FISCAL AND MONETARY POLICY INTERACTIONS IN A LIQUIDITY TRAP WHEN GOVERNMENT DEBT MATTERS

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DISCUSSION PAPER | 2020 / 33

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Fiscal and Monetary Policy Interactions in a Liquidity Trap when Government Debt Matters*

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November 10, 2020

Abstract

What does central bank independence imply for the optimal conduct of time-consistent fiscal and monetary policy in a liquidity trap? To provide an answer, I consider a stochastic noncooperative game in which the lower bound on nominal rates is an occasionally binding constraint and in which government debt serves as a tool to influence future policy trade-offs. I show that a transitory consolidation of debt in the liquidity trap optimally reduces expected real rates and stimulates current consumption and inflation via an expectation channel. The reaction function of the independent central bank outside the lower bound is pivotal in obtaining this result—considering instead coordinated policy produces the opposite effect of an optimal increase in debt. Lengthening the debt maturity allows to mitigate issues related to lack of coordination.

Keywords: Optimal Time-Consistent Policy, Distortionary Taxation, Liquidity Trap, Fiscal and Monetary Policy Interactions

*The views expressed in this paper are those of the author and do not necessarily represent those of the National Bank of Belgium.

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‡I am grateful to my thesis advisors, Rigas Oikonomou and Raf Wouters, and to participants at the Belgian Macroeconomic Workshop, seminar at Université Catholique de Louvain and the National Bank of Belgium for useful comments. All remaining errors are mine.

"Moreover, if central banks were to enter into a form of coordination with fiscal authorities that reduced their independence, it would ultimately be self-defeating."

– Draghi (October 2018)

1 Introduction

Recent policy responses to economic crises included substantial interest rate cuts in combination with debt-financed fiscal stimuli. In the aftermath of the Great Recession, this resulted in the nominal rate being stuck at the lower bound for an extended period of time. Moreover, structural deficits remained high even after the automatic cyclical response of revenues and spending had reversed due to economic recovery. At the same time, prominent economists advocated for a progressive return to lower debt levels claiming that it could help steering expectations in a desired direction.[1]

Standard New-Keynesian models are however silent about the role of government debt in a liquidity trap because they assume lump-sum taxes and Ricardian fiscal policy. Hence, households are indifferent between tax-financing and debt-financing. According to this literature, the fiscal multiplier is superior to one at the lower bound (Christiano et al., 2011; Erceg and Lindé, 2014) and optimal fiscal policy is therefore expansionary (Schmidt, 2013; Nakata, 2016).

In the presence of labor taxes, debt-financed fiscal stimuli may however distort households consumption and saving decisions across time. In particular, higher labor taxes push up the marginal cost of production and thus create an endogenous trade-off between output gap and inflation stabilisation. This trade-off persists so long as debt is above steady state.[2]

[1] For example, this was suggested by Ben Bernanke (June 2011): "*...At the same time, acting now to put in place a credible plan for reducing future deficits would not only enhance economic performance in the long run, but could also yield near-term benefits by leading to lower long-term interest rates and increased consumer and business confidence...*"

[2] In my model, there is a subsidy which offsets the steady state distortions from nominal rigidities and monopolistic competition. Hence, steady state debt is consistent with a non-distortionary tax rate and stable inflation.

Consequently, whenever a time-consistent government inherits a large stock of nominal debt, it has an incentive to erode its real value with surprise inflation.[3] Leeper et al. (2020) have labeled this tendency the *inflationary bias*.

Since households anticipate this bias, inflation expectations (and inflation itself) rise until the temptation is removed and debt is on the path of returning to its efficient steady state level. Bai et al. (2017) point out that this *debt stabilisation bias* is suboptimal because it creates excessive inflation volatility with respect to the Ramsey outcome. As demonstrated by Schmitt-Grohé and Uribe (2004), the debt issued by a Ramsey planner resembles to a random walk because of the (time-inconsistent) commitment not to raise future inflation when debt increases, and instead to spread the burden through time with labor taxes.

In a liquidity trap, the debt stabilisation bias is nevertheless instrumental in steering inflation expectations. With one-period bonds, accommodating the inflationary bias brings large benefits for the joint fiscal and monetary policy because the roll-over-cost of government liabilities is more sensitive to short-term rate variations. Hence, stabilising debt at a higher pace prevents inflation to rise further in the future. This accomodative stance of monetary policy was first identified by Burgert and Schmidt (2014) as being at the heart of the optimal increase in debt when the nominal rate hits the lower bound. As the economy exits the liquidity trap, the nominal rate stabilises debt by remaining below the level warranted by considerations of inflation and output gap stabilisation alone. In effect, the lower expected real rates stimulate consumption and output, and thereby mitigate the inflation shortfall stemming from the interest rate gap. In this respect, this policy may be seen as a substitute to forward guidance when the policymaker lacks commitment. It is effectively signaling that debt stabilisation considerations will push monetary policy to keep its rate under target after the lift-off from the lower bound.

This paper shows that the effectiveness of this channel and, the way it plays out, tightly depends on the assumption about the institutional set-up. The existing literature usually

[3] As explained by Niemann et al. (2013), surprise inflation is akin a lump-sum tax on the financial wealth of the households that eases future policy trade-offs.

assumes that monetary and fiscal policy are jointly optimal: the policymaker is allowed to choose the policy instruments simultaneously while facing a consolidated budget constraint. Empirically, there is however little reason to think that monetary policy would be bound to help public finances when the situation normalises on the grounds that it would alleviate inflationary biases. Such a claim would ex-ante provide perverse incentives to the fiscal authority regarding its fiscal sustainability. It is precisely this sort of considerations which underlie the independence granted to central banks in most advanced economies to pursue their goals, namely inflation stability and output stabilisation.[4]

A growing strand of the literature thus models the interactions between fiscal and monetary policy as a noncooperative game. Some contributions, such as Dixit and Lambertini (2003); Adam and Billi (2008); Blake and Kirsanova (2011) have focused on the gains of appointing a conservative central bank when the lack of coordination generates policy biases. Gnocchi (2013), and more recently Camous and Matveev (2020), have introduced asymmetry of commitment in a noncooperative game to study the welfare gains from having a central bank which disciplines the fiscal authority.

In line with the above papers, I consider here a central bank and a fiscal authority that play a noncooperative game. I focus on the Markov-Perfect Equilibria under Stackelberg leadership of the non-Ricardian fiscal policy. This institutional set-up is consistent with a central bank that is transparent about its goals and conducts its policy independently from fiscal considerations. There are three main results stemming from this analysis.

First, considering a noncooperative game overturns the classical policy insight (see e.g. Burgert and Schmidt, 2014; Matveev, 2020) that a government with short-term liabilities should provide a large debt-financed fiscal stimulus in the liquidity trap. Instead, in my model, a transitory consolidation of debt financed with labor taxes becomes optimal. To understand this result, I study a simplified example and show analytically that the inflation-

[4] This was emphasised by Mario Draghi (October 2018): "*...if central banks were to enter into a form of coordination with fiscal authorities... Fiscal authorities would have an incentive to use monetary policy to achieve other objectives. And this would end up with monetary policy becoming fiscally dominated, which history shows is inconsistent with price stability in the long run...*"

ary bias created by a debt increase leads the independent central bank to *lean against the wind* when the nominal rate lifts-off the lower bound. Since this contractionary monetary policy is anticipated by the rational households, issuing more debt depresses consumption in the liquidity trap and exacerbates the inflation shortfall. In this context, numerical simulations under plausible calibration reveal that the government provides only a tame—tax-financed—fiscal stimulus and reduces debt to keep inflation under target at the exit of the lower bound. This strategy supports consumption of forward-looking households who expect labor tax cuts and expansionary monetary policy during the recovery. Nevertheless, the subdued response of government spending with respect to the joint policy causes a deeper recession in the near-term.

Second, liquidity trap episodes arise more often when fiscal and monetary policy are conducted noncooperatively. This can be explained by the persistently low inflation expectations stemming from the optimal debt consolidation undertaken at the outset of the liquidity trap. To mitigate this adverse effect, the government accumulates more debt in the risky steady state and sustains a higher nominal rate and inflation rate relative to the coordinated policy.

Third, the maturity of debt has little effect on the outcome of the noncooperative game because the central bank remains adamant in defending its inflation target. However, as demonstrated by Matveev (2020), coordinated policy involves a reduction of debt when maturity is longer. Therefore, increasing the maturity of debt alleviates coordination issues.

The remainder of the paper is organised as follows. Section 2 describes the key ingredients of the model and outlines the equilibrium conditions in log-linear form. The policy problem and the structure of the noncooperative game are also laid out. Section 3 derives the optimality conditions under the benchmark of coordination while section 4 treats the noncooperative game under Stackelberg leadership of the fiscal authority. Section 5 presents an analytical example characterising the response of consumption and government spending to a debt increase in the liquidity trap. Section 6 solves the model numerically and discusses the main results of the paper. Finally, section 7 concludes.

2 The model

The aim of this paper is to study the strategically optimal behaviour of monetary and fiscal policy at the zero lower bound (ZLB). The model is a cashless New Keynesian economy in which fiscal policy is non-Ricardian due to the presence of distortionary taxation. The non-policy block of the model is broadly similar to Leeper et al. (2020); Matveev (2020). For the sake of clarity, I provide an overview of the main ingredients here.

The private sector is composed of an infinitely lived representative household, a representative aggregate good producer and intermediate good producers which compete monopolistically and are subject to costly price adjustments. The public sector is represented by two institutions, a central bank (CB) and a fiscal authority (FA), which I consider independently from one another.

2.1 Households and firms

The representative household derives utility from consuming the private good c_t and the public good G_t while it dislikes hours worked h_t. I assume a separable utility function leading to the following expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \xi_t \Big[\frac{c_t^{1-\gamma_c}}{1-\gamma_c} + \nu_g \frac{G_t^{1-\gamma_g}}{1-\gamma_g} - \nu_h \frac{h_t^{1+\gamma_h}}{1+\gamma_h} \Big] \tag{1}$$

where $\mathbb{E}_t$ is the rational expectations operator conditional on information in period t, β is the time discount factor. Parameters γ_c and γ_g are respectively the intertemporal eslasticy for private and government consumption and, γ_h is the inverse of the Frisch elasticity. I also attach utility weights ν_g and ν_h to characterise preference for government consumption and hours, relatively to private consumption.

The variable ξ_t is an exogenous process characterising the time preference. Under this specification, time preference between states of two consecutive periods evolves according to $\xi_t/(\beta\xi_{t+1})$. Since this process is the only source of fluctuations in this economy, I write it directly in terms of the natural real rate

$$r_t^n = \rho_r r_{t-1}^n + \epsilon_t \tag{2}$$

where $r_t^n \equiv \xi_{t+1}/\xi_t$ and $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$ is a normally distributed exogenous shock that will be the source of liquidity trap episodes.

The household sells hours to intermediate firms for a wage W_t net of labor taxes τ_t and may save via two nominal and non state-contingent assets: a one-period bond B_t^s and a perpetual bond B_t. Following Woodford (2001), the perpetuity yields a coupon with payoff decaying at exponential rate ρ. Consequently, when $\rho = 0$, the short-term bond and the perpetuity have the same one-period maturity.[5] Firm profits yield a dividend $\Pi_{i,t}$ and lump-sum transfers T_t are collected from the government. The household budget constraint (in real terms) is:

$$c_t + \frac{b_t^s}{R_t} + q_t B_t = (1-\tau_t) w_t h_t + \frac{b_{t-1}^s}{\pi_t} + (1+\rho q_t)\frac{b_{t-1}}{\pi_t} + \int_0^1 \frac{\Pi_{i,t}}{P_t} di + \frac{T_t}{P_t} \tag{3}$$

where R_t is the gross nominal interest rate, q_t is the price of the perpetual bond and π_t is the gross inflation rate. The household chooses $\{c_t, h_t, B_t^s, B_t\}_{t=0}^\infty$ to maximise expected lifetime utility (1) subject to (3) and the no-Ponzi scheme conditions on the two bonds

$$\lim_{j\to\infty} \beta^j \left(\left(\prod_{k=0}^{t+j} r_k^n \right) \frac{B_{t+j}^s}{c_{t+j+1}^{\gamma_c} \pi_{t+j+1}} \right) \geqslant 0 \quad \text{and} \quad \lim_{j\to\infty} \beta^j \left(\left(\prod_{k=0}^{t+j} r_k^n \right) \frac{B_{t+j}(1+\rho q_{t+j+1})}{c_{t+j+1}^{\gamma_c} \pi_{t+j+1}} \right) \geqslant 0$$

Intermediate firms operate under monopolistic competition and seek to maximise profits subject to quadratic price adjustment costs à la Rotemberg (1982). From the profit maximisation problem of the final producer, the demand function of the generic firm producing

[5] I only introduce the short-term bond B_t^s because I want to be able to refer to the short-term yield even when the maturity of the perpetuity is superior to one period.

i is given by

$$y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\theta} y_t \tag{4}$$

where θ is the marginal rate of substitution between varieties. The program of the firm i is

$$\max_{P_{i,t}} \mathbb{E}_0 \sum_{t=0}^{\infty} \lambda_t \left[\left(\frac{P_{i,t}}{P_t}\right)^{-\theta} y_t \left(\frac{P_{i,t}}{P_t} - (1-s)w_t\right) - \frac{\iota}{2}\left(\frac{P_{i,t}}{P_{i,t-1}} - 1\right)^2 \right] \tag{5}$$

where λ_t is the multiplier of budget constraint from the household problem and ι is the price adjustment cost factor.

Parameter s is an employment subsidy which offsets steady state distortions stemming from monopolistic competition and distortionary taxation. For the rest of this paper, this subsidy is kept constant over time and lump-sum transfers are restricted to the sole purpose of financing it.[6] The reason for introducing this subsidy is to make a positive amount of debt sustainable at the steady state. Absent of the subsidy, a time-consistent policymaker wants to reduce any positive amount of liabilities that successive policymakers will inherit. This debt consolidation removes the incentives to inflate debt away in the future and lowers inflation expectations to their efficient level.[7]

2.2 Public authorities

There are two authorities exercising policy in this economy: a monetary authority (CB) and a fiscal authority (FA). Each authority uses its own instruments to pursue its objective:

- The CB chooses the sequence of short-term nominal interest rates $\{R_t\}_{t=0}^{\infty}$, while being constrained by a zero lower bound (ZLB).
- The FA chooses the sequence of labor taxes and government expenditures, $\{\tau_t, G_t\}_{t=0}^{\infty}$, to finance its net debt position.

[6] This implies that distortions from labor taxation do occur outside the steady state. See Leith and Wren-Lewis (2013) for a similar use of a steady state subsidy.

[7] For a detailed analysis of those dynamics in a real economy and Markov Perfect Equilibrium, see e.g. Debortoli and Nunes (2013).

Assuming that the short-bond is in zero net supply, the budget constraint of the FA reads

$$\frac{b_{t-1}}{\pi_t} = \frac{b_t}{R_t} + \tau_t w_t h_t - G_t - s(w_t h_t - wh)$$

The last term implies that the subsidy is not rebated at the steady state.

2.3 Log-linear approximation

Since the optimality conditions of the household are standard, I proceed by showing the log-linear equations directly.[8] Variables without time-subscript represent steady-state values and hated variables are log-deviations from the steady-state.

$$y\hat{y}_t = c\hat{c}_t + G\hat{G}_t \qquad \text{(Resource Constraint)} \qquad (6)$$

$$\hat{\pi}_t = \kappa_t \hat{\tau}_t + \kappa_c \hat{c}_t + \kappa_y \hat{y}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1} \qquad \text{(Phillips Curve)} \qquad (7)$$

$$\hat{c}_t = -\frac{1}{\gamma_c}(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - r_t^n) + \mathbb{E}_t \hat{c}_{t+1} \qquad \text{(Euler equation)} \qquad (8)$$

$$\hat{i}_t = \rho\beta \mathbb{E}_t \hat{q}_{t+1} - \hat{q}_t \qquad \text{(No arbitrage)} \qquad (9)$$

where $\kappa_t = w\tau(\theta - 1)/\iota$, $\kappa_c = \gamma_c(\theta - 1)/\iota$ and $\kappa_y = \gamma_h(\theta - 1)/\iota$.

The log-linear budget constraint reads

$$\Omega(\hat{b}_t + (1-\rho)\hat{q}_t - \beta^{-1}(\hat{b}_{t-1} - \hat{\pi}_t)) - G\hat{G}_t + w\tau y \frac{\theta - 1}{\theta}\tau_t - \frac{y}{\theta}(\gamma_c \hat{c}_t + (1 + \gamma_y)\hat{y}_t) \qquad (10)$$

where $\Omega \equiv qb$ is the steady state market value of debt.

A private-sector rational expectations equilibrium consists of a sequence $x_t \equiv \{\hat{c}_t, \hat{\pi}_t, \hat{b}_t, \hat{y}_t\}$ satisfying equations (6)–(10), given the policies $p_t \equiv \{\hat{i}_t > -r^*, \hat{G}_t, \hat{\tau}_t\}$, exogenous process $\{\epsilon_t\}$, and initial conditions $\hat{b}_{-1}$.

[8] see Matveev (2020) for a description of the non-linear optimality conditions

2.4 The policy problem

The society delegates the same objective to the two authorities which have to pursue it using their respective instruments. I derive this social objective by taking a second-order approximation of the utility function of the household (1) around the non-stochastic steady-state. Leaving the details of the derivation to Appendix A, the loss function reads

$$\mathcal{L}_t = \frac{1}{2}\left[\gamma_c c\hat{c}_t^2 + \gamma_g G\hat{G}_t^2 + \gamma_h y\hat{y}_t^2 + \iota y\pi_t^2\right] \tag{11}$$

In minimising this loss function, the policymaker is constrained to choose time-consistent actions. Consequently, it cannot commit beyond the repayment of its debt obligations: any promise made to influence agent expectations and ease current policy trade-off would only have a grip if it is optimal not to deviate when the promise has to be met. In other words, we focus on Markov-Perfect Equilibria both between the government and the private sector and between the government in the current period and successive governments.

Notice that, in this framework, time-consistency does not mean that the policymaker has to take expectations as given. When taxes are distortionary, debt policy matters for consumption and saving decisions of households. Hence, the FA can influence the expectations of the households by choosing the adequate stock of debt that the next policymakers will inherit. To account for this possibility, I write expectations in the optimisation problem with the following notation:

$$\mathbb{E}_t\pi_{t+1} \equiv \mathbb{E}_t\Pi(s_{t+1})$$

$$\mathbb{E}_t c_{t+1} \equiv \mathbb{E}_t\mathcal{C}(s_{t+1})$$

$$\mathbb{E}_t q_{t+1} \equiv \mathbb{E}_t\mathcal{Q}(s_{t+1})$$

where $s_t \equiv \{\hat{b}_{t-1}, r_t^n\}$ is the state vector.

This section considers two different policy set-ups: the coordinated policy, which is the case usually assumed in the literature and a noncooperative policy game with Fiscal Leadership:

- **Coordination** assumes that the policymaker minimises the utility loss stemming from the exogenous shock on the natural real rate by choosing the policy instruments available in the economy simultaneously under a consolidated budget constraint. In other words, public authorities internalise the effects of their policy decisions on the constraints and choices of one another. Given Markov-perfection, the joint policymaker is represented by a sequence of authorities with identical preferences, each one leading its future selves. This set-up generally allows to reach the best possible outcome, all other things being equal. This is probably the reason for its broad application in the optimal policy literature.

- **Noncooperative game** assumes that the CB and the FA play a noncooperative Markov Perfect game in which the FA is the Stackelberg leader. The game is described formally as follows:

  Timing of the game—For every period $t \geqslant 0$, the timeline of events is: a) exogenous shock on the natural real rate realises and is observed by both the authorities and the private sector; b) the FA chooses its fiscal instruments; c) the CB chooses the nominal rate; d) economic variables realise. Let vector $\upsilon_t \equiv (r_t^n, \hat{G}_t, \hat{\tau}_t, \hat{i}_t, x_t)$ gather chronologically the intraperiod events. Then, I can define the history of the game as $\phi_t \equiv (\upsilon_t, \phi_{t-1})$ for $t > 0$ and $\phi_0 \equiv (\upsilon_0, \hat{b}_{-1})$ for $t = 0$.

  Strategy of the players—The FA leads the CB and private sector within each period with histories $\phi_t^{FA} \equiv (\hat{b}_{t-1}, r_t^n)$.[9] I denote its strategy by $\omega_{FA} = \{\hat{G}_t(\phi_t^{FA}), \hat{\tau}_t(\phi_t^{FA})\}_{t\geqslant 0}$. The CB then follows with histories $\phi_t^{CB} \equiv (\hat{b}_{t-1}, r_t^n, \hat{G}_t, \hat{\tau}_t)$ and chooses the policy rate according to strategy $\omega_{CB} = \{\hat{i}_t(\phi_t^{CB})\}_{t\geqslant 0}$. Finally, private agents, which face histories $\phi_t^x \equiv (\hat{b}_{t-1}, r_t^n, p_t)$, take decisions according to $\omega_x = \{x_t(\phi_t^x)\}_{t\geqslant 0}$.

[9] I restrict here histories of the FA to the inherited debt and exogenous shock. Hence, I avoid issues related to multiple reputational equilibria as in King et al. (2008).

Notice that, since the FA is the first intraperiod mover, it takes strategy ω_{CB} as a constraint in its optimisation problem. The reason for considering this leadership structure is that the policy pursued by the CB is usually more transparent and can be anticipated by the FA.[10] Moreover, this lack of coordination is more consistent with empirical evidence about institutional set-ups in the US and other advanced economies because the CB is *independent* and not constrained by fiscal variables when choosing its strategy ω_{CB}. Since I want to focus on the interactions between the CB and the FA, I do not consider households and price setters as a player of the game interacting strategically. Instead, private agents optimality conditions constitute a constraint delimiting the set of implementable solutions.

I start by presenting the coordinated problem and then shows how it differs from the noncooperative game.

3 Optimal coordinated policy

The Bellman equation of the coordinated policymaker reads

$$V(s_t) = \min_{\{\hat{c}_t, \hat{G}_t, \hat{y}_t, \hat{\pi}_t, \hat{\tau}_t, \hat{i}_t, \hat{b}_t\}} \frac{1}{2}(\gamma_c c \hat{c}_t^2 + \gamma_g G \hat{G}_t^2 + \gamma_h y \hat{y}_t^2 + \iota y \pi_t^2) + \beta V(s_{t+1})$$

[10] For additional details about the rationale behind this leadership structure, see Bai et al. (2017) and Chen et al. (2019)

such that

$$y\hat{y}_t = c\hat{c}_t + G\hat{G}_t \tag{12}$$

$$\hat{\pi}_t = \kappa_t \hat{\tau}_t + \kappa_c \hat{c}_t + \kappa_y \hat{y}_t + \beta \mathbb{E}_t \Pi(s_{t+1}) \tag{13}$$

$$\hat{c}_t = -\frac{1}{\gamma_c}(\hat{i}_t - \mathbb{E}_t \Pi(s_{t+1}) - r_t^n) + \mathbb{E}_t \mathcal{C}(s_{t+1}) \tag{14}$$

$$\hat{i}_t = \rho\beta \mathbb{E}_t \mathcal{Q}(s_{t+1}) - \hat{q}_t \tag{15}$$

$$\hat{i}_t > -r^* \tag{16}$$

$$0 = \Omega(\hat{b}_t + (1-\rho)\hat{q}_t - \beta^{-1}(\hat{b}_{t-1} - \hat{\pi}_t)) - G\hat{G}_t + w\tau y \frac{\theta - 1}{\theta}\hat{\tau}_t - \frac{y}{\theta}(\gamma_c \hat{c}_t + (1+\gamma_y)\hat{y}_t) \tag{17}$$

where $r^* \equiv ln(\beta^{-1})$.

I attach multipliers $\Lambda_t^r, \Lambda_t^p, \Lambda_t^q, \Lambda_t^i, \Lambda_t^{zlb}$ and Λ_t^b to constraints (12)-(17), respectively. The first order necessary conditions of the program are delegated in Appendix B.

4 Optimal noncooperative policy

4.1 Central bank reaction function

Since the CB acts as a intraperiod follower in the sequential policy set-up, it optimises independently from fiscal policy. Moreover, this paper abstracts from Quantitative Easing policy such that the CB issues zero reserves and holds zero bonds while still setting the interest rate. Hence, its Bellman equation reads

$$U(s_t) = \min_{\{\hat{c}_t, \hat{y}_t, \hat{\pi}_t, \hat{i}_t\}} \frac{1}{2}(\gamma_c c\hat{c}_t^2 + \gamma_g G\hat{G}_t^2 + \gamma_h y\hat{y}_t^2 + \iota y\pi_t^2) + \beta U(s_{t+1})$$

with $\{\hat{c}_{t+j}, \hat{y}_{t+j}, \hat{\pi}_{t+j}, \hat{i}_{t+j} > -r^*, \hat{G}_{t+j-1}, \hat{\tau}_{t+j-1}\}$ given for $j \geq 1$ and such that

$$
\begin{aligned}
&y\hat{y}_t = c\hat{c}_t + G\hat{G}_t \\
&\hat{\pi}_t = \kappa_t \hat{\tau}_t + \kappa_c \hat{c}_t + \kappa_y \hat{y}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1} \\
&\hat{c}_t = -\frac{1}{\gamma_c}(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - r_t^n) + \mathbb{E}_t \hat{c}_{t+1} \\
&\hat{i}_t > -r^*
\end{aligned}
$$

Rearranging the first order necessary conditions give the following reaction function of the CB (RFCB)

$$\gamma_h \hat{y}_t + \gamma_c \hat{c}_t = \gamma_c c^{-1} \mu_t^{zlb} - \Psi \hat{\pi}_t \tag{18}$$

where μ_t^{zlb} is the lagrange multiplier associated with the ZLB constraint and $\Psi \equiv \iota y(c^{-1}\kappa_c + y^{-1}\kappa_y)$. Outside the lower bound, this rule corresponds to a usual "leaning against the wind" policy of a time-consistent CB[11] whereby any rise in inflation is dampened by creating a negative consumption and output gap. Notice that, since taxes are distortionary, it would not be possible for the CB to reach a perfect stabilisation outcome (the so-called "divine coincidence") by closing both the inflation and output gap. Moreover, the trade-off is exacerbated by the presence of lower bound constraint.[12]

4.2 Optimisation problem of the fiscal leader

The fiscal leader optimises taking the reaction function of the CB (18) as a constraint. Its Bellman equation reads

$$W(s_t) = \min_{\{\hat{c}_t, \hat{G}_t, \hat{y}_t, \hat{\pi}_t, \hat{\tau}_t, \mu_t^{zlb}, \hat{b}_t\}} \frac{1}{2}(\gamma_c c \hat{c}_t^2 + \gamma_g G \hat{G}_t^2 + \gamma_h y \hat{y}_t^2 + \iota y \pi_t^2) + \beta W(s_{t+1})$$

such that (12)-(17) as well as the reaction function (18) and $\mu_t^{zlb} < 0$.

[11]See e.g. Chapter 5 of Galí (2015)

[12]Adam and Billi (2007) show that the cost of facing a binding ZLB is higher under discretion than under commitment because of a reinforcement loop between discretionary policy and private expectations.

The FONCs for the policy problem are detailed below

$$\partial\mathcal{L}/\partial\hat{c}_t \equiv \qquad 0 = \gamma_c c\hat{c}_t + \lambda_t^r c + \lambda_t^p \kappa_c - \lambda_t^q \gamma_c - \lambda_t^b \frac{y}{\theta}\gamma_c - \lambda_t^m \gamma_c \tag{19}$$

$$\partial\mathcal{L}/\partial\hat{G}_t \equiv \qquad 0 = \gamma_g G\hat{G}_t + \lambda_t^r G - \lambda_t^b G \tag{20}$$

$$\partial\mathcal{L}/\partial\hat{y}_t \equiv \qquad 0 = \gamma_h y\hat{y}_t - \lambda_t^r y + \lambda_t^p \kappa_y - \lambda_t^b \frac{y}{\theta}(1+\gamma_h) - \lambda_t^m \gamma_h \tag{21}$$

$$\partial\mathcal{L}/\partial\hat{\pi}_t \equiv \qquad 0 = \iota y\hat{\pi}_t - \lambda_t^p + \lambda_t^b \Omega\beta^{-1} - \lambda_t^m \Psi \tag{22}$$

$$\partial\mathcal{L}/\partial\hat{\tau}_t \equiv \qquad 0 = \lambda_t^p \kappa_\tau + \lambda_t^b w\tau y\frac{\theta-1}{\theta} \tag{23}$$

$$\partial\mathcal{L}/\partial\hat{q}_t \equiv \qquad 0 = \lambda_t^q + \lambda_t^b \Omega(1-\rho) - \lambda_t^{zlb} \tag{24}$$

$$\partial\mathcal{L}/\partial\mu_t^{zlb} \equiv \qquad 0 = \lambda_t^m \gamma_c c^{-1} + \lambda_t^s \tag{25}$$

$$\partial\mathcal{L}/\partial\hat{b}_t \equiv \qquad 0 = \lambda_t^p \beta\Theta_{1,t} + \Omega(\lambda_t^b - \mathbb{E}_t\lambda_{t+1}^b) + \lambda_t^q\Theta_{2,t} + \lambda_t^{zlb}(\Theta_{3,t} - \Theta_{2,t}) \tag{26}$$

and the lower bound constraints: $\lambda_t^s \mu_t^{zlb} = 0$, $\mu_t^{zlb} < 0$, $(\rho\beta\mathbb{E}_t\mathcal{Q}(s_{t+1}) - \hat{q}_t)\lambda_t^{zlb} = 0$ and $\rho\beta\mathbb{E}_t\mathcal{Q}(s_{t+1}) - \hat{q}_t > -r^*$. Moreover, the following definitions apply

$$\Theta_{1,t} \equiv \frac{\partial\mathbb{E}_t\Pi(s_{t+1})}{\partial\hat{b}_t}$$
$$\Theta_{2,t} \equiv \frac{\gamma_c\partial\mathbb{E}_t\mathcal{C}(s_{t+1})}{\partial\hat{b}_t} + \frac{\partial\mathbb{E}_t\Pi(s_{t+1})}{\partial\hat{b}_t} - \rho\beta\frac{\gamma_c\partial\mathbb{E}_t\mathcal{Q}(s_{t+1})}{\partial\hat{b}_t}$$
$$\Theta_{3,t} \equiv \frac{\gamma_c\partial\mathbb{E}_t\mathcal{C}(s_{t+1})}{\partial\hat{b}_t} + \frac{\partial\mathbb{E}_t\Pi(s_{t+1})}{\partial\hat{b}_t}$$

Those terms summarise the derivative of expectations with respect to debt and imply that, despite not being able to commit directly, the FA can nevertheless influence expectations of the households through an appropriate choice of debt. Moreover, the derivatives vary across time because of the lower bound and the non-linearity of the policy functions.

