhibits decreasing returns to scale. This technically corresponds to the following assumption:

Assumption 1 $p < \pi < 2p$.

Assumption 1 states that the additional effect on the probability of success of an effort is decreasing from p to $\pi - p$.

2.2.3 Signals and monitoring

Prior to the decision of continuation, the principal receives a signal correlated with output's value. The signal can be either high (H) or low (L). After observing the signal and before observing the true value of output, the principal decides to continue or to stop.

The investment in monitoring changes the accuracy of the signal. Without monitoring, the signals are noisy. They reflects imperfectly the true output's value.

The probabilities of observing H and L are conditional on S and F . These probabilities are changed by the investment in monitoring. Without monitoring, the conditional probability of observing H , given that the surplus is S is equal to $\pi_H = \text{prob}(H|S) < 1$.

Monitoring increases this probability from π_H to one. And it conversely decreases the probability of observing L conditional on S from $(1 - \pi_H)$ to zero. We assume that the probability of observing L conditional on failure is in any cases equals to one. Table 2 summarizes the conditional probability of signals H and L given S and F .

	Monitoring	No monitoring
$prob(H S)$	1	π_H
$prob(L S)$	0	$1 - \pi_H$
$prob(H F)$	0	0
$prob(L F)$	1	1

Table 2: Conditional probability of observing H and L given S and F .

There is an asymmetry in the production of signals. A bad result cannot lead to a high signal, while in the absence of monitoring, a high result can lead to a low signal. When the principals monitors, she sure that if she has observed a signal L , the relation will end with a failure. While if she doesn't monitor, a low signal doesn't necessary means that the surplus will be F . In the case of a high signal, whatever the monitoring decision, the principal is sure that the relation will end with a success.

We assume that the monitoring decision and the signal are private information to the principal. Private nature of monitoring and signals.

The cost of monitoring output is equals to μ . To have perfectly informative signals, the principal has to pay μ .

2.2.4 Contracts

At first stage, the principal offers a contract to the agents. We restrict the set of feasible contracts by assuming that:

Assumption 2 *The value of the surplus (S or F) cannot be observed by a party outside the relationship.*

This assumption imply together with the private nature of effort, monitoring and signals that the remaining contracting variable is the continuation decision . Then the wages offered to agents will be contingent on this decision. We denote the wage paid to agent i contingent on continuation W_i^c and the wage contingent on stop W_i^s .

The aim of the principal is to design contracts and choose the monitoring and continuation decisions that maximizes her expected profits.

3 Continuation, effort and monitoring decisions

In this section, the describe the decisions token by the principal and the agents at each stage of the game.

3.1 Ex post stage: continuation decision

At the final stage of the game, after observing the signals, the principal decides if she sells or not the product. This decision will be contingent on the observed signal. The principal decides to continue only if after observing signals, she expects a positive profit. This decision therefore depends on the informational content of the signals. Before analyzing the decision we assume that S and F are such that:

Assumption 3

$$\frac{\pi(1 - \pi_H)}{1 - \pi\pi_H}S + \frac{1 - \pi}{1 - \pi\pi_H}F < 0 \quad (1)$$

This assumption means that when the principal doesn't monitor, her expected gross surplus after observing a low signal is negative, even if she's sure that both agents have done a high effort⁴. $\frac{\pi(1-\pi_H)}{1-\pi\pi_H}$ is the conditional probability of S given L, e_1^h, e_2^h and no monitoring and $\frac{1-\pi}{1-\pi\pi_H} = \text{prob}(F|L, e_1^h, e_2^h, \text{no monitoring})$. These probabilities are constructed with the Baye's rule and using the probabilities from tables 1 and 2. Assumption 3 implies that the principal takes a continuation decision which is different when the signal is different. In section 5, we discuss the results when this assumption is removed.

The continuation decision is given by the following rules:

- If the principal has monitored, the signal she has observed is perfectly informative. And thus, she continues only if she observes H and if the wages W_i^c, W_i^s are such that: $S - W_1^c - W_2^c \geq 0 - W_1^s - W_2^s$. She never continues after a signal L because she's sure to make a negative profit because the limited liability constraint rules out negative wages.
- If the principal hasn't monitored, the observed signal is imperfect. She continues after a signal H if $S - W_1^c - W_2^c \geq 0 - W_1^s - W_2^s$. After a signal L , her gross profit is (by assumption) in any cases negative. Then as in the case of monitoring, she stops.

Assumption 3 implies that without perfectly informative signals (no monitoring case), the potential losses are sufficiently high to refrain the principal to continue when there is a doubt about output's value (a low signal is observed). Principal's continuation decision is simplified by assumption 3: the continuation decision is independent of efforts and monitoring decision, and just depends on the observed signal: when a low signal is observed it is optimal to stop and to continue if a high signal is observed (at least for wages that keep the net profit positive⁵).

With the description of the continuation decision, we can compute the ex ante probability of continuation. These probabilities determine the value of the ex ante profit, and therefore affect the contract, monitoring and effort decisions. From the above discussion, it is clear that the ex ante probability of continuation is equals to the probability of observing H which is a function of the effort and monitoring choices by the principal and the agents. Table 3 summarize this ex ante probability of continuation ϕ .

⁴A fortiori, it will be the case when only one agent did a high effort

⁵We suppose that such wages exist.

	Monitoring		No monitoring	
	e_1^h	e_1^l	e_1^h	e_1^l
e_2^h	π	p	$\pi\pi_H$	$p\pi_H$
e_2^l	p	0	$p\pi_H$	0

Table 3: Ex ante probabilities of continuation $\phi(e_1^h, e_2^h, m)$

The principal's ex ante payoffs have the following form:

$$\phi(e_1^h, e_2^h, m)(S - W_1^c - W_2^c) + (1 - \phi(e_1^h, e_2^h, m))(-W_1^s - W_2^s) - \mu(m) \quad (2)$$

Where m represents the monitoring decision. m can equal M (monitoring) or NM (no monitoring). The associated costs $\mu(m)$ are respectively μ and 0 . And ϕ is given by table 3.

The investment in monitoring is complementary with agents' effort. As table 3 shows, the gain from monitoring (which is the increase in the probability of continuation when the surplus equals S) is related to probability of success which depends on agents' effort. If there's no effort by the agents, the monitoring is useless.

Without monitoring, there is an ex post inefficiency: If the principal stops after a signal L , while the surplus equals S , the continuation decision is inefficient. Because she lacks of proper signals, the principal may be refrained to continue while it is (ex post) optimal to do so. Because $\pi\pi_H < \pi$ and $p\pi_h < p$, the principal stops too often in the absence of monitoring. The only way to remove this ex post inefficiency is to control the output. This is exactly the role played by monitoring. Monitoring is an output control, which suppress by giving precise signals the ex post inefficiency. However, as it is costly, it is not always used.

We now turn to the study of monitoring and effort decisions. As these decisions are private, the game played by the principal and the agents can be represented as a simultaneous move game. This game is represented in figure 1. In figure 1, the payoffs have the form (agent 1, agent 2). The payoffs of the principal can be computed with equation 2 and table 3.

3.2 Choice of information technology

The principal monitors output if the cost (μ) is smaller than the benefits. The benefits are function of agents' efforts. The monitoring decision will therefore depends on principal's beliefs about agents' efforts. When the principal believes that agents i and j have done a high effort with probability δ_i and δ_j , it is optimal to monitor if the profits with monitoring:

$$q(S - W_1^c - W_2^c) + (1 - q)(-W_1^s - W_2^s) - \mu \quad (3)$$

where where $q = \delta_i\delta_j\pi + (1 - \delta_i)\delta_jp + (1 - \delta_j)\delta_ip$ is the ex ante probability of success, are greater than the profit without monitoring:

$$q\pi_H(S - W_1^c - W_2^c) + (1 - q\pi_H)(-W_1^s - W_2^s) \quad (4)$$

Combining (3) and (4), at wages (W_i^c, W_i^s) , it is optimal to monitor if:

$$\mu \leq (1 - \pi_H)q(S - W_1^c - W_2^c + W_1^s + W_2^s) \quad (5)$$

		Monitoring		
		e_2^h	e_2^l	
e_1^h		$(\pi W_1^c + (1-\pi)W_1^s - \Delta e_1^h,$ $\pi W_2^c + (1-\pi)W_2^s - \Delta e_2^h)$	$(pW_1^c + (1-p)W_1^s - \Delta e_1^h,$ $pW_2^c + (1-p)W_2^s)$	
e_1^l		$(pW_1^c + (1-p)W_1^s,$ $pW_2^c + (1-p)W_2^s - \Delta e_2^h)$	$(0,0)$	

		No monitoring		
		e_2^h	e_2^l	
e_1^h		$(\pi\pi_H W_1^c + (1-\pi\pi_H)W_1^s - \Delta e_1^h,$ $\pi\pi_H W_2^c + (1-\pi\pi_H)W_2^s - \Delta e_2^h)$	$(p\pi_H W_1^c + (1-p\pi_H)W_1^s - \Delta e_1^h,$ $p\pi_H W_2^c + (1-p\pi_H)W_2^s)$	
e_1^l		$(p\pi_H W_1^c + (1-p\pi_H)W_1^s,$ $p\pi_H W_2^c + (1-p\pi_H)W_2^s - \Delta e_2^h)$	$(0,0)$	

Figure 1: Effort-monitoring game

3.3 Agents choice of effort

At the wages (W_i^c, W_i^s) , the agents will choose the level of effort that maximize their expected utility. The choice of agent i depends on principal monitoring decision and on agent j 's choice of effort. The following relations describe the best response of agent i to agent j 's choice of effort and principal's monitoring decision.

e_i^h is a best response to $(e_j^h, \text{monitoring})$ if W_i^c, W_i^s are such that:

$$(\pi - p)(W_i^c - W_i^s) \geq \Delta e_i^h \quad (6.1)$$

e_i^h is a best response to $(e_j^h, \text{no monitoring})$ if W_i^c, W_i^s are such that:

$$(\pi - p)\pi_H(W_i^c - W_i^s) \geq \Delta e_i^h \quad (6.2)$$

e_i^h is a best response to $(e_j^l, \text{monitoring})$ if W_i^c, W_i^s are such that:

$$p(W_i^c - W_i^s) \geq \Delta e_i^h \quad (7.1)$$

e_i^h is a best response to $(e_j^l, \text{no monitoring})$ if W_i^c, W_i^s are such that:

$$p\pi_H(W_i^c - W_i^s) \geq \Delta e_i^h \quad (7.2)$$

Agent i randoms his effort choice and chooses e_i^h with probability δ_i in response to a random choice of e_j^h by agent j with a probability δ_j , and monitoring by the principal if W_i^c, W_i^s are such that:

$$(q - \delta_j p)(W_i^c - W_i^s) \geq \delta_i \Delta e_i^h \quad (8.1)$$

The value⁶ of $\delta_i = \frac{p(W_j^c - W_j^s) - \Delta e_j^h}{(W_j^c - W_j^s)(2p - \pi)}$

⁶Note that to have non degenerated mixed strategies, $W_i^c - W_i^s \in [\frac{\Delta e_i^h}{p}, \frac{\Delta e_i^h}{\pi - p}]$

Agent i randoms his effort choice and chooses e_i^h with probability δ'_i in response to a random choice of e_j^h by agent j with a probability δ'_j , and no monitoring by the principal if W_i^c, W_i^s are such that:

$$(q' - \delta'_j p)\pi_h(W_i^c - W_i^s) \geq \delta'_i \Delta e_i^h \quad (8.2)$$

The value⁷ of $\delta'_i = \frac{p\pi_H(W_j^c - W_j^s) - \Delta e_i^h}{(W_j^c - W_j^s)(2p - \pi)\pi_H}$

3.4 Summary: equilibria in the effort-monitoring game

From the best response functions, we can construct the map of Nash equilibria The fol-

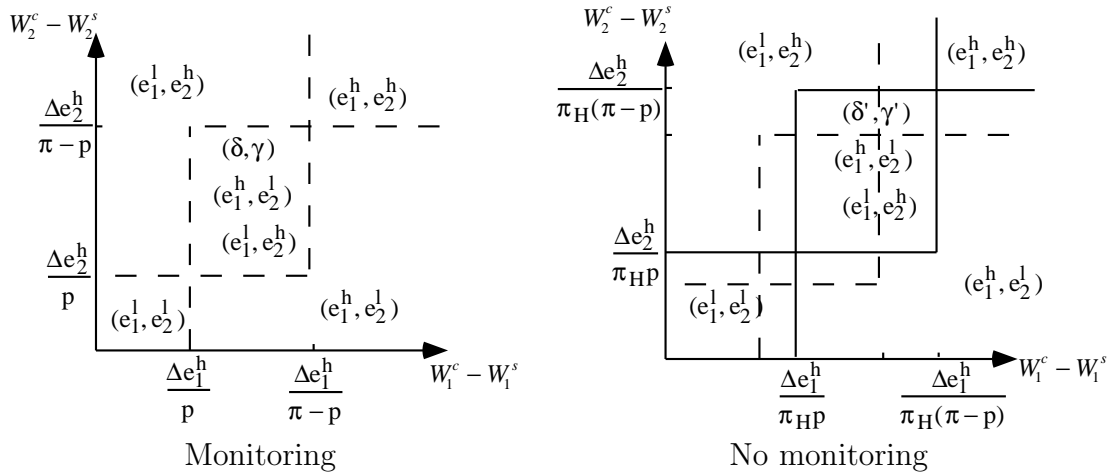


Figure 2: Nash equilibria in the effort-monitoring game

lowing figures represent for all possible wages the Nash equilibria in the effort-monitoring game. The left figure shows the equilibria when the principal monitors. The right figure represents the equilibria when the principal doesn't. The dotted lines on the right figure are a reproduction of equilibria in the case of monitoring (as depicted in the left figure). From these figures, it appears that there are wages for which it is not possible to have a certain effort behaviour, unless the principal monitors. It is also important to note that for some wages, many equilibria may exist at the same time. The equations (5) and (6), (7), (8) represents how the principal and the agents respond to wages offered.

4 Choice of a contract

In this section, we look at the contract selected by the principal. We first derive the set of optimal contracts, and after, we make some comparative static analysis.

⁷Note that to have non degenerated mixed strategies, $W_i^c - W_i^s \in [\frac{\Delta e_i^h}{\pi_H p}, \frac{\Delta e_i^h}{\pi_H(\pi - p)}]$

4.1 Set of feasible contract

The principal maximize her profit:

$$\max_{W_1^c, W_2^c, W_1^s, W_2^s} \phi(e_1^h, e_2^h, m)(S - W_1^c - W_2^c) + (1 - \phi(e_1^h, e_2^h, m))(-W_1^s - W_2^s) - m(\mu) \quad (2)$$

The maximization problem must respect the following constraints: individual rationality: $EU_i \geq 0$, $i = 1, 2$ and limited liability: $W_i^c \geq 0, W_i^s \geq 0$, $i = 1, 2$.

The choices of m by the principal is given by equation (5), the choices of e_1 and e_2 by agents 1 and 2 are given by equations 6 to 8 for $i = 1, 2$.

Before giving the solution to this problem, we give two definitions and establish two lemmas. These lemmas help us to prove proposition 1 and its corollary.

Definition 1: We define an equilibrium in the game as a pair of wages and the associated behaviour of the agents and the principal: $(W_1^c, W_1^s, W_2^c, W_2^s, e_1, e_2, MorNM)$.

Definition 2: When the principal prefers that the two agents do a high effort, we will refer to this situation as *team production*. When she prefers only one high effort, which means that agents' effort are perfect substitute, we will call this situation *delegation of production*.

Lemma 1 *The principal offers wages W_i^s always equal to zero.*

This result is obvious to establish. As only the gap between $W_i^c - W_i^s$ matters for incentive purpose, the value of W_i^s will be given by the limited liability constraint.

Lemma 2 *If there exist some wages W_i^c, W_i^s such that, given the behaviour of the agents, it is optimal for the principal to monitor (equation 5 is satisfied), the principal will never offer wages \bar{W}_i^c, \bar{W}_i^s such that: (i) the behaviour of the agents remains the same and (ii) it is not optimal to monitor (equation 5 is not satisfied).*

Lemma 1 means that if there exist wages such that, given the behaviour of the agents it is optimal to monitor, the principal has no interest in deviating to a contract for which it is not optimal to monitor and the behaviour of the agents is unchanged. To prove the lemma, we show that such a deviation requires higher wages and that the increase in wages doesn't compensate the benefits of saving on monitoring cost. A proof of lemma 2 is given in the appendix.

Proposition 1 (i) *There are seven equilibria:
Two where there is team production:*

$$\left(\frac{\Delta e_1^h}{\pi - p}, \frac{\Delta e_2^h}{\pi - p}, e_1^h, e_2^h, M \right)$$

$$\left(\frac{\Delta e_1^h}{\pi_H(\pi - p)}, \frac{\Delta e_2^h}{\pi_H(\pi - p)}, e_1^h, e_2^h, NM \right)$$

Four where there is delegation to agent i :

$$\begin{aligned} & \left(\frac{\Delta e_i^h}{p}, 0, e_i^h, e_j^l, M \right) \\ & \left(\frac{\Delta e_i^h}{\pi_{HP}}, 0, e_i^h, e_j^l, NM \right) \\ & i, j = 1, 2 \end{aligned}$$

One where there is no production:

$$(0, 0, e_1^l, e_2^l, NM)$$

(ii) These equilibria are unique if $\Delta e_1^h \neq \Delta e_2^h$.

(iii) The principal never offers wages for which the agents random their effort choices.

A complete proof of proposition 1 is given in the appendix, we just give here the intuitions behind these results. To derive the set of contract, we proceed case by case: for any behaviour of the principal and the agents, we search the wages that maximize the profit. i.e. the lowest wages that induce the selected behaviour. As only the gap ($W_i^c - W_i^s$) is important for incentive purpose, and as the limited liability constraint binds for W_i^s , the wages W_i^c are those who satisfy equations 6 to 8 with equality. To complete the first part of the proof, we have to show that the principal has no interest in deviating from these contracts to other where her monitoring behaviour is changed. This result is established by lemma 2.

Note that the individual rationality constraint binds only for the equilibria when at least one agent does a low effort. This point is developed further in remark 1.

The third part of proposition 1 tell us that the principal never selects wages that induce random choice of effort by agents. To understand this, we must compare the ex ante costs and benefits of random choices of efforts. Ex post, four situations can occur: both agents have selected a high effort, only one agent (1 or 2) has selected a high effort, or none has selected a high effort. The first situation is beneficial: there is team production at a smaller cost than with pure strategies. The total cost is smaller than with the wages associated with team production. The other three situations are prejudicial: when only one manager has done a high effort, there is cost for the principal. In case of continuation, she must pay wages to the two agents. and the total wages are greater than the total wage she has to pay to have delegation of production with pure strategies. When there is no high effort, there is a cost only if the firm monitors. Proposition 1 says that the ex ante benefit associated with the good situation (2 high effort) are always smaller than the ex ante costs of the bad situations⁸. The advantages of the mixed strategies are more than offset by the increase in cost. Ex ante, at the time of contract, the costs and benefits

⁸Actually our proof shows that the upper bound of profit is always smaller than the profit with only one manager doing a high effort.

associated with the wages that induce agents to random their choices of effort are such that, it is always possible to find a wage contract for which they agents don't random and that give a higher profit to the principal. Then the principal will not offer wages for which the agents random their choices of effort.

The following corollary is derived from proposition 1:

Corollary 1 *If a monitoring equilibrium exists, the principal will never select a contract for which it is not optimal to monitor.*

This corollary extends the result of lemma 2 for the optimal contracts. It states that if μ is small enough to guarantee the existence of at least one of the equilibrium contract with monitoring as defined in proposition 1, the principal never selects a contract in which she doesn't monitor. The second lemma established that it is not optimal to deviate from a monitoring contract to a no monitoring contract with the same agent behaviour. The corollary states that a deviation from an optimal monitoring contract to an (optimal) no monitoring contract with any agent behaviour is costly.

4.2 Comparison between equilibria

We compare the equilibria within the set of proposition 1. Depending on parameters value, the principal selects one of these seven contracts. In this subsection, we make comparative static analysis to understand what drive the choice of contract.

The comparative static can be summarize as follow: When the principal monitors, she prefers team production to delegation to agent i if:

$$(\pi - p)S \geq \frac{p\Delta e_i^h}{\pi - p} + \frac{\pi\Delta e_j^h}{\pi - p} \quad (9)$$

When she doesn't monitor, she prefers team production if:

$$(\pi - p)\pi_H S \geq \frac{p\Delta e_i^h}{\pi_H(\pi - p)} + \frac{\pi\Delta e_j^h}{\pi_H(\pi - p)} \quad (10)$$

When the principal prefers delegation, she delegates to agent i if $\Delta e_i^h \leq \Delta e_j^h$ and otherwise to agent j . The left hand side of (9) is the increase in expected surplus, when the principal offers $(W_1^c, W_2^c) = (\frac{\Delta e_1^h}{\pi - p}, \frac{\Delta e_2^h}{\pi - p})$ instead of $(W_i^c, W_j^c) = (\frac{\Delta e_i^h}{p}, 0)$. The right hand side is the increase in expected wages. Equation (10) has the same interpretation when the principal doesn't monitor. It describes when it is optimal to offer the contract $(W_1^c, W_2^c) = (\frac{\Delta e_1^h}{\pi_H(\pi - p)}, \frac{\Delta e_2^h}{\pi_H(\pi - p)})$ instead of $(W_i^c, W_j^c) = (\frac{\Delta e_i^h}{\pi_H p}, 0)$. The corollary 1 limits the comparison: if a monitoring contract exists, we just have to check at equation (9) to know which monitoring contract will be selected by the principal. If no monitoring contract exists, we just look at equation (10).

For our comparisons, we use as benchmark the first best choice of contract. We call the first best contract the contract selected by the principal when he has symmetric information about efforts i.e. she observes the level of efforts. The first best wage is to

give to agent i an expected wage equals to Δe_i^{h9} . The first best choice of production mode is represented in figure 3 . The first best is to choose team production when

$$(\pi - p)S \geq \Delta e_i^h \quad (11)$$

if the principal monitors and

$$(\pi - p)\pi_H S \geq \Delta e_i^h \quad (12)$$

if she doesn't.

The choice of contract is represented in the following figures:

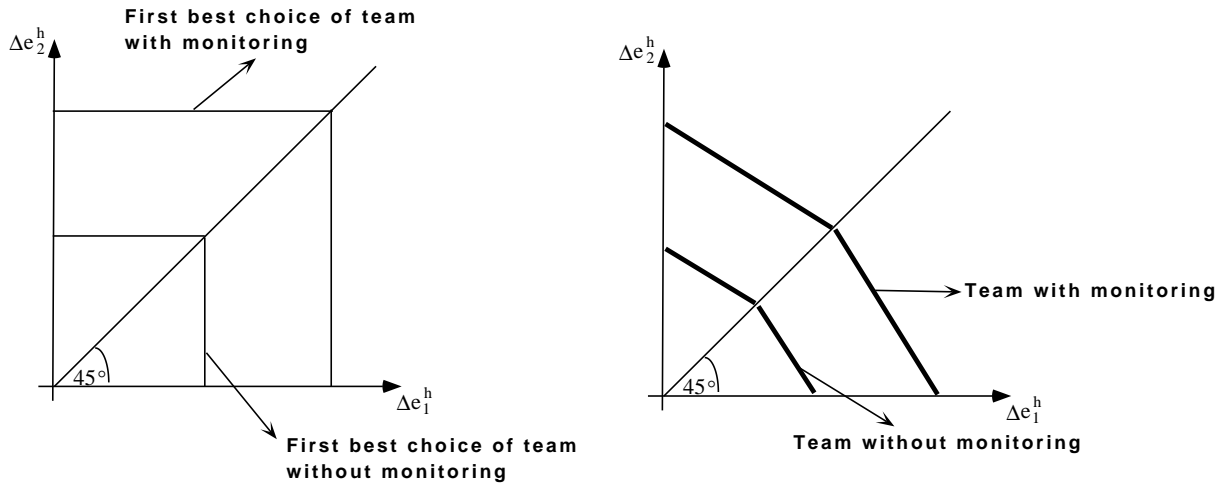


Figure 3: First best and second best choice of contract as a function of Δe_1^h and Δe_2^h

In figure 3, the contract chosen by the principal is represented as a function of the disutility of effort Δe_1^h and Δe_2^h , keeping S and μ constant. In this figure, we do not represent the monitoring conditions. The left figure represents the first best choice of contract (equations (11) and (12)). The right figure represents the second best choice (described by equations (9) and (10)).

The choice between monitoring and no monitoring contract is given by the corollary. As soon as μ is small enough to have an equilibrium with monitoring, we know that an equilibrium of this class will be chosen. The choice of contract as a function of μ and the surplus' value is represented in figure 4.

In figure 4, the bold lines represent the separations between the contracts. The condition (9) and (10) are represented by the horizontal lines (they are independent of the value of μ). The other lines are the monitoring conditions for $(\frac{\Delta e_1^h}{\pi-p}, \frac{\Delta e_2^h}{\pi-p})$ and $(\frac{\Delta e_i^h}{p}, 0)$. To simplify the graph we assume that $\Delta e_i^h < \Delta e_j^h$. So the productive effort is never delegated to agent j .

We now develop in few remarks the interesting points:

⁹For example, the principal can pay Δe_i^h if the effort is e^h and zero if the effort is e^l or she can pay $\frac{\Delta e_i^h}{q}$, if the decision is continue and the ex ante probability of success is q and zero if she stops.

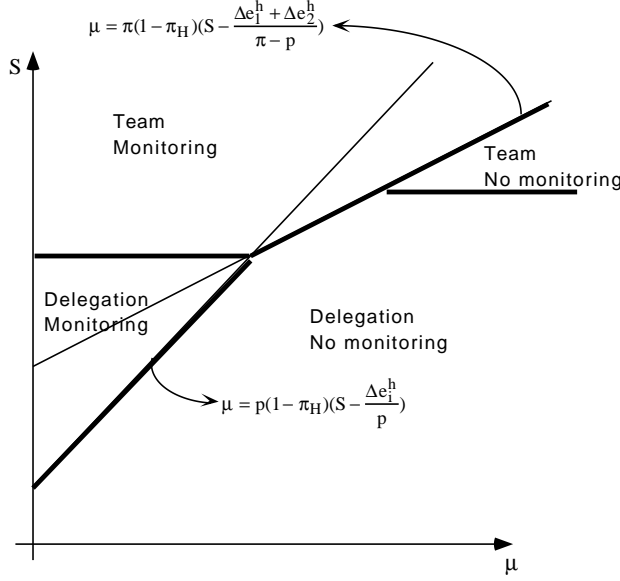


Figure 4: Contracts as a function of μ and S

Remark 1 *Even with monitoring, the first best choice of production mode cannot be achieved.*

We develop our argument in three points: first we show that when the principal chooses team production, the agents receive a rent. Second, as a direct consequence of the payment of rents, the team production suffers a cost disadvantage compared to delegation and the principal delegates too often compared with the first best. Third, we look how the monitoring affects the ex ante distortions in the choice of contract.

When the agents work in team, they benefit from the effort of the other. If agents i and j are both doing a high effort, they have a strictly positive expected utility¹⁰: $EU_i = \frac{p\Delta e_i^h}{\pi - p} > 0$. Two high efforts imply the payment of rents to the agents. While when the production is delegated, the agents have an expected utility equals to their reservation level. The source of rent is the inability of the principal to give proper incentives to both agents. To have two high effort, the principal has to pay wages that satisfy the following set of constraints¹¹:

$$(\pi - p)(W_i^c - W_i^s) \geq \Delta e_i^h \quad (6.1)$$

$$W_i^c, W_i^s \geq 0 \quad (LL)$$

$$\pi W_i^c + (1 - \pi)W_i^s - \Delta e_i^h \geq 0 \quad (IR)$$

It is easy to see that it is impossible to find wages that satisfy the three constraints with equality. These rents have two sources: the non observable of effort and the limited liability of the agents. To avoid the payment of rents, either the efforts must be observable or the agents not be wealth constrained. We examine now the effect of each of these

¹⁰The level of the rent is not linked to principal's monitoring decision.

¹¹We give an argument valid for the case of monitoring. It translates easily to the no monitoring case.

assumptions. (i) The agents do not have wealth constraint. In this case, the principal can construct an incentive scheme that satisfy (6.1) and (IR). The wages satisfying these two constraints are $(W_i^c, W_i^s) = (\frac{(1-p)\Delta e_i^h}{\pi-p}, \frac{-p\Delta e_i^h}{\pi-p})$. In this case¹², the interests of the principal and the agents are aligned. They still receive an expected wage that correspond to their disutility of effort but now their choices of effort reflect the interest of the principal in the continuation decision. When the signal is low, the output is not sold but the agents have to pay a compensation to the principal (wages W_i^s are negative). With this incentive scheme, the agents are responsible for the failure of the project. It is possible to have two high effort without paying any rent to the agents. (ii) The efforts are observable. In this case the principal compensates the agents for their disutility of effort. In our model, the principal observes only a signal reflecting output's quality. From this, the principal cannot infer agents' effort. This creates a "free riding" problem: the agents can receive a positive wage without doing a high effort. An incentive scheme that compensate the agents just for their disutility of effort is not enough to have team production.

This inability to give proper incentives imply the following distortion ex ante in the choice of contracts: compared to the first best, the principal delegates to often. The first best choice¹³ of production mode is represented in figure 3 and described by equations (11) and (12). The principal is not able to replicate the first best. This is a direct consequence of the payment of rents to the agents. It creates a distortion that favors the delegation of investment. This result is independent of the monitoring decision. The payment of rent increases the cost of the team. This cost disadvantage explains why delegation of production is for more parameter values compared to first best.

When the principal monitors, the level of rent paid to the agents under team production is unchanged whatever the monitoring decision. Then, even with monitoring there is a distortion in the choice of contract. There is a bias favoring delegation. The control of the output is not enough to implement the first best. However, the fact that the principal monitors reduced the global inefficiency. This point is developed in our second remark.

Remark 2 *The monitoring increases the global efficiency by (i) decreasing the cost of incentives and (ii) removing the ex post inefficiency.*

The second point is obvious. It was postulated in assumption 3. The first point can be observed from proposition 1. The effect of monitoring on team's incentives is: a decrease in wages from $\frac{\Delta e_i^h}{\pi_H(\pi-p)}$ to $\frac{\Delta e_i^h}{\pi-p}$ and an increase in the probability of being paid. These two effects compensate and the expected wage of the agents is constant. But the wages paid ex post are smaller if she monitors. This point is clear in figure 3. For middle values of efforts' cost, the principal can have team production only if she monitors. When the costs of effort increase, to have team production, the principal must decrease the wages, in order to keep her profit at a sufficiently high level. The decrease of wages is feasible only if μ is such that it is efficient to monitor. When μ is too high, it is not possible to decrease the cost of incentives and the principal has to switch from team production to delegation. The lack of proper signals which corresponds to a situation where it is not

¹²With this incentive scheme, the monitoring and continuation decisions are unchanged.

¹³We called the first best the choice made under symmetric information about efforts.

optimal to monitor introduces a new distortion in the ex ante decision of contract. The wage must be increased to compensate the diminution in the probability of continuation and this introduce a second bias in favor of delegation of production. But this second bias can be removed if the principal monitors. A direct consequence of the decrease in wages, is that the monitoring favors team production. We can give a similar argument for delegation of investment. When the principal monitors, the wage is decreased from $\frac{\Delta e_i^h}{\pi_H p}$ to $\frac{\Delta e_i^h}{p}$. This decrease in wage has a positive impact on the efficiency. When the cost Δe_i^h increases, the principal switches to no production sooner if she doesn't monitor¹⁴.

Note that there is also a distortion in the monitoring decision compared. At equilibrium, when the principal observes the efforts, she doesn't pay any rent to the agents and just compensated for their disutility of effort. Therefore, as wage are smaller in the case of team production, the principal monitors too few when she doesn't observe the effort.

There is a trade off between the cost of information and the level of investment. There is more investment in productive activities when the cost of information is low. For the same costs of effort, the firm decision may change from team to delegation or from delegation to no production if the cost of monitoring increases. The incentives to perform effort are positively correlated with the cost of information μ .

4.3 Conclusion: the role of information

Information plays a different role in the two production mode. Under team production, the observation of ex post joint output does not reveal information about efforts. If it is sometimes profitable for the principal to monitor, it doesn't make her information about agents effort more precise. This impossibility for the principal to infer, even with monitoring, the individuals efforts is the source of rent for the agents. While when production is delegated, we are in a more classical moral hazard problem. There is an agent who receives a constant wage (equals to zero). He isn't incite to do effort. The final result signals the other agent's effort. The monitoring is in this case the suppression of one source of uncertainty. If she monitors, she suppress the uncertainty surrounding signals.

In the model we described, there are two sources of inefficiency. First, ex ante the contract may not be efficient. The principal chooses the wrong production mode, which means in our context that the principal delegates the production too often. And second, ex post the continuation decision may be inefficient. The ex post inefficiency takes its source in the absence of precise signals. The monitoring can remove (at a cost μ) the ex post inefficiency. In all the monitoring equilibria, the continuation decision is optimal. But ex post efficiency is not the sole role of monitoring. It also affects the ex ante contract decision. The choice of production mode is affected by the preciseness of information about output. We have shown in remark 2 that the production mode depends on the availability of information (together with other factors). Availability of information (represented by

¹⁴When the principal doesn't monitor, she switches to no production as soon as $\pi_H p S - \Delta e_i^h < 0$ When she monitors, no production starts when $p S - \Delta e_i^h - \mu < 0$. A necessary condition for the existence of the monitoring equilibrium is: $\mu \leq p(1 - \pi_h)(S - \frac{\Delta e_i^h}{p})$. Combining these three inequalities, it is easy to show that the principal doesn't produce for lower values of Δe_i^h when she doesn't monitor.

monitoring cost) affects the choice of a production technology. Ex ante inefficiency is not completely restored by the monitoring. The absence of proper incentives imply the payments of rents to team members. These rents distort the choice of production. If monitoring decreases (at equilibrium) the wages paid to the agents, it does not decrease the rents. And therefore if monitoring restores ex post efficiency, it does not solve the ex ante inefficiency problem.

5 Extension

In section 3, we assumed that it is not efficient for the principal to continue if she had observed a low signal (assumption 3). In this section, we first present an alternative assumption which is less restrictive than assumption 3 but who has the same implication in term of continuation decision. After, we look at the cases where this alternative assumption doesn't hold. And we show that the principal has to choose a contract that commit herself to a given behaviour.

Assumption 4 S and F are such that:

$$\frac{\pi(1 - \pi_H)}{1 - \pi\pi_H} \left(S - \frac{\Delta e_1^h + \Delta e_2^h}{\pi_H(\pi - p)} \right) + \frac{1 - \pi}{1 - \pi\pi_H} \left(F - \frac{\Delta e_1^h + \Delta e_2^h}{\pi_H(\pi - p)} \right) \leq 0 \quad (13)$$

and

$$\frac{p(1 - \pi_H)}{1 - p\pi_H} \left(S - \frac{e_i^h}{\pi_H p} \right) + \frac{1 - p}{1 - p\pi_H} \left(F - \frac{e_i^h}{\pi_H p} \right) \leq 0 \quad (14)$$

Equation number (13) says that when the principal offers the contract $(W_1^c, W_2^c) = \left(\frac{\Delta e_1^h}{\pi_H(\pi - p)}, \frac{\Delta e_2^h}{\pi_H(\pi - p)} \right)$, which corresponds to an equilibrium $(e_1^h, e_2^h, \text{no monitoring})$, the expected profit after the signal L is negative. Equation (14) means the same for the contract $(W_i^c, W_j^c) = \left(\frac{\Delta e_i^h}{\pi_H(\pi - p)}, 0 \right)$. The results of the paper are unchanged when the less restrictive assumption 4 holds.

We now turn to the study of the cases when at least one of these equations is not verified. In this case, the continuation decision won't be contingent on the observed signal (as it was previously) but will be contingent on the offered contract. When assumption 4 does not hold, it means that the losses¹⁵ F are not high enough compared to the gains to stop after a low signal or alternatively that without monitoring, the signal are too noisy (π_H is low) and an observation of L is not a good signal of the future result. The violation of one of the inequality of assumption 4 has dramatic results on the incentive problem. Without monitoring, it becomes impossible to provide incentives to the agents, unless the principal can commit herself to stop after a low signal. Without monitoring or this commitment to stop, the continuation decision is independent of the signal and therefore, the wage received by the agent is constant and equals to W_i^c . But at a constant wage, the agents always do a low effort. Then the contract may contain a commitment device (that takes the form of an increase in wages), in order to credibilize principal's strategy.

¹⁵remember that $F < 0$.

In the next subsections, we describe the changes in continuation, monitoring, efforts and contracts decisions.

5.1 Continuation decision:

When (13) doesn't hold, the expected profit after a low signal and two high effort is positive, if the principal doesn't monitor. Then in this case, it is optimal to continue whatever is the observed signal¹⁶. The violation of the inequality (14) has the same implication for the case of one high effort and no monitoring. When the principal monitors, the continuation decision is unchanged compared to the result under assumption 3: it is optimal to continue after H and to stop after L .

5.2 Monitoring condition

After offering wages W_i^c and agents' effort, it is optimal to monitor if the expected increase in benefit is greater than the cost. The profit with monitoring is:

$$q(S - W_1^c - W_2^c) - \mu \quad (15)$$

Where q is the probability of success given by agents' behaviour. The profit without monitoring, when assumption 4 is violated, is:

$$q(S - W_1^c - W_2^c) + (1 - q)(F - W_1^c - W_2^c) \quad (16)$$

Subtracting (16) from (15), it is optimal to monitor at wages W_i^c if:

$$\mu \leq -(1 - q)(F - W_1^c - W_2^c) \quad (17)$$

5.3 Contracts

We assumed that the only contracting variable is the continuation decision. But if the principal takes a continuation decision which is, in the absence of monitoring, independent of the signals, it is not anymore possible to offer a contract to the agent that incite them to perform effort. Then, the principal must offer wages that commit herself either to monitor or to stop after a signal L . The maximization program of the principal is:

$$\max_{W_1^c, W_2^c} \phi(e_1, e_2)(S - W_1^c - W_2^c) - \mu$$

s.t. $EU_i \geq 0, W_i^c \geq 0, i = 1, 2$

$$q(S - W_1^c - W_2^c) - \mu \geq 0 \quad (18)$$

$$\mu \leq -(1 - q)(F - W_1^c - W_2^c) \quad (17)$$

¹⁶If the principal stops, the profit is zero (cfr. lemma 1).

or

$$\frac{q(1 - \pi_H)}{1 - q\pi_H}S + \frac{1 - q}{1 - q\pi_H}F - (W_1^c + W_2^c) \leq 0 \quad (19)$$

$$\mu \geq q\pi_H(S - W_1^c - W_2^c) \quad (20)$$

The first constraints are the standard individual rationality and limited liability. The constraints (18) is a positive profit requirement, and (17) ensures that it is optimal for the principal to monitor. If the constraints (17) and (18) are satisfied, it is optimal to monitor. Alternatively, the principal can choose wages such that (20) is satisfied and commit herself to stop after a low signal by choosing wages that respect the fifth constraint (19). The constraint (20) ensures that it is not optimal to monitor. (19) ensures that it is optimal to stop after L without monitoring. The behaviour of the agents at wages W_i^c is described by equations (6) to (8). The only change for the agents is their expectations about principal's monitoring behaviour. The solution to this problem is given in proposition 2.

