Is Workforce Diversity Good for Efficiency? An Approach Based on the Degree of Concavity of the Technology

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Is Workforce Diversity Good for Efficiency?

An Approach Based on the Degree of Concavity of the Technology

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Abstract

To answer the question of workforce diversity and efficiency, this paper departs from the approach used in most recent empirical papers exploiting firm-level evidence, where output is regressed on traditional inputs plus an index of diversity (Parrotta et al., 2012). We suggest addressing the question by adopting a more structural framework. The idea is to root the empirical strategy applied to firm-level data in the theoretical literature on population heterogeneity/stratification and growth (Bénabou, 1994). Essentially, what that literature suggests is that diversity is optimal when the technology displays concavity in the share of workers considered (e.g. decreasing marginal contribution of rising shares of more productive/skilled workers). What is also shown in this paper is that a production function à-la-Hellerstein-Neumark — where workforce diversity is captured via an index of labour shares — is suitable for estimating the concavity of the technology, and thus for assessing the case for/against workforce diversity. Finally, the paper contains an application of this Bénabou-Hellerstein-Neumark framework to two panels of Belgian firms covering the 1998-2012 period. The main result is that of an absence of strong evidence that age, gender or educational diversity is good or bad for efficiency.

Keywords: efficiency, labour diversity, concavity.

JEL Codes: J11, J14, J21

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1. Introduction

The popular press usually discusses workforce diversity as being beneficial for efficiency. How do economists address this topical question?

A first stream of the economic literature adopts a rather micro and within-firm perspective. It has its roots in personnel economics and human resources management theory. Some authors active in that field argue that diversity can create negative effects due to poor communication, lower social ties and trust, and also poor cooperation among workers (Becker, 1957; Lazear, 1998, 1999). Others posit that diversity can be beneficial to firm performance due to better decision making, improved problem solving, enhanced creativity, or a better ability to interact with clients that are themselves very diverse (Hong and Scott 2001, 2004; Glaeser et. al. 2000). Empirically, economists try to assess which of these two antagonist forces prevail by examining how (within firm) workforce diversity translates into firm-level efficiency gains/losses. The most recent contributions exploit the potential of firm-level longitudinal (i.e. panel) data to explore how within firm changes of the degree of diversity of the workforce affect output. Recent examples are Kurtulus (2011), Ilmakunnas & Ilmakunnas, (2011), Garnero et al., (2014) or Parrotta et al. (2012). Compared to studies based on cross-sectional material, these provide evidence and results that are much more robust and trustworthy. Findings generally show that educational diversity is beneficial for firm productivity. In contrast, age and gender (i.e. demographic) diversity are found to hamper firm-level added value per worker *ceteris paribus*.

We would argue that one of weaknesses of the above empirical papers resides in the rather *ad hoc* specification of the underlying technology. The authors basically regress productivity[2] on labour, capital[3] and descriptive indicators of labour diversity (i.e. standard deviation, dissimilarity or Herfindhal/Simpson indices). The reduced-form equations that are estimated do not explicitly derive from the standard textbook production functions (Cobb-Douglas, CES…). What is more, they do no connect with another stream of the economic literature assessing the benefits/losses of diversity. That literature is more structural. It has developed concepts like super[sub]modularity of production (Milgrom & Roberts, 1990; Iranzo et al.,

2 Generally the log of value added per worker.
3 And not all of them have information on capital stock.

2008) [4], the O-ring theory (Kremer, 1993), that of assortative matching (Becker, 1981; Durlauf & Seshadriand, 2003), or has examined the relationship between local stratification and growth (Bénabou, 1994; 1996a,b). Also, it takes a more macro stance. Diversity/homogeneity is discussed in terms of its impact on aggregate output (i.e. that of the different neighbourhoods/regions forming a city/country….), and results carry very specific implications in terms of how diverse/heterogeneous individuals[5] should be allocated across entities.[6] This said, we would argue here that both literatures ultimately address the same key question, which is to determining — using Grossman & Maggi (2000) terminology — whether crossmatching (all entities comprise a diversified set of individuals) is preferable than selfmatching (each type of individuals is concentrated in one distinct entity).

In this paper, we suggest exploring the diversity/efficiency nexus, in the context of private-economy firms, using a branch of that second literature; more specifically, the framework of authors who have studied stratification/diversity and growth in the context of cities (Bénabou, 1993) and/or educational systems (Vandenberghe, 1999). Referring to the discussion above, that literature presents the advantage that it has developed a structural and encompassing view on efficiency, and it deals explicitly with the issue of optimal allocation of diverse individuals. What it essentially shows is that crossmatching (i.e. diversity) is effective when the 'local' technology (i.e. the one characterising neighbourhoods, schools or firms) displays concavity; in other words, decreasing marginal contribution to total output of rising shares of individuals of the most productive type (e.g. highly educated).

The second methodological contribution of this paper is to show that a slightly "augmented" version of the Hellestein-Neumark framework (Hellerstein & Neumark, 1995) [HN hereafter]

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4 The latter narrowly corresponds to what is commonly considered as the cost/benefit of skill diversity (Grossman & Maggi, 2000). Super[sub] modularity carry very specific implications for the optimal organization of production. If a technology is supermodular, efficiency requires self-matching. An example is the O-ring technology imagined by Kremer (1993), where output critically depends each individual's correct execution of his/her task. In that case, workers should be sorted so that those with similar skills work together. In contrast, when a technology is submodular, crossmatching (diversity) is indicated.