Finally, notice that the lagrange multiplier of the ZLB constraint is not necessarily the same for the CB and the FA i.e. $\mu_t^{zlb} \neq \lambda_t^{zlb}$. The shadow value of the ZLB constraint depends on the hurdle it represents for each authority. Since the short-rate is the only instrument of monetary policy and the lower bound undermines its effectiveness, this hurdle is higher for the CB. The FA on the other hand may still influence expectations about future bond prices when debt is long-term and/or uses the other instruments at its disposal—the labor tax rate and the government spending. When the FA leads, it can thus choose the ZLB multiplier of the CB as long as it remains negative (otherwise the ZLB constraint is binding neither for the CB nor for the FA).

5 Analytical results for a special case

I present here an analytical example with short-term one period debt to demonstrate the main mechanisms under the following simplifying assumptions:

Assumption 1 *The shock is discrete with two states* $\{r_H^n, r_L^n\}$ *and occurs only once and for all in the initial period. We have* $r_t^n = r_L^n$ *if* $t = 0$ *and* $r_t^n = r_H^n$ *otherwise. Moreover,* $r_L^n << 0$ *such that the ZLB binds.*

Assumption 2 *An outstanding amount of debt above the steady state level tightens the budget constraint of the government at positive interest rates. Formally, if* $\hat{b}_{t-1} > 0$ *and* $\hat{i}_t > r^*$*, then* $\lambda_t^b < 0$.

Assumption 3 *The following parameter restrictions apply:*

1. $(\gamma_c G + \gamma_g c)\eta_y > -\gamma_g y \eta_c$

2. $(\gamma_h G + \gamma_g y)\eta_c > -\gamma_g c \eta_y$

where $\eta_c \equiv \frac{\iota y}{\theta}\kappa_c + \frac{y}{\theta}\gamma_c - c - \gamma_c \Omega < 0$ *and* $\eta_y \equiv y + \frac{\iota y}{\theta}\kappa_y + \frac{y}{\theta}(1 + \gamma_h) > 0$.

Notice that Assumption 1 is without loss of generality since the initial period could be interpreted as the last one in which the ZLB is binding. Moreover, considering an i.i.d shock has no qualitative effects. Assumption 2 is intuitive when looking at the budget constraint (17) and, like Assumption 3, holds under baseline calibration.[13]

[13] See Table 1 for the baseline calibration

After the initial period, the environment becomes deterministic. Hence, the decision rules for the set of endogenous variables are a function of the inherited level of debt only. Given the linear-quadratic structure of the problem, this function is known to be linear with constant coefficient and, for each variable, will be denoted as follows: $\hat{\pi}_t = \Pi_b \hat{b}_{t-1}$, $\hat{c}_t = \mathcal{C}_b \hat{b}_{t-1}$, $\hat{y}_t = \mathcal{Y}_b \hat{b}_{t-1}$, etc.

5.1 The role of inflation expectations and the CBRF

When the objective function of the two authorities is the same, the first order necessary conditions *in the liquidity trap* are the same for the coordinated and uncoordinated problem. The sequential policy problem can be seen as a coordinated policy problem in which the instrument of the CB is taken as given and equal to the lower bound. However, this does not mean that the equilibrium is the same, since control variables are chosen taking into account expectations outside the ZLB which are determined by the outcome of the game when the CB regains control of its instrument. Outside the ZLB, the following proposition applies

Proposition 1 *Issuing more debt in the initial period leads to more inflation at the exit of the liquidity trap. Formally, if $\hat{\pi}_t = \Pi_b \hat{b}_{t-1}$ for $t > 0$, then we have $\Pi_b > 0$.*

Proof. Outside the ZLB, the target rule for the budget constraint multiplier is given by[14]

$$\lambda_t^b = -\Phi_b \hat{\pi}_t \tag{27}$$

with $\Phi_b \equiv \Delta^{-1} D(yc\Psi + (\gamma_h c + \gamma_c y)\frac{\iota y}{\Psi}) > 0$, $\Delta \equiv \gamma_h c[\gamma_g y \zeta_c + (\gamma_c G + \gamma_g c)\zeta_y] + \gamma_c y[\gamma_g c \zeta_y + (\gamma_h G + \gamma_g y)\zeta_c]$, $\zeta_y \equiv \eta_y + \frac{\gamma_h}{\Psi}(\frac{\iota y}{\theta} + \Omega\beta^{-1})$, $\zeta_c \equiv \eta_c + \frac{\gamma_c}{\Psi}(\frac{\iota y}{\theta} + \Omega\beta^{-1})$ and $D \equiv \gamma_h \gamma_c G + \gamma_h \gamma_g c + \gamma_c \gamma_g y$. According to Assumption 2, if $\hat{b}_0 > 0$, we have that $\lambda_1^b < 0$ and hence, it must be that $\hat{\pi}_1 > 0$. ■

A government issuing more debt in the liquidity trap e.g. to finance a fiscal stimulus mitigating the decline in output, will generate higher inflation when the ZLB stops biding.

[14]See Appendix C.1 for derivation

This inflation may come from higher labor taxes needed to finance the additional debt burden and/or from surprise inflation to erode its real value. How beneficial is this strategy depends on the RFCB and the impact of future real interest rates on consumption both inside and outside the liquidity trap as described in the following proposition

Proposition 2 *In the liquidity trap, the marginal effect of an increase in government debt on consumption depends on the reaction function of the CB. Qualitatively, we have that*

- *if* $\Phi_c(\Psi) < \gamma_c^{-1}$*, a marginal increase in government debt has a positive effect on current consumption. Formally,* $\frac{\partial \hat{c}_0}{\partial \hat{b}_0} > 0$*.*
- *if* $\Phi_c(\Psi) > \gamma_c^{-1}$*, a marginal increase in government debt has a negative effect on current consumption. Formally,* $\frac{\partial \hat{c}_0}{\partial \hat{b}_0} < 0$*.*

where $\Phi_c(\Psi) = \gamma_h G c^{-1} \Phi_b(\Psi) + \gamma_g y \Psi - \gamma_c \gamma_h G c^{-1} \frac{\iota y}{\Psi}$

Proof. For $t > 0$, the target rule for consumption is given by

$$\hat{c}_t = -\Phi_c \hat{\pi}_t \tag{28}$$

where Φ_c is defined as in the proposition (see Appendix C.2 for derivation). Substituting this target rule for the expectation term in the time zero Euler equation (8) gives

$$\hat{c}_0 = (\gamma_c^{-1} - \Phi_c)\Pi_b \hat{b}_0 - \gamma_c^{-1}(r^* - \hat{d}_0)$$

Taking the partial derivative of this expression with respect to debt gives $\frac{\partial \hat{c}_0}{\partial \hat{b}_0} = (\gamma_c^{-1} - \Phi_c)\Pi_b$. The fact that $\Pi_b > 0$ according to Proposition 1 completes the proof. ■

As it turns out, $\Phi_c(\Psi) < \gamma_c^{-1}$ is verified only with a very restrictive (and unlikely) set of parameters. Figure 1 shows for which value of the inflation weight in the loss function of the CB (ιy) this condition is verified under baseline calibration.

[Figure 1 about here.]

The star corresponds to the socially optimal weight derived from the second-order approximation of the utility around the efficient steady state. Hence, under fiscal leadership, it is generally the case that, increasing the debt level in the liquidity trap reduces consumption. Moreover, appointing a more conservative CB increases the negative sensitivity of consumption to debt. The intuition is simple. A CB attaching a higher weight than society to inflation stabilisation leans more against the inflationary bias stemming from high government debt. Hence, the anticipation of higher real rates at the exit of the lower bound weigh on the consumption of forward-looking households.

5.2 The role of endogenous government spending

In what follows, consider the common case where $\Phi_c(\Psi) > \gamma_c^{-1}$. So far, we have seen that increasing debt in the liquidity trap boosts inflation expectations but depresses current consumption. The reason is that the CB is expected to respond to excess inflation by setting a negative output gap at the exit of the liquidity trap. What is then the net impact on output and inflation in the liquidity trap?

The answer to this question crucially depends on the marginal response of government spending to a variation of government debt. Totally differentiating the resource constraint gives

$$\underbrace{\frac{\partial \hat{y}_0}{\partial \hat{b}_0}}_{?} = y^{-1}\left(c\underbrace{\frac{\partial \hat{c}_0}{\partial \hat{b}_0}}_{<0} + G\underbrace{\frac{\partial \hat{G}_0}{\partial \hat{b}_0}}_{?}\right)$$

The following proposition characterises the sign of the last term

Proposition 3 *In the liquidity trap, the marginal effect of an increase in government debt on government spending depends on its relative effect on consumption and on the tightness of the ZLB constraint. Issuing more debt stimulates (depresses) government spending if the positive effect from the ZLB constraint tightening outweighs (falls short of) the negative effect from the drop in consumption.*

Proof. The target rule for choosing government spending in the liquidity trap reads (see Appendix C.3 for derivation)

$$\hat{G}_0 = \frac{\gamma_c \eta_y - \gamma_c \eta_c}{D} \psi \pi_0 - \frac{\gamma_h}{D} \lambda_0^{zlb} \tag{29}$$

Solving the consumption target rule (38) for inflation gives

$$\psi_0 \pi_0 = \frac{Dc\hat{c}_0 - (\gamma_h G + \gamma_g y)\gamma_c \lambda_0^{zlb}}{\Sigma} \tag{30}$$

where $\Sigma \equiv \gamma_g c \eta_y + (\gamma_h G + \gamma_g y)\eta_c > 0$.

