

Regional Equilibrium Unemployment Theory at the Age of the Internet

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Abstract

This paper studies equilibrium unemployment in a two-region economy where homogeneous workers and jobs are free to move and the housing market clears. Because of the Internet, searching for a job in another region without first migrating there is nowadays much simpler than in the past. Search-matching externalities are amplified by this possibility and by the fact that some workers can simultaneously receive a job offer from each region. The rest of the framework builds on Moretti (2011). We study numerically the impacts of various local shocks in a stylized US economy. Contrary to what could be expected, increasing matching effectiveness in the other region yields growing regional unemployment rates. We characterize the optimal allocation and conclude that the Hosios condition is not sufficient to restore efficiency. In the efficient allocation, the regional unemployment rates are much lower than in the decentralized economy and nobody searches in the other region.

Keywords: Matching; Search then move; Spatial equilibrium; Regional economics; Unemployment differentials.

JEL: J61, J64, R13, R23.

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1 Introduction

Regional differences in unemployment rates are a pervasive phenomenon in many European countries (Overman and Puga (2002); Elhorst (2003)), while the evidence in North America is more mixed (Topel (1986), Blanchard and Katz (1992), OECD (2005), p.81). Some papers explain these disparities using a general equilibrium approach with inter-regional migration. Migration occurs when workers are unemployed and decide to relocate. Thanks to the development of the Internet, long-distance job-search is nowadays largely accessible: Job seekers can easily look for a job in several regions simultaneously and only migrate in case of successful search. We develop a general equilibrium search-matching framework that takes this feature into account. One could think that this opportunity of searching all over the country would reduce regional unemployment rates and hence increase output. Our theoretical and numerical analyses show that a rise in unemployment and a loss in efficiency are instead very plausible.

Regional unemployment differentials are usually important. Focusing on large regions¹ in 2009, the unemployment rate in Michigan is almost 9 percentage points higher than in North Dakota. Comparing to the recent crisis, the US national unemployment rate in 2010 is only 5 percentage points higher than in 2006. Kline and Moretti (2013) argue that substantial differences in unemployment across Metropolitan areas remain after controlling for education, age, gender and race. These differentials are moreover long lasting. At the state level, we find that regressing 2007 unemployment rates on the corresponding rates in 1990 exhibits a high degree of persistence, with a significant regression coefficient of 0.47 (0.099) and a R^2 of 0.32. Similar results are found for European countries. In 2009, the difference between the highest and the lowest French unemployment rates is 8 percentage points. This differential reaches 11 percentage points in Italy. There is also evidence of persistence (see e.g. Elhorst, 2003).

Especially in the US, inter-regional migration is a widespread phenomenon.² However, it seems to be insufficient to balance regional unemployment rates in the long run. Among the determinants that may lead to migration, better labor market prospects are important (see e.g. Beaudry *et al.*, 2013). The Internet nowadays allows both sides of the labor market to find more rapidly potential partners, even faraway, thanks to job-search and meta-search engines. Moreover, the recruitment process can now also be conducted on-line through virtual recruiting tools.³ These tools therefore reduce the necessity to physically move to the new region before starting to seek a job. In 2010, 25% of the interviewed Americans who use the Internet declared to do so to find a position. In Europe, in 2005, among the unemployed workers, 25% used the Internet to search for a job. This share has increased to almost 50% in 2011.⁴ According to Autor

¹Large regions correspond to the NUTS 2 level in Eurostat and State level in the U.S.

²Two percent of the Americans surveyed in 2011 declared to live in another US state the year before. Source: U.S. Census Bureau, 2011 American Community Survey.

³See e.g. <http://variousthingslive.com/virtual-open-house/> and the links on <http://hiring.monster.com/hr/hr-best-practices/recruiting-hiring-advice/acquiring-job-candidates/virtual-recruitment-strategies.aspx>.

⁴Sources: U.S. Census Bureau (2012), Survey of Income and Program Participation, 2008 Panel and

(2001), the change in the matching process (that we model below) is one the three main consequences of the development of the Internet. The aim of this paper is thus to better understand how the possibility of seeking a job in the whole country and of migrating only in case of success affects regional unemployment rate and efficiency.

To introduce this possibility in a spatial equilibrium setting with genuine unemployment, we build upon the synthesis of Moretti (2011) under pure labor market competition and introduce search-matching frictions and wage bargaining within this framework (Mortensen and Pissarides (1999); Pissarides (2000)). Moretti (2011) builds a two-region spatial equilibrium model *à la* Rosen (1979)-Roback (1982). Contrary to these authors however, Moretti (2011) assumes that the regional supply of housing is not perfectly inelastic and the supply of labