# Inefficient equilibrium unemployment in a duocentric economy with matching frictions

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# Inefficient equilibrium unemployment in a duocentric economy with matching frictions*


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**Abstract**

This article examines unemployment disparities and efficiency in a densely populated economy with two job centers and workers distributed between them. We introduce commuting costs and search-matching frictions to deal with the spatial mismatch between workers and firms. In equilibrium, there exists a unique threshold location where job-seekers are indifferent between job centers. In a decentralized economy job-seekers do not internalize a composition externality they impose on all the unemployed. Their decisions over job-search is thus typically not optimal and hence the equilibrium unemployment rates are inefficient. We calibrate the model for Los Angeles and Chicago Metropolitan Statistical Areas. Simulations exercises suggest that changes in the workforce distribution have non-negligible effects on unemployment rates, wages and net output.



Keywords: Spatial mismatch, commuting, urban unemployment, externality, United States.
JEL codes: J64, R13, R23.



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## 1. Introduction

Does the spatial structure of a city affect labor markets outcomes? How do job-seekers organize their search activity along the space dimension? What are the implications of this activity on equilibrium unemployment and efficiency? These questions have already been studied in monocentric cities (see Zenou, 2009, for an overview) or in the case of uniformly distributed agents and jobs around the circle (see Marimon and Zilibotti, 1999; Hamilton, Thisse, and Zenou, 2000; Decreuse, 2008). Cities with similar population sizes are often believed to be comparable. Fig. 1 illustrates that this is however not true for major cities.[1] Paris and Shanghai have around 7.5 millions of people, yet Shanghai is 3.5 times more crowded than Paris. London and Moscow seem to have a more uniform population distribution compared to Jakarta, Berlin or New York which are more populated in the center. In the U.S. Los Angeles Metropolitan Statistical Area (MSA) has twice the population density of Chicago MSA. Moreover for the U.S. the monocentric view seems old-dated: "America changed from a nation of distinct cities separated by farmland, to a place where employment and population density is far more continuous" according to Glaeser (2007).

The aim of the paper is to better understand disparities in unemployment rates in metropolitan areas, in particular in the U.S. We consider a densely populated city with two business districts and a possibly non-uniform distribution of workers located along a line connecting them. Each business district is a distinct labor market characterized by search-matching frictions. An endogenous number of firms choose to set up in either of these centers. The unemployed use their time endowment to look for vacant jobs in the business districts. Constrained by high relocation costs, workers maintain their residence even if their employment status changes.[2] Employed workers commute to the job center where they have been recruited until the match is exogenously destroyed.

In equilibrium, unemployed workers specialize their search in only one job center. The closer a job-seeker resides to a job center, the lower are the commuting costs, so the higher is the total surplus created if a firm located in this job center matches with this job-seeker. As we assume individual Nash bargaining over the wage, commuting costs are shared between the employer and the employee. Moreover, the vacancies open in a job center are generic in the sense of being

[1]Figure 1 presents a three dimensional perspective where the boundaries of a city are the result of overlaying population density and built-up areas. For instance, London is limited to its 52 boroughs, Shanghai to "the city proper" and Paris to the municipal area and "la petite couronne." Jakarta is represented by the Jabotabek area which is Jakarta municipality plus Tangerang, Bekasi, and Bogor. Moscow is limited to the area within its municipal boundary.

[2]Rupert and Wasmer (2009) provide evidence about the reasons for moving within the county. In the U.S., 65.4% of the intra-county residential mobility is house related and only a small 5.6% move within a county for job related reasons (other motives being family or personal reasons). High relocation costs have also been assumed by Raphael and Riker (1999), Brueckner and Zenou (2003), Zenou (2006, 2009a,c, 2013).

accessible to any job-seeker wherever she lives. The expected profit made in a job center is higher when those seeking a job there are concentrated in the neighborhood of this center. When an additional individual joins the queue of job-seekers in a center, she ignores the consequences of this decision on expected profits and hence on vacancy creation. So doing, decentralized agents overlook an externality their search decision imposes on all job-seekers in the same center. If agents chose where they search in an efficient way (i.e. so as to maximize net output), the so-called Hosios condition would be sufficient to internalize standard search-matching externalities. This condition which is familiar in the search-matching literature expresses that agents' shares of the total surplus created by a match equal respectively the elasticities of the matching function with respect to the stocks of buyers (vacant jobs) and sellers (job-seekers) in the labor market. However, as search decisions have no reason to be optimal, the decentralized economy is typically not efficient and so the equilibrium unemployment rates are typically inefficient as well, even if the Hosios condition is met.

A numerical analysis provides orders of magnitude of the impacts of changes in the shape of the workforce distribution on unemployment rates and on efficiency. A first exercise considers a uniformly distributed workforce of mass lower than one and a complementary mass of workers located in the central business district (CBD). Simulations show that letting this mass rise lowers the unemployment rate everywhere. According to the size of the mass of workers in the CBD, the decentralized value of net output can be up to 2% lower then its efficient counterpart. Next, we consider Los Angeles and Chicago MSAs. We calibrate the model in both MSAs with census data for the year 2000. Then, we develop several counterfactual exercises either interchanging the two workforce distributions or replacing the actual ones by some standard parametric distributions. The counterfactual assumptions we consider can cause changes in unemployment rates up to about half a percentage point and in net output up to 5% when the workforce is more concentrated far from the job centers. These are non-negligible effects.

Because of our focus on duo-centric cities, this article is mainly related to Coulson, Laing, and Wang (2001).[3] They show that, in the presence of heterogeneous commuting ability, the unemployment rate will be higher in the region with the highest vacancy costs. Although we are also interested in unemployment disparities we depart from their approach in the following ways. First, while in Coulson et al. (2001) workers are located only in the job centers, we spatially distribute the workforce between the job centers (with commuting costs increasing with distances). This assumption seems to us more in accordance with the stylized fact described in

[3] In addition to the already-mentioned monocentric city and circular models, Rupert and Wasmer (2012) have recently developed a new framework under the isotropy assumption according to which space looks the same wherever an agent is located.

Glaeser (2007)'s quote above. Second, we conduct a welfare analysis. Third, we provide orders of magnitude of the importance of the shape of the population distribution thanks to a numerical analysis. Regarding the composition externality at the origin of our inefficiency result, we are close to the mechanism put forward by Decreuse (2008) in another context with search-matching frictions.

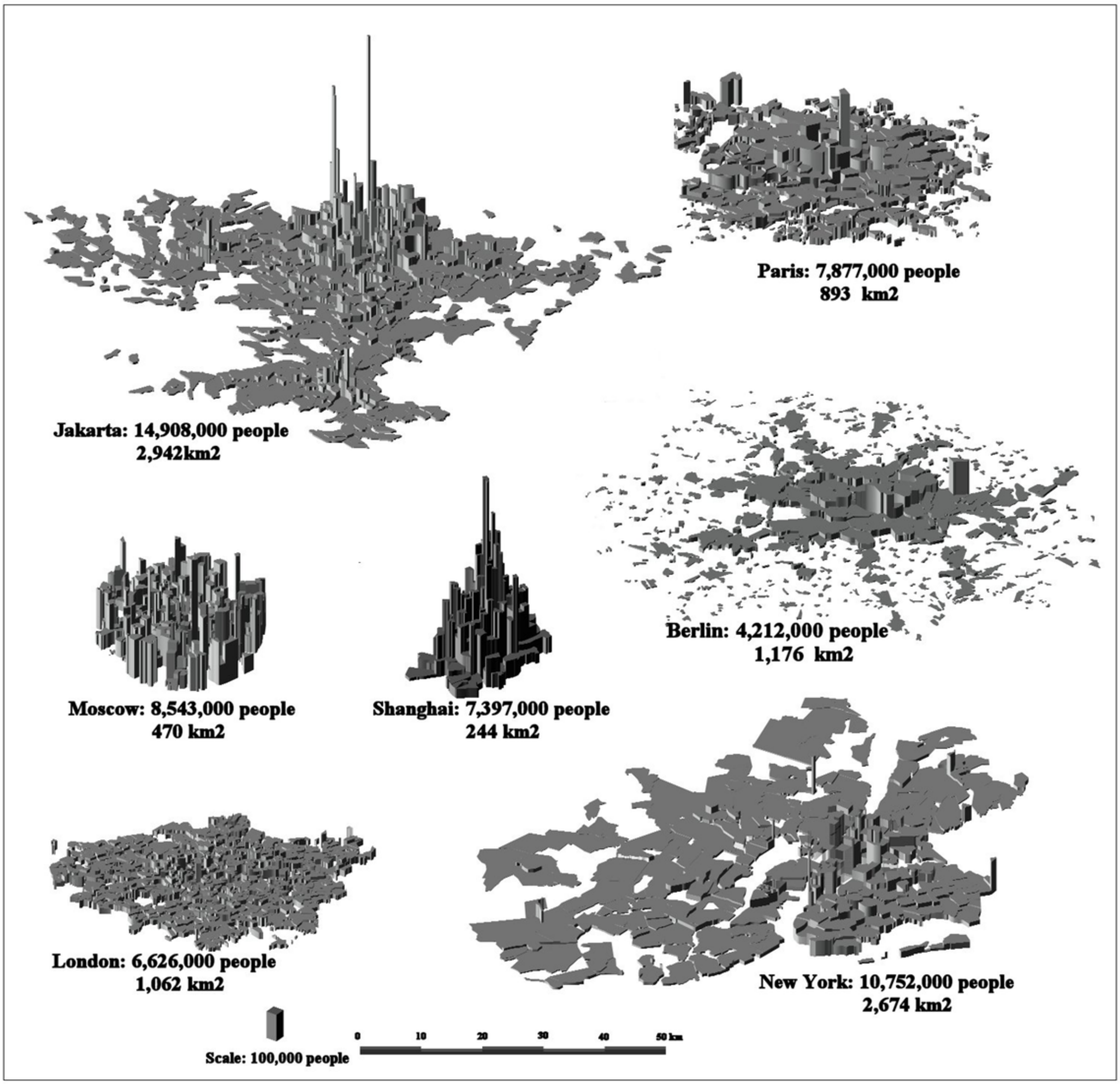



**Fig. 1.** Spatial distribution of population in 7 major metropolis, represented at the same scale
Source: Bertaud (2008)

The next section presents the model and the welfare analysis. Section 4 discusses two numerical analyses. We first study the decentralized economy versus the social optimum and as a second step we calibrate the model for Los Angeles and Chicago MSAs and conduct counterfactual simulations. Section 6 concludes.

## 2. The model

A decentralization of jobs from the Central Business District (CBD) to the suburbs has occurred in many countries (see e.g. Gobillon, Selod, and Zenou, 2007). This motivated the duo-centric structure in Coulson et al. (2001) among others. There, individuals and jobs are located in either a central city or a suburb. Instead, as Brueckner and Zenou (2003), we consider two job centers (indexed by $j = \{A, B\}$) in a densely populated area where people are distributed along a straight line joining the two centers. The workforce is normalized to unity and homogeneous in every sense but their location, denoted $x$. The workforce is distributed according to an exogenous and continuous density function $f : x \in [0, 1] \mapsto f(x) \in \mathbb{R}_0^+$, with CDF $F$.

We build a dynamic two-good model (a consumption good and labor) in continuous time and in steady state. There are two types of economic agents: workers and firms. Risk-neutral and infinitely-lived agents discount the future at a common rate $r$. Each job center presents a distinct labor market where an endogenous number of firms choose to set up. Firms open job-center specific vacancies. Workers supply inelastically one unit of labor and firms produce under perfect competition and constant returns to scale the consumption good (the numéraire). Individuals are either unemployed or employed. Job-seekers share their time endowment between looking for a job in $A$ and in $B$. If employed, they can be occupied in either of the job centers and commute at a unit cost $\tau > 0$.[4]

The matching process is represented by a standard differentiable matching function $M_j(V_j, U_j)$ specific to each job center $j \in \{A, B\}$. This function yields the number of matches per unit of time in $j$, $M_j$, as a function of the number of vacant jobs in job center $j$, $V_j$, and the number of active job-seekers, $U_j$, in market $j$.[5] As is widely accepted, we assume $M_j$ to be increasing and concave in both of its arguments, exhibiting Constant Returns to Scale. The rate at which a vacancy is filled in $j$ is:

$$\frac{M_j}{V_j} = M_j\left(1, \frac{U_j}{V_j}\right) = M_j\left(1, \frac{1}{\theta_j}\right) = \mu_j(\theta_j), \text{ with } \mu_j'(\theta_j) < 0$$

where $\theta_j \equiv \frac{V_j}{U_j}$ is named the labor market tightness in $j$. A more tight labor market makes it more difficult to recruit workers due to a congestion effect. Similarly, the rate at which a job-seeker finds a vacancy in $j$ is:

$$\frac{M_j}{U_j} = \theta_j\mu_j(\theta_j) = \psi_j(\theta_j), \text{ with } \psi_j'(\theta_j) > 0$$

A more tight labor market increases the rate at which job-seekers find a job (the so-called thick

[4]More precisely, $\tau$ denotes the pecuniary and time cost per unit of distance commuted to the job center.

[5]It should be noticed that by assumption firms in any center $j$ do not open vacancies that are only accessible to job-seekers in a specific location $x$. This realistic assumption plays a major role in the model.

market externality). As is standard, we assume the following Inada conditions:

$$\lim_{\theta_j \to 0}[\psi_j(\theta_j)] = \lim_{\theta_j \to +\infty}[\mu_j(\theta_j)] = 0$$

$$\lim_{\theta_j \to +\infty}[\psi_j(\theta_j)] = \lim_{\theta_j \to 0}[\mu_j(\theta_j)] = +\infty$$

When a match is formed in job center $j$, $y_j$ units of output are produced.[6] In the presence of search-matching frictions, when a vacancy and a job-seeker have matched, a surplus is created. For, if they separate, each partner has to start again a new search process. Let $\Upsilon(x)$ denote the present-discounted value of the expected utility of an unemployed worker located in $x$. $W_j(x)$ has the same meaning for a worker employed in job center $j$. Let $\Pi_j(x)$ be the present-discounted value of expected profit from job in $j$ occupied by a worker located in $x$. These functions verify the Bellman equations introduced below. Two standard assumptions are made in the matching literature (see Mortensen and Pissarides, 1999, and Pissarides, 2000). First, there is free entry of vacancies. Second, the (ex-post) surplus created by a matched is shared. Under free entry, the (total) surplus of a match in $j$ with a worker located in $x$, denoted $S_j(x)$, is defined by:

$$S_j\left(x\right) = \Pi_j\left(x\right) + W_j\left(x\right) - \Upsilon\left(x\right).$$

### 2.1. Wage formation

If as will be assumed later the surplus $S_j\left(x\right)$ is positive for all $x$ and $j$, each contact between a worker and a vacancy leads to a contractual relationship and workers have no incentive to quit. These features are taken into account when writing the various Bellman equations.

Jobs are destroyed at an exogenous rate $\delta_j$ also called the separation rate. In that case, both parties search for a new suitable partner. Under free entry, the present-discounted expected profit made on a vacant position is nil. So, $\Pi_j(x)$ verifies the following Bellman equation:

$$r\,\Pi_j(x) = y_j - w_j(x) - \delta_j\,\Pi_j(x) \tag{1}$$

Let $z_j(x)$ denote the worker's commuting distance to her workplace:

$$z_j(x) = \begin{cases} x & \text{for } \; j = A \\ 1 - x & \text{for } \; j = B. \end{cases}$$

The inter-temporal value of having a job, $W_j(x)$, solves the following Bellman equation:

$$r\,W_j(x) = w_j(x) - \tau\,z_j(x) - \delta_j[W_j(x) - \Upsilon(x)] \tag{2}$$

[6] We do not assume that commuting affects the productivity of workers. The average commuting time in the U.S. is rather short according to Gobillon et al. (2007) and the latest American Community Survey by the U.S. Census. If productivity was negatively affected by the time devoted to commuting (as suggested e.g. by van Ommeren and i Puigarnau, 2011, for Germany), the model could be adapted by introducing a weakly decreasing relationship between $y_j$ and the commuting distance $x$. The model developed below should then be adapted (in particular Assumptions 1 and 2). However, the qualitative conclusions would remain unaffected.

Using Eqs. (1) and (2), the surplus writes:

$$(r+\delta_j)\ S_j\,(x) = y_j - \tau\, z_j\,(x) - r\,\Upsilon\,(x) \tag{3}$$

Wages $w_j(x)$ are specific to each job center and depend on worker's location. During the negotiation, agents take tightness and $\Upsilon(x)$ as given. They are determined by a Nash bargaining solution that satisfies:

$$w_j\,(x) = \arg\max_w\,[W_j\,(x) - \Upsilon\,(x)]^{\beta_j}\ [\Pi_j\,(x)]^{1-\beta_j}$$

where $\beta_j \in (0,1)$ denotes the exogenous worker's bargaining power. Then, the first-order condition of this problem can be written as:

$$\beta_j\,\Pi_j(x) = (1-\beta_j)\ [W_j\,(x) - \Upsilon\,(x)] \tag{4}$$

Hence, the surplus accruing to the worker (resp., the employer) verifies:

$$W_j\,(x) - \Upsilon\,(x) = \beta_j\,S_j(x) \qquad \text{resp.,} \qquad \Pi_j(x) = (1-\beta_j)\ S_j(x) \tag{5}$$

The wage equation is solved by plugging Eqs. (1) and (3) into Eq. (5):

$$w_j\,(x) = \beta_j y_j + (1-\beta_j)\ [\tau\, z_j\,(x) + r\,\Upsilon(x)] \tag{6}$$

### 2.2. The supply side

The unemployed only commute from time to time for an interview. Hence, they incur commuting costs that we neglect.[7] The inter-temporal value in unemployment $\Upsilon(x)$ solves the following Bellman equation:

$$\begin{aligned} r\,\Upsilon(x) \quad &= \quad b + \max\Big\{0, \max_{\varepsilon(x)\in[0,1]}\Big[\varepsilon(x)\,\psi_A(\theta_A)[W_A(x)-\Upsilon(x)] \\ &\qquad + (1-\varepsilon(x))\,\psi_B(\theta_B)[W_B(x)-\Upsilon(x)]\Big]\Big\} \end{aligned} \tag{7}$$

where $b$ is the instantaneous value in unemployment. Eq. (7) tells that an unemployed has first to decide whether she searches for a job or not. If she does, as in Coulson et al. (2001), she optimizes the use of her unit time endowment to search for work in $A$ and $B$. The rate at which a job offer is found in $A$ (respectively, in $B$) is $\varepsilon(x)\,\psi_A(\theta_A)$ (respectively, $(1-\varepsilon(x))\,\psi_B(\theta_B)$). These rates are multiplied by the job-center-specific gain of becoming employed, $W_j(x) - \Upsilon(x)$.

[7] Their main search activity is made from where they live or close to it via the reading of newspapers, surfing on the web, visiting the nearest one-stop career center, sending out resumes, contacting friends and relatives and the like (for descriptive evidence, see Kuhn and Mansour, 2011, for the U.S. and Longhi and Taylor, 2011, for Great Britain). The assumption of absence of commuting cost can easily be relaxed to the case where the unemployed commute a non negligible amount of time, but anyway less than employed individuals (see e.g. Zenou, 2009).

Let the expected returns to search in location $j$ for a worker located in $x$ be defined by:

$$\Sigma_j(\theta_j, x) = \psi_j(\theta_j)\left[W_j(x) - \Upsilon(x)\right]$$

Using Eqs. (5) and (3), $\Sigma_j(\theta_j, x)$ can be expressed as:

$$\Sigma_j(\theta_j, x) = \beta_j\,\psi_j(\theta_j)\,S_j\,(x) = \beta_j\,\psi_j(\theta_j)\,\frac{y_j - \tau\,z_j(x) - r\,\Upsilon(x)}{r + \delta_j} \tag{8}$$

Remembering Bellman Eq. (7), the optimal $\varepsilon(x) = 1$ if $\Sigma_A\,(\theta_A, x) > \Sigma_B\,(\theta_B, x)$ (case *(i)*) and conversely $\varepsilon(x) = 0$ if $\Sigma_A\,(\theta_A, x) < \Sigma_B\,(\theta_B, x)$ (case *(ii)*). In addition, we need to consider the possibility of no search at all (case *(iii)*). The analysis of these three cases will lead to the following property:

**Lemma 1.** *For any location* $x \in [0, 1]$,

$$\frac{\partial \Sigma_A}{\partial x}\,(\theta_A, x) \leqslant 0 \leqslant \frac{\partial \Sigma_B}{\partial x}\,(\theta_B, x)\,, \tag{9}$$

*with strict inequalities if the unemployed seek jobs, and*

$$\frac{\partial \Sigma_j}{\partial \theta_j} > 0,\; j \in \{A, B\}.$$

*Proof.* Case *(i)*

If $\Sigma_A(\theta_A, x) > \Sigma_B(\theta_B, x)$ and $\Sigma_A(\theta_A, x) > 0$, as $\varepsilon(x) = 1$, combining Eqs. (7) and (8) yields:

$$r\,\Upsilon(x) = \frac{\beta_A\,\psi_A(\theta_A)(y_A - \tau\,x) + (r + \delta_A)b}{r + \delta_A + \beta_A\,\psi_A(\theta_A)}$$

which is decreasing in $x$ conditional on tightness. Now, considering Eq. (8) in $A$ leads to

$$\Sigma_A\,(\theta_A, x) = \beta_A\,\psi_A(\theta_A)\,S_A(x) = \beta_A\,\psi_A(\theta_A)\frac{y_A - \tau\,x - b}{r + \delta_A + \beta_A\,\psi_A(\theta_A)} \tag{10}$$

which is decreasing in $x$ (conditional on tightness) because the surplus of a match $S_A(x)$ is decreasing. Then, substituting $r\,\Upsilon(x)$ into Eq. (8) evaluated in $j = B$ yields:

$$\Sigma_B\,(\theta_B, x) = \beta_B\;\psi_B(\theta_B)\;\frac{(y_B - \tau(1-x) - b)\,(r + \delta_A) + \beta_A\psi_A(\theta_A)(y_B - y_A - \tau)}{(r + \delta_B)\,(r + \delta_A + \beta_A\psi_A(\theta_A))}.$$

Therefore, in Case *(i)*, we have the following property:

$$\frac{\partial \Sigma_A}{\partial x}\,(\theta_A, x) < 0 < \frac{\partial \Sigma_B}{\partial x}\,(\theta_B, x)\,.$$

Case *(ii)*

If $\Sigma_B\,(\theta_B, x) > \Sigma_A\,(\theta_A, x)$ and $\Sigma_B\,(\theta_B, x) > 0$, as $\varepsilon(x) = 0$, combining Eqs. (7) and (8) yields:

$$r\,\Upsilon\,(x) = \frac{\beta_B\,\psi_B\,(\theta_B)\;(y_B - \tau\,(1 - x)) + (r + \delta_B)\,b}{r + \delta_B + \beta_B\,\psi_B\,(\theta_B)}$$

which is increasing in $x$ conditional on tightness. Now, considering Eq. (8) in $B$ leads to

$$\Sigma_B(\theta_B, x) = \beta_B\,\psi_B(\theta_B)\,S_B(x) = \beta_B\,\psi_B(\theta_B)\frac{y_B - \tau(1-x) - b}{r + \delta_B + \beta_B\,\psi_B(\theta_B)} \tag{11}$$

which is increasing in $x$ (conditional on tightness) because the surplus $S_B(x)$ is increasing. Then, considering Eq. (8) also in $A$ yields:

$$\Sigma_A(\theta_A, x) = \beta_A\ \psi_A(\theta_A)\ \frac{(y_A - \tau x - b)(r + \delta_B) + \beta_B\psi_B(\theta_B)(y_A - y_B - 2\tau x - \tau)}{(r + \delta_A)(r + \delta_B + \beta_B\psi_B(\theta_B))}.$$

Therefore, in Case *(ii)*, we have the following property:

$$\frac{\partial \Sigma_A}{\partial x}(\theta_A, x) < 0 < \frac{\partial \Sigma_B}{\partial x}(\theta_B, x).$$

Case *(iii)*

If $\Sigma_A(\theta_A, x) < 0$ and $\Sigma_B(\theta_B, x) < 0$, then the unemployed workers do not search and $r\,\Upsilon(x) = b$, so (9) again applies.

Finally, from (10) (resp. 11), the partial derivative with respect to tightness can be written as:

$$\frac{\partial \Sigma_j}{\partial \theta_j} = \frac{(1-\eta_j)(r+\delta_j)}{r + \delta_j + \beta_j\psi_j(\theta_j)}\,\frac{\Sigma_j}{\theta_j} > 0, \qquad j \in \{A, B\}$$

where $\eta_j = \eta_j(\theta_j) = -\frac{\theta_j \mu_j'(\theta_j)}{\mu_j(\theta_j)} \in (0,1)$. □

To guarantee that the surplus is positive for all $x$ and $j$, we henceforth rule out Case *(iii)* by fixing an upper-bound on $\tau$:[8]

**Assumption 1.** $\tau < \min\{y_A, y_B\} - b$

Lemma 1 shows that conditional on the level of tightness in each center the expected return to search in $A$ (respectively, in $B$) shrinks (respectively, grows) as the distance to $A$ rises (and hence the one to $B$ shrinks). These effects are entirely driven by the evolution of the surpluses, which in turn vary as commuting costs do. Moreover, the relationships $x \mapsto \Sigma_j(\theta_j, x), j \in \{A, B\}$, are differentiable except at the threshold $\tilde{x}$ such that $\Sigma_A(\theta_A, \tilde{x}) = \Sigma_B(\theta_B, \tilde{x})$, if any.

Since by Assumption 1 all jobless individuals are searching for a job, Lemma 1 directly implies the existence of a threshold location denoted $\tilde{x}$ that separates the pool of unemployed in two groups defined by the center in which they are looking for an occupation.

**Lemma 2.** *For any $\theta_A$ and $\theta_B$, either*

[8] Relaxing this assumption would somewhat complicate the model as there would be a reservation distance above which surplus in $A$ would become negative and hence matches in $A$ would not be formed and there would be a reservation distance below which matches in $B$ would not be formed for the same reason. This would not add much insight into our analysis.

- *there exists a unique* $\tilde{x} \in [0,1]$ *such that*

$$\Sigma_A(\theta_A, \tilde{x}) = \Sigma_B(\theta_B, \tilde{x}) \tag{12}$$

- *or* $\Sigma_A(\theta_A, 1) > \Sigma_B(\theta_B, 1)$ *in which case all unemployed workers search in A*

- *or* $\Sigma_A(\theta_A, 0) < \Sigma_B(\theta_B, 0)$ *in which case all unemployed workers search in B.*

Henceforth, we concentrate on the case where $\tilde{x} \in [0,1]$ verifies Eq. (12). Assumption 2 below will later guarantee such a configuration.

On the basis of these two lemmas, we can now return to wage formation by substituting $\Upsilon(x)$ into (6). The wage-setting rule becomes:

$$w_j(x) = \beta_j y_j + (1-\beta_j)\,[b + \tau\, z_j(x) + \psi_j(\theta_j)\;\beta_j\, S_j(x)] \tag{13}$$

where $S_j(x) = (y_j - \tau\, z_j(x) - b)/(r + \delta_j + \beta_j\, \psi_j(\theta_j))$. In equilibrium, the relationship $x \mapsto w_A(x)$ exists below $\tilde{x}$ and $x \mapsto w_B(x)$ above.

### 2.3. The labor demand side

We adopt a one-job-one-firm setting. Each vacancy can be either filled of vacant. As is standard in the search-matching literature, opening a vacant job costs $k_j$ per unit of time. Under free-entry of vacancies, firms open vacancies in $j$ until the expected cost of hiring a worker equals the expected profit made on a filled position. In job centers $A$ and $B$, this condition is respectively:

$$\frac{k_A}{\mu_A(\theta_A)} = \int_0^{\tilde{x}} \max\{\Pi_A(x), 0\}\frac{f(x)}{F(\tilde{x})}dx, \qquad \frac{k_B}{\mu_B(\theta_B)} = \int_{\tilde{x}}^1 \max\{\Pi_B(x), 0\}\frac{f(x)}{1-F(\tilde{x})}dx \tag{14}$$

The LHS of Eqs. (14) is the expected cost of opening a vacancy respectively in $A$ and $B$. The RHS measures the expected profit from a new job respectively in $A$ and $B$, conditional on having met an applicant (i.e. an unemployed in respectively $[0, \tilde{x}]$ and $[\tilde{x}, 1]$). By assumption 1 and the surplus sharing rule (5), $\Pi_j(x)$ is always positive ($j \in \{A, B\}$). Let

$$\Gamma_A(\tilde{x}) = \int_0^{\tilde{x}} x\,\frac{f(x)}{F(\tilde{x})}\,dx \qquad \text{and} \qquad \Gamma_B(\tilde{x}) = \int_{\tilde{x}}^1 (1-x)\,\frac{f(x)}{1-F(\tilde{x})}\,dx \tag{15}$$

denote the conditional expected commuting distance respectively to job centers $A$ and $B$. For any density $f(x)$, these functions verify the following properties:

$$0 \leqslant \Gamma_A(\tilde{x}) \leqslant \tilde{x} \qquad \text{and} \qquad \Gamma_A'(\tilde{x}) = \frac{f(\tilde{x})}{F(\tilde{x})}(\tilde{x} - \Gamma_A(\tilde{x})) \geqslant 0 \tag{16}$$

$$0 \leqslant \Gamma_B(\tilde{x}) \leqslant 1-\tilde{x} \qquad \text{and} \qquad \Gamma_B'(\tilde{x}) = -\frac{f(\tilde{x})}{1-F(\tilde{x})}(1 - \tilde{x} - \Gamma_B(\tilde{x})) \leqslant 0 \tag{17}$$

Using Eqs. (5), (1) and the expressions for $S_j(x)$ found in (10) and (11), the free entry conditions (14) become:

$$\frac{k_j}{\mu_j(\theta_j)} = (1-\beta_j)\frac{y_j - \tau\,\Gamma_j\,(\tilde{x}) - b}{r + \delta_j + \beta_j\,\psi_j\,(\theta_j)}, \quad j \in \{A, B\}$$

where conditional on $\tilde{x}$, tightness in job center $j$ is only affected by parameters specific to this center. After some rearrangement and recalling that $\psi_j(\theta_j) = \theta_j\mu_j(\theta_j)$, the free-entry condition can be rewritten in the following way:

$$\frac{r+\delta_j}{1-\beta_j}\frac{\theta_j}{\psi_j\,(\theta_j)} + \frac{\beta_j}{1-\beta_j}\theta_j = \frac{y_j - \tau\,\Gamma_j\,(\tilde{x}) - b}{k_j}, \quad j \in \{A, B\} \tag{18}$$

**Lemma 3.** *For any value of the threshold $\tilde{x}$, under Assumption 1, the equilibrium value of tightness is unique. In $j = A$ (respectively, $j = B$), tightness decreases (respectively, increases) with the threshold $\tilde{x}$.*

*Proof.* Under the matching functions' Inada conditions, the LHS of Eq. (18) is increasing in $\theta_j$ from 0 to $+\infty$. By Assumption 1, the RHS is positive for all value of $\tilde{x}$. Hence, for any $\tilde{x} \in [0, 1]$ and any $j \in \{A, B\}$, Eq. (18) implicitly defines a unique level of tightness $\theta_j = \Theta_j\,(\tilde{x})$ with $\Theta'_A\,(\tilde{x}) < 0$ and $\Theta'_B\,(\tilde{x}) > 0$. $\square$

A rise in $\tilde{x}$ raises the conditional commuting distance to job center $A$ and hence reduces the expected surplus of a match. This induces a decline in the number of vacant jobs created per job-seeker in $A$. A similar phenomenon applies in $B$ with the opposite implication on tightness. A composition effect is at work. When a firm decides whether to open a vacancy, it compares the expected cost to the expected profit made when the position is filled. Since vacant jobs are specific to the job center but accessible to individuals located anywhere, and as workers' commuting costs are partly reimbursed by the employer,[9] this expected profit shrinks when job-seekers living further away enter the queue of unemployed in the job center. In other contexts, similar composition effects have been emphasized by e.g. Decreuse (2008) and Albrecht, Navarro, and Vroman (2010).

### 2.4. The equilibrium

The steady-state equilibrium can be defined recursively. First, we need to characterize the 3-tuple $\left\{\tilde{\theta}_A, \tilde{\theta}_B, \tilde{x}\right\}$. Second, the size of the population in unemployment and the number of vacancies are then determined.

**Definition 1.** *An equilibrium is 3-tuple $\left\{\tilde{\theta}_A, \tilde{\theta}_B, \tilde{x}\right\}$ that verifies the free-entry condition (18) on each labor market and search indifference condition (12).*

[9] This compensation, which is well documented in the empirical literature (see Zenou, 2009, for an overview), is an immediate consequence of Eq. (5).

**Existence and Uniqueness.**

Let us define

$$\mathcal{S}_j(x) = \Sigma_j(\Theta_j(x), x),$$

that is the expected return to search in $j$ for someone located at any value of the threshold $x$, once the effect of commuting distance on tightness is taken into account. Figure 2 illustrates Lemma 4 and Proposition 1 introduced below.