5 Mainly in terms of their skills.

6 The results of the more empirical and firm-centric literature implicitly carry similar implications about optimal allocation. If for instance a representative firm is less effective when age heterogeneity (as captured by the standard deviation of age) rises, the inevitable implication is that maximising overall productivity requires age selfmatching.

can be used to assess the degree concavity/convexity of the technology in the share of a particular type of worker. The key idea of HN is to estimate a production function where an heterogeneous/diverse labour input appears as a sum of shares ; and where different worker types (e.g. educated/uneducated; men/women, young/old…) potentially differ in terms of marginal product. Most authors have used the HN framework to measure productivity/skills difference across different types of workers; with the aim of comparing them to wage differences (and assess the degree of alignment of wage and productivity/skills). Our objective here is rather to show that an HN framework, that allows for imperfect substitutability across labour types, is suitable to address the question of concavity/convexity in the share of types of workers, and thus that of the relationship between diversity and efficiency.

The rest of the paper is organized as follows. Section 2 exposes our analytical framework in details. Section 3 presents our data as well as the econometric strategy. Section 4 contains the results of its application to the analysis of Belgian firm-level data where workers differ in terms of educational attainment, age and gender. Section 5 concludes.

2. Framework

i) Concavity/convexity and overall efficiency

Imagine an economy that consists of i=1…N firms, each of them potentially employing two (unequally productive) types of workers. The economy-wide output (W) is the sum of output of the N firms. The proportion of (high/low) productive workers in firm i is x_i ;while the corresponding proportion of the same type of workers in firm N is x_N

[1.] $W=Y(x_1) +Y(x_2)+ \ldots + Y(x_N)$

Starting from a situation synonymous with crossmatching $x_1=x_2=\ldots=x_N=\theta$ — where θ is the share of the workers of the type considered in the whole population — consider the effect of raising their share in firm 1, at the expense of, say, firm N.

[2.] $\delta W/\delta x_1= \delta Y(.)/\delta x_1 + \delta Y(.)/\delta x_N\ [\delta x_N/\delta x_1]$

By assumption the rise of the type's share in firm 1 translates (leaving aside the question of size differences across firms) into a reduction of their share in firm N. Logically thus $[\delta x_N/\delta x_1]=-1$

[3.] $\delta W/\delta x_1= \delta Y(.)/\delta x_1 - \delta Y(.)/\delta x_N$

In $x_1=x_N=\theta$ the two derivatives are equal, and expression [3] is equal to 0, meaning that that point corresponds to an extremum. Whether it defines a maximum or a minimum depends on the second-order condition

[4.] $\delta^2 W/\delta x_1\ \delta x_1 = \delta^2 Y(.)/\delta x_1\ \delta x_1 – \delta^2 Y(.)/\delta x_N\ \delta x_N.[\delta x_N/\delta x_1]$

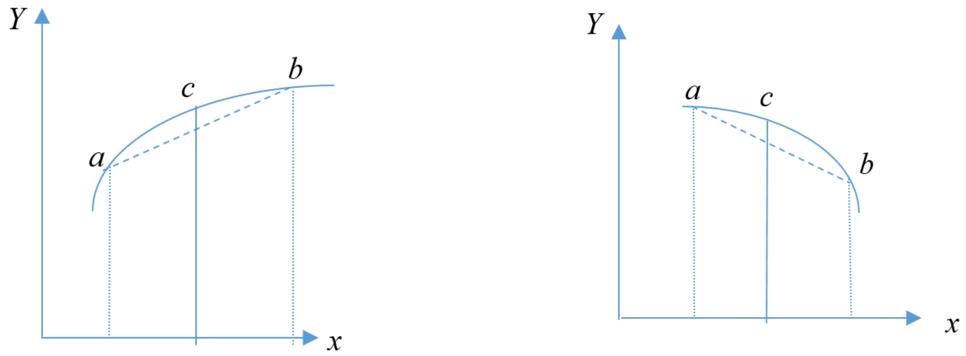
Or equivalently as, again, $[\delta x_N/\delta x_1]=-1$

[5.] $\delta^2 W/\delta x_1\ \delta x_1 = \delta^2 Y(.)/\delta x_i \delta x_1 + \delta^2 Y(.)/\delta x_N\ \delta x_N$

Thus if $\delta^2 Y(.)/\delta x_i\ \delta x_i >0$ (ie. the firm-level technology is convex in the share of the high productive type) optimality requires adopting corner solutions (i.e. selfmatching/minimal diversity). By contrast, if $\delta^2 Y(.)/\delta x_i\ \delta x_i<0$ (the firm-level technology is concave), the optimum is interior and symmetric ($x_1=x_N=\theta$). Maximizing ouput requires crossmatching/maximal diversity.

Figure 1 illustrates the idea of concavity in x (*i.e.* the share of high(low) productive workers) being good for efficiency. Of course, Figure 1a shows that a higher share of the high productive type (say in firm a) translates into a higher firm-level output. But, if we assume that such a move translates into a reduction of the equivalent share elsewhere in the economy (say in firm *b*), the question of the net impact amounts to verifying that output in *c* is higher than the *a* and *b* average. The point to bear in mind is that intra-firm diversity is higher if the economy consists of firms in *c* rather than *a* or *b*.

Figure 1 – Concavity of production technology in a given worker type and overall efficiency



a. High productive type

b. Low productive type

Finally for this section, we would like to talk about the apparent contrast beween the framework of this paper and the one underpinning most existing works by empirical economists on diversity. This paper focuses on workforce diversity and its impact on *aggregate* efficiency, while the latter works generally care about *firm-level* efficiency. Our view is that there is fundamentally no opposition between what matters for a representative firm (and its managers) and what holds for the whole economy.

Assume for a moment that we exclusively consider the point of view of the firm and its managers. They decide to increase the proportion of presumably more productive workers[7] (*x* goes up in Figure 2*)*. That move (say from *a* to *c*) has two consequences. First, it mechanically (i.e. linearly) increases the average of the individual productivities characterizing the workers.

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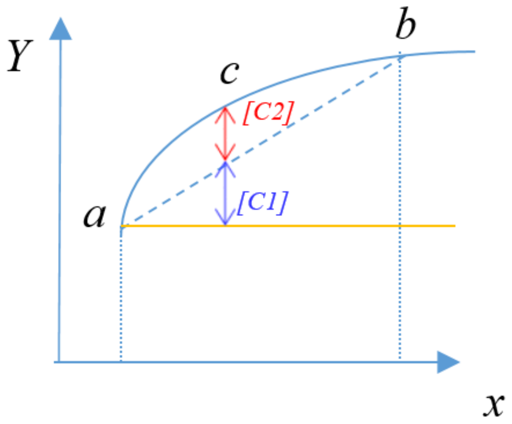
[7] The reasoning is the same with a move synonymous with a rising share of the less productive type.

The second consequence is that the firm becomes more diverse. In order to determine whether diversity matters, managers need to determine whether output Y is affected beyond what mechanically derives from the change of the average of individual productivities. In Figure 2, that mechanical/linear effect corresponds to segment [C1]. And what comes on top to the segment [C2] to the contribution of diversity.[8] That decomposition can be done using a traditional HN log-linear model — where the labour shares appear as a simple *sum* — to which one adds an Herfindahl index.[9] The HN (productivity weighted) sum of labour shares will capture the mechanical/linear output consequence of a higher x (in other words [C1]), and the coefficient of the Herfindahl index will reflect [C2]. What we propose in this paper is to detect [C2] simply by allowing for non-linearties in the labour-quality index, in other words by replacing the traditional HN linear expression by a CES index $[x^{\rho}+\lambda(1-x)^{\rho}]^{1/\rho}$ where $\rho\neq 1$ informs the managers (or the social planner) that diversity matters for efficiency (more on this in the next subsection).

Figure 2 – Concavity and the point of view of the firm's managers



[8] We could have considered a case where [C2] is negative. The important thing is to detect any deviation form the dashed line.

[9] When the information about firms' workforce takes the form of a vector of $j=1\ldots n$ shares, economists generally resort to the Herfindahl/Simpson index to quantify diversity at firm level and assess its impact on output. The index writes $H_i= 1- \sum_j^n (x_{ij})^2$, and in the case of n=2 types $H_i= 1- x_i^2 -(1-x_i)^2$

ii) Concavity and the Hellerstein-Neumark framework

The next step is to specify a realistic (and econometrically tractable) firm-level production function that is function of x_i. The one we retain here owes a lot to Hellerstein & Neumark (1995), but also to the literature on productivity and skill diversity (Duffy et al., 2004; Iranzo et al. 2008), or the one studying the relationship between age and productivity (Vandenberghe et al., 2013).[10] In these works, the production function of a representative firm (from now on, for simplicity of exposure, we drop index *i*) writes as a standard Cobb-Douglas

[6.] $Y=A\,K^{\alpha}\,QL^{\beta}$

where *Y* is output (or productivity), *K* is the stock of capital. The key variable is what is called *the quality of labour aggregate QL*. Total labour is *L*. But what matters is its decomposition into different types. Without loss of generality, we consider a situation with two types (*h*,*l*) where L^h is the number of (presumably) high productive workers in the firm. Parameters μ^h, represents the types' contribution to output (or actual skills).[11]

[7.] $QL=[\mu^h(L^h)^{\rho}+\mu^l(L-L^l)^{\rho}]^{1/\rho}$

We suggest specifying the quality aggregate as a CES index, where labour types are not perfectly substituable and contribute to output non-linearly. The latter assumption is essential for assessing concavity/convexity of the technology in a worker's type, and answering the question of the desirablity of diversity in terms of overall efficiency. By contrast, HN assume perfect substitutability ($\rho=1$) meaning the CES collapses to a simple sum, and also, (as will become clearer after) that diversity does not matter for the economy's efficiency, as the firm-level technology is neither convave nor convex in a worker's type.

Expression [7] can be easily be rewritten in terms of labour shares, with $x\equiv L_h/L$ the proportion of workers with contribution μ_h. By definition, in a two-type setting, (1-x) is the share of the

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[10] Which is relatively more developed than the literature on workforce diversity, and better connected to the standard economic theory of production.

[11] Note that here, contrary to Iranzo et al. (2008), workers' skills are not available or measured *ex ante*. But estimates of the relative productivity (i.e. skills) by type can be estimated econometrically.

other type of workers present in the firm.