Combining (29) and (30) and, totally differentiating the resulting equation with respect to $\hat{b}_t$ gives

$$\frac{\partial \hat{G}_0}{\partial \hat{b}_0} = \underbrace{\frac{c(\gamma_c \eta_y - \gamma_c \eta_c)}{\Sigma} \frac{\partial \hat{c}_0}{\partial \hat{b}_0}}_{\text{budget constraint effect } (<0)} \underbrace{- \frac{(\gamma_c \eta_y - \gamma_c \eta_c)(\gamma_h G + \gamma_g y)\gamma_c + \Sigma \gamma_h}{D\Sigma} \frac{\partial \lambda_0^{zlb}}{\partial \hat{b}_0}}_{\text{ZLB constraint effect } (>0)} \tag{31}$$

The negative sign of the second term stems from Corollary 2 in Appendix C.4. Hence, the marginal effect of debt on government spending is positive if the second term is smaller than the first term (the effect of the ZLB constraint tightening outweighs the effect of the drop in consumption) and negative otherwise. ■

The drop in consumption stemming from a marginal increase in debt has two opposite effects on government spending. On the one hand, the room for a fiscal expansion is reduced due to the shrinkage of the tax base which tightens the budget constraint. On the other hand, additional fiscal support is warranted to mitigate the inflation shortfall. If the second effect dominates, the government reacts to the low state of the natural real rate by increasing its own consumption. Whether this fiscal stimulus is strong enough to generate a positive effect on the output gap determines the desirability of issuing more debt in the liquidity trap. In the next section, I investigate this trade-off quantitatively.

6 Numerical results

The model is calibrated on the US economy before the Great Recession. Time is discrete and a period represents one quarter. Since calibration follows Matveev (2020), I do not expand on the economic rationale behind the parameter values and instead directly provide a summary in Table 1.

[Table 1 about here.]

The market value of debt in the efficient steady state corresponds to 40% of output which itself amounts to one quarter of the unitary time endowment of the households. This section studies the properties of the optimal time-consistent equilibrium with one-period debt and with longer maturity. A exogenous shock of four unconditional standard deviations on the natural real rate constitutes the baseline scenario to motivate the liquidity trap episode.[15]

Since the main objective of this paper is to study the optimal use of debt to stabilise the economy in a liquidity trap, it is crucial to properly account for the role of uncertainty in shaping private sector expectations about output and inflation. To this aim, methodological approaches that compute optimal policies along a deterministic path are inadequate because they assume that the duration and depth of the recession are known once the shock has realised. Hence, I instead solve the model using a collocation method on a finite domain for the states. This method not only fully internalises the effect of uncertainty on the decision rules of the agents but also allows to deal with issues related to non-linearity and time-consistency. More details about the algorithm are provided in Appendix D.

6.1 Deflationary bias at positive nominal rates

Under time-consistent policy, it is well-known (see e.g. Nakov, 2008) that the risk of entering a liquidity trap in the future induces a *deflationary bias* at positive nominal rates i.e.

[15] Since the economy starts in the unstable deterministic steady state, the shock is assumed to occur after a burn-in of 200 periods to phase out the influence of initial conditions

inflation remains persistently below target. The reason is the following. At positive rates, forward-looking households and firms anticipate that inflation and output may be lower in the future because of the binding lower bound and therefore have an incentive to reduce their consumption and prices. When the policymaker cannot commit to higher inflation, this incentive creates a trade-off between inflation and output gap stabilisation which results in a deflation and a positive output gap outside the lower bound.

In the context of the present model, the deflationary bias induces monetary policy to lower its nominal rate in normal times. The government, in turn, supports this effort to overcome the inflation shortfall by issuing more debt.[16] Under coordination, the optimal response of monetary policy is to accomodate the upward pressure on inflation expectations. Hence, by issuing only a moderate amount of debt above the steady state, the policymaker lowers expected real rates sufficiently to relax the deflation bias.[17]

In comparison, overcoming the deflation bias is more tedious under noncooperative game. When the government issues additional debt, the induced inflationary bias triggers a contractionary response from monetary policy and thus pushes up expected real rates. As a result, the debt level compatible with the desired inflation in the risky steady state is much higher than under coordination and associated output gap falls short of its deterministic counterpart. Table 2 shows the difference for selected variables in the risky steady state between coordination and noncooperative game.

[Table 2 about here.]

What is then the risk of a liquidity trap once the economy has settled to the risky steady state? Despite starting from a higher nominal rate at the risky steady state, the stabilisation properties of the noncooperative equilibrium make it more likely to visit the lower bound. In particular, the persistent drop in inflation (expectations) stemming from the debt consolidation in the liquidity trap increases the threshold value for the natural real rate

[16] See Matveev (2020) for a description of a similar response of debt.

[17] Those lower expected real rates are akin the ones described by Nakata and Schmidt (2019) with the appointment of a conservative central bank.

below which the lower bound becomes binding, relative to coordination. Simulating a long sample of $100,000$ periods from the risky steady state gives an unconditionnal probability of liquidity trap equal to 13.2% under noncooperative game and to 8.6% under coordination.[18]

6.2 Optimal response with short-term liabilities

This subsection emphasises the role of debt in the optimal time-consistent response to the shock on the natural real rate when debt maturity is one period. To this aim, I set the parameter $\rho = 0$. Figure 2 compares the outcome of coordinated policy and non-cooperative game by showing the trajectory of nine key variables: the market value of government debt, the nominal interest rate, the output gap, the labor tax rate, inflation, consumption, government spending, the real interest rate and the price of the government bond.

[Figure 2 about here.]

Looking at the first panel, the noncooperative FA reduces its debt at the outset of the liquidity trap by increasing labor taxes and returns it only very slowly to its steady state value. As a result, inflation remains under target long after the nominal rate has lifted-off from the lower bound. This subdued inflation, in turn, causes the independent CB to adopt an expansionary monetary stance by keeping its nominal rate at a lower level than the one which would have prevailed absent of the consolidation. This monetary policy response improves economic outlook as can be seen by the sustained overshooting of the output gap in the top right panel. Since households anticipate real rates to stay below their natural level at the exit of the lower bound, the expectation channel mitigates the drop of consumption in the liquidity trap. This consumption effect, together with the initial increase in inflationary labor taxes, results in a mild inflation shortfall following the negative shock on the natural real rate. Moreover, the depressive effect of labor taxes is weaker because the lower bound prevents a monetary policy tightening to dampen the resulting inflation.

[18] The first $5,000$ observations are discarded to phase out the influence of the initial conditions.

This debt consolidation contrasts markedly with the short-lived increase in debt observed under coordination. When the CB coordinates with the FA, it internalises the effect of its policy rate decisions on the government budget constraint. Since the roll-over-cost of debt is more sensitive to the policy rate when the maturity is short, the CB optimally accommodates the inflationary bias stemming from higher debt. Hence, issuing more debt increases inflation (expectations) and relaxes the lower bound constraint. Furthermore, it finances both a tax cut and a fiscal expansion stimulating the economy sufficiently to create a positive output gap in the liquidity trap. The fiscal stimulus is nevertheless slightly overturned at the exit of the ZLB.

Finally, notice that the short-term impact on consumption is very similar under both institutional set-ups but the output loss is smaller under coordination. The accommodative monetary policy creates the ground for a much larger fiscal stimulus which allows the economy to leave the liquidity trap after a shorter period.

6.3 Enhanced coordination with longer debt maturity

To investigate the role of debt maturity in the equilibrium outcome, I increase the value of the parameter ρ from 0 to 0.9434 which corresponds to an average debt maturity of 16 quarters. Figure 3 displays the same variables as for the short-term case.

[Figure 3 about here.]

The performance of both institutional set-ups in stabilising the economy is now strikingly similar. In particular, the transition path features an initial small increase in debt, followed by a persistent drop which is slightly more pronounced, and recovers more slowly, when coordination is lacking. This also explains the small difference in inflation. Overall, the dynamics resemble to the noncooperative policy with short-term debt.

What explains this alignment of coordinated and noncooperative policy when the maturity of debt is lengthened? By lengthening the debt maturity, the roll-over-cost of government debt is made less sensitive to the short-term rate. Hence, the coordinated CB focuses less on the fiscal cost of raising the policy rate and more on the benefits in terms of inflation and output gap stabilisation. In other words, it acts more independently from the FA. In this sense, issuing debt of longer maturity provides the authorities with a hedge against coordination issues in dealing with liquidity traps.

6.4 Discussion of the results

The analysis in this paper helps to shed light on several dimensions of the optimal time-consistent fiscal and monetary policy when debt matters for the expectations of the agents.

First, the optimality of a deficit-financed fiscal stimulus in the liquidity trap lies on the assumption of monetary and fiscal policy coordinating. This assumption may seem reasonable because it allows the joint policymaker to reach the best outcome in terms of stabilisation policies. However, while fruitful from a conceptual point of view, this institutional set-up is inconsistent with empirical evidence about central bank independence. In practice, central bank independence safeguards the long-term stability of prices, perhaps at the cost of forsaken shorter-term benefits from enhanced coordination (especially in a liquidity trap). This paper shows that if private agents are convinced that monetary policy will never become subservient to fiscal policy, a large accumulation of government debt may be perilous. In the absence of default, a large stock of debt necessarily implies higher interest charges—and thus taxation burden—when the nominal rate lifts-off the lower bound. Should firms pass on the increase in their marginal costs to consumers, the resulting inflation would call for a contractionary monetary policy. Of course, if households are not myopic, they will already raise their savings in the liquidity trap in anticipation of the higher real rates and labor taxes, a scenario that public policy desperately tries to avoid.

Second, coordination issues worsen with shorter debt maturity. When the roll-over-cost of debt is more sensitive to the policy rate, the negative effects of monetary tightening arise in the nearer-term and the expectation channel just described is magnified. On the contrary, when debt is long-term, fiscal policy is more insulated from monetary policy decisions and the precise institutional set-up becomes less decisive for stabilisation outcomes. In this respect, the recent shortening of the privately held government debt stemming from quantitative easing policies may constitute a cause of concerns.

7 Conclusion

I have demonstrated, both analytically and numerically, that deficit-financed stimuli in a liquidity trap are undesirable when fiscal and monetary policy are conducted noncooperatively in a time-consistent manner. Large stocks of government debt entail higher inflationary labor taxes and contractionary monetary policy at the exit of the lower bound. Anticipation of higher real rates and lower output gives forward-looking households incentives to reduce consumption and thus jeopardises the recovery. In this context, long-lasting consolidation of debt in the liquidity trap turns out to be optimal.

This policy insight raises a number of questions regarding the current macroeconomic situation in the United States and elsewhere. First, the high stock of government debt may weigh negatively on the duration of lower bound episodes if households expect the central bank to remain adamant in defending its inflation target when the economy recovers. Second, the maturity shortening of privately held debt observed those last years because of quantitative easing tends to amplify this expectation channel and to aggravate coordination issues in stabilising the economy. Of course, some of those results may be sensitive to the central bank being able to influence private sector expectations through government debt purchases. Future research should therefore study the impact of quantitative easing on the strategies played by the two authorities.

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Appendices

Appendix A Derivation of the loss function

The loss function is obtained with a second order approximation of the utility of the households around the efficient steady-state. The utility function comprises three components:

1. The first component of utility is $F = \frac{(y_t(1-\frac{\iota}{2}(\pi_t-1)^2)-G_t)^{1-\gamma_c}}{1-\gamma_c}\xi_t$

$$
\begin{array}{ll}
F_y = c^{-\gamma_c}(1-\frac{\iota}{2}(\pi-1)^2)\xi & F_g = -c^{-\gamma_c}\xi \\
F_\pi = -\iota(\pi-1)yc^{-\gamma_c}\xi & F_{yy} = -\gamma_c c^{-\gamma_c-1}(1-\frac{\iota}{2}(\pi-1)^2)\xi \\
F_{gg} = -\gamma_c c^{-\gamma_c-1}\xi & F_{\pi\pi} = (\gamma_c c \iota(\pi-1)c^{-\gamma_c-1} - \iota c^{-\gamma_c})y\xi \\
F_{yg} = \gamma_c c^{-\gamma_c-1}(1-\frac{\iota}{2}(\pi-1)^2)\xi & F_{gy} = \gamma_c c^{-\gamma_c-1}(1-\frac{\iota}{2}(\pi-1)^2)\xi \\
F_{\pi y} = 0 & F_{y\pi} = 0 \\
F_{g\pi} = 0 & F_{\pi g} = 0 \\
F_\xi = \frac{c^{1-\gamma_c}}{1-\gamma_c} & F_{y\xi} = c^{-\gamma_c}(1-\frac{\iota}{2}(\pi-1)^2) \\
F_{\xi\xi} = 0 & F_{g\xi} = -c^{-\gamma_c} \\
F_{\xi y} = (1-\frac{\iota}{2}(\pi-1)^2)c^{-\gamma_c} & F_{\xi g} = -c^{-\gamma_c}
\end{array}
$$

given that $\xi = 1$, we have

$$
F \simeq (y\hat{y}_t - G\hat{G}_t - \frac{1}{2}\gamma_c c^{-1}y^2\hat{y}_t^2 + \gamma_c c^{-1}G\hat{G}_t y\hat{y}_t + y\hat{y}_t\hat{\xi}_t - G\hat{G}_t\hat{\xi}_t - \frac{1}{2}\gamma_c c^{-1}G^2\hat{G}_t^2 \\
- \frac{1}{2}\iota\hat{\pi}_t^2 + t.i.p = y\hat{y}_t - G\hat{G}_t - \frac{1}{2}\gamma_c c^{-1}(c\hat{c}_t)^2 + y\hat{y}_t\hat{\xi}_t - G\hat{G}_t\hat{\xi}_t - \frac{1}{2}\iota y\hat{\pi}_t^2)c^{-\gamma_c} + t.i.p
$$

2. The second component of utility is $G = \frac{G_t^{1-\gamma_g}}{1-\gamma_g}\nu_g\xi_t$

$$
\begin{array}{ll}
G_g = G^{-\gamma_g}\nu_g\xi & G_{gg} = -\gamma_g G^{-\gamma_g-1}\nu_g\xi \\
G_\xi = \frac{G^{1-\gamma_g}}{1-\gamma_g}\nu_g & G_{\xi\xi} = 0 \\
G_{g\xi} = G^{-\gamma_g}\nu_g & G_{\xi g} = G^{-\gamma_g}\nu_g
\end{array}
$$

given that $\xi = 1$ and $\nu_g = G^{\gamma_g}c^{-\gamma_c}$, we have

$$
G \simeq (G\hat{G}_t - \frac{1}{2}\gamma_g G\hat{G}_t^2 + G\hat{G}_t\hat{\xi}_t)c^{-\gamma_c} + t.i.p
$$

3. The third component of utility is $H = \frac{y_t^{1+\gamma_h}}{1+\gamma_h}\nu_h\xi_t$

$$
\begin{array}{ll}
H_y = y^{\gamma_h}\nu_h\xi & H_{hh} = \gamma_h y^{\gamma_h - 1}\nu_h\xi \\
H_\xi = \frac{y^{1+\gamma_h}}{1+\gamma_h}\nu_h & H_{\xi\xi}\nu_h = 0 \\
H_{y\xi} = y^{\gamma_h}\nu_h & G_{\xi y} = y^{\gamma_h}\nu_h
\end{array}
$$

given that $\xi = 1$ and $\nu_h = y^{-\gamma_h}c^{-\gamma_c}$, we have

$$
H \simeq (y\hat{y}_t + \frac{1}{2}\gamma_h y^2\hat{y}_t^2 + y\hat{y}_t\hat{\xi}_t)c^{-\gamma_c} + t.i.p
$$

Putting all the components together and scaling with c^{γ_c}, the second-order approximation is given by

$$
\begin{aligned}
U \simeq F + G - H = y\hat{y}_t - G\hat{G}_t - \frac{1}{2}\gamma_c c^{-1}(c\hat{c}_t)^2 + y\hat{y}_t\hat{\xi}_t - G\hat{G}_t\hat{\xi}_t - \frac{1}{2}\iota y\hat{\pi}_t^2 \\
+ G\hat{G}_t - \frac{1}{2}\gamma_g G\hat{G}_t^2 + G\hat{G}_t\hat{\xi}_t - y\hat{y}_t - \frac{1}{2}\gamma_h y^2\hat{y}_t^2 - y\hat{y}_t\hat{\xi}_t + t.i.p
\end{aligned}
$$

Hence, the social objective corresponds to

$$
U \simeq -\frac{1}{2}\Big[\big(\gamma_c c\hat{c}_t^2 + \gamma_g G\hat{G}_t^2 + \gamma_h y\hat{y}_t^2 + \iota y\hat{\pi}_t^2\Big] + t.i.p
$$

Appendix B FONCs of the coordinated problem

The FONCs for the policy problem are detailed below

$$
\begin{aligned}
\partial\mathcal{L}/\partial\hat{c}_t \equiv &\quad 0 = \gamma_c c\hat{c}_t + \Lambda_t^r c + \Lambda_t^p\kappa_c - \Lambda_t^q\gamma_c - \Lambda_t^b\frac{y}{\theta}\gamma_c \\
\partial\mathcal{L}/\partial\hat{G}_t \equiv &\quad 0 = \gamma_g G\hat{G}_t + \Lambda_t^r G - \Lambda_t^b G \\
\partial\mathcal{L}/\partial\hat{y}_t \equiv &\quad 0 = \gamma_h y\hat{y}_t - \Lambda_t^r y + \Lambda_t^p\kappa_y - \Lambda_t^b\frac{y}{\theta}(1+\gamma_h) \\
\partial\mathcal{L}/\partial\hat{\pi}_t \equiv &\quad 0 = \iota y\hat{\pi}_t - \Lambda_t^p + \Lambda_t^b\Omega\beta^{-1} \\
\partial\mathcal{L}/\partial\hat{\tau}_t \equiv &\quad 0 = \Lambda_t^p\kappa_\tau + \Lambda_t^b w\tau y\frac{\theta-1}{\theta} \\
\partial\mathcal{L}/\partial\hat{q}_t \equiv &\quad 0 = \Lambda_t^q + \Lambda_t^b\Omega(1-\rho) - \Lambda_t^{zlb} \\
\partial\mathcal{L}/\partial\hat{b}_t \equiv &\quad 0 = \Lambda_t^p\beta\Theta_{1,t} + \Omega(\Lambda_t^b - \mathbb{E}_t\Lambda_{t+1}^b) + \Lambda_t^q\Theta_{2,t} + \Lambda_t^{zlb}(\Theta_{3,t} - \Theta_{2,t})
\end{aligned}
$$

After some algebra, the set of first order necessary conditions can be reduced to the following five equations

$$
\begin{aligned}
y\hat{y}_t &= \frac{\gamma_g y\eta_c + (\gamma_c G + \gamma_g c)\eta_y}{D}\psi\pi_t + \frac{\gamma_g\gamma_c y}{D}\Lambda_t^{zlb} \\
c\hat{c}_t &= \frac{\gamma_g c\eta_y + (\gamma_h G + \gamma_g y)\eta_c}{D}\psi\pi_t + \frac{(\gamma_h G + \gamma_g y)\gamma_c}{D}\Lambda_t^{zlb} \\
&(\Omega - \beta\Theta_{1,t} - \Omega(1-\rho)\Theta_{2,t})\psi\pi_t = \Omega\psi\mathbb{E}_t\pi_{t+1} + \Theta_{3,t}\Lambda_t^{zlb} \\
&\Lambda_t^{zlb}(\hat{i}_t - r^*) = 0 \\
&\hat{i}_t > -r^*
\end{aligned}
$$

where $\eta_y \equiv y + \frac{\iota y}{\theta}\kappa_y + \frac{y}{\theta}(1+\gamma_h)$, $\eta_c \equiv \frac{\iota y}{\theta}\kappa_c + \frac{y}{\theta}\gamma_c - c - \gamma_c\Omega(1-\rho)$ and $D \equiv \gamma_h\gamma_c G + \gamma_h\gamma_g c + \gamma_c\gamma_g y$. The parameter $\psi \equiv \frac{y\iota}{y\frac{\iota}{\theta} + \beta^{-1}\Omega} > 0$ determines the inflation response to a change in the shadow value of the budget constraint. The response to a tightening is always positive and the magnitude depends on the relative cost of inflation with respect to its benefits. Finally, $\Theta_{1,t}$, $\Theta_{2,t}$ and $\Theta_{3,t}$ are defined as in the text.

Appendix C Analytical example

C.1 Derivation of the target rule for the budget constraint multiplier in the proof of proposition 1

From equations (20), (23) and (24), we know respectively that outside the ZLB $\lambda_t^r = \lambda_t^b - \gamma_g G^{-1}(y\hat{y}_t - c\hat{c}_t)$, $\lambda_t^p = -\frac{\iota y}{\theta}\lambda_t^b$ and $\lambda_t^q = -\Omega\lambda_t^b$.