**Lemma 4.** *The expected return to search in market $A$ decreases with the value of the threshold $x$. The opposite is true in $B$.*

*Proof.* From Lemmas 1 and 3,

$$\frac{\partial \mathcal{S}_j(x)}{\partial x} = \frac{\partial \Sigma_j(\Theta_j(x), x)}{\partial \theta_j} \frac{\partial \Theta_j(x)}{\partial x} + \frac{\partial \Sigma_j(\Theta_j(x), x)}{\partial x} < 0 \text{ if } j = A, > 0 \text{ if } j = B.$$

□

Taking the effect of the threshold on the level of tightness under free entry, Eq. (12) can then be rewritten as

$$\mathcal{S}_A(\tilde{x}) = \mathcal{S}_B(\tilde{x}) \tag{19}$$

which admits at most one solution. Hence, if an equilibrium exists, it is unique. To ensure existence, one needs

$$\mathcal{S}_A(0) > \mathcal{S}_B(0) \qquad \text{and} \qquad \mathcal{S}_A(1) < \mathcal{S}_B(1). \tag{20}$$

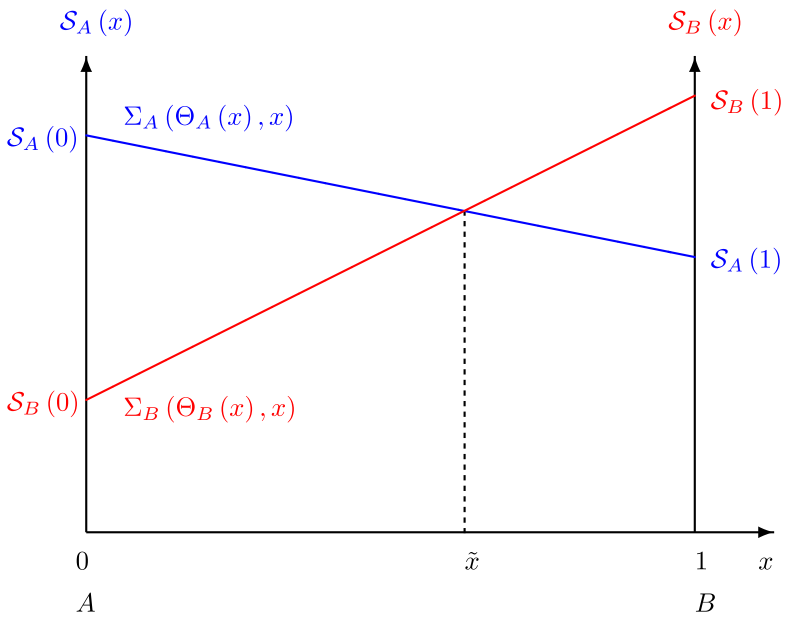



**Fig. 2.** The equilibrium

Let us define $\underline{\psi}_A = \psi_A(\Theta_A(1))$ the exit rate to a job in CBD A when everybody on the segment $[0,1]$ seeks a job there since $\tilde{x} = 1$. At the other extreme, we denote $\overline{\psi}_A = \psi_A(\Theta_A(0))$. Obviously, we have $0 < \underline{\psi}_A < \overline{\psi}_A$. Similarly in B, let $\underline{\psi}_B = \psi_B(\Theta_B(0))$, $\overline{\psi}_B = \psi_B(\Theta_B(1))$, with $0 < \underline{\psi}_B < \overline{\psi}_B$. From the definition of $\mathcal{S}(.)$ and (8), Inequalities (20) are equivalent to the following assumption:

**Assumption 2.** *Parameter $\tau$ is such that:*

$$\tau > \max\left\{y_A - b - \frac{\overline{\nu}_B}{\underline{\nu}_A}(y_B - b), y_B - b - \frac{\overline{\nu}_A}{\underline{\nu}_B}(y_A - b)\right\}$$

*where we define*

$$\overline{\nu}_A = \frac{\beta_A \overline{\psi}_A}{r + \delta_A + \beta_A \overline{\psi}_A},\ \overline{\nu}_B = \frac{\beta_B \overline{\psi}_B}{r + \delta_B + \beta_B \overline{\psi}_A},\ \underline{\nu}_A = \frac{\beta_A \underline{\psi}_A}{r + \delta_A + \beta_A \underline{\psi}_A},\ \underline{\nu}_B = \frac{\beta_B \underline{\psi}_B}{r + \delta_B + \beta_B \underline{\psi}_B}.$$

Assumption 2 puts a lower bound on the commuting cost $\tau$. Intuitively, if $\tau$ is too low and $\mathcal{S}_A(1) > \mathcal{S}_B(1)$ (resp. $\mathcal{S}_B(0) > \mathcal{S}_A(0)$), even workers located very close to $B$ (resp. $A$) are better of searching a job in $A$ ($B$). Search costs should then be high enough to prevent that a single labor market exists. It can be checked that Assumptions 1 and 2 can be incompatible if the two marginal products $y_A$ and $y_B$ are too different.[10] In sum, a direct consequence of the two previous lemmas, we conclude:

**Proposition 1.** *An equilibrium exists and is unique under Assumptions 1 and 2.*

Knowing the steady-state equilibrium $\left\{\tilde{\theta}_A, \tilde{\theta}_B, \tilde{x}\right\}$, the wage in any location $x$ on the left of $\tilde{x}$ is obtained by plugging $\tilde{\theta}_A$ in Eq. (13) (resp. $\tilde{\theta}_B$ on the right of $\tilde{x}$.)

The equilibrium population sizes in each state and the number of vacancies are also easily computed. Let

$$G_j(x) = \begin{cases} F(x) & \text{for } j = A \\ 1 - F(x) & \text{for } j = B. \end{cases}$$

In job center $j$, the steady-state equilibrium numbers of unemployed $\tilde{U}_j$, of employed $\tilde{L}_j$, and of vacancies are given by:

$$\tilde{U}_j = \frac{\delta_j G_j(\tilde{x})}{\delta_j + \psi_j(\tilde{\theta}_j)},\quad \tilde{L}_j = G_j(\tilde{x}) - \tilde{U}_j \qquad \text{and} \qquad \tilde{V}_j = \tilde{\theta}_j \tilde{U}_j \tag{21}$$

The equilibrium density of employed (respectively, unemployed) in any location $x$ is:

$$\begin{aligned} L_A(\tilde{x}) &= \frac{\psi_A\left(\tilde{\theta}_A\right) f(x)}{\delta_A + \psi_A\left(\tilde{\theta}_A\right)}, && \left(\text{resp., } U_A(\tilde{x}) = \frac{\delta_A f(x)}{\delta_A + \psi_A(\tilde{\theta}_A)}\right) && \text{for} \quad x \leqslant \tilde{x} \qquad (22) \\ L_A(\tilde{x}) &= 0, && (\text{resp., } U_A(\tilde{x}) = 0) && \text{for} \quad x > \tilde{x} \end{aligned}$$

[10] For instance, one can imagine that $y_A$ is so low compared to $y_B$, so that: $0 < y_A - b < y_B - b - \frac{\overline{\nu}_A}{\underline{\nu}_B}(y_A - b) < y_B - b$. In this example, if the marginal commuting cost $\tau$ verifies Assumption 1 (i.e. is lower than $y_A - b$), it can obviously not be compatible with Assumption 2 at the same time. In this example, $\mathcal{S}_A(0) < \mathcal{S}_B(0)$, which amounts to saying that job-center A ceases to exist due to a lack of productivity compared to B. Henceforth, we neglect such uninteresting cases where the two-center model collapses to a one-business-district setting.

$$
\begin{aligned}
L_B(\tilde{x}) &= \frac{\psi_B\left(\tilde{\theta}_B\right) f(x)}{\delta_B + \psi_B\left(\tilde{\theta}_B\right)}, \quad \left(\text{resp., } U_B(\tilde{x}) = \frac{\delta_B f(x)}{\delta_B + \psi_B(\tilde{\theta}_B)}\right) \quad \text{for} \quad x > \tilde{x} \\
L_B(\tilde{x}) &= 0, \quad (\text{resp., } U_B(\tilde{x}) = 0) \quad \text{for} \quad x \leqslant \tilde{x}
\end{aligned} \tag{23}
$$

In segment $(0, \tilde{x})$ (respectively,$(\tilde{x}, 1)$), the equilibrium unemployment rate is:

$$
\frac{\delta_A}{\delta_A + \psi_A(\tilde{\theta}_A)}, \qquad \left(\text{resp., } \frac{\delta_B}{\delta_B + \psi_B(\tilde{\theta}_B)}\right). \tag{24}
$$

### 2.5. Comparative static analysis

Conditional on $\tilde{x}$, the comparative statics of the model is fully standard (see e.g. Pissarides, 2000). By looking at the free-entry conditions (18), it is easily seen that for a given value of the threshold firms post less vacancies and hence equilibrium tightness falls in any job center after a marginal rise in the cost of opening a vacancy, the job destruction rate or the workers' bargaining power in this center. The same holds if the instantaneous value in unemployment or the discount rate rises. Moreover, a rise in the marginal product $y_j$ increases $\theta_j$.

Turning to the comparative statics on the threshold $\tilde{x}$, Appendix A shows that the *partial* effect of a rise in $k_j, \delta_j, b$ or $r$ or a decline in $y_j$ on the expected return of search in $j$, $\Sigma_j(\theta_j, x)$, reinforces the above-mentioned effect through equilibrium tightness (see the summary in Table 6 of this appendix). This is however not true for the bargaining power $\beta_j$, for reasons explained later on. Fig. 3 illustrates the total effect of parameter changes on the $\Sigma_j(\Theta_j(x), x)$ (i.e. $\mathcal{S}_j(x)$) schedules for any value of the threshold $x$ when this effect has a clear sign.

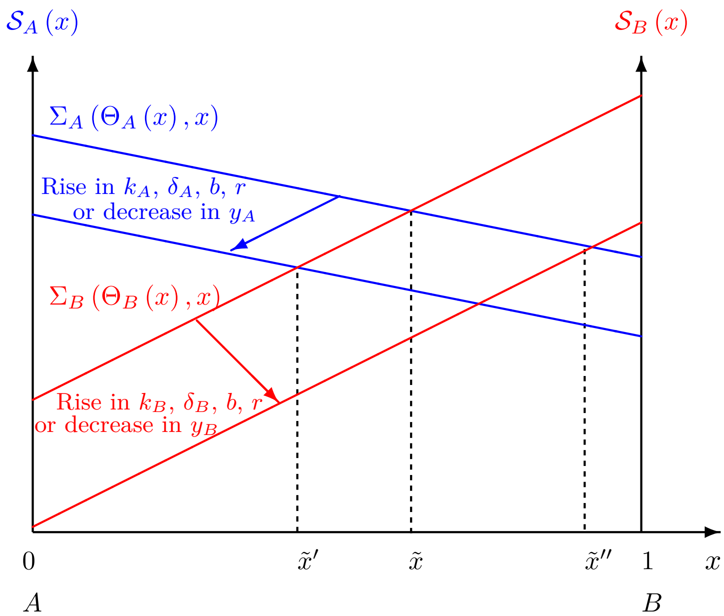



**Fig. 3.** Comparative statics

In Fig. 3, when a rise in $k_A$ or in $\delta_A$ (or a decline in $y_A$) shifts the whole curve $\Sigma_A(\Theta_A(x), x)$

downwards without affecting the same curve in B, the equilibrium threshold declines (see $\tilde{x}'$). The equilibrium value of $\theta_B$ then declines because $\Theta_B'(\tilde{x}) > 0$ and the $\Theta_B(x)$ schedule is not *directly* affected by changes in any of the parameters $k_A, \delta_A$ or $y_A$. On the contrary, the total effect on equilibrium tightness in A is ambiguous for the direct effect of a rise in $k_A$ or in $\delta_A$ (or a decline in $y_A$) on tightness and the effect through the threshold $\tilde{x}$ go in opposite directions (see Appendix A.3 for more explanations). The case where $k_B$ or $\delta_B$ rises (or $y_B$ declines) is symmetric. It induces a rise in the equilibrium threshold value (see $\tilde{x}''$ on Fig. 3). The equilibrium value of $\theta_A$ then declines because $\Theta_A'(\tilde{x}) < 0$ and the $\Theta_A(x)$ schedule is not *directly* affected by changes in any of the parameters $k_B, \delta_B$ or $y_B$. Furthermore, opposite forces lead as above to an ambiguous impact on equilibrium tightness in B.

We obtain ambiguous marginal impacts of the instantaneous value in unemployment and of the interest rate on the equilibrium threshold value since both $\Sigma_j\left(\Theta_j(x), x\right)$ curves shift in the same direction (namely downwards in Fig. 3). Then, the total effects on equilibrium tightness levels are obviously ambiguous as well. The impacts of a change in the bargaining power depend on the magnitude of this power. Rising the bargaining power in center $j$ has a positive partial effect on $\Sigma_j\left(\theta_j, x\right)$ but, as we have seen at the beginning of this subsection, it also has a negative direct effect on tightness $\theta_j$ for any $x$ and hence a negative effect on the expectation $\Sigma_j$. Appendix A shows that a rise in any bargaining power $\beta_j$ does not affect the $\Sigma_j\left(\Theta(x), x\right)$ curve when the Hosios condition (i.e. $\beta_j = \eta_j$ defined as $-\theta_j \mu_j'\left(\theta_j\right)/\mu_j\left(\theta_j\right)$[11]) is verified. In this particular case, the positive partial effect and the negative induced effect through tightness cancel out. Therefore, the threshold $\tilde{x}$ remains unaffected.[12] Under the Hosios condition, a rise in workers' bargaining power in center $j$ has therefore only a direct negative effect on equilibrium tightness in the same center along (18) (and no effect in the other one). If now the workers' bargaining powers are both "too high" (i.e. $\beta_j > \eta_j, j \in \{A, B\}$), a rise in $\beta_A$ lowers $\tilde{x}$ and a rise in $\beta_B$ increases $\tilde{x}$. This occurs because the negative direct effect of a rise in $\beta_j$ on tightness outweighs the positive *partial* effect on the expected return of search $\Sigma_j(\Theta_j(\tilde{x}), \tilde{x})$. So, the latter curve shifts downwards. The opposite effects are observed when the workers' bargaining powers are both "too low". The total impact of rise in the workers' bargaining power can sometimes be signed if the Hosios condition does not apply. See Table 1.

[11] $\eta_j$ is also the elasticity of the matching function with respect to the stock of unemployment.

[12] Pissarides (2000) shows a related result in a setting without explicit spatial heterogeneity but endogenous participation decisions. The participation rate reaches a maximum when the Hosios condition is met. Then, a marginal rise in the workers' bargaining power does not modify participation decisions.

**Table 1**
Comparative statics: The case of workers' bargaining powers $\beta_j$

| | $\eta_A \geqslant \beta_A$ | $\eta_A < \beta_A$ | | $\eta_B \geqslant \beta_B$ | $\eta_B < \beta_B$ |
|---|---|---|---|---|---|
| $d\tilde{x}/d\beta_A$ | $\geqslant 0$ | $< 0$ | $d\tilde{x}/d\beta_B$ | $\leqslant 0$ | $> 0$ |
| $d\theta_A/d\beta_A$ | $< 0$ | ? | $d\theta_A/d\beta_B$ | $\geqslant 0$ | $< 0$ |
| $d\theta_B/d\beta_A$ | $\geqslant 0$ | $< 0$ | $d\theta_B/d\beta_B$ | $< 0$ | ? |

## 3. The social optimum and its decentralization

The social planner chooses the market tightness levels and the allocation of workers in $j \in \{A, B\}$ subject to the available matching technology. As is often the case in this literature, we limit the analysis to the case where $r \mapsto 0$.[13] Then, the social welfare function maximizes aggregate net output in steady state (ignoring the transitional dynamics). Aggregate net output, $\Omega$, is defined as output produced net of commuting costs plus the value of time in unemployment minus the cost of creating vacant jobs. Using (15) and (21), $\Omega$ can be written as:

$$\begin{aligned} \Omega\left(\theta_A, \theta_B, x\right) &= \sum_{j \in \{A,B\}} \left(y_j - \tau \Gamma_j(x)\right) \frac{\psi_j(\theta_j) G_j(x)}{\delta_j + \psi_j(\theta_j)} + b \frac{\delta_j G_j(x)}{\delta_j + \psi_j(\theta_j)} - k_j V_j & (25) \\ &= b + \sum_{j \in \{A,B\}} G_j\left(x\right) \mathcal{W}_j\left(\theta_j, x\right) & (26) \end{aligned}$$

where

$$\mathcal{W}_j\left(\theta_j, x\right) = \frac{\psi_j\left(\theta_j\right)}{\delta_j + \psi_j\left(\theta_j\right)}\left(y_j - \tau\,\Gamma_j\left(x\right) - b\right) - \frac{\theta_j\,\delta_j}{\delta_j + \psi_j\left(\theta_j\right)} k_j, \tag{27}$$

designates the net aggregate surplus created in job center $j$. We proceed in two steps. First, for each level of $x$, we determine the optimal values of tightness, $\theta_A^*(x)$ and $\theta_B^*(x)$, in each job center. Second, we choose $x$ to maximize $b + \sum_{j \in \{A,B\}} G_j\left(x\right) \mathcal{W}_j\left(\theta_j^*(x), x\right)$

The first-order condition with respect to $\theta_j$ is:

$$\frac{\delta_j}{1 - \eta_j(\theta_j^*(x))} \frac{\theta_j^*(x)}{\psi_j(\theta_j^*(x))} + \frac{\eta_j(\theta_j^*(x))}{1 - \eta_j(\theta_j^*(x))} \theta_j^*(x) = \frac{y_j - \tau\,\Gamma_j\left(x\right) - b}{k_j} \tag{28}$$

In the standard matching literature, if the worker's bargaining power $\beta_j$ happens to be equal to the above-defined elasticity $\eta_j$, the surplus sharing rule (5) internalizes search-matching externalities. This equality is the already mentioned Hosios condition (see Hosios, 1990, and

[13] As explained e.g. by Cahuc and Zylberberg (2004) in the case of the basic matching model, the social planner problem can be studied in two ways. First, in the more general approach, the planner solves a dynamic optimal control problem subject to the law of motion of the unemployment rate. The second approach sets aside the problem of dynamic optimization by looking directly at the maximization of net output in a steady state subject to the equation characterizing the steady-state unemployment rate. Both approaches lead to the same equation characterizing optimal tightness in steady state when $r \mapsto 0$. In our setting, optimal control techniques cannot be applied since we would have a continuum of law of motions (namely, one in each location $x$). So, we adopt the second approach and assume $r \mapsto 0$.

Pissarides, 1990). Put differently, the decentralized economy is efficient (i.e. maximizes net output) despite the presence of these externalities. In the present setting, the next lemma shows that the Hosios condition is necessary and the following discussion will show that this condition is typically not sufficient.

**Lemma 5.** *The Hosios condition $\beta_j = \eta_j(\theta_j^*(x^*))$, $\forall j \in \{A, B\}$, is necessary to decentralize the optimal level of tightness, i.e. to guarantee that $\tilde{\theta}_j = \theta_j^*$.*

*Proof.* The right-hand sides of Eqs. (18) and (28) are identical provided that $\tilde{x} = x^*$ and the Hosios condition holds. This threshold being given, they are moreover not a function of tightness. So, let these right-hand sides be equal and take any positive value. Then, the unique decentralized level of tightness $\tilde{\theta}_j$ and the unique optimal one $\theta_j^*$ can only be equal if the left-hand sides of Eqs. (18) and (28) are identical when $r \mapsto 0$. This equality however can only hold if the Hosios condition is met. So, this condition is needed to decentralize the optimal level of tightness. □

The intuition for the *partial* optimality result of the Hosios condition is straightforward (See also Decreuse, 2008). When the decision of where to search $x$ is fixed, the average commuting cost $\Gamma_j(x)$ in each job center is fixed. The problem of determining the optimal tightness in each market takes the same form as in the basic matching model, with an additional cost that is exogenous. Therefore, as in the basic matching model, the Hosios condition ensures that the decentralized equilibrium generates the social optimal allocation. In the present setting however, the Hosios condition is only necessary and sufficient if the decentralized value of the threshold $\tilde{x}$ is the efficient one $x^*$. Otherwise, the right-hand-sides of (28) and of (18) are different. Consequently, we may have $\tilde{\theta}_j \neq \theta_j^*$ despite the Hosios condition is met.

We now turn to the optimality condition with respect to $x$. To this end, we get using (28):

$$k_j = (y_j - \tau\, \Gamma_j\,(x) - b)\, \frac{(1 - \eta_j(\theta_j^*(x)))\; \mu_j\left(\theta_j^*(x)\right)}{\delta_j + \eta_j(\theta_j^*(x))\; \psi_j\left(\theta_j^*(x)\right)}$$

so, from Eq. (25)

$$\mathcal{W}_j\left(\theta_j^*(x), x\right) = (y_j - \tau\, \Gamma_j\,(x) - b)\, \frac{\eta_j(\theta_j^*(x))\; \psi_j\left(\theta_j^*(x)\right)}{\delta_j + \eta_j(\theta_j^*(x))\; \psi_j\left(\theta_j^*(x)\right)} \tag{29}$$

Remembering (7) and (10), this way of expressing $\mathcal{W}_j$ makes clear that under the Hosios condition, $\beta_j = \eta_j\left(\theta_j^*(x)\right)$, the product $G_j(x)\left(b + \mathcal{W}_j\left(\theta_j^*(x), x\right)\right)$ is the expected discounted utility of an unemployed searching in $j$. The optimal $x$ therefore maximizes the expected utility of an unemployed,[14] namely:

$$\Omega\left(\theta_A^*(x),\theta_B^*(x),x\right)=b \quad + \quad \int_0^x \left(y_A-\tau\,\zeta-b\right)\frac{\eta_A\left(\theta_A^*(x)\right)\ \psi_A\left(\theta_A^*(x)\right)}{\delta_A+\eta_A\left(\theta_A^*(x)\right)\ \psi_A\left(\theta_A^*(x)\right)}\ f(\zeta)\ d\zeta$$
$$+ \quad \int_x^1 \left(y_B-\tau\left(1-\zeta\right)-b\right)\frac{\eta_B\left(\theta_B^*(x)\right)\ \psi_B\left(\theta_B^*(x)\right)}{\delta_B+\eta_B\left(\theta_B^*(x)\right)\ \psi_B\left(\theta_B^*(x)\right)}\ f(\zeta)\ d\zeta$$

Let us define

$$\mathscr{E}_j(\theta)=\frac{\delta_j}{\left(\delta_j+\eta_j(\theta)\ \psi_j(\theta)\right)^2}\ \frac{\partial(\eta(\theta)\ \psi(\theta))}{\partial\theta}\ \left.\frac{\partial\theta_j}{\partial\Gamma_j}\right|_{\text{Eq. (28)}}<0 \tag{30}$$

the marginal change of $\frac{\eta_j\left(\theta_j^*(x)\right)\ \psi_j\left(\theta_j^*(x)\right)}{\delta_j+\eta_j\left(\theta_j^*(x)\right)\ \psi_j\left(\theta_j^*(x)\right)}$ when $\Gamma_j$ increases by one unit along Eq. (28). This term is negative as a rise in average transportation cost $\Gamma$ decreases tightness. The first-order condition with respect to $x$ writes:

$$\left(y_A-\tau x-b\right)\frac{\eta_A\left(\theta_A^*(x)\right)\ \psi_A\left(\theta_A^*(x)\right)}{\delta_A+\eta_A\left(\theta_A^*(x)\right)\ \psi_A\left(\theta_A^*(x)\right)}+\frac{F(x)}{f(x)}\ \Gamma_A'(x)\ \mathscr{I}_A\left(x,\theta_A^*(x)\right) \tag{31}$$
$$=\quad \left(y_B-\tau(1-x)-b\right)\frac{\eta_B\left(\theta_B^*(x)\right)\ \psi_B\left(\theta_B^*(x)\right)}{\delta_B+\eta_B\left(\theta_B^*(x)\right)\ \psi_B\left(\theta_B^*(x)\right)}-\frac{1-F(x)}{f(x)}\ \Gamma_B'(x)\ \mathscr{I}_B\left(x,\theta_B^*(x)\right)$$

in which $\mathscr{I}_j\left(x,\theta_j^*(x)\right)=\left(y_j-\tau\Gamma_j(x)-b\right)\mathscr{E}_j\left(\theta_j^*(x)\right)$. Under the Hosios condition, the first terms on both sides of Eq. (31) equal $\Sigma_j\left(\theta_j^*(x),x\right)$. Remembering the value of the derivatives $\Gamma_j'(x)$ in (16) and (17), the first-order condition (31) can be rewritten as:

$$\Sigma_A\left(\theta_A^*(x),x\right)+\left(x-\Gamma_A(x)\right)\mathscr{I}_A\left(x,\theta_A^*(x)\right)=\Sigma_B\left(\theta_B^*(x),x\right)+\left(1-x-\Gamma_B(x)\right)\mathscr{I}_B\left(x,\theta_B^*(x)\right) \tag{32}$$

Let us define $I_j(x)=\left(z_j(x)-\Gamma_j(x)\right)\mathscr{I}_j\left(x,\theta_j^*(x)\right)\ j\in\{A,B\}$.

Under the Hosios condition $\beta_j=\eta_j\left(\theta_j(x^*)\right)$, the optimal threshold $x^*$ then verifies:

$$\Sigma_A(\theta_A^*(x^*),x^*)+I_A(x^*)=\Sigma_B(\theta_B^*(x^*),x^*)+I_B(x^*) \tag{33}$$

Each expression $I_j(x^*)$, henceforth $I_j$ for short, has no reason to be nil unless $\eta_j\mapsto 0$,[15] which is a degenerate case where the number of vacancies has no influence the number of hirings. If $I_A$ and $I_B$ happen to be equal, the Hosios condition is sufficient to guarantee the equality between the decentralized and the optimal triple $(\theta_A,\theta_B,x)$. This would be the case if the two job centers were identical and the distribution of the population was uniform on $[0,1]$. In general, $I_A$ and $I_B$ have however no reason to be equal and hence the Hosios condition is not sufficient to guarantee that the steady-state equilibrium is efficient.

The negative effects $I_j$ omitted by decentralized agents have a clear interpretation. In the decentralized economy, the threshold location verifies Eq. (19), which expresses that the private

[14] As $r\mapsto 0$, this is also the expected utility of a member of the labor force.

[15] When $\eta\mapsto 0$ the term $\frac{\partial(\eta(\theta)\psi(\theta))}{\partial\theta}\mapsto 0$ (see (30)), hence $\mathscr{E}_j(\theta)\mapsto 0$ and $\mathscr{I}_j\left(x,\theta_j(x)\right)\mapsto 0$.

marginal gains $\Sigma_j\left(\Theta_j(x), x\right)$ of searching in $A$ and in $B$ are equal in equilibrium. This indifference condition (19) overlooks that a change in the threshold affects the conditional expected commuting distance, $\Gamma_j$, of all workers and thereby the expected surplus accruing to employers. This in turn modifies the number of vacancies created in both job centers under free entry and eventually the levels of tightness. Finally, this induced change in both levels of tightness has an impact on the expected utility of all job-seekers. This externality is different from the standard search-matching externalities that are internalized under the Hosios condition. At the root of this additional externality, one finds the composition effect introduced in Subsection 2.3. Since vacant jobs are specific to the job center but accessible to individuals located anywhere and as workers' commuting costs are shared through the wage bargain, the expected profit of opening a vacancy shrinks when job-seekers further away enter the queue of unemployed seeking an occupation in the job center. We henceforth talk about the composition externality. This externality is made of two opposite effects $I_A < 0$ and $I_B < 0$. If $I_A < I_B$, Eq. (33) implies that at a social optimum, the return to search of the pivotal job-seeker (i.e. someone located in $x^*$) is higher in job center A than in B: $\Sigma_A(\theta_A^*(x^*), x^*) > \Sigma_B(\theta_B^*(x^*), x^*)$. Therefore, the social optimum needs to instruct some job seekers to search in market B rather than in A. In other words, in the decentralized economy too many job-seekers are searching for a job in business district A.

With a Cobb-Douglas matching function on each labor market, a popular functional form used in the numerical analysis below, $\eta_j(.)$ becomes a parameter. Then, differentiating Eq. (28) gives:

$$\left.\frac{\partial \theta_j}{\partial \Gamma_j}\right|_{\text{Eq. (28)}} = -\frac{\tau}{k_j}\ \frac{1-\eta_j}{\eta_j}\ \frac{\psi_j(\theta_j)}{\delta_j + \psi_j(\theta_j)}$$

Using Eq. (28):

$$\left(y_j - \tau\Gamma_j(x) - b\right)\left.\frac{\partial \theta_j}{\partial \Gamma_j}\right|_{\text{Eq. (28)}} = -\tau\ \frac{\theta_j}{\eta_j}\ \frac{\delta_j + \eta_j\ \psi_j(\theta_j)}{\delta_j + \psi_j(\theta_j)}$$

Hence, by (30), $\mathscr{I}_j(x, \theta_j^*(x))$ can be rewritten as:

$$-\tau\frac{(1-\eta_j)\delta_j\psi_j(\theta_j^*(x))}{\left(\delta_j + \eta_j\psi_j(\theta_j^*(x))\right)\left(\delta_j + \psi_j(\theta_j^*(x))\right)} = -\tau\left[\frac{\psi_j(\theta_j^*(x))}{\delta_j + \psi_j(\theta_j^*(x))} - \frac{\eta_j\psi_j(\theta_j^*(x))}{\delta_j + \eta_j\psi_j(\theta_j^*(x))}\right].$$

So,

$$I_j(x) = -\tau\ \left(z_j(x) - \Gamma_j(x)\right)\ \left[\frac{\psi_j(\theta_j^*(x))}{\delta_j + \psi_j(\theta_j^*(x))} - \frac{\eta_j\ \psi_j(\theta_j^*(x))}{\delta_j + \eta_j\ \psi_j(\theta_j^*(x))}\right] < 0. \tag{34}$$

The $I_j$ terms capture the effects of the composition externality. Recalling Eq. (31), $I_j$ does not only depend on how sensitive the conditional expected commuting distance $\Gamma_j(x)$ is to the value of the threshold $x$, but also on the mass of individuals $F(x)$ (respectively, $1 - F(x)$).

Given (16) and (17), $I_j$ is also the product of the positive difference between the marginal commuting cost $\tau\, z_j(x)$ and the (conditional) average one $\tau\, \Gamma_j(x)$ and a second positive difference, $\left[\frac{\psi_j(\theta_j^*(x))}{\delta_j+\psi_j(\theta_j^*(x))} - \frac{\eta_j\ \psi_j(\theta_j^*(x))}{\delta_j+\eta_j\ \psi_j(\theta_j^*(x))}\right]$, which depends on tightness $\theta_j^*(x)$ only. The latter difference is increasing in tightness if and only if $\sqrt{\eta_j}\psi_j(\theta_j^*(x)) < \delta_j$,[16] which corresponds to an unemployment rate lower than $\frac{\sqrt{\eta_j}}{1+\sqrt{\eta_j}}$ . For $\eta_j = 0.1$, this corresponds to an unemployment rate of 24%, while for $\eta_j = 0.9$, it corresponds to an unemployment rate of 48%. Therefore, as most empirical analyses find a value of $\eta \in [0.4, 0.7]$, we can take for granted that the latter difference is increasing in tightness.

This section can be summarized as follows:

**Proposition 2.** *The Hosios condition is necessary but typically not sufficient to guarantee that the decentralized equilibrium is efficient. The decentralized threshold $\tilde{x}$ can be lower or above the efficient one $x^*$ depending on the relative importance of the composition externalities in the two job centers on the expected utility of job-seekers searching in $A$ and in $B$.*

As explained above, a typical counter-example, where the Hosios condition is sufficient, is the case where the two centers are symmetric and the population is uniformly distributed.

## 4. Numerical analyses

These analyses aim at illustrating how the shape of distribution of the workforce influences the decentralized allocation, in particular in terms of equilibrium (un)employment levels. They also intend to provide orders of magnitude about the gap between the efficient and the decentralized allocations. The first numerical analysis compares the decentralized equilibrium and the optimal allocation for a uniform distribution combined with a parametrized mass of workers. Second, we look at two U.S. Metropolitan Statistical Areas (MSA), Los Angeles and Chicago, and after calibrating the model for two major job centers of these MSAs we develop a counterfactual analysis with respect to the distribution of the workforce.