Proposition 2 *When assumption 4 is violated, there are seven equilibria: (e_1^h, e_2^h, M) ; (e_i^h, e_j^l, M) , $i = 1, 2$; (e_1^h, e_2^h, NM) ; (e_i^h, e_j^l, NM) , $i = 1, 2$; (e_1^l, e_2^l, NM) . The wages in the monitoring equilibria are either equal to those of proposition 1, if at these wages, the monitoring condition (17) is satisfied or by the monitoring condition. The wages in no monitoring equilibria are the lowest wages that satisfy (19) and (20). In all the case, the wages must be at least greater than those of proposition 1 to extract the desired level of effort from the agents.*

The set of contract and a proof of proposition 2 are given in the appendix. What is clear is that, to extract effort from the agents, the principal must commit herself to a given monitoring and continuation decision. The goal of this commitment is to have a continuation decision that is contingent on the observed signal. Given our restrictions in the contracting variables, the only way to commit to a given behaviour is to increase the wages in order to make a deviation from the prescribed behaviour harmful. The principal may either commit to monitor. In this case, she must increase the wages only if the optimal wages of proposition 1 do not satisfy the monitoring condition (17). Or she commits to stop after a low signal by increasing the wages above their optimal level to have a an null or negative expected profit after a low signal if she continues. The expected profit will be null if at these wages the no monitoring condition (20) is satisfied. If not, the principal has to increase the wage further in order to fulfill the no monitoring condition.

The following remark analyses the global efficiency of this solution.

Remark 3 *When the principal needs to credibilize her monitoring and continuation decisions, it decreases global efficiency.*

There are new distortions in the contract and monitoring decisions. First, the additional wage that is necessary to credibilize a strategy increases the distortions in the ex ante choice of contract described in remark 1. This introduces a new bias that favors delegation of investment and no production. Second, the monitoring decision is inefficient. The decision is based on equation (17) instead of equation (5). Previously, when the

wages increased, it became more difficult to monitor. Under this new setting, when the wages increase, it becomes easier to monitor. This difference comes from the benefit of monitoring. When it is optimal to stop after L , the benefit of monitoring is an increase by $q(1 - \pi_H)$ in the number of cases where the relation ends with a success. When it is not optimal to stop after L , the benefit is a decrease by $(1 - q)$ in the number of cases where the relation ends with a failure. The losses associated with a failure are: $F - W_1^c - W_2^c$. This explain why the wages affect differently the incentive to monitor. It is not clear that the principal monitors less when assumption 4 is violated. She monitors less if: $(1 - q)(-F + W_1^c + W_2^c) \leq q(1 - \pi_H)(S - W_1^c - W_2^c)$

6 Conclusion

The main message of this paper is that without input or output observability, it is not possible to implement the efficient team production, even if the principal observes a signal reflecting perfectly the output's quality. To achieve the first best, the principal needs to make payment to the agents contingent either on the observed input or on the observed output.

If we suppress the limited liability constraint, and we deal with a team of risk neutral agents, it doesn't guarantee that the first best could be achieved. An incentive scheme as the one described in remark 1, where the agents and principal's interest are aligned by making the agent responsible of the losses of the project may leads to the collapse of the incentive system if with the lower W_i^c , the fourth assumption is not satisfied.

This raise the question of the credibility of the strategies. When the signal are too noisy or the loss are not high enough in absolute value, the incentive system loses its credibility and to restore the credibility, the principal needs to introduce new distortions in her incentive scheme.

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A Proof of lemma 1

The proof is quite simple. If at wages $W_i^c, W_i^s = 0$ the agents choose an effort level that give an ex ante probability of continuation \hat{q} and if it is optimal to monitor, it implies that:

$$\hat{q}(S - W_1^c - W_2^c) - \mu \geq \hat{q}\pi_H(S - W_1^c - W_2^c) \quad (21)$$

At wages \overline{W}_i^c , if the behaviour of the agents is not changed, and if it is not optimal to monitor, it implies:

$$\mu \geq \hat{q}(1 - \pi_h)(S - \overline{W}_1^c - \overline{W}_2^c) \quad (22)$$

Combine (21) and (22) we have:

$$\hat{q}(1 - \pi_h)(S - W_1^c - W_2^c) \geq \hat{q}(1 - \pi_h)(S - \overline{W}_1^c - \overline{W}_2^c)$$

Who implies:

$$\overline{W}_1^c + \overline{W}_2^c \geq W_1^c + W_2^c \quad (23)$$

Then if (21) is true, the following must be true:

$$\hat{q}(S - W_1^c - W_2^c) - \mu \geq \hat{q}\pi_H(S - \overline{W}_1^c - \overline{W}_2^c)$$

And this proves the lemma.

B Proof of proposition 1

To prove proposition 1, we first show how we compute the set of contract then we show that the contract that imply a random choice of effort by manager are always dominated. Finally, we proof the uniqueness of the equilibrium.

B.1 Determination of the set of contracts

We compute the set of contract simply by solving principal's maximization problem. She chooses the wages $(W_1^c, W_1^s, W_2^c, W_2^s)$ that maximize her profits (given by equation 2). Subject to the following constraints: First, agents must receive at least their reservation utility normalized to zero (individual rationality). Second, wages must be non negative (limited liability) and third, if the principal wants the agent to follow a certain behaviour, wages must be such that agents freely chooses this behaviour (incentive compatibility).

The first lemma says that the limited liability constraint binds at W_i^s .

Given this, we look for all agents and principal behaviour, excluding for the moment those who imply random choices of effort, the lowest wages W_i^c that satisfy the corresponding incentive compatibility constraints (these constraints are given for each behaviour by equations 6 to 8). In the next subsection, we proof that principal never selects contract for which agents random their choices.

We proceed cases by cases:

For $e_1 = e_1^l, e_2 = e_2^l$ and no monitoring, the principal maximizes her profit by offering $W_1^c = W_2^c = 0$. For no effort, the principal pays no wages. Because of the complementarity between productive effort and monitoring, the principal never monitors when both agents select a low effort.

For $e_1 = e_1^h, e_2 = e_2^l$ and no monitoring, the lowest wages are $W_2^c = 0$ and $W_1^c = \frac{e_1^h}{p}$. This wage is the wage that makes agent 1 indifferent between e_1^h and e_1^l , given that agent 2 does no effort (e_2^l) and principal monitors which requires that: $\mu \leq p(S - \frac{e_1^h}{p})$.

For $e_1 = e_1^h, e_2 = e_2^l$ and monitoring, the lowest wages are $W_2^c = 0$ and $W_1^c = \frac{e_1^h}{\pi_H p}$. This wage is the wage that makes agent 1 indifferent between e_1^h and e_1^l , given that agent 2 does no effort (e_2^l) and principal doesn't monitor (which requires that: $\mu \geq p(S - \frac{e_1^h}{\pi_H p})$).

The cases $e_1 = e_1^l, e_2 = e_2^h$, no monitoring, $e_1 = e_1^l, e_2 = e_2^h$, monitoring, are symmetric to the two previous one.

For $e_1 = e_1^h, e_2 = e_2^h$ and monitoring, the principal selects the wages that make both agents indifferent between a high and a low effort, given that she monitors and that the other agent is doing a high effort. These wages are $(W_1^c, W_2^c) = (\frac{e_1^h}{\pi-p}, \frac{e_2^h}{\pi-p})$. It requires that: $\mu \leq \pi(S - \frac{e_1^h}{\pi-p} - \frac{e_2^h}{\pi-p})$.