Using those expressions in equation (19) and rearranging gives

$$c\hat{c}_t = \left(\gamma_g c G^{-1} y\hat{y}_t + \zeta_c\lambda_t^b + \gamma_c\frac{\iota y}{\Psi}\hat{\pi}_t\right)\left(\gamma_c + \gamma_g c G^{-1}\right)^{-1} \tag{32}$$

and doing the same with equation (21), we have

$$y\hat{y}_t = \left(\gamma_g y G^{-1} c\hat{c}_t + \zeta_y\lambda_t^b + \gamma_h\frac{\iota y}{\Psi}\hat{\pi}_t\right)\left(\gamma_h + \gamma_g y G^{-1}\right)^{-1} \tag{33}$$

Combining those two expressions gives a target rule for $\hat{y}_t(\hat{\pi}_t, \lambda_t^b)$ and $\hat{c}_t(\hat{\pi}_t, \lambda_t^b)$ with $t > 0$

$$\hat{y}_t = \left(\frac{\iota y}{\Psi}\hat{\pi}_t + [\gamma_g y\zeta_c + (\gamma_c G + \gamma_g c)\zeta_y]D^{-1}\lambda_t^b\right) \tag{34}$$

$$\hat{c}_t = \left(\frac{\iota y}{\Psi}\hat{\pi}_t + [\gamma_g c\zeta_y + (\gamma_h G + \gamma_g y)\zeta_c]D^{-1}\lambda_t^b\right) \tag{35}$$

Next, use equations (34) and (35) to substitute respectively for $\hat{y}_t$ and $\hat{c}_t$ in the RFCB (18) (with $\mu_t^{zlb} = 0$). Rearranging the resulting expression gives the target rule in proposition 1.

C.2 Derivation of the target rule for consumption in the proof of proposition 2

The derivation is straightforward. Rearrange the RFCB (18) to obtain: $\hat{y}_t = -\gamma_h^{-1}(\Psi\hat{\pi}_t + \gamma_c\hat{c}_t)$. Next, use this expression, together with equation (27), to respectively get rid of $\hat{y}_t$ and λ_t^b in equation (32). Then, rearranging gives target rule (28).

C.3 Derivation of the target rule for government spending at the ZLB in the proof of proposition 3

At the ZLB, we have that $\mu_0^{zlb} < 0$. Then, given the complementary slackness condition, it must be that $\lambda_0^s = 0$ and from equation (25), it implies that $\lambda_0^m = 0$. From equations (20), (23) and (24), we know respectively that inside the ZLB $\lambda_0^r = \lambda_0^b - \gamma_g G^{-1}(y\hat{y}_0 - c\hat{c}_0)$, $\lambda_0^p = -\frac{\iota y}{\theta}\lambda_0^b$ and $\lambda_0^q = -\Omega\lambda_0^b + \lambda_0^{zlb}$. Substituting these in equation (22) gives

$$\lambda_0^b = -\psi\hat{\pi}_0 \tag{36}$$

Using those expressions in equation (19) and rearranging gives

$$c\hat{c}_0 = (\gamma_g c G^{-1} y\hat{y}_0 - \eta_c\psi\hat{\pi}_0 - \gamma_c\lambda_0^{zlb})(\gamma_c + \gamma_g c G^{-1})^{-1}$$

and doing the same with equation (21), we have

$$y\hat{y}_0 = (\gamma_g y G^{-1} c\hat{c}_0 - \eta_y\psi\hat{\pi}_0)(\gamma_h + \gamma_g y G^{-1})^{-1}$$

Combining the two previous equations, we obtain two targeting rules

$$y\hat{y}_0 = \frac{\gamma_g y\eta_c + (\gamma_c G + \gamma_g c)\eta_y}{D}\psi\hat{\pi}_0 + \frac{\gamma_g\gamma_c y}{D}\lambda_0^{zlb} \tag{37}$$

$$c\hat{c}_0 = \frac{\gamma_g c\eta_y + (\gamma_h G + \gamma_g y)\eta_c}{D}\psi\hat{\pi}_0 + \frac{(\gamma_h G + \gamma_g y)\gamma_c}{D}\lambda_0^{zlb} \tag{38}$$

Taking the difference between the first and second one gives target rule (29).

C.4 Two corollaries used in the proof of proposition 3

Corollary 1 *For equilibria exhibiting monotone dynamics outside the liquidity trap, we have that* $\Omega - \beta\frac{\iota y}{\theta}\Theta_1 - \Omega\Theta_3 > 0$.

Proof. *Consider two consecutive periods outside the liquidity trap. Then, using target rule* (39) *in equation* (26)*, we get* $\mathbb{E}_t\hat{\pi}_{t+1} = \frac{\Gamma}{\Omega}\hat{\pi}_t$ *where* $\Gamma \equiv \Omega - \beta\frac{\iota y}{\theta}\Theta_1 - \Omega\Theta_3$*. This is an AR(1) process with auto-regressive parameter* $\frac{\Gamma}{\Omega}$*. Hence, for any monotonic dynamics, it must be that* $0 < \frac{\Gamma}{\Omega} < 1$ *and so,* $\Gamma > 0$. ■

Corollary 2 *In the liquidity trap, a marginal increase in debt tightens the ZLB constraint. Formally,* $\frac{\partial\lambda_0^{zlb}}{\partial\hat{b}_0} < 0$.

Proof. *Using target rules* (39) *and* (36) *in equation* (26) *and rearranging*

$$\psi\hat{\pi}_0 = \frac{\Omega\Phi_b}{\Gamma}\mathbb{E}_0\hat{\pi}_1 + \frac{\Theta_3}{\Gamma}\lambda_0^{zlb} \tag{39}$$

where $\Gamma \equiv \Omega - \beta\frac{\iota y}{\theta}\Theta_1 - \Omega\Theta_3 > 0$ *(see corollary 1) and* $\Theta_3 \equiv (\gamma_c\mathcal{C}_b + \Pi_b) = \gamma_c\Pi_b(\gamma_c^{-1} - \Phi_c) < 0$ *(see equation* (28)*).*

Combining (39) *and* (38) *and, totally differentiating the resulting equation with respect to* $\hat{b}_t$ *gives*

$$\frac{\partial\lambda_0^{zlb}}{\partial\hat{b}_0} = \underbrace{\frac{D\Gamma c}{\Xi}\frac{\partial\hat{c}_0}{\partial\hat{b}_0}}_{\text{consumption effect } (<0)} - \underbrace{\frac{(\gamma_g c\eta_y + (\gamma_h G + \gamma_g y)\eta_c)\Omega\Phi_b}{\Xi}\Pi_b}_{\text{inflation expectation effect } (>0)}$$

where $\Xi \equiv (\gamma_g c\eta_y + (\gamma_h G + \gamma_g y)\eta_c)\Theta_{3,0} + \Gamma(\gamma_h G + \gamma_g y)\gamma_c > 0$. ■

Appendix D Solution method

The non-linearities introduced by the occasionally binding ZLB constraint precludes traditional local methods. Instead, I approximate the linear policy functions using a projection method (collocation) on a finite number of points in the domain. Since the policy functions are linear with a kink, I rely on cubic splines for the basis function. In solving the model, the noncooperative policy game involves some additional steps which are emphasised distinctly. The algorithm proceeds by checking convergence on two nested loops as follows:

1. Construct the grid for the state variables. Use a Gaussian quadrature scheme to discretise the normally distributed innovations to the natural real rate.

2. Use the linear policy functions of the commitment solution from Dynare (outside the ZLB) to obtain an initial guess for the basis coefficients.

3. *Outer loop*: With the current guess of the basis coefficients, approximate the partial derivative of the expectation terms with respect to government debt.

4. *Inner loop*: To solve the system of equilibrium equations, I proceed as follows.

   (i) Use the guess of the basis coefficients to recover government debt from the budget constraint at the collocation nodes.

   (ii) Build the new grid and approximate the expectation terms associated with next period's decisions.

   (iii) Solve the linear system outside the ZLB with matrix inversion.

   (iv) **(only for policy game**) Verify KKT condition on the ZLB constraint of the CB. If $\lambda_t^s < 0$, KKT is violated and the CBRF is not a binding constraint for the Fiscal Leader. Hence, solve the alternative system with equation (18) muted.

   (v) Verify whether ZLB constraint binds. In the affirmative, solve the alternative system at the ZLB and check KKT conditions.

   (vi) Update the guess for the basis coefficients based on the decision rules for the current period. If the difference between the old and the new guess is smaller than $1.49e^{-8}$, the inner loop has converged. Otherwise, go back to step (i).

5. Update the guess for the the partial derivative of the expectation terms with respect to government debt based on basis coefficients obtained from (4). If the difference between the old and the new guess is smaller than $1.49e^{-8}$, the outer loop has converged. Otherwise, go back to step (3).

I implement this procedure in Matlab by relying on the CompEcon toolbox of Miranda and Fackler (2002) for the Gaussian quadrature and function evaluation at the collocation nodes.

Tables

Table 1: Baseline calibration.

Symbol	Description	Value
β	Subjective discount factor	0.99
θ	Elasticity of substitution among goods	11
ϵ	Price adjustment cost	117.805
γ_c	Intertemporal elasticity of c	1
γ_g	Intertemporal elasticity of G	1
γ_h	Intertemporal elasticity of h	1
ν_h	Utility weight on labor	20
ν_g	Utility weight on labor	0.25
ρ_r	AR coefficient on natural real rate shock	0.77
σ	S.D. of natural real rate shock (%)	0.4

Table 2: Effect of institutional set-up on the risky steady state

Variable	Deterministic	Risky	
		Coord.	Noncoop.
Mrkt val debt	40	52.8	103.8
Nominal rate	2.48	1.89	2.60
Real rate	2.48	1.81	2.40
Inflation	0	0.08	0.20
Output	0	0.007	-0.187
Tax rate	17.35	17.32	17.35
Gov. spending	20	19.9	19.8

Note: Market value of debt is in percentages of annual output. Nominal, real rates and inflation are in annual percentages. Output is in relative difference from deterministic steady state. Tax rate is in absolute percentages. Government spending is in percentages of output.

Figures

Figure 1: Sensitivity of consumption to a marginal increase in debt for different values of the inflation weight in the loss function of the CB (under baseline calibration).