This section assumes the following Cobb Douglas matching functions:

$$M_j \quad = \quad m_j V_j^{1-\eta_j} U_j^{\eta_j} \qquad j \in \{A, B\},$$

where $m_j$ is the matching function scale parameter. The job finding rate and the rate of filling a vacancy respectively are:

$$\psi_j(\theta_j) \quad = \quad m_j \theta_j^{1-\eta_j} \tag{35}$$

$$\mu_j(\theta_j) \quad = \quad m_j \theta_j^{-\eta_j} \tag{36}$$

[16] The derivative of $\frac{\psi}{\delta+\psi} - \frac{\eta\ \psi}{\delta+\eta\ \psi}$ with respect to $\psi$ is $\frac{\delta}{(\delta+\psi)^2} - \frac{\eta\ \delta}{(\delta+\eta\ \psi)^2}$. This term is positive whenever $(\delta + \eta\ \psi)^2 > \eta\ \ (\delta+\psi)^2$ or $\sqrt{\eta}\psi < \delta$.

### 4.1. A numerical illustration

In the first exercise we take symmetric job centers and we set the discount rate to zero. In both business districts, productivity $y_j$ is normalized to 1, the value of leisure is normalized to $b = 0.4$ (Shimer, 2005), the quarterly separation rate in 2000[17] is $\delta_A = \delta_B = 0.03$ (Shimer, 2012) and we set $m_A = m_B = 1$. The vacancy costs $k_j$ are set to match an unemployment rate of 4% in the U.S. in 2000. Finally, we take $\tau = 0.4$ (Zenou, 2009b, p. 40). The Hosios condition is assumed by imposing $\beta_j = \eta_j = 0.5$. The labor force is distributed according to a uniform distribution whose total mass is $1 - \alpha_A$ and a mass point of $\alpha_A \in [0, 1)$ is located at $x = 0$. So, the CDF is $F(x) = \alpha_A + (1 - \alpha_A)x$, implying that as $\alpha_A$ tends to 1, the population becomes more and more concentrated at $x = 0$. The model is calibrated for $\alpha_A = 0$. The average commuting costs towards job centers $A$ and $B$ for $x \in (0, 1)$ respectively are,

$$\Gamma_A(x) = \frac{(1-\alpha_A)\frac{x^2}{2}}{\alpha_A + (1-\alpha_A)x}, \quad \text{with} \quad \frac{\partial \Gamma_A(x)}{\partial \alpha_A} < 0, \quad \text{and} \quad \Gamma_B(x) = \frac{1-x}{2}.$$

The simulation results in Fig. 4 depicts how the allocation at the decentralized equilibrium (blue solid curves) and the optimal one (red dashed curves) are modified when $\alpha_A$ increases from 0 to 1. The two allocations coincide when $\alpha_A = 0$ as the two job centers are symmetric and the workforce is uniformly distributed. Then, we know from the previous section that the Hosios condition guarantees efficiency. When the mass point $\alpha_A$ increases, vacant jobs in $A$ have a larger probability of meeting job-seekers close to them. So, for any threshold value $x$ the conditional expected commuting distance $\Gamma_A(x)$ decreases. Therefore, the schedule $\Theta_A(x)$ shifts upwards when $\alpha_A$ rises and so does the expected returns to search in $A$, $\mathcal{S}_A(x)$ or equivalently $\Sigma_A(\Theta_A(x), x)$. On the contrary, the schedule $\Theta_B(x)$ and hence $\Sigma_B(\Theta_B(x), x)$ are unaffected by $\alpha_A$. From (19) and Fig. 3, $\tilde{x}$ has to rise with $\alpha_A$. The bottom left panel of Fig. 4 quantifies this effect. On the contrary, notice that the optimal threshold $x^*$ slightly *declines* with $\alpha$. To see why, we need to understand how changes in $\alpha_A$ modify the $I_j$ terms (34) in both job centers. Notice first that because the schedule $\Gamma_A(x)$ shifts downwards when $\alpha_A$ increases, the difference between the marginal and the average commuting distance to job center $A$, $x - \Gamma_A(x)$, increases whatever the value of $x$, while the corresponding term in $B$ is not a function of $\alpha_A$. Furthermore, the last term defining $I_j$ shifts upwards for any $x$ when $\alpha$ increases (because the schedule $\Theta_A(x)$ shifts upwards) while, again the corresponding term in $B$ remains unaffected. For these two reasons, as $\alpha_A$ rises, the schedule $I_A(x)$ shifts downwards while nothing changes in $B$. So, compared to the decentralized equilibrium, the efficient threshold will be lower (more job-seekers should search in $B$ instead of $A$). This does not however explain why the efficient

[17] This data was constructed by Robert Shimer. For additional details, see Shimer (2007) and his webpage `http://sites.google.com/site/robertshimer/research/flows`.

$x^*$ shrinks with $\alpha_A$. Two opposite movements affect the left-hand side of (33): the one we have just explained and the upward shift of the schedule $\Sigma_A(\Theta_A(x), x)$ discussed earlier. From the profile of $x^*$ in the bottom left panel of Fig. 4, we deduct that the former slightly outweighs the latter.

As far as the decentralized equilibrium value of $\tilde{\theta}_A$ is concerned, two forces are here also at work. First, when $\alpha_A$ rises, for the reason explained earlier, the $\Theta_A(x)$ schedule shifts upwards, inducing a rise in $\tilde{\theta}_A$ for any threshold $x$. However, the decentralized $\tilde{x}$ increases as well. This in turn induces a decline in the decentralized value $\tilde{\theta}_A$ since $\Theta_A'(\tilde{x}) < 0$. Fig. 4 shows that the former effect dominates. Because the efficient $x^*$ shrinks with $\alpha_A$, the schedule $\alpha_A \mapsto \theta_A^*$ dominates the corresponding one in the decentralized equilibrium.[18] In job center $B$, only one mechanism is at work since the conditional expected commuting distance to $B$, $\Gamma_B(x)$, is not affected by $\alpha_A$. As $\tilde{x}$ rises with $\alpha_A$, $\Gamma_B(\tilde{x})$ declines and hence the decentralized value $\tilde{\theta}_B$ increases as well. For the same reason, the efficient value $\theta_B^*$ somewhat declines because $x^*$ slightly declines with $\alpha_A$. Both the decentralized and the efficient levels of total employment increase, changes in $A$ being naturally dominant when the population gets concentrated around this job center. Similarly, the aggregate unemployment level decreases with $\alpha_A$, from 4% when $\alpha_A = 0$ to 3.65% when $\alpha_A = 1$. The largest difference between the decentralized and the efficient unemployment rates amounts to 0.031 percentage points when $\alpha_A = 0.69$.

Finally, the gap in social welfare between the efficient and the decentralized economy has an inversed U-shaped profile and is nil at the two extreme values of $\alpha_A$. At the lower bound, this is obvious under the Hosios condition. When $\alpha_A$ tends to 1, as explained above, $\tilde{\theta}_A$ tends to the efficient value, commuting costs tend to disappear and the model converges towards the standard Mortensen-Pissarides setting without explicit geographical dispersion. For intermediate values of $\alpha_A$, we know that the gap in social welfare level is positive. On Fig. 4, the largest relative welfare loss of 1.93% is observed at $\alpha_A = 0.69$.

## 4.2. Los Angeles and Chicago MSAs

### 4.2.1. The data

In the second part of the numerical exercise we calibrate our model with data on MSAs in the U.S. The leading MSAs, ranked by population, are also those with the highest mean travel time to work i.e. New York (34 minutes), Los Angeles (29 minutes) and Chicago (31 minutes) (see McGuckin and Srinivasan, 2003; Rapino, McKenzie, and Marlay, 2011). We calibrate the model using data from the U.S. 2000 census.

[18]When $\alpha_A$ tends to 1, $\Gamma_A(x)$ tends to zero whatever the value of $x$. So, wherever the marginal worker is located, the decentralized and the efficient $\theta_A$'s must converge to each other.

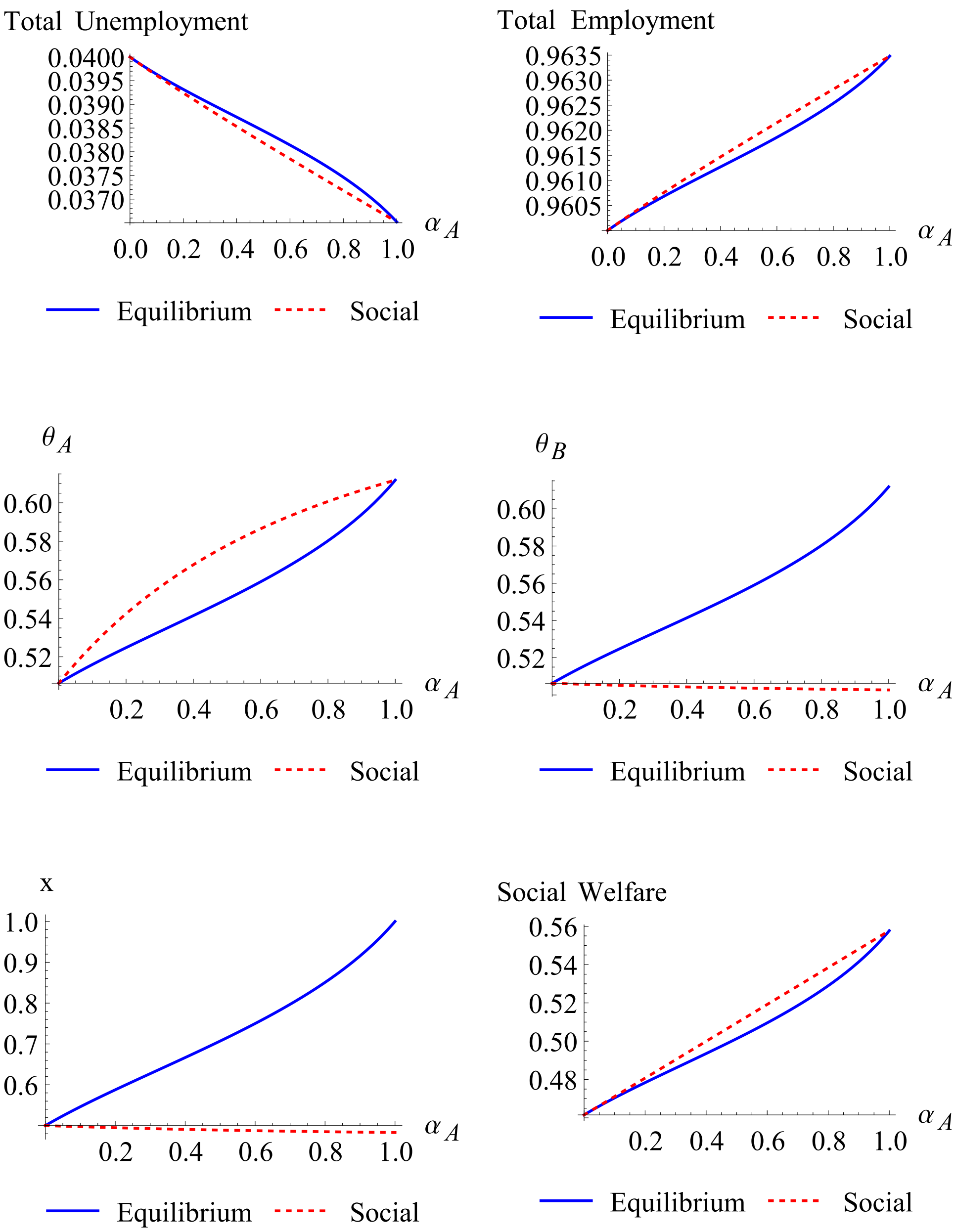
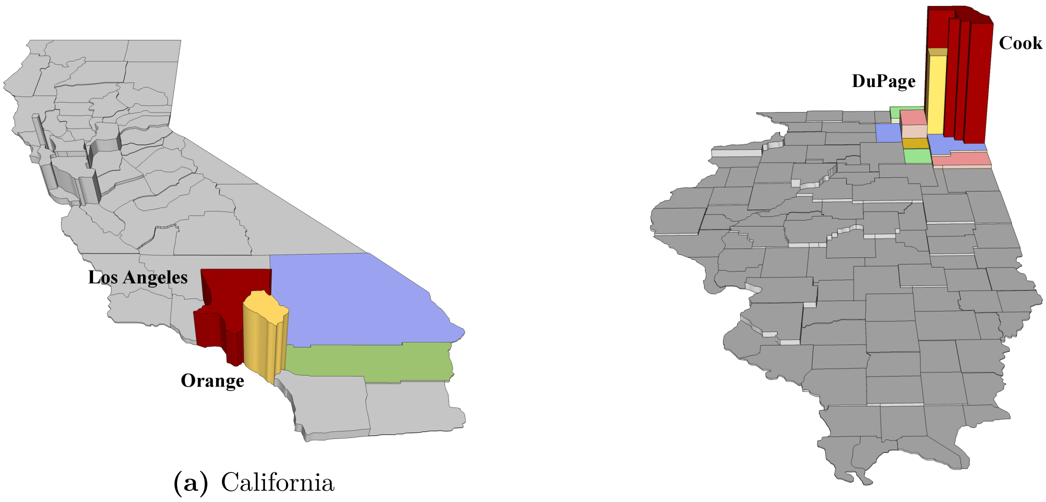



**Fig. 4.** Decentralized versus Optimal allocation

We have access to data of the total workforce, the number of employed, unemployed and commuters at the zip code and county levels on the basis of the location of residence. Average wages and the number of employees are available only at the county level on the basis of the location of the job. We collect information for Los Angeles and Chicago MSAs,[19] which we respectively take as representative of “new” and “old” cities.[20]




**(a)** California

**(b)** Illinois

**Fig. 5.** Paid employees by county, 2000
(Thousands per sq mi)
Source: U.S. Census Bureau, Department of commerce.

We assume there is one job center per county and determine its size by the number of paid employees per square mile.[21] Fig. 5 depicts the states of California and Illinois by county and each county’s size is measured by the height on the map. According to the U.S. 2000 census definition, Los Angeles MSA sprawls over the counties of Los Angeles, Orange, Riverside and San Bernardino, that are colored on Fig. 5a. Two job centers exceed the others in size: Los Angeles county (in red) and Orange county (in yellow) respectively account for 63% and 23% of the total paid employees in Los Angeles MSA. Chicago MSA is made of a number of counties that are colored on Fig. 5b.[22] The largest job centers in Chicago MSA, see Fig. 5b, are in Cook county (in red) with a share of 60% and DuPage county (in yellow) with 14% of paid employees.

[19]New York MSA’s configuration is out of the scope of this model since it presents four important job centers on a row: New York, Queens, Nassau and Suffolk counties. San Francisco MSA is a multicentric MSA and hence out of the scope of our model.

[20]Old cities used to be the ten most populated in 1900, i.e. New York, Chicago, Philadelphia, Detroit, Boston and San Francisco. In contrast, new cities like Los Angeles, Atlanta, Houston, Dallas, Miami and Nassau-Suffolk had much smaller populations during that century.

[21]Data available only at the county level. Source: U.S. Census Bureau, Department of Commerce, 2000 and the National Association of Counties.

[22]Chicago MSA spreads over the states of Illinois, Indiana (Lake and Porter counties) and Wisconsin (Kenosha county). The counties included from the state of Illinois are: Cook, DeKalb, DuPage, Grundy, Kane, Kendall, Lake, McHenry, Will and Kankakee. Metropolitan areas defined by the Office of Management and Budget, June 30th, 1999. Source: Population division, U.S. Census Bureau. Released online on July 1999.

Interestingly for our study the highway Route 5 links Los Angeles and Orange counties, see Fig. 6. We only take into account the active population with residence along this highway. In most cases Route 5 passes through a zip code, while in others it is at the border of two zip code areas, in which case we average their populations. Job center $A$ or CBD is assumed to be located in Los Angeles city center and job center $B$ or SBD in Santa Ana city center. They are separated by 33.9 miles. In Chicago MSA, we only consider the active population with residence along Routes 290 and 88, which connect Cook and DuPage counties, see Fig. 7. The CBD is assumed to be located in Chicago city center and the SBD in Naperville city center. They are separated by 33.6 miles.