[8.] $QL = L\ [\mu_h x^\rho + \mu_l (1-x)^\rho]^{1/\rho}$

or equivalently, picking type h workers as reference category

[9.] $QL = L\ \mu_h^{1/\rho}\ [x^\rho + \lambda(1-x)^\rho]^{1/\rho}$

with $\lambda \equiv \mu_l/\mu_h$ reflect the relative contribution of type l workers to output

The point is that expression [9] now appears as function of the share of type h workers (x). This means that the key question raised in this paper (i.e. is diversity good/bad for efficiency), amounts to determining whether QL is concave or convex in x.

Back to the full production function, one needs to inject [9] into [6]

[10.] $Y = \tilde{A}\ K^\alpha\ L^\beta\ [x^\rho + \lambda(1-x)^\rho]^{\beta/\rho}$

where $\tilde{A} = A\ \mu_h^{\beta/\rho}$

Noting $f(x) \equiv [x^\rho + \lambda(1x)^\rho]^{1/\rho}$ the part of the labour quality aggregate that consists of a CES index, the firm-level output's second-order derivative with respect to x is

[11.] $\frac{\partial^2 Y(x)}{\partial x \partial x} = A\mu_h^{\beta/\rho} K^\alpha L^\beta\ \beta f(x)^{\beta-1} \Big[(\beta-1)(\frac{\partial f(x)}{\partial x}^2 / f(x) + \frac{\partial^2 f(x)}{\partial x \partial x}\Big]$

where

[12.] $\frac{\partial f(x)}{\partial x} = [x^\rho + \lambda(1-x)^\rho]^{1/\rho-1}(x^{\rho-1} - \lambda(1-x)^{\rho-1}) = [x^\rho + \lambda(1-x)^\rho]^{1/\rho-1} x^{\rho-1}(1-\lambda(\frac{1-x}{x})^{\rho-1})$

and, most importantly, the 2nd order derivative of the CES index is

[13.] $\frac{\partial^2 f(x)}{\partial x \partial x} = (\rho-1)[x^\rho + \lambda(1-x)^\rho]^{1/\rho-2} \lambda (1-x)^{\rho-2} x^{\rho-2}$

The sign of [11] is entirely determined by those of parameters β and ρ. The first parameter is nothing but the output elasticity with respect to total labour of the Cobb-Douglas part of the prodution function. And, presumably, in the presence of capital, it is inferior to 1. This means a diminishing marginal productivity for total labour (L).[12]And, by extension, that law also applies to any quality-adjusted labour aggregate à-la HN. Assuming that labour types' marginal productivity differ significantly (ie. in expression [12] $\lambda(\frac{1-x}{x})^{\rho-1}\neq 1$ or just that $\lambda\neq 1$ in case of perfect substitutablity ($\rho=1$)), then changes in the value of x amounts to changing the overall level of (quality-ajusted) labour. That logically translates into a fall of marginal productivity, that is captured in expression [11] by the term premultiplied by β-1.

The more interesting question is what happens with parameter ρ conditional on a certain value of β ; or, said differently, to determine whether the law of diminishing marginal productivity is positively (or negatively) affected by the diversity of the labour force. And that amounts to determining if $\rho\neq 1$. If $\rho<1$ we would conclude that diversity is good for efficiency *ceteris paribus*. If $\rho>1$ then diversity is a bad thing for efficiency. And if $\rho=1$ diversity becomes irrelevant.

3. Econometric analysis

i) Data

The empirical results of this paper derive from the analysis of two panels. The first one contains around 8,000+ firms with more than 20 employees. These firms are largely representative of the Belgian private economy in terms of sector/industry, and are well documented as to the capital they used and, their productivity performance.[13] Using firm identifiers, we have been able to add social security information[14] on the age and gender of (all) workers employed by these firms, for a period running from 1998 to 2006. Table 1 presents the descriptive statistics.

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12 With the standard HN-Cobb-Douglas $Y=A\,K^{\alpha}\,QL^{\beta}$, one has that $\frac{\partial^2 Y(x)}{\partial L \partial L}= A\,K^{\alpha}\beta(\beta\text{-}1)\,L^{\beta-2}<0$ if $\beta<1$

13 These observations come from the Bel-first database. Most for-profit firms located in Belgium must feed that database to comply with legal prescriptions.

14 Compiled in the so-called Carrefour database.

Of particular importance are the ones describing age and gender. Note in that age and gender diversity (as captured by the Herfindahl index) seems to have risen between 1998 and 2006.

The second panel contains information about the educational attainment of the workforce. It comprises a slightly smaller number of firms (4000+); also from all sectors forming the Belgian private economy. It runs from 2008 to 2012. Firms are also well documented in terms of sector, overall size of the labour force, capital used, and productivity (value added). But there is no information on the age and gender of the workforce that would allow a more refined breakdown of educational categories.

Descriptive statistics, are reported in Table 2. Of prime interest in this paper is the breakdown by educational attainment. Table 2 shows that, during the observed period (2008–2012), more than 75% of the workforce of private for-profit firms located in Belgium have, at most, an upper secondary school degree. This means less than 25% of workers are in possessing of a tertiary education background; clearly less than the percentage among the current generation of school leavers.[15] This discrepancy logically reflects the lower propensity of older generations to stay on beyond secondary education, and complete a tertiary degree. But given the focus of this paper, perhaps the most important point worth observing is that educational diversity inside firms located in Belgium (as reflected by the Herfindahl index reported in the last column) has seemingly increased between 2008 and 2012.