For $e_1 = e_1^h, e_2 = e_2^h$ and no monitoring, the principal selects the wages that make both agents indifferent between a high and a low effort, given that she doesn't monitor and that the other agent is doing a high effort. This gives wages $(W_1^c, W_2^c) = (\frac{e_1^h}{\pi_H(\pi-p)}, \frac{e_2^h}{\pi_H(\pi-p)})$. It requires that: $\mu \geq \pi(S - \frac{e_1^h}{\pi_H(\pi-p)} - \frac{e_2^h}{\pi_H(\pi-p)})$.

To complete the proof, we must show that there is no profitable deviations from these contracts. If one contract is optimal, the only potentially profitable deviation is to change the wages in order to modify the monitoring behaviour but not the agents' choices of effort. Consider first a contract where the principal monitors. We know from lemma 2, that it is not optimal to change the wages (it is necessary an increase in total wages) to have no monitoring¹⁷. From a contract with monitoring, there is no profitable deviation.

Second, consider a deviation from a contract where the principal doesn't monitor. This means that at these wages, the monitoring condition (5) isn't satisfied. Then to have monitoring, the principal must decrease wages. But if there exist wages smaller than the initial one where the monitoring condition is satisfied and where the agent behaviour is unchanged, the initial contract cannot be optimal. If so, it would contradict lemma 1.

B.2 Elimination of contracts that induce mixed strategies

To show that the center never wants to offer a contract that induce random choices of efforts by agents, we proceed in two steps: we first compute an upper bound of profit that can be reached with mixed strategies in effort game and after show that this upper bound is always dominated by other contracts.

¹⁷As the monitoring decision is unobservable, there is no credible commitment for the principal to a non monitoring behaviour. Then the only way to have no monitoring is to change the wages, in order to have condition 5 unsatisfied.

We first concentrate on wages $W_1^c \in [\frac{\Delta e_1^h}{p}, \frac{\Delta e_1^h}{\pi-p}]$ and $W_2^c \in [\frac{\Delta e_2^h}{p}, \frac{\Delta e_2^h}{\pi-p}]$, and we assume that for all these wages, the monitoring condition is satisfied. We after extend the proof (trivially) to the cases where it isn't satisfied.

We define $q = \delta_i \delta_j \pi + (1 - \delta_i) \delta_j p + (1 - \delta_j) \delta_i p$ as the ex ante probability of success with mixed strategies. Replacing δ_i and δ_j by their value, given in subsection 3.3 we have: $q = \frac{p^2 W_1^c W_2^c - \Delta e_1^h \Delta e_2^h}{(2p - \pi) W_1^c W_2^c}$ We now compute, for each $q \in [0, \pi]$ the smallest total wage that gives a probability of success equals to q by solving the following program

$$\min_{W_1^c, W_2^c} W_1^c + W_2^c$$

s.t.

$$q = \frac{p^2 W_1^c W_2^c - \Delta e_1^h \Delta e_2^h}{(2p - \pi) W_1^c W_2^c}$$

This give wages equal to:

$$W_1^c = W_2^c = \frac{\sqrt{\Delta e_1^h \Delta e_2^h}}{\sqrt{p^2 - (\pi - 2p)q}} \quad (24)$$

For the rest of this subsection, we assume (for notational simplicity) that $\Delta e_1^h = \Delta e_2^h$ and we call after the disutility of effort Δe^h . The proof trivially extends to other cases.

We define q^* as the solution of the following program:

$$\max_q q \left(S - \frac{2\Delta e^h}{\sqrt{p^2 + (\pi - 2p)q}} \right) - \mu$$

q^* correspond to the upper bound of profit with mixed strategy contract. The solution q^* satisfies the following equality:

$$S = \frac{2\Delta e^h}{\sqrt{p^2 + (\pi - 2p)q^*}} + \frac{q^*(2p - \pi)\Delta e}{(p^2 + (\pi - 2p)q^*)^{(3/2)}} \quad (25)$$

The right hand side of equality (25) is the marginal cost of q , the left hand side is the marginal benefit. The marginal cost is increasing and convex. By convexity of the marginal cost, q^* is unique.

If q^* is smaller than p , This solution is always dominated by the contracts $(\frac{\Delta e_1^h}{p}, 0)$ and $(0, \frac{\Delta e_2^h}{p})$ that gives a higher probability of success ($= p$) at a lower cost.

If q^* is greater than π , the maximal profit with mixed strategies is (by convexity of the marginal cost) when $W_i^c = \frac{\Delta e_i^h}{\pi-p}$, which gives degenerated mixed strategies: $\delta = \gamma = 1$.

If q^* is in $[p, \pi]$, it implies that the marginal cost at $q = p$ must be smaller than the marginal benefit S :

$$\frac{\pi \Delta e^h}{\sqrt{p(\pi - p)(\pi - p)}} \leq S \quad (26)$$

and the marginal cost at $q^* = \pi$ must be greater than S :

$$\frac{\Delta e^h}{\pi - p} + \frac{p^2 \Delta e}{(\pi - p)^3} \geq S \quad (27)$$

We proof then that if (26) is satisfied, q^* is dominated by a pure strategy with one effort. If the existence condition (26) is satisfied, it doesn't satisfy the dominance condition (28):

$$p\left(S - \frac{\Delta e^h}{p}\right) \geq q^*\left(S - \frac{2\Delta e^h}{\sqrt{p^2 + (\pi - 2p)q^*}}\right) \quad (28)$$

Rewriting (26) and (28) we have:

$$\frac{\pi \Delta e^h}{\sqrt{p(\pi - p)}} \leq (\pi - p)S \quad (29)$$

$$(\pi - p)S \leq \frac{\pi - p}{p - q} \left(\Delta e^h - \frac{q2\Delta e^h}{\sqrt{p^2 + (\pi - 2p)q}} \right) \quad (30)$$

Combining (17) and (18),to proof the claim, we have to show that:

$$\frac{\pi \Delta e^h}{\sqrt{p(\pi - p)}} \leq \frac{\pi - p}{p - q} \left(\Delta e^h - \frac{q2\Delta e^h}{\sqrt{p^2 + (\pi - 2p)q}} \right) \quad (31)$$

Simplifying (31):

$$\frac{\pi(q - p)}{\sqrt{p(\pi - p)}} \leq (\pi - p) \left(\frac{2q}{\sqrt{p^2 + (\pi - 2p)q}} - 1 \right) \quad (32)$$

And this inequality is satisfied strictly for all $q \in [p, \pi]$: First note that (32) is satisfied with inequality for $q = p$. And the rate of growth of right hand side is positive and increasing while the rate of growth of left hand side is positive but constant. For $q = p$, rate of growth are equal. It implies that (30) is always satisfied.

As we shown the contracts that induce mixed strategies are dominated by $(W_i^c, W_j^c) = (\frac{\Delta e_i^h}{p}, 0)$. to be valid, it must be that if the monitoring condition is satisfied for the wages that induce mixed strategies, the monitoring condition is also satisfied for $(W_i^c, W_j^c) = (\frac{\Delta e_i^h}{p}, 0)$. We just proved that:

$$p\left(S - \frac{\Delta e_h}{p}\right) > q(S - W_1^c - W_2^c), \forall q$$

It implies that if:

$$q(1 - \pi_H)(S - W_1^c - W_2^c) \geq 2\mu$$

then

$$p(1 - \pi_h)\left(S - \frac{\Delta e_h}{p}\right) \geq 2\mu$$

This last equation is the monitoring condition for $(W_i^c, W_j^c) = (\frac{\Delta e_i^h}{p}, 0)$.

Up to now, we considered that the monitoring conditions were satisfied for all the wages that induced mixed strategies. When it is not the case and for some $W_1^c \in [\frac{\Delta e_1^h}{\pi_H p}, \frac{\Delta e_1^h}{\pi_H(\pi-p)}]$ and $W_2^c \in [\frac{\Delta e_2^h}{\pi_H p}, \frac{\Delta e_2^h}{\pi_H(\pi-p)}]$, we can show that these contracts are dominated by $(W_i^c, W_j^c) = (\frac{\Delta e_i^h}{\pi_H p}, 0)$. The proof is the same replacing p by $p' = p\pi_h$ and π by $\pi' = \pi\pi_H$.