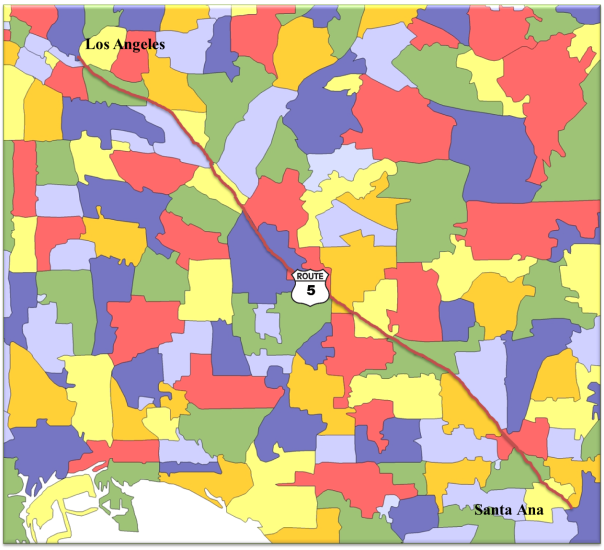



**Fig. 6.** California, job centers connected through Route 5
Source: `http://www.zipmap.net/California.htm`

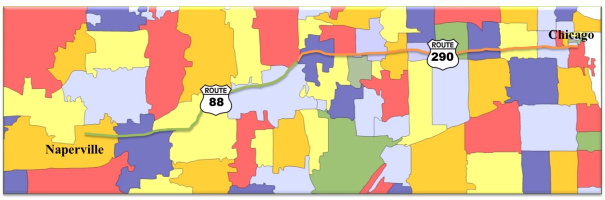
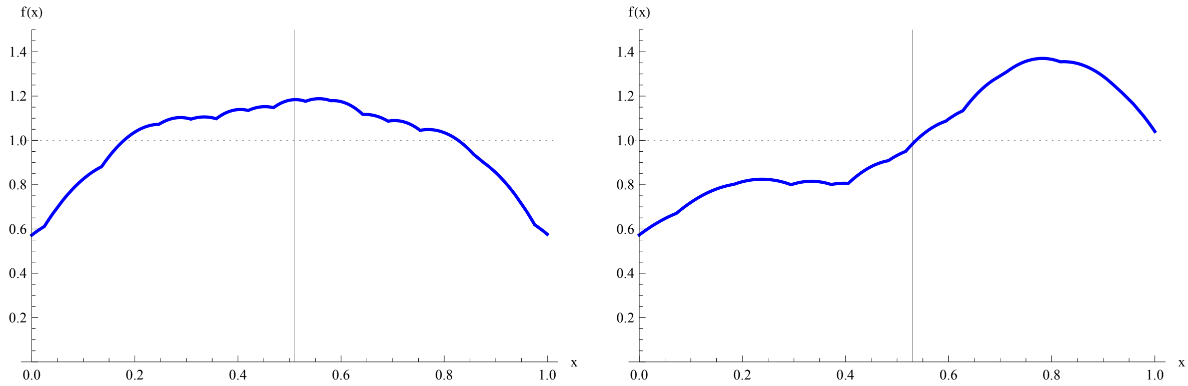



**Fig. 7.** Chicago, job centers connected through Route 88 and 290
Source: `http://www.zipmap.net/Illinois.htm`

The labor force in each zip code area located between the specified job centers forms a discrete workforce distribution, which we transform into a continuous density $f(x)$ and CDF $F(x)$ on the segment $[0, 1]$. For this purpose, for both MSAs, we use the Kernel procedure, with smoothing factor 5 which yields a bandwidth of 0.27 and 0.31 for Los Angeles and Chicago MSA, respectively.[23] The densities are displayed in Fig. 8. We denote $x_0$ as the geographical boundary

[23] We use a quadratic Kernel (Epanechnikov) $k_2(x) = \frac{3}{4}\left(1 - x^2\right)$. We calculate the bandwidth using the Silverman rule-of-thumb: $h = 5 \cdot \hat{\sigma}^2 \cdot C_\nu(k) \cdot N^{-1/(2\nu+1)}$ where the bandwidth $h$, equals the product of the




**(a)** Los Angeles MSA **(b)** Chicago MSA

**Fig. 8.** Kernel distributions of the labor force

between the two counties. Now that we have the continuous distribution $F$ of the workforce with residence along the selected routes, we can fix $x_0$ such that $F(x_0)$ matches the observed share of the workforce living in the county on the left ("LC" for short). Then of course, $1-F(x_0)$ matches the share in the county to the right ("RC" for short). In Los Angeles MSA, Los Angeles (resp. Orange) county spreads over the segment $[0, 0.51]$ (resp. $(0.51, 1]$). In Chicago MSA, Cook (resp. DuPage) county spreads over the segment $[0, 0.53]$ (resp. $(0.53, 1]$). In Los Angeles MSA, see Fig. 8a, the population density is at its highest level on the border between counties, whereas at the extremes, i.e. $x = 0$ and $x = 1$, we observe the lowest densities. In Chicago MSA, however, Fig. 8b, the distributed is skewed to the right. We also observe an inverted-U-shape density within DuPage county. To summarize these differences by two numbers, $F(x_0) = 0.5$ in Los Angeles MSA while it is close to 0.4 in the other one.

#### 4.2.2. The calibration

The parameters of the model are: $y_j$, $\delta_j$, $\eta_j$, $\beta_j$, $m_j$ for $j = \{A, B\}$ and $r$, $b$ and $\tau$, the unknowns being $\tilde{\theta}_j$ and $\tilde{x}$. The reference year is 2000. We match the *means* of the unemployment rates in the selected zip codes along the indicated connecting routes respectively within the left and the right counties. Since average wages are only available for the year 2000 at the county level, we also match the average wages respectively in the left and the right counties.[24] We use a quarter as the unit of time. The real interest rate in the U.S. in 2000 was 4%,[25] thus we take $r = 0.98\%$. Following Petrongolo and Pissarides (2001) we choose an elasticity of the matching function $\eta_A = \eta_B = 0.5$, and as common practice we assume the Hosios condition, $\beta_A = \beta_B = 0.5$. Due

smoothing factor 5, times the sample standard deviation, $\hat{\sigma}^2$, a constant, $C_\nu(k) = 2.34$, the sample $N$ and the order of the kernel $\nu = 2$.

[24] Wage estimates are calculated from data collected from employers in all industry divisions by Occupational Employer Statistics (OES), Bureau of Labor Statistics. The information. Source for California: Employment Development Department. Source for Illinois: The Workforce Information center.

[25] Source: Federal Reserve, Daily Treasury Real Long-Term Rates in 2000 (average).

to the small gap between unemployment rates in Los Angeles MSA we assume the scale factor of the matching function to be equal across job centers (see Table 2). Indeed, along Route 5, the average unemployment rate of the zip codes areas that belong to Los Angeles (resp. Orange) county is 8% (resp. 7%). For Chicago MSA, however, we assume $m_A < m_B$ due to an average unemployment rate gap of 6 percentage points (see Table 3). The Unemployment insurance (UI) replacement rate in the states of California and Illinois is around 0.5 (Taylor, 2011).

As we observe different unemployment rates across counties, it is natural to think that different separation rates $\delta_j$'s might be part of the explanation. We do not have this information at the county level for the year 2000. However, we have found more recent data about the number of initial claims for UI at the county level (this includes new, additional and transitional claims).[26] We select the year 2007[27] and compute the ratio between these initial claims in 2007 and employment in the same year for the counties under scrutiny. This proxy for separation rates is not exactly what we need to calibrate separation rates. So, we don't use their levels in each MSA. We only use the ratio between these proxies for the *county to the right* (RC) and the *county to the left* (LC). We set the separation rate to 0.036 (Pissarides, 2009) for the LCs (Los Angeles and Cook counties) and this ratio is only used to scale the separation rate in the RCs (Orange and DuPage counties). In both MSAs, this leads to $\delta_B < \delta_A$ (see Table 2 and Table 3).

Due to the importance of commuting by car,[28] to calibrate $\tau$ we first use the mileage reimbursement rate for privately owned automobile (POA). This information is provided by the U.S. General Service administration (GSA) and for the year 2000 it was calculated to be 0.325 USD/mile.[29] Second, $\tau$ takes into account the opportunity cost of time spent commuting. We find that the commuting times during peak hours between the two job centers are 60 and 53 minutes, respectively in Los Angeles and Chicago MSA. Since in Los Angeles (Chicago) MSA the average hourly wage is 16 USD (21 USD), the opportunity cost component of the commuting costs equals 27 USD (29 USD respectively). For the final commuting cost parameter we add the mileage reimbursement and the opportunity cost, double it to take into account a round trip

[26] This data is provided by the State of California Employment Development Department and the Illinois Department of Employment Security. A *"new claim"* is the first claim for a benefit year period (e.g., for the regular UI program it is 52 weeks). An individual would only have one new claim during a benefit year period. An *"additional claim"* is when another claim is filed during the same benefit year and there is intervening work between the first claim and the second claim. An individual can have multiple additional claims during the same benefit year if she meets the eligibility requirements. A *"transitional claim"* is when a claimant is still collecting benefits at the end of their benefit year period and had sufficient wage earnings during that year to start up a new claim once the first benefit year period ends.

[27] This is the first year where the relevant information is available. Moreover the years 2000 and 2007 share similar economic conditions, both being characterized by an unemployment rate reaching a local minimum.

[28] In 2000, in Los Angeles MSA around 86% of the labor force resident in the zip code zones around Route 5 commuted by car. In Chicago MSA, for the same year, 80% of the labor force resident in zip code zones around Route 88 and 290 were car-commuters.

[29] It includes (i) gasoline and oil (excluding taxes), (ii) depreciation of original vehicle cost, (iii) maintenance, accessories, parts, and tires, (iv) insurance and (v) state and Federal taxes.

and multiply it by 66 working days in a quarter.

Recall that for each MSA we aim to match in 2000 the average unemployment rates of the zip codes areas that belong to a county to the left and to the right, denoted $\bar{u}_{LC}$ and $\bar{u}_{RC}$, and the corresponding average wages, denoted $\bar{w}_{LC}$ and $\bar{w}_{RC}$.

Hence, for a given value of the threshold $\tilde{x}$, we have four unknowns: $y_A$, $y_B$, $\tilde{\theta}_A$ and $\tilde{\theta}_B$. We define a system of four equations according to the relative position of the boundary of the two counties ($x_0$) and the threshold ($\tilde{x}$). We equate the observed share[30] of employed workers in LC and respectively RC and the formulas coming from the model:

$$e_{LC} = \frac{\psi_A}{\delta_A + \psi_A} F(\underline{x}) + \frac{\psi_B}{\delta_B + \psi_B} (F(x_0) - F(\underline{x})) \quad (37)$$

$$e_{RC} = \frac{\psi_A}{\delta_A + \psi_A} (F(\overline{x}) - F(x_0)) + \frac{\psi_B}{\delta_B + \psi_B} (1 - F(\overline{x})) \quad (38)$$

where $\psi_j$ stands for $\psi_j(\theta_j)$, $\underline{x} = \min\{x_0, \tilde{x}\}$ and $\overline{x} = \max\{x_0, \tilde{x}\}$. The system of four equations is then:

$$\bar{w}_{LC} = \int_0^{\underline{x}} w_A(x) \frac{\psi_A}{\delta_A + \psi_A} \frac{f(x)}{e_{LC}} dx + \int_{\underline{x}}^{x_0} w_B(x) \frac{\psi_B}{\delta_B + \psi_B} \frac{f(x)}{e_{LC}} dx \quad (39)$$

$$\bar{w}_{RC} = \int_{x_0}^{\overline{x}} w_A(x) \frac{\psi_A}{\delta_A + \psi_A} \frac{f(x)}{e_{RC}} dx + \int_{\overline{x}}^{1} w_B(x) \frac{\psi_B}{\delta_B + \psi_B} \frac{f(x)}{e_{RC}} dx \quad (40)$$

$$\bar{u}_{LC} = \frac{\psi_A}{\delta_A + \psi_A} \frac{F(\underline{x})}{F(x_0)} + \frac{\psi_B}{\delta_B + \psi_B} \frac{(F(x_0) - F(\underline{x}))}{F(x_0)} \quad (41)$$

$$\bar{u}_{RC} = \frac{\psi_A}{\delta_A + \psi_A} \frac{(F(\overline{x}) - F(x_0))}{1 - F(x_0)} + \frac{\psi_B}{\delta_B + \psi_B} \frac{(1 - F(\overline{x}))}{1 - F(x_0)} \quad (42)$$

where $\frac{\psi_A}{\delta_A+\psi_A} \frac{f(x)}{e_{LC}}$ and $\frac{\psi_B}{\delta_B+\psi_B} \frac{f(x)}{e_{RC}}$ are the conditional employed population density in LC and RC, respectively. Then, Eq. (13) is used to express $w_j(x)$ in terms of unknowns and parameters (see Appendix B for more details).

We set the initial condition $\tilde{x} = x_0$[31] and then solve Eqs. (39) to (42). Next, we compute the two expected returns to search $\Sigma_j(\tilde{\theta}_j, \tilde{x})$ respectively locations A and B and according to the sign and the magnitude of the difference between the two $\Sigma_j$'s, a new value of $\tilde{x}$ is computed and the system Eqs. (39) to (42) is solved again. This iterative procedure is applied until equality (12) is verified. Finally, we compute the cost of opening a vacancy, $k_j$, using the free entry condition (18).

From the calibration in Table 2, the threshold $\tilde{x}$ separating job seekers in two groups is very close to the boundary, $x_0$, between the two counties. We do not have data about commuters for the zip codes we consider. Still, information *at the county level* is worth to look at. According to the U.S. Census Bureau, 97% of work-commuters in Los Angeles and Orange counties

[30]Since in the model the total labor force is normalized to one, we match the share and not the level of employment.

[31]We have however checked that the calibration is robust to changes in this initial condition.

travel within and between these counties.[32] In line with the calibration property $\tilde{x} \approx x_0$, the commuters' flow within each of these two counties is much higher than between them. The share of inner-county commuters in Los Angeles and Orange counties in 2000 was 96% and 85%, respectively. The higher average unemployment rate in Los Angeles county compared to Orange county is the consequence of a bigger separation rate, despite a slightly higher productivity level and lower unit cost of opening vacancies in job center $A$. All in all, tightness is higher in job center $A$ than in the job center $B$.

In the calibration for Chicago MSA, Table 3, the threshold $\tilde{x}$ separating job seekers in two groups is again close to the boundary, $x_0$, between the two counties. With the same caveat as in LA, let us look at the commuting patterns *at the county level.* According to the U.S. Census Bureau in 2000, 94% of work-commuters in Cook and DuPage counties travel within and between these counties. A 93% of work-commuters in Cook county are inner-county commuters. This is higher than in DuPage county where 65% of commuters travel to work within the county and the rest go to work in Cook county. The average unemployment rate is considerably higher in Cook county. This is first due to a higher separation rate. Next, a lower scale factor of the matching function in the CBD and a much higher vacancy cost lead to lower tightness and lower probability of being recruited in the CBD despite a higher productivity than in the SBD.

[32] Data of commuting patterns in the state of California, at the county level (U.S. Census Bureau).