Table1- Descriptive statistics. Age, Gender. Main variables (weighted (£))/1998-2006

	Value added[log] k EUR	N. of empl.[log]	Capital[log]	Share 50+	Herf age	Share female	Herf gender
1998	10.072	6.146	8.111	0.132	0.213	0.249	0.277
1999	10.095	6.088	8.146	0.136	0.217	0.256	0.284
2000	10.140	6.056	8.198	0.139	0.223	0.262	0.289
2001	10.122	6.148	8.130	0.143	0.226	0.271	0.298
2002	10.353	6.356	8.428	0.154	0.240	0.280	0.306
2003	10.356	6.268	8.503	0.167	0.256	0.281	0.306
2004	10.424	6.270	8.522	0.174	0.264	0.284	0.308
2005	10.435	6.280	8.486	0.179	0.269	0.289	0.310
2006	10.510	6.263	8.665	0.188	0.278	0.294	0.314
N	75393						

£:Weights are equal to the firm's number of workers Source: Belfirst-Carrefour

[15] Statistics Belgium estimates that they now represent between 35 and 40% of a cohort.

Table2 - Descriptive statistics. Education. Main variables (weighted(£))/2008-2012

	Value added [log]	N. of empl.[log]	Capital[log]	secondary or less	more than secon.	Herf. educ
2008	9.515	5.313	10.248	0.784	0.215	0.181
2009	9.262	5.080	10.035	0.761	0.239	0.200
2010	9.340	5.133	10.095	0.762	0.238	0.199
2011	9.373	5.170	10.111	0.755	0.245	0.199
2012	9.391	5.171	10.119	0.751	0.249	0.203
N	227,838					

£:Weights are equal to the firm's number of workers Source: Belfirst-Carrefour

ii) Identification strategy

The econometric version of our model, that we apply to a panel of firms writes

[14.] $Y_{it}=A_{i0}\, K_{it}^{\alpha}\, (QL_{it})^{\beta}\, e^{\tau.t\, +\, \omega_{it}}$

And its log equivalent is

[15.] $ln\ Y_{it}= B_{i0} + \alpha ln(K_{it})+ \beta ln(L_{it}) + \beta ln\ f(x_{it})+ \tau.t + \omega_{it}$

with $B_{i0}\equiv ln(A_{i0}) + \beta/\rho ln(\mu_h)$, $f(x_{it})=(x_{it}^{\rho} + \lambda(1\text{-}x_{it})^{\rho})^{1/\rho}$ the CES index as a function of labour shares (2-types case), and τ the constant rate of TFP growth, common to all firms.

We assume a three-component error term.

[16.] $\omega_{it}= \Theta_i + \gamma_{it} + \delta_{it}$

meaning that the linear (or non linear) least squares sample-error term potentially consists of *i)* an unobservable firm fixed effect Θ_i; *ii)* a short-term shock γ_{it} (whose evolution may correspond to a first-order Markov chain, causing a simultaneity bias), and is observed by the firm (but not by the econometrician) and (partially) anticipated by managers, and, *iii)* a purely random shock δ_{it}.

The panel structure of our data allows for the estimation of models that eliminate the fixed effects (Θ_i). For instance, resorting to the growth-equivalent of [15] (i.e. lag *T* differences of logs, or log of ratio of Y_{it} to its lagged *T* values) leads to

[17.] $ln\ (Y_{it}/Y_{it\text{-}T}) = \tau T + \alpha ln(K_{it}/K_{it\text{-}T})+\ \beta ln(L_{it}/L_{it\text{-}T}) + \beta/\rho ln(f(x_{it})/f(x_{it\text{-}T}))\ +\ \omega_{it}\text{-}\ \omega_{it\text{-}T}$

where ω_{it}- $\omega_{it\text{-}T}$ = γ_{it} - γ_{iT} + δ_{it}- $\delta_{it\text{-}T}$

This said, another challenge is to go around the simultaneity bias caused by short-term shock γ_{it}. Equation [17] suggests estimating a model where the dependent variable is the (estimated) TFP, following a two-step strategy.[16] The first stept consists of estimating the log of TFP as the residual of the regression of output on capital and total labour:

[18.] $ln(\frac{\widehat{TFP_{\iota t}}}{TFP_{\iota t-T}}) = ln\ (Y_{it}/Y_{it\text{-}T})\ \text{-}\ \hat{\alpha}ln(K_{it}/K_{it\text{-}T})\text{-}\ \hat{\beta}ln(L_{it}/L_{it\text{-}T})$

It is when estimating that first-step equation that we control for the presence of γ_{it}, using the strategy developed by Levinsohn & Petrin (2003) (LP henceforth) and, more recently by Ackerberg, Caves & Fraser (2006) (ACF henceforth). Both LP and ACF estimations involve assumptions about the time of the choice of inputs. Capital is assumed quasi-fixed (in the short- to medium run), whereas labour is more flexible and partially chosen after the (unilaterally) observed productivity chock γ_{it}. This makes least square estimates for labour inputs endogeneous. To go around this problem, LP assumes that γ_{it} can be proxied by a 3rd order polynom in the use of intermediate inputs (i.e. purchases of raw materials, services, electricity…) and also in capital.[17] The sole presence of this proxy/polynom at step 1 makes it possible to consistently estimate β using OLS or non-linear least squares (NLLSQ). By extension, the residuals of that first-step LP-ACF equation are also clear of γ_{it} and can be used at step two to consistently estimate λ and ρ (ie. the parameters of the CES index $f(x_{it})$) using NLLSQ.