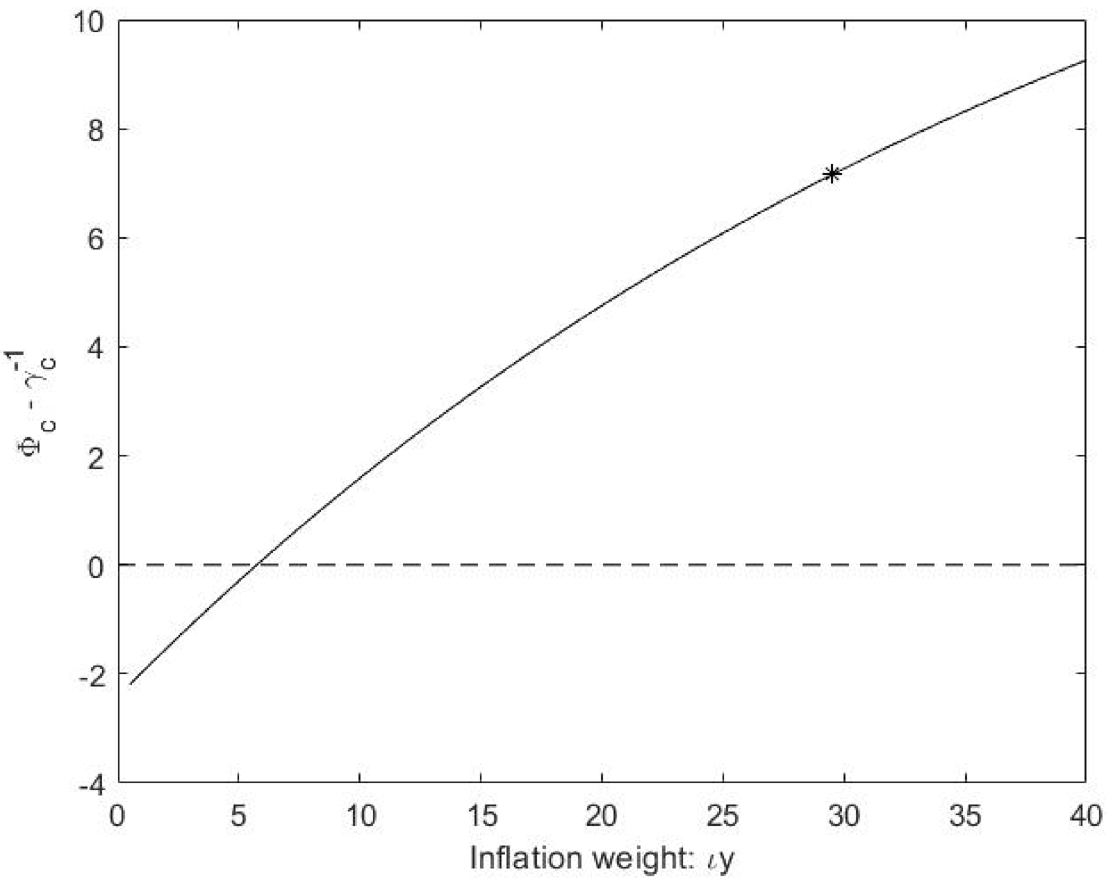


Figure 2: IRFs of key variables to a negative shock on the natural real rate driving the economy in a liquidity trap: short-term debt

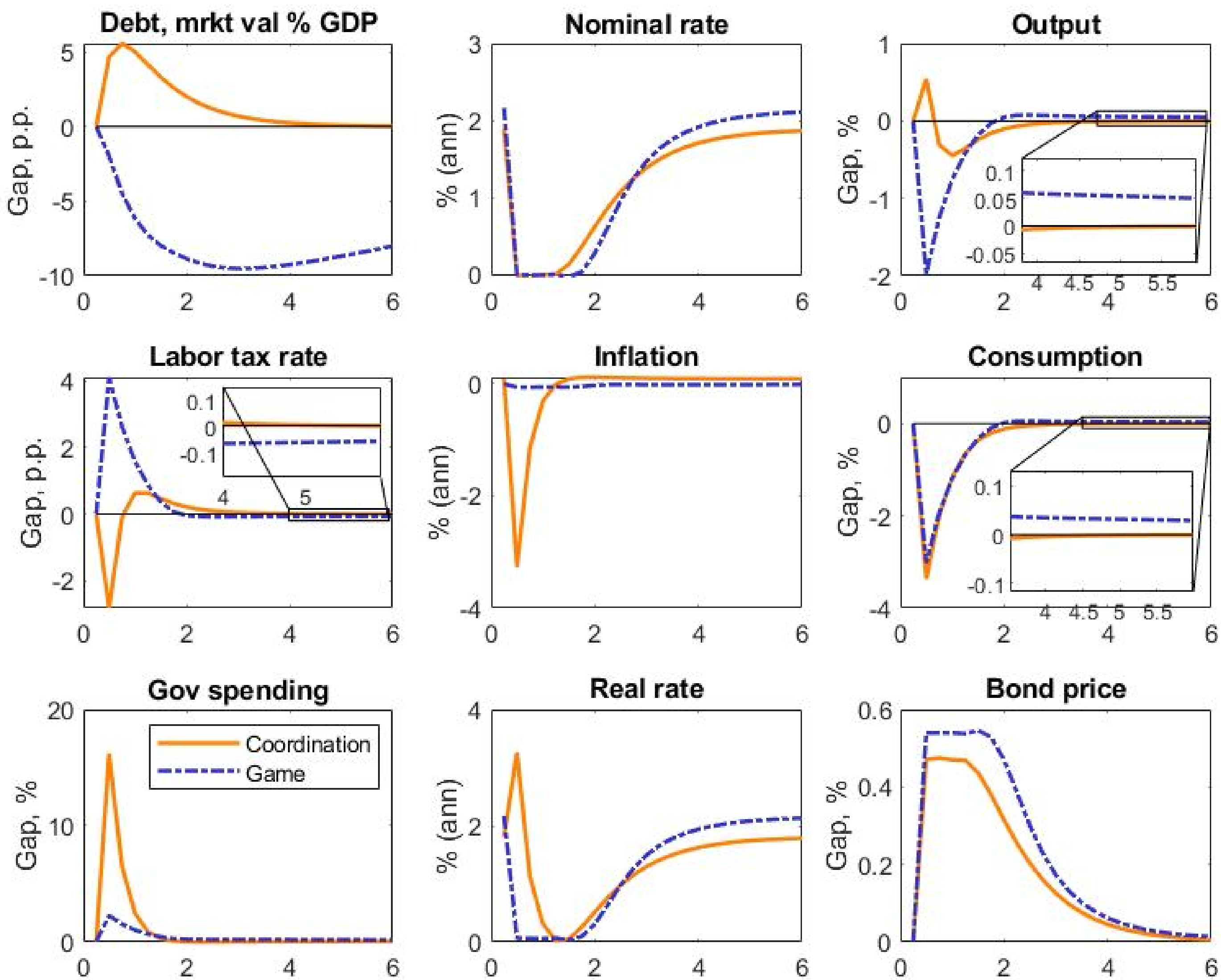
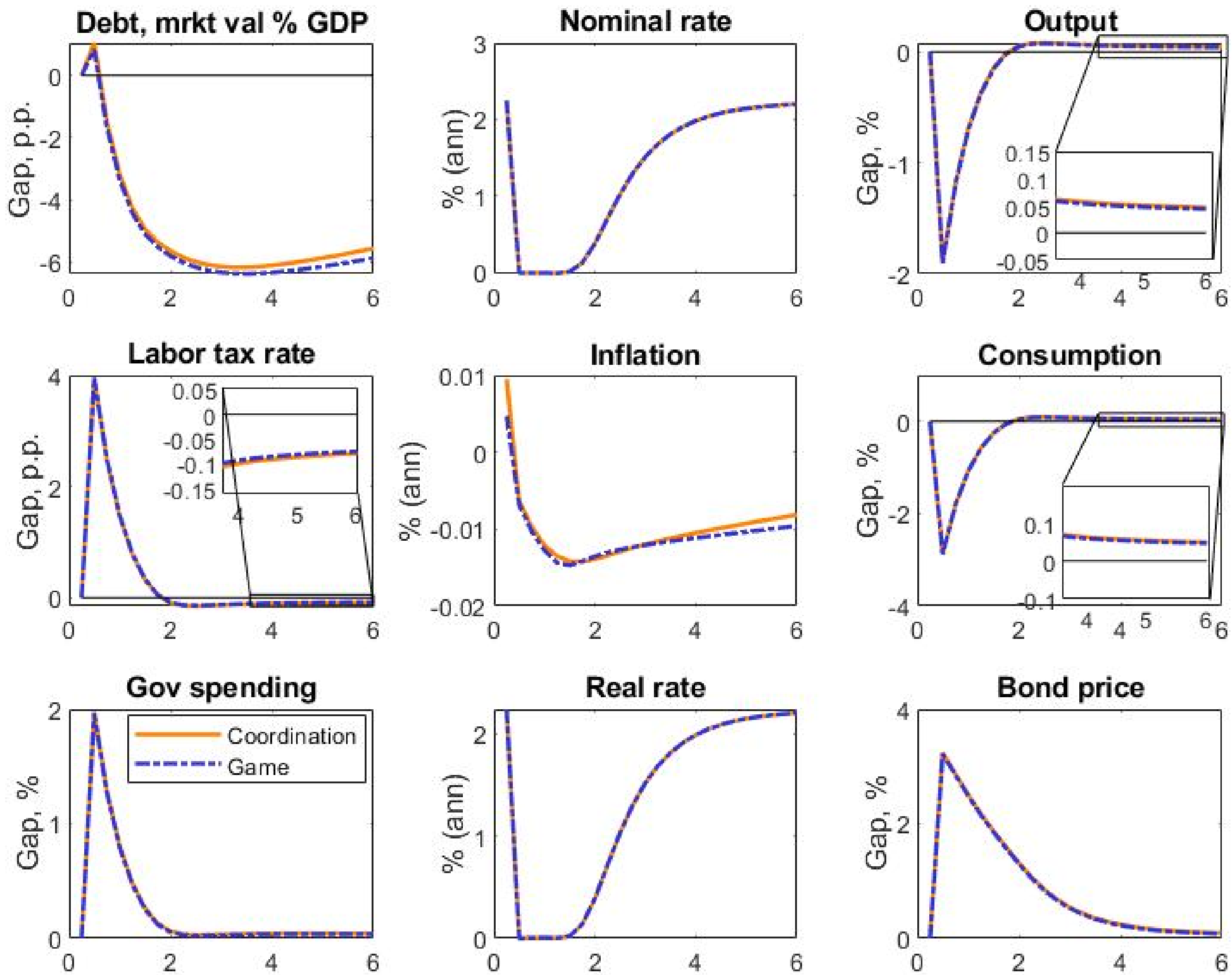


Figure 3: IRFs of key variables to a negative shock on the natural real rate driving the economy in a liquidity trap: long-term debt



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ISSN 1379-244X D/2020/3082/33