**Table 2**
Calibration of Los Angeles MSA, quarterly data

| | Value | Description | Source/Target |
|---|---|---|---|
| **1. Parameters** | | | |
| **1.1. From the literature, data and assumptions** | | | |
| $r$ | 0.98 | Interest rate (%) | Federal Reserve |
| $\eta_A = \eta_B$ | 0.5 | Matching fn. Elasticity | Petrongolo and Pissarides (2001) |
| $\beta_A = \beta_B$ | 0.5 | Workers' bargaining power | Hosios condition $\eta_j = \beta_j$ |
| $b$ | $4,452$ | Unemployment Insurance | Bureau of Labor Statistics |
| $\delta_A$ | 0.036 | Separation rate LC | Pissarides (2009) |
| $\delta_B$ | 0.023 | Separation rate RC | Data |
| $m_A = m_B$ | 0.5 | Matching fn. scale factor of LC and RC | Unemployment rates |
| $\tau$ | $3,542$ | Commuting cost (USD per unit of scaled distance) | U.S. General Service Administration and hourly wage |
| $x_0$ | 0.51 | Boundary between Los Angeles - Orange counties | Data |
| $F(x_0)$ | 0.51 | CDF for Los Angeles county | Data |
| **1.2. Computed by the model (USD/quarter)** | | | |
| $y_A$ | $9,322$ | Labor productivity LC | Eqs. (39) - (42) |
| $y_B$ | $9,190$ | Labor productivity RC | Eqs. (39) - (42) |
| $k_A$ | $4,825$ | Vacancy cost LC | Eqs. (39) - (42) |
| $k_B$ | $9,340$ | Vacancy cost RC | Eqs. (39) - (42) |
| **2. Outcomes** | | | |
| **2.1. Matched labor market outcomes** | | | |
| $\bar{w}_{LC}$ | $8,964$ | Average wages Los Angeles county (USD/quarter) | State of California EDD [b] |
| $\bar{w}_{RC}$ | $8,843$ | Average wages Orange county (USD/quarter) | State of California EDD [b] |
| $\bar{u}_{LC}$ | 8.18 | Average unempl. rate Los Angeles county (%) | U.S. 2000 census |
| $\bar{u}_{RC}$ | 7.31 | Average unempl. rate Orange county (%) | U.S. 2000 census |
| **2.2. Endogenous variables computed by the model [a]** | | | |
| $\tilde{x}$ | 0.52 | Location of the marginal worker | Eq. (12) |
| $\tilde{\theta}_A$ | 0.65 | Market tightness LC | Eq. (18) |
| $\tilde{\theta}_B$ | 0.33 | Market tightness RC | Eq. (18) |
| $\psi_A\left(\tilde{\theta}_A\right)$ | 0.40 | Exit rate of unempl. LC | Eq. (35) |
| $\psi_B\left(\tilde{\theta}_B\right)$ | 0.29 | Exit rate of unempl. RC | Eq. (35) |
| $\mu_A\left(\tilde{\theta}_A\right)$ | 0.62 | Vacancy filling rate LC | Eq. (36) |
| $\mu_B\left(\tilde{\theta}_B\right)$ | 0.87 | Vacancy filling rate RC | Eq. (36) |
| $k_A/\mu_A\left(\tilde{\theta}_A\right)$ | $7,801$ | Exp. cost of opening a vacancy in LC | |
| $k_B/\mu_B\left(\tilde{\theta}_B\right)$ | $10,770$ | Exp. cost of opening a vacancy in RC | |

[a] Different initial values for $\tilde{x}$ do not affect the calibrated results.
[b] Employment Development Department. www.ca.gov

**Table 3**
Calibration of Chicago MSA, quarterly data

| | Value | Description | Source/Target |
|---|---|---|---|
| **1. Parameters** | | | |
| **1.1. From the literature, data and assumptions** | | | |
| $r$ | 0.98 | Interest rate (%) | Federal Reserve |
| $\eta_A = \eta_B$ | 0.5 | Matching fn. Elasticity | Petrongolo and Pissarides (2001) |
| $\beta_A = \beta_B$ | 0.5 | Workers' bargaining power | Hosios condition $\eta_j = \beta_j$ |
| $b$ | $5,438$ | Unemployment Insurance | Bureau of Labor Statistics |
| $\delta_A$ | 0.036 | Separation rate LC | Pissarides (2009) |
| $\delta_B$ | 0.024 | Separation rate RC | Data |
| $m_A$ | 0.6 | Matching fn. scale factor of | Unemployment rate Cook county |
| $m_B$ | 0.8 | LC and RC | Unemployment rate DuPage county |
| $\tau$ | $3,890$ | Commuting cost | U.S. General Service Administration |
| | | (USD per unit of scaled distance) | and hourly wage |
| $x_0$ | 0.53 | Boundary between Cook - DuPage counties | Data |
| $F(x_0)$ | 0.42 | CDF for Cook county | Data |
| **1.2. Computed by the model (USD/quarter)** | | | |
| $y_A$ | $11,686$ | Labor productivity LC | Eqs. (39) - (42) |
| $y_B$ | $10,841$ | Labor productivity RC | Eqs. (39) - (42) |
| $k_A$ | $11,056$ | Vacancy cost LC | Eqs. (39) - (42) |
| $k_B$ | $4,671$ | Vacancy cost RC | Eqs. (39) - (42) |
| **2. Outcomes** | | | |
| **2.1. Matched labor market outcomes** | | | |
| $\bar{w}_{LC}$ | $11,686$ | Average wage Cook county (USD/quarter) | IDES [b] |
| $\bar{w}_{RC}$ | $10,841$ | Average wage DuPage county (USD/quarter) | IDES [b] |
| $\bar{u}_{LC}$ | 8.96 | Unemployment rate Cook county (%) | U.S. 2000 census |
| $\bar{u}_{RC}$ | 3.19 | Unemployment rate DuPage county (%) | U.S. 2000 census |
| **2.2. Endogenous variables computed by the model** [a] | | | |
| $\tilde{x}$ | 0.54 | Location of the marginal worker | Eq. (12) |
| $\tilde{\theta}_A$ | 0.37 | Market tightness LC | Eq. (18) |
| $\tilde{\theta}_B$ | 0.89 | Market tightness RC | Eq. (18) |
| $\psi_A\left(\tilde{\theta}_A\right)$ | 0.37 | Exit rate of unempl. LC | Eq. (35) |
| $\psi_B\left(\tilde{\theta}_B\right)$ | 0.75 | Exit rate of unempl. RC | Eq. (35) |
| $\mu_A\left(\tilde{\theta}_A\right)$ | 0.98 | Vacancy filling rate LC | Eq. (36) |
| $\mu_B\left(\tilde{\theta}_B\right)$ | 0.85 | Vacancy filling rate RC | Eq. (36) |
| $k_A/\mu_A\left(\tilde{\theta}_A\right)$ | $11,233$ | Exp. cost of opening a vacancy in LC | |
| $k_B/\mu_B\left(\tilde{\theta}_B\right)$ | $5,506$ | Exp. cost of opening a vacancy in RC | |

[a] Different initial values for $\tilde{x}$ do not affect the calibrated values.
[b] Illinois Department of Employment Security. `www.illinois.gov`

#### 4.2.3. The counterfactual simulations

In Table 4 (resp., 5) we take the calibrated parameters of Tables 2 (resp., 3) and simulate the implications of substituting counterfactual distributions of the workforce. In Table 4 (resp., 5), column (1) reproduces the key endogenous indicators of Table 2 (resp., 3). In column (2), we swap the active population distribution between MSAs. Next, we look at the consequences of a uniform distribution in column (3). Finally, in columns (4) to (6) we do the same for three truncated normal distributions on a support $[0, 1]$: A symmetric density ($\mathcal{N}(0.5, 0.15)$), a normal distribution positively skewed because of the truncation ($\mathcal{N}(0.25, 0.5)$), and a negatively skewed ($\mathcal{N}(0.75, 0.5)$). The latter density functions are illustrated in Fig. 9.

**Table 4**
Simulations for Los Angeles MSA, quarterly data

| | Los Angeles MSA [a] (From Table 2) | Counterfactual population distributions: Chicago MSA | Uniform $unif(0,1)$ | Normal $\mathcal{N}(mean, st.dev.)$ truncated at $(0,1)^b$: $\mathcal{N}(0.5, 0.15)$ | $\mathcal{N}(0.25, 0.5)$ | $\mathcal{N}(0.75, 0.5)$ |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $x_0$ | 0.51 | 0.51 | 0.51 | 0.51 | 0.51 | 0.51 |
| $F(x_0)$ | 0.51 | 0.40 | 0.51 | 0.53 | 0.63 | 0.40 |
| $\bar{w}_{LC}$ | $8,964$ | $8,964$ | $8,961$ | $8,988$ | $8,962$ | $8,969$ |
| $\bar{w}_{RC}$ | $8,843$ | $8,837$ | $8,839$ | $8,865$ | $8,847$ | $8,838$ |
| $\bar{u}_{LC}$ | 8.18 | 8.15 | 8.10 | 8.65 | 8.10 | 8.26 |
| $\bar{u}_{RC}$ | 7.31 | 7.21 | 7.23 | 7.72 | 7.36 | 7.23 |
| $e_{RC} + e_{LC}$ | 92.25 | 92.41 | 92.32 | 91.78 | 92.18 | 92.36 |
| $\tilde{x}$ | 0.518 | 0.518 | 0.518 | 0.582 | 0.520 | 0.517 |
| $\tau\,\Gamma_A(\tilde{x})$ | $1,002$ | 967 | 918 | $1,388$ | 918 | $1,062$ |
| $\tau\,\Gamma_B(\tilde{x})$ | 933 | 833 | 853 | $1,307$ | 983 | 858 |
| $\tilde{\theta}_A$ | 0.65 | 0.66 | 0.67 | 0.58 | 0.67 | 0.64 |
| $\tilde{\theta}_B$ | 0.33 | 0.34 | 0.34 | 0.30 | 0.33 | 0.34 |
| $\psi_A\left(\tilde{\theta}_A\right)$ | 0.40 | 0.41 | 0.41 | 0.38 | 0.41 | 0.40 |
| $\psi_B\left(\tilde{\theta}_B\right)$ | 0.29 | 0.29 | 0.29 | 0.27 | 0.29 | 0.29 |
| $\mu_A\left(\tilde{\theta}_A\right)$ | 0.62 | 0.62 | 0.61 | 0.66 | 0.61 | 0.62 |
| $\mu_B\left(\tilde{\theta}_B\right)$ | 0.87 | 0.85 | 0.86 | 0.92 | 0.87 | 0.86 |
| $k_A/\mu_A\left(\tilde{\theta}_A\right)$ | $7,801$ | $7,840$ | $7,894$ | $7,361$ | $7,895$ | $7,727$ |
| $k_B/\mu_B\left(\tilde{\theta}_B\right)$ | $10,770$ | $10,925$ | $10,894$ | $10,173$ | $10,692$ | $10,877$ |
| $\Omega\left(\tilde{\theta}_A, \tilde{\theta}_B, \tilde{x}\right)$ | $7,749$ | $7,818$ | $7,825$ | $7,398$ | $7,781$ | $7,769$ |

[a] Different initial values for $\tilde{x}$ do not affect the final results of the calibration.
[b] A normal distribution that is truncated at 0 on the left and at 1 on the right is defined in density form as $f(x) = \frac{\phi(x)I(x)}{\Phi(1)-\Phi(0)}$ where $\phi$ (resp. $\Phi$ designates the Normal density (resp. cumulative density) function and $I_{(0,1)}(x) = 1$ if $0 \leqslant x \leqslant 1$, $I_{(0,1)}(x) = 0$ otherwise.

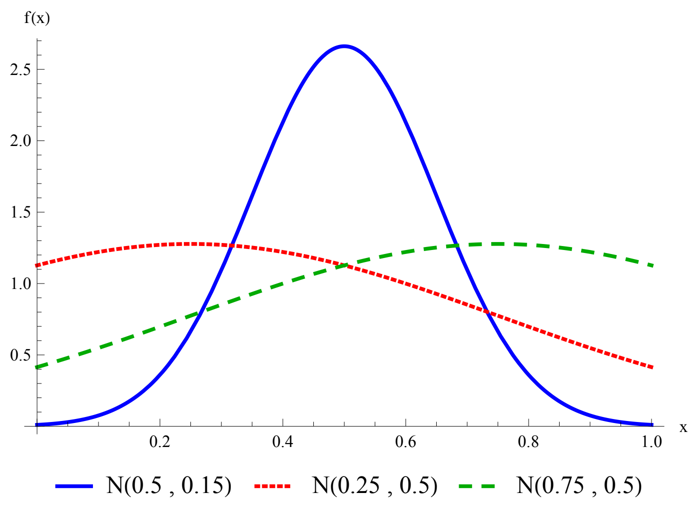



**Fig. 9.** Distributions for simulations
Columns (4) to (6)

Substituting the counterfactual distribution of the workforce in Chicago into Los Angeles MSA consists of concentrating more population around job center $B$. Comparing columns (1) and (2) of Table 4 we observe that the threshold location does not change at the two-digit level. Tightness turns out to be higher in both job centers and net output is 1% higher when we take the workforce distribution of Chicago MSA. This is due to a lower average commuting cost towards both job centers (see Table 4). Therefore, firms get a higher surplus from a match and are induced to post more vacancies. This explains 0.16 percentage points rise in the MSA's employment rate and a decline in both average unemployment rates, especially $\bar{u}_{RC}$ in Orange county.

Since the actual distribution of Los Angeles MSA's workforce is not too far from uniform (see Fig. 8a), it is natural to consider this assumption in column (3) of Table 4. The same mechanisms as in column (2) are at work. Eventually, net output is 1% higher than in column (1). The most dramatic change appears in column (4) where we assume that the labor force is concentrated around the boundary between the two counties ($\mathcal{N}(0.5, 0.15)$). The substantial increase in $\tau\,\Gamma_j(\tilde{x})$ in both counties eventually leads to a drop in tightness levels by more than 10% and of net output by 4.5%. Lower tightness levels cause a decline in the total employment rate and a rise of both unemployment rates by nearly half a percentage point. Finally, the distribution $\mathcal{N}(0.75, 0.5)$ is interesting because it looks similar to the actual distribution in Chicago MSA. However, even if $F(x_0)$ is the same in both cases, the workforce near the CBD is less important (e.g. $F[0.25] = 0.18$ in column (2) versus 0.15 in column (6)). In addition in the RC (Orange county), the inversed-U shape profile of the workforce in the RC is more pronounced in column (2) than in column (6). These differences are sufficient to induce that, compared to the actual distribution in column (1), $\tau\,\Gamma_A(\tilde{x})$ rises in column (6) while it decreases in column (2). In column (6), $\tau\,\Gamma_B(\tilde{x})$ is lower than in column (1) but the decline is less pronounced than in column (2). These differences lead to lower tightness in job center $A$ under the assumption $\mathcal{N}(0.75, 0.5)$ while it rises with the counterfactual distribution of Chicago MSA. Additionally, the average unemployment rate in the left county, $\bar{u}_{LC}$, is higher than in column (1) while it was lower with the Chicago distribution. So, limited differences in the distribution of the workforce turn out to have opposite effects on the average unemployment rates in the LC.

Comparing columns (1) and (2) of Table 5 we observe that differences of the outcomes of the model are small at the two-digit level (see $\tilde{x}$ and $\tilde{\theta}_A$). Substituting Los Angeles MSA workforce distribution implies however a drop in average wages (4.3% for the LC and 1.6% for the RC) and a rise in average unemployment rates (+0.15 percentage points in the LC and +0.08 in the RC). Moreover, total employment rate falls by 0.8 percentage points and net output is reduced by 0.6%. The more concentrated distribution around the boundary between counties explains these differences. Hence, higher average commuting costs towards both job centers diminish welfare. Comparing a uniform distribution with Chicago MSA's actual distribution of the active population, see Fig. 8b, one can expect a stronger effect in job center $A$ since the workforce density of DuPage county is strictly below one. With equal density all along the MSA the average commuting costs to job center $A$ ($B$) fall (rise) by 5.6% (1.6%). However, total employment rate and net output drop by 0.7 percentage points and 0.3%, respectively. We observe larger differences particularly in unemployment rates once we introduce a normal distribution positively or negatively skewed. The normal distribution of column (4) is specially worth considering since commuting costs to both job centers rise substantially. Consequently, net output shrinks by 4% and the average unemployment rate in the LC (resp., RC) is 0.6 (resp. 0.3) percentage points higher than in column (1). Finally, in column (6) net output drops by 0.6%.

**Table 5**
Simulations for Chicago MSA, quarterly data

| | | **Counterfactual population distributions** | | | | |
|---|---|---|---|---|---|---|
| | **Chicago MSA**[a] | **Los Angeles MSA** | **Uniform** | **Normal $\mathcal{N}(mean, st.dev.)$ truncated at $(0,1)$**[b] | | |
| | (From Table 2) | | $unif(0,1)$ | $\mathcal{N}(0.5, 0.15)$ | $\mathcal{N}(0.25, 0.5)$ | $\mathcal{N}(0.75, 0.5)$ |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $x_0$ | 0.53 | 0.53 | 0.53 | 0.53 | 0.53 | 0.53 |
| $F(x_0)$ | 0.42 | 0.53 | 0.53 | 0.58 | 0.65 | 0.42 |
| $\bar{w}_{LC}$ | $11,686$ | $11,184$ | $11,177$ | $11,213$ | $11,176$ | $11,189$ |
| $\bar{w}_{RC}$ | $10,841$ | $10,670$ | $10,666$ | $10,687$ | $10,676$ | $10,664$ |
| $\bar{u}_{LC}$ | 8.96 | 9.11 | 9.01 | 9.57 | 9.00 | 9.19 |
| $\bar{u}_{RC}$ | 3.19 | 3.27 | 3.22 | 3.53 | 3.34 | 3.20 |
| $e_{RC} + e_{LC}$ | 94.38 | 93.62 | 93.70 | 92.95 | 92.99 | 94.27 |
| $\tilde{x}$ | 0.540 | 0.540 | 0.541 | 0.537 | 0.542 | 0.539 |
| $\tau\,\Gamma_A(\tilde{x})$ | $1,114$ | $1,146$ | $1,052$ | $1,569$ | $1,046$ | $1,220$ |
| $\tau\,\Gamma_B(\tilde{x})$ | 879 | 979 | 893 | $1,384$ | $1,028$ | 902 |
| $\tilde{\theta}_A$ | 0.37 | 0.37 | 0.38 | 0.34 | 0.38 | 0.36 |
| $\tilde{\theta}_B$ | 0.89 | 0.87 | 0.89 | 0.79 | 0.86 | 0.88 |
| $\psi_A\left(\tilde{\theta}_A\right)$ | 0.37 | 0.36 | 0.37 | 0.35 | 0.37 | 0.36 |
| $\psi_B\left(\tilde{\theta}_B\right)$ | 0.75 | 0.75 | 0.75 | 0.71 | 0.74 | 0.75 |
| $\mu_A\left(\tilde{\theta}_A\right)$ | 0.98 | 0.99 | 0.98 | 1.04 | 0.98 | 1.00 |
| $\mu_B\left(\tilde{\theta}_B\right)$ | 0.85 | 0.86 | 0.85 | 0.90 | 0.86 | 0.85 |
| $k_A/\mu_A\left(\tilde{\theta}_A\right)$ | $11,233$ | $11,193$ | $11,308$ | $10,667$ | $11,315$ | $11,103$ |
| $k_B/\mu_B\left(\tilde{\theta}_B\right)$ | $5,506$ | $5,442$ | $5,497$ | $5,176$ | $5,411$ | $5,491$ |
| $\Omega\left(\tilde{\theta}_A, \tilde{\theta}_B, \tilde{x}\right)$ | $9,714$ | $9,660$ | $9,744$ | $9,278$ | $9,718$ | $9,660$ |

[a] Different initial values for $\tilde{x}$ do not affect the final results of the calibration.
[b] A normal distribution that is truncated at 0 on the left and at 1 on the right is defined in density form as $f(x) = \frac{\phi(x)I(x)}{\Phi(1)-\Phi(0)}$ where $\phi$ (resp. $\Phi$ designates the Normal density (resp. cumulative density) function and $I_{(0,1)}(x) = 1$ if $0 \leqslant x \leqslant 1$, $I_{(0,1)}(x) = 0$ otherwise.