[19.] $ln\ (\widehat{TFP_{\iota t}/\ TFP_{\iota t\text{-}T}}) = \tau T + \beta/\rho ln(f_{it}(x_{it})/f_{it\text{-}T}(x_{it\text{-}T})) + \delta_{it}\ \text{-}\ \delta_{it\text{-}T}$

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[16] Not to be confounded with the two-stage estimation characterizing the method of Levinsohn & Petrin (2003) or Ackerberg, Caves & Fraser (2006), to estimate the parameters of a production function.

[17] The actual assumption made by LP is that the use of intermediates inputs is a monotonic function of γ_{it} and k_{it} that can be inverted. And the inverse function can be approximated by a 3rd-order polynom in intermediates and capital.

iii) Econometric results

We report the key results of our analysis in Tables 3 (age), 4 (gender) and 5(education). In each of them we report the results for the level [1] and the growth specification [2],[3],[4]. The advantage of the growth specification is that it accounts for firm-level fixed effects, known for being very important across firms (Syverson, 2011). Among our growth specification, we distinguish one-step [2] and two-step models [3][4]. The first step of the latter implements the LP [3] or the ACF[4] strategy to control for endogeneity/simultaneity and delivers unbiased estimates of (total factor) productivity which can then (in the second step) be regressed on labour shares. Alongside each of these 4 specifications, we also report the results obtained with the traditional model used by empirical economists that consists of regressing output on total capital and labour, the HN sum of shares for the different types of labour (bar the reference one)[18] plus the firm-level Herfindahl index capturing workforce diversity.

A first result is that we find evidence of (marginal) productivity differences across all the estimated models (i.e. $\lambda(\frac{1-x}{x})^{\rho-1}\neq 1)$. Younger workers appear more productive than older workers, educated workers more than less educated ones, and in all cases except one (Table 5, model [4]), men seem more productive than women.

Second, as to the degree of concavity/convexity of the production function, our main result is that of an absence of strong evidence that age, gender or educational diversity is good or bad for efficiency. In Tables 3 (age) and 4 (gender), the probability that $\rho<1$ (i.e. concavity/diversity being good for efficiency) seems reasonably high when estimating models [1][2], but no longer when turning to the models that account for endogeneity/simultaneity [3][4]; in particular ACF where for both age and gender ρ*'s* appear very close to 1. This is also what we find for education, but this time for all the econometric models estimated.

Third, our results match up with those delivered by using the traditional HN + Herfindahl index approach. In Table 3 (age) and in Table 4 (gender), the coefficient of the Herfindhal index in models [1] [2] — akin parameter ρ — hints at diversity-related efficiency gains. But these gains

18 Meaning here, given our two-type setting, that the equation only contains one share.

invariably vanish in the models [3][4] that account for endogeneity/simultaneity. As to education (Table 5), we also conclude that the coefficient of the Herfindahl index, like parameter ρ, and whatever the model used [1][2][3][4], points at an absence of any significant impact of diversity on efficiency.

Table 3- Age diversity

	Level		Growth specification (FE)					
	[1] CES	[1'] HN-Herf	[2] CES	[2'] HN-Herf	[3] CES LP	[3'] HN-Herf LP	[4] CES ACF	[4'] HN-Herf ACF
α [K]	0.0856^{***}	0.0853^{***}	0.0237^{***}	0.0237^{***}				
	(0.00117)	(0.004)	(0.00271)	(0.003)				
β [L]	0.923^{***}	0.921^{***}	0.651^{***}	0.651^{***}				
	(0.00227)	(0.007)	(0.00477)	(0.005)				
ρ	0.594^{***}		0.855^{***}		0.960^{***}		1.095^{***}	
	(0.0405)		(0.0804)		(0.0947)		(0.232)	
λ	1.450^{***}		1.478^{***}		1.225^{***}		1.233^{***}	
	(0.0844)		(0.102)		(0.0866)		(0.168)	
η [1-x]		0.627^{***}		0.333^{***}		0.194^{***}		0.270^{**}
		(0.140)		(0.054)		(0.0543)		(0.0950)
δ [Herf]		**0.567^{***}**		**0.149^{***}**		**0.0612**		**0.0818**
		(0.113)		(0.044)		(0.0446)		(0.0821)
Controls	Share part-time work, share blue-collar workers							
Nobs	73,738	73,738	63,792	63,792	63,792	63,792	21,671	21,671
λ	1.450		1.478		1.225		1.233	
Pr(λ=1)	0.000		0.000		0.009		0.166	
RMP$^{\$}$	1.176	1.680	1.368	1.512	1.198	1.248	1.297	1.353
Pr(RMP=1)	0.000		0.000		0.000		0.000	
ρ	**0.594^{***}**		**0.855**		**0.960**		**1.095**	
Pr(ρ =1)	0.000		0.072		0.670		0.683	

$^{\$}$: Implied relative marginal productivity (ref:workers aged 50+) = $\lambda(\frac{1-x}{x})^{\rho-1}$ or $\lambda = \eta/\beta + 1$ in the case of the HN-Herfindhal model. Standard errors in parentheses
Source: Bel-first; Carrefour
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 4- Gender diversity