B.3 Uniqueness of equilibrium

The only problematic case is for the contract $(\frac{\Delta e_1^h}{\pi-p}, \frac{\Delta e_2^h}{\pi-p})$ when the monitoring condition is satisfied if agents select (e_1^h, e_2^h) and the monitoring condition isn't satisfied if agents random their effort choice. In this case two equilibria may exist but if $\Delta e_1^h \neq \Delta e_2^h$, there is a possibility of reaching the same ex ante probability of success given by the mixed strategy at a lower total cost by giving equal wages determined by equation (24). The mixed strategy contract is any more an equilibrium in the entire game.

C Proof of corollary 1

The proof is simplified by the results of lemma 2. In this lemma, we proved that a no monitoring equilibrium cannot dominate a monitoring equilibrium in which the agents select the same level of effort. Then to prove the corollary, we just need to prove that a no monitoring equilibrium can not dominate a monitoring equilibrium with different effort levels. There are only two cases to consider:

Case 1: Suppose that (e_1^h, e_2^h, NM) dominates (e_i^h, e_j^l, M) .

Our proof is simple, we just show that the condition for the existence of monitoring equilibrium are not compatible with the conditions for the dominance of a no monitoring equilibrium.

The monitoring equilibrium exist if at wages $(W_i^c, W_j^c) = (\Delta e_i^h p, 0)$, the following monitoring condition is satisfied:

$$\mu \leq p(1 - \pi_H)(S - \frac{\Delta e_i^h}{p}) \quad (33)$$

If the equilibrium (e_1^h, e_2^h, M) exists, it cannot dominate (e_i^h, e_j^l, M) , otherwise lemma 2 applies. So, either this equilibrium does not exist:

$$\mu \geq \pi(1 - \pi_H)(S - \frac{\Delta e_i^h + \Delta e_j^h}{\pi - p}) \quad (34)$$

or it is dominated by (e_i^h, e_j^l, M) :

$$(\pi - p)S \leq \frac{p\Delta e_i^h + \pi\Delta e_j^h}{\pi - p} \quad (35)$$

The no monitoring equilibrium dominates the monitoring one if:

$$\pi\pi_h(S - \frac{\Delta e_i^h + \Delta e_j^h}{\pi_H(\pi - p)}) \geq p(S - \frac{\Delta e_i^h}{p}) - \mu$$

Who can be rewritten as:

$$\mu \geq \frac{p\Delta e_i^h + \pi\Delta e_j^h}{\pi - p} + (p - \pi\pi_H)S \quad (36)$$

Now, we show that there is an incompatibility between the conditions (33), (34) and (35) or alternatively between (33), (34) and (36). A necessary condition for (33) and (34) is:

$$\pi(1 - \pi_H)(S - \frac{\Delta e_i^h + \Delta e_j^h}{\pi - p}) \geq p(1 - \pi_H)(S - \frac{\Delta e_i^h}{p})$$

And this equation is exactly equation (36).

A necessary condition for (33) and (35) is:

$$\begin{aligned} \frac{p\Delta e_i^h + \pi\Delta e_j^h}{\pi - p} + (p - \pi\pi_H)S &\leq p(1 - \pi_H)(S - \frac{\Delta e_i^h}{p}) \\ \iff (\pi - p)S &\geq \frac{\pi\Delta e_i^h + \pi\Delta e_j^h}{(\pi - p)\pi_H} - \Delta e_i^h \end{aligned} \quad (37)$$

the two necessary conditions are compatible if:

$$\Delta e_i^h \leq -\Delta e_j^h$$

Which is impossible and prove that our initial supposition was wrong.

Case 2: Suppose that (e_i^h, e_j^l, NM) dominates (e_1^h, e_2^h, M) .

We proceed as in the previous case: The monitoring equilibrium exists if:

$$\mu \leq \pi(1 - \pi_H)(S - \frac{\Delta e_i^h + \Delta e_j^h}{\pi - p}) \quad (38)$$

The monitoring equilibrium (e_i^h, e_j^l, M) doesn't exist if:

$$\mu \geq p(1 - \pi_H)(S - \frac{\Delta e_i^h}{p}) \quad (39)$$

Or is dominated by (e_1^h, e_2^h, M) if:

$$(\pi - p)S \geq \frac{p\Delta e_i^h + \pi\Delta e_j^h}{\pi - p} \quad (40)$$

The equation (38) and (39) are compatible only (40) is true. The no monitoring equilibrium dominates if:

$$p\pi_H(S - \frac{\Delta e_i^h}{\pi_H p}) \geq \pi(S - \frac{\Delta e_i^h + \Delta e_j^h}{\pi - p}) - \mu$$

$$\iff \mu \geq (\pi - p\pi_H)S - \frac{p\Delta e_i^h + \pi\Delta e_j^h}{\pi - p} \quad (41)$$

(38) and (41) are compatible only if:

$$(\pi - p)S \leq \frac{\pi\Delta e_i^h + \pi\Delta e_j^h}{\pi - p} - \frac{\Delta e_i^h}{\pi_H} \quad (42)$$

(40) and (42) are compatible only if:

$$\pi_H \geq 1$$

And this is impossible and prove the corollary.

D Proof of proposition 2

To proof this second proposition, we proceed case by case. The argument given in proposition 1 to eliminates contracts that induce mixed strategies is still valid. For each equilibrium, we look at the wages that satisfy the relevant constraints.

Team production and monitoring(e_1^h, e_2^h, M)

The wages W_1^c, W_2^c are equal to:

$$W_i^c = \frac{\Delta e_i^h}{\pi - p} \quad i = 1, 2$$

If $\mu \leq -(1 - \pi)(F - \frac{\Delta e_1^h + \Delta e_2^h}{\pi - p})$

Otherwise, W_1^c, W_2^c are such that:

$$W_1^c + W_2^c = \frac{\mu}{1 - \pi} + F \leq S - \frac{\mu}{\pi}$$

$$W_i^c \geq \frac{\Delta e_i^h}{\pi - p} \quad i = 1, 2$$

Delegation to agent i , and monitoring (e_i^h, e_j^l, M), $i, j = 1, 2$

The wages W_i^c, W_j^c are equal to

$$(W_i^c, W_j^c) = \left(\frac{\Delta e_i^h}{p}, 0\right)$$

If $\mu \leq -(1 - p)(F - \frac{\Delta e_i^h}{p})$

Otherwise, W_i^c, W_j^c are such that:

$$W_1^c + W_2^c = \frac{\mu}{1 - p} + F \leq S - \frac{\mu}{p}$$

and $W_i^c \geq \frac{\Delta e_i^h}{p}$ and $0 \leq W_j^c \leq \frac{\Delta e_j^h}{p}$

Team production and no monitoring (e_1^h, e_2^h, NM)
The wages must respect the two constraints:

$$W_1^c + W_2^c \geq \frac{\pi(1 - \pi_H)}{1 - \pi\pi_H}S + \frac{1 - \pi}{1 - \pi\pi_H}F$$

$$W_1^c + W_2^c \geq S - \frac{\mu}{\pi(1 - \pi_H)}$$

And must respect the following constraint: $W_i^c \geq \frac{\Delta e_i^h}{\pi_H(\pi - p)}$ $i = 1, 2$

Delegation to agent i and no monitoring (e_i^h, e_j^l, NM) The wages must respect the two constraints:

$$W_1^c + W_2^c \geq \frac{p(1 - \pi_H)}{1 - p\pi_H}S + \frac{1 - p}{1 - p\pi_H}F$$

$$W_1^c + W_2^c \geq S - \frac{\mu}{p(1 - \pi_H)}$$

And must respect the following constraint: $W_i^c \geq \frac{\Delta e_i^h}{\pi_H p}$, $0 \leq W_j^c \leq \frac{\Delta e_j^h}{\pi_H p}$

No production (e_1^l, e_2^l, NM)

$$W_1^c = W_2^c = 0$$