## 5. Conclusions

Urban development and more specifically the way the population is distributed within metropolitan areas generate various externalities. Glaeser and Kahn (2010) emphasize environmental externalities through the emission of greenhouse gases. Part of them are due to car usage and commuting. This paper emphasizes another externality due to commuters, namely on job creation. For this purpose, we develop a stylized representation of a densely populated metropolitan area where the workforce lives between two job centers characterized by search-matching frictions. Because wages compensate partly for commuting costs, the expected value of opening a vacancy in a job center shrinks when the pool of job-seekers spreads over longer distances. When jobless people decide where to seek jobs, they do not internalize that their decision affects job creation and hence has an impact on all the unemployed. Because of this composition externality, we show that the regional unemployment rates are typically inefficient even if the Hosios condition is met. In addition to this normative statement, this paper measures how changes in the distribution of the workforce affects net output and equilibrium unemployment rates.

We conduct a numerical analysis to see how the shape of the population distribution affects unemployment rates, average wages and efficiency. In a first step, we measure the gap between the efficient and the decentralized allocations when a mass of workers located in the CBD grows while the mass of workers uniformly spread along the rest of the line joining the CBD to the SBD shrinks. In this illustrative example, we conclude that the gap in unemployment rates is at most 0.03 percentage point (the unemployment averaging 4%) while net output is more affected (the gap reaching here 2%). In a second step, we calibrate the model for Los Angeles and Chicago MSAs. We simulate the impacts of substituting counterfactual distributions of the workforce. It turns out that the location of the population has non-negligible effects on unemployment and net output. Within the range of distributions we have considered for Los Angeles, changes in unemployment rates (respectively, in net output) can reach 0.6 percentage points (resp., 4%). The order of magnitude in Chicago is the same.

Considering the role of public policies (subsidies to job creation, reductions in commuting costs and the like) is a natural complement to this paper. A useful extension would be to develop the same analysis in a two-dimensional space with a larger number of job centers.

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# A. Comparative statics

We define $\psi_j'(\theta_j) = (1-\eta_j(\theta_j))\,\psi_j(\theta_j)/\theta_j$, where $\eta_j$ is the elasticity of the matching rate $\mu_j(\theta_j)$ (i.e. $\psi_j(\theta_j)/\theta_j$) with respect to tightness.

## A.1. Equilibrium tightness for a given level of the threshold $\tilde{x}$

For $j \in \{A, B\}$ we can rewrite Eq. (18) as $\mathcal{F}_j(\theta_j, \tilde{x}) = 0$ where we define

$$\mathcal{F}_j(\theta_j, \tilde{x}) = \frac{\theta_j k_j}{\psi_j(\theta_j)} - (1-\beta_j)\frac{y_j - \tau\,\Gamma_j(\tilde{x}) - b}{r + \delta_j + \beta_j\,\psi_j(\theta_j)}$$

Let $\zeta_j$ denote any of the parameters in $\{k_j, \delta_j, y_j, \beta_j, b, r\}$. Using the implicit function theorem,

$$\frac{\partial\theta_j}{\partial\zeta_j} = -\frac{\partial\mathcal{F}_j/\partial\zeta_j}{\partial\mathcal{F}_j/\partial\theta_j} \tag{A.1}$$

where $\partial\theta_j/\partial\zeta_j$ can also be written $\partial\Theta_j(\tilde{x})/\partial\zeta_j$ (see the proof of Lemma 3) and in which

$$\frac{\partial\mathcal{F}_j}{\partial\theta_j} = \frac{k_j}{\psi_j(\theta_j)}\,\frac{\eta_j(\theta_j)(r+\delta_j) + \beta_j\psi_j(\theta_j)}{r+\delta_j+\beta_j\psi_j(\theta_j)} > 0.$$

The sign of $\partial\theta_j/\partial\zeta_j$ is therefore given by the one of $\partial\mathcal{F}_j/\partial\zeta_j$. So, along $\mathcal{F}_j(\theta_j, \tilde{x}) = 0$,

- $\frac{\partial\theta_j}{\partial k_j} < 0$, because $\frac{\partial\mathcal{F}_j}{\partial k_j} = \frac{\theta_j}{\psi_j(\theta_j)} > 0$.
- $\frac{\partial\theta_j}{\partial\delta_j} = \frac{\partial\theta_j}{\partial r} < 0$, because $\frac{\partial\mathcal{F}_j}{\partial\delta_j} = \frac{1}{r+\delta_j+\beta_j\psi_j(\theta_j)}\,\frac{k_j\theta_j}{\psi_j(\theta_j)} > 0$.
- $\frac{\partial\theta_j}{\partial y_j} > 0 > \frac{\partial\theta_j}{\partial b}$, because $\frac{\partial\mathcal{F}_j}{\partial y_j} = -\frac{\partial\mathcal{F}_j}{\partial b} = -\frac{1-\beta_j}{r+\delta_j+\beta_j\psi_j(\theta_j)} < 0$.
- $\frac{\partial\theta_j}{\partial\beta_j} < 0$, because $\frac{\partial\mathcal{F}_j}{\partial\beta_j} = \frac{r+\delta_j+\psi_j(\theta_j)}{(1-\beta_j)(r+\delta_j+\beta_j\psi_j(\theta_j))}\,\frac{k_j\theta_j}{\psi_j(\theta_j)} > 0$.

## A.2. Equilibrium threshold

By totally differentiating Eq. (19), on gets:

$$\frac{d\tilde{x}}{d\zeta_j} = \frac{\frac{d\mathcal{S}_B(\tilde{x})}{d\zeta_j} - \frac{d\mathcal{S}_A(\tilde{x})}{\partial\zeta_j}}{\frac{d\mathcal{S}_A(\tilde{x})}{d\tilde{x}} - \frac{d\mathcal{S}_B(\tilde{x})}{d\tilde{x}}} \tag{A.2}$$

where $\zeta_j$ denotes any of the parameters in job center $j$. By Lemma 4 the denominator of Eq. (A.2) is negative, and

$$\frac{d\mathcal{S}_j}{d\zeta_{j'}} = \frac{\partial\Sigma_j(\Theta_j(\tilde{x}), \tilde{x})}{\partial\theta_j}\,\frac{\partial\Theta_j(\tilde{x})}{\partial\zeta_{j'}} + \frac{\partial\Sigma_j(\Theta_j(\tilde{x}), \tilde{x})}{\partial\zeta_{j'}} \tag{A.3}$$

is nil when $j \neq j'$ except for $\zeta_{j'} \in \{r, b\}$. By Lemma 1, we know that $\partial\Sigma_j/\partial\theta_j > 0$. So, in order to study the sign of the numerator of Eq. (A.2) we need to sign the second term on the right-hand side of (A.3):

- $\frac{\partial\Sigma_j}{\partial k_j} = 0$.

- $\frac{\partial \Sigma_j}{\partial \delta_j} = \frac{\partial \Sigma_j}{\partial r} = -\Sigma_j \left(\Theta_j \left(\tilde{x}\right), \tilde{x}\right) \frac{1}{r+\delta_j+\beta_j\, \psi_j(\theta_j)} < 0.$
- $\frac{\partial \Sigma_j}{\partial y_j} = -\frac{\partial \Sigma_j}{\partial b} = \frac{\beta_j\, \psi_j(\theta_j)}{r+\delta_j+\beta_j\, \psi_j(\theta_j)} > 0.$
- $\frac{\partial \Sigma_j}{\partial \beta_j} = \Sigma_j \left(\Theta_j \left(\tilde{x}\right), \tilde{x}\right) \frac{r+\delta_j}{\beta_j\, [r+\delta_j+\beta_j\, \psi_j(\theta_j)]} > 0.$

The comparative static analysis is summarized in Table 6.

**Table 6**
Comparative statics

| $\zeta_{j'}$ | $\frac{\partial \mathcal{S}_B}{\partial \zeta_{j'}}$ | $= \frac{\partial \Sigma_B(\Theta_B(\tilde{x}),\tilde{x})}{\partial \theta_B}$ | $\frac{\partial \Theta_B(\tilde{x})}{\partial \zeta_{j'}}$ | $+ \frac{\partial \Sigma_B(\Theta_B(\tilde{x}),\tilde{x})}{\partial \zeta_{j'}}$ | $\frac{\partial \mathcal{S}_A}{\partial \zeta_{j'}}$ | $= \frac{\partial \Sigma_A(\Theta_A(\tilde{x}),\tilde{x})}{\partial \theta_A}$ | $\frac{\partial \Theta_A(\tilde{x})}{\partial \zeta_{j'}}$ | $+ \frac{\partial \Sigma_A(\Theta_A(\tilde{x}),\tilde{x})}{\partial \zeta_{j'}}$ | $\frac{\partial \mathcal{S}_B(\tilde{x})}{\partial \zeta_j} - \frac{\partial \mathcal{S}_A(\tilde{x})}{\partial \zeta_j}$ | $\frac{d\tilde{x}}{d\zeta_j}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $k_A$ | 0 | + | 0 | 0 | − | + | − | 0 | + | − |
| $\delta_A$ | 0 | + | 0 | 0 | − | + | − | − | + | − |
| $y_A$ | 0 | + | 0 | 0 | + | + | + | + | − | + |
| $\beta_A$ | 0 | + | 0 | 0 | ?† | + | − | + | ? | ? |
| $b$ | − | + | − | − | − | + | − | − | ? | ? |
| $r$ | − | + | − | − | − | + | − | − | ? | ? |
| $k_B$ | − | + | − | 0 | 0 | + | 0 | 0 | − | + |
| $\delta_B$ | − | + | − | − | 0 | + | 0 | 0 | − | + |
| $y_B$ | + | + | + | + | 0 | + | 0 | 0 | + | − |
| $\beta_B$ | ?† | + | − | + | 0 | + | 0 | 0 | ? | ? |

† It can be checked that $\frac{\partial \mathcal{S}_j}{\partial \beta_j} \gtreqqless 0 \Leftrightarrow \eta_j \gtreqqless \beta_j$.

Given that $\frac{\partial \mathcal{S}_j}{\partial \beta_j} \gtreqqless 0 \Leftrightarrow \eta_j \gtreqqless \beta_j$, the signs of the partial derivatives with respect to the workers' bargaining power verify:

$$\frac{d\tilde{x}}{d\beta_A} \gtreqqless 0 \Leftrightarrow \eta_A \gtreqqless \beta_A \qquad \text{and} \qquad \frac{d\tilde{x}}{d\beta_B} \gtreqqless 0 \Leftrightarrow \eta_B \lesseqqgtr \beta_B.$$

So, an ambiguity remains for $d\tilde{x}/db$ and $d\tilde{x}/dr$ only.

### A.3. Equilibrium tightness

The total effect of a marginal change in parameter $\zeta_j$ on equilibrium tightness in $j$ is

$$\frac{d\theta_j}{d\zeta_j} = \frac{\partial \theta_j}{\partial \zeta_j} + \frac{\partial \theta_j}{\partial \tilde{x}} \frac{d\tilde{x}}{d\zeta_j}, \qquad j \in \{A, B\} \tag{A.4}$$

where on the right-hand side, the first term is given by (A.1), the second one equals $-\left(\partial \mathcal{F}_j / \partial \tilde{x}\right) / \left(\partial \mathcal{F}_j / \partial \theta_j\right)$ (negative for $j = A$ and positive for $j = B$), while the third one is given by by (A.2). This total effect (A.4) has an ambiguous sign for all parameters.

## B. Computation of the wage equations for the program

Using the wage Eq. (13), Eqs. (37) and (38) respectively become:

$$
\begin{aligned}
\bar{w}_{LC} &= \int_0^{\underline{\mathsf{x}}} w_A(x)\,\frac{\psi_A}{\delta_A+\psi_A}\,\frac{f(x)}{e_{LC}}dx + \int_{\underline{\mathsf{x}}}^{x_0} w_B(x)\,\frac{\psi_B}{\delta_B+\psi_B}\,\frac{f(x)}{e_{LC}}dx \\
&= \frac{1}{e_{LC}}\left\{\frac{\psi_A}{\delta_A+\psi_A}\int_0^{\underline{\mathsf{x}}} w_A(x)f(x)dx + \frac{\psi_B}{\delta_B+\psi_B}\int_{\underline{\mathsf{x}}}^{x_0} w_B(x)f(x)dx\right\} \\
&= \frac{1}{e_{LC}}\left\{\frac{\psi_A}{\delta_A+\psi_A}\left[\left(\beta_A y_A + (1-\beta_A)\left(b+\frac{\beta_A\psi_A(y_A-b)}{r+\delta_A+\beta_A\psi_A}\right)\right)F(\underline{\mathsf{x}}) + \frac{(1-\beta_A)(r+\delta_A)}{r+\delta_A+\beta_A\psi_A}\,\tau\int_0^{\underline{\mathsf{x}}} x f(x)dx\right]\right. \\
&\quad \left. +\frac{\psi_B}{\delta_B+\psi_B}\left[\left(\beta_B y_B + (1-\beta_B)\left(b+\frac{\beta_B\psi_B(y_B-b)}{r+\delta_B+\beta_B\psi_B}\right)\right)(F(x_0)-F(\underline{\mathsf{x}})) + \frac{(1-\beta_B)(r+\delta_B)}{r+\delta_B+\beta_B\psi_B}\,\tau\int_{\underline{\mathsf{x}}}^{x_0}(1-x)f(x)dx\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
\bar{w}_{RC} &= \int_{x_0}^{\overline{\mathsf{x}}} w_A(x)\,\frac{\psi_A}{\delta_A+\psi_A}\,\frac{f(x)}{e_{RC}}dx + \int_{\overline{\mathsf{x}}}^{1} w_B(x)\,\frac{\psi_B}{\delta_B+\psi_B}\,\frac{f(x)}{e_{RC}}dx \\
&= \frac{1}{e_{RC}}\left\{\frac{\psi_A}{\delta_A+\psi_A}\int_{x_0}^{\overline{\mathsf{x}}} w_A(x)f(x)dx + \frac{\psi_B}{\delta_B+\psi_B}\int_{\overline{\mathsf{x}}}^{1} w_B(x)f(x)dx\right\} \\
&= \frac{1}{e_{RC}}\left\{\frac{\psi_A}{\delta_A+\psi_A}\left[\left(\beta_A y_A + (1-\beta_A)\left(b+\frac{\beta_A\psi_A(y_A-b)}{r+\delta_A+\beta_A\psi_A}\right)\right)(F(\overline{\mathsf{x}})-F(x_0)) + \frac{(1-\beta_A)(r+\delta_A)}{r+\delta_A+\beta_A\psi_A}\,\tau\int_{x_0}^{\overline{\mathsf{x}}} x f(x)dx\right]\right. \\
&\quad \left. +\frac{\psi_B}{\delta_B+\psi_B}\left[\left(\beta_B y_B + (1-\beta_B)\left(b+\frac{\beta_B\psi_B(y_B-b)}{r+\delta_B+\beta_B\psi_B}\right)\right)(1-F(\overline{\mathsf{x}})) + \frac{(1-\beta_B)(r+\delta_B)}{r+\delta_B+\beta_B\psi_B}\,\tau\int_{\overline{\mathsf{x}}}^{1}(1-x)f(x)dx\right]\right\}
\end{aligned}
$$

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