	Level		Growth specification (FE)					
	[1] CES	[1'] HN-Herf	[2] CES	[2'] HN-Herf	[3] CES LP	[3'] NH-Herf LP	[4] CES ACF	[4'] HN-Herf ACF
α [K]	0.0825***	0.0806***	0.0235***	0.0235***				
	(0.00116)	(0.00403)	(0.00271)	(0.00271)				
β [L]	0.924***	0.922***	0.652***	0.652***				
	(0.00225)	(0.00738)	(0.00478)	(0.00478)				
ρ	0.644***		0.840***		0.966***		1.050***	
	(0.0104)		(0.0401)		(0.0487)		(0.168)	
λ	1.529***		1.245***		1.147***		0.959***	
	(0.0153)		(0.0426)		(0.0372)		(0.0974)	
η [1-x]		0.462***		0.151***		0.112***		-0.00218
		(0.0292)		(0.0257)		(0.0258)		(0.0599)
δ [Herf]		**0.526*****		**0.0982*****		**0.0363**		**-0.0300**
		(0.0385)		(0.0272)		(0.0273)		(0.0617)
Controls	Share part-time work, share blue-collar workers							
Nobs	73,736	73,736	63,788	63,788	63,788	63,788	21,671	21,671
λ	1.529		1.245		1.147		0.959	
Pr(λ=1)	0.000		0.000		0.000		0.675	
RMP$	1.758	1.501	1.304	1.232	1.156	1.144	0.951	0.997
Pr(RMP=1)	0.000		0.000		0.000		0.000	
ρ	**0.644*****		**0.840*****		**0.966**		**1.050**	
Pr(ρ =1)	0.000		0.000		0.483		0.765	

$: Implied relative marginal productivity (ref: male workers) = $\lambda(\frac{1-x}{x})^{\rho-1}$ or $\lambda = \eta/\beta + 1$ in the case of the HN-Herfindhal model

Standard errors in parentheses

Source: Bel-first; Carrefour

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 5 - Educational diversity

	Level		Growth specification (FE)					
	[1] CES	[1'] HN-Herf	[2] CES	[2'] Herf	[3] CES LP	[3'] Herf LP	[4] CES ACF	[4'] Herf ACF
α [K]	0.310***	0.310***	0.265***	0.265***				
	(0.000832)	(0.00279)	(0.00292)	(0.00292)				
β [L]	0.712***	0.711***	0.559***	0.559***				
	(0.00120)	(0.00310)	(0.00316)	(0.00316)				
ρ	0.996***		0.968***		0.999***		0.942***	
	(0.00862)		(0.0153)		(0.0151)		(0.0485)	
λ	1.386***		1.009***		1.012***		1.051***	
	(0.00933)		(0.00855)		(0.00800)		(0.0277)	
η [1-x]		0.224***		0.00491		0.00732		0.0466*
		(0.00802)		(0.00496)		(0.00500)		(0.0220)
δ [Herf]		**0.0571*****		**0.0143**		**0.00260**		**0.0286**
		(0.0120)		(0.00817)		(0.00822)		(0.0374)
Controls	Share females, share blue-collar workers							
Nobs	227,564	227,564	172,816	172,816	172,816	172,816	7,536	7,536
λ	1.386		1.009		1.012		1.051	
Pr(λ=1)	0.000		0.288		0.130		0.0658	
RMP[$]	1.395	1.315	1.057	1.009	1.014	1.012	1.145	1.060
Pr(RMP=1)	0.000		0.000		0.000		0.000	
β	0.712		0.559		0.624		0.778	
Pr(β =1)	0.000		0.000		0.000		0.000	
ρ	**0.996**		**0.968***		**0.999**		**0.942**	
Pr(ρ =1)	0.616		0.036		0.943		0.236	

[$]: Implied relative marginal productivity (ref: workers with an upper secondary degree or less) = $\lambda(\frac{1-x}{x})^{\rho-1}$ or $\lambda = \eta/\beta + 1$ in the case of the HN-Herfindhal model. Standard errors in parentheses
Source: Bel-first; Carrefour
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

5. Final comments

The key message of the paper is that looking at the degree of concavity/convexity of the production function is useful to assess the efficiency costs/benefits of labour diversity. The inspiration comes from the economic literature on (social) heterogeneity, stratification and growth (Bénabou, 1994, 1996a,b ; Vandenberghe, 1999). By focusing on concavity, this paper departs from the approach used by most recent empirical economics papers, that consists of regressing output on descriptive indices of workforce diversity. We think that our approach is more structural. It explicitly addresses the question underpinning most of the empirical works done by economists about workforce diversity and efficiency; namely whether crossmatching (all entities have a diversified set of individuals) is more/less effective than selfmatching (one type prevails in each entity). And although it takes a more macro stance, the key issue remains the one that matters for firm-level efficiency.

We show mathematically that if the technology used by individual firms is concave in the share of a given worker's type (e.g. old, female or educated), crossmatching/diversity of the types is synonymous with efficiency. We then show that a generalised version of Hellerstein-Neumark labour-quality index — that has been extensively used by empirical economists to analyse productivity-related issues — is suitable to assess the degree of concavity of the technology. What HN have shown is that labour heterogeneity/diversity can be represented, within a Cobb-Douglas function, as a sum of labour shares. To all those interested in analysing the diversity-efficiency nexus, we simply propose to aggregate these shares non-linearly as a CES index.

In the second part of the paper, we implement our innovate framework using two panels of firms located in Belgium for which we have information on age/gender (panel 1) and educational attainment (panel 2). We apply various treatments that are aimed at controlling for the two main (potential) sources of bias: firm unobserved heterogeneity and simultaneity. We address the first problem by resorting to a growth/fixed-effect specification of our HN-with-CES-index production function. And we cope with simultaneity by implementing both the Levinsohn & Petrin and the Ackerberg, Caves & Fraser (2006) idea of using observed intermediate input decisions (i.e. purchases of raw materials, services, electricity…) to control for/proxy unobserved short-term productivity shocks causing simultaneity.

The main results of the paper is an absence of strong and systematic evidence that age or gender or educational diversity is good/bad for efficiency.

References

Ackerberg, D.A, K. Caves and G. Frazer (2006), Structural Identification of Production Functions, Department of Economics, *Working Paper*, UCLA.

Becker, G. (1957), *The Economics of Discrimination*, Chicago, University of Chicago Press

Becker, G. (1981), *Treatise on the Family*, Cambridge, Harvard University Press.

Bénabou, R. (1994), Education, Income Distribution and Growth: The Local Connection, *NBER Working Paper Series*, No. 4798, NBER, Ma.

Bénabou, R. (1996a), Heterogeneity, Stratification, and Growth: Macroeconomic Implications of Community Structure and School Finance, *American Economic Review*, 86(3), pp. 584-609.

Bénabou, R. (1996b), Equity and Efficiency in Human Capital Investment: The Local Connection, *Review of Economic Studies*, 63(2), pp. 237-64.

Duffy, J., C. Papageorgiou and F. Perez-Sebastian (2004), Capital-Skill Complementarity? Evidence from a Panel of Countries, *The Review of Economics and Statistics*, MIT Press, 86(1), pp. 327-344.

Durlauf, S.N. & A. Seshadriand (2003), Is assortative matching efficient?, *Economic Theory*, 21, pp. 475-493

Garnero, A, S. Kampelmann and F. Rycx (2014), The Heterogeneous Effects of Workforce Diversity on Productivity, Wages, and Profits, *Industrial Relations: A Journal of Economy and Society*, 53(3), pp. 430-477.

Glaeser, E., D.I. Laibson, J.A. Scheinkman and C.L. Soutter (2000), Measuring Trust, *The Quarterly Journal of Economics*, 115, pp. 811-846.

Grossman, G. and G. Maggi (2000), Diversity and Trade, *The American Economic Review*, 90(5), pp. 1255-1275.

Hellerstein, J.K. and D. Neumark (1995), Are Earning Profiles Steeper than Productivity Profiles: Evidence from Israeli Firm-Level Data, *The Journal of Human Resources*, 30 (1). pp. 89-112.

Hong, L., and E.P. Scott (2001), Problem Solving by Heterogeneous Agents. *Journal of Economic Theory*, 97, pp. 123-163.

Hong, L., and Scott E. P. (2004), Groups of Diverse Problem Solvers Can Outperform Groups of High-Ability Problem Solvers, *Proceedings of the National Academy of Sciences*, 101, pp. 123-139.

Ilmakunnas, P. and S. Ilmakunnas (2011), Diversity at the Workplace: Whom Does it Benefit?, *De Economist*, Springer, 159(2), pp. 223-255.

Iranzo, S., F. Schivardi and E. Tosetti (2008). Skill Dispersion and Firm Productivity: An Analysis with Employer-Employee Matched Data, *Journal of Labor Economics*, 26, pp. 247-285.

Kurtulus, A. F. (2011), What Types of Diversity Benefit Workers? Empirical Evidence on the Effects of Co-Worker Dissimilarity on the Performance of Employees. *Industrial Relations: A Journal of Economy and Society*, 50, pp. 678-712.

Kremer, M. (1993), The O-Ring Theory of Economic Development, *The Quarterly Journal of Economics*, 108(3), pp. 551-575.

Lazear, E. P. (1998), *Personnel Economics for Managers*, New York, John Wiley & Sons.

Lazear, E. P. (1999), Globalisation and the Market for Team-Mates, *Economic Journal*, 109(454), pp. 15-40

Levinsohn, J. and A. Petrin (2003), Estimating production functions using inputs to control for unobservables. *Review of Economic Studies*. 70 (2), pp. 317-341.

Milgrom. P. and J. Roberts, (1990), The Economics of Modern Manufacturing: Technology, Strategy, and Organization, *The American Economic Review*, 80(3), pp. 511-528.

Parrotta, P., D. Pozzoli and M. Pytlikova (2012), Does Labor Diversity Affect Firm Productivity?, *IZA Discussion Papers 6973*, Institute for the Study of Labor (IZA).

Syverson, C. (2011), What Determines Productivity?, *Journal of Economic Literature*, 49(2), pp. 326–365.

Vandenberghe, V. (1999), Cost Efficiency and Feasibility of Education Policy in the Presence of Local Social Externalities, IRES-Discussion paper No 1999-021, UCL, Louvain-la-Neuve.

Vandenberghe, V., M. Rigo and F. Waltenberg (2013), Ageing and Employability. Evidence from Belgian Firm-Level Data, *Journal of Productivity Analysis*, 40(1), pp. 111-136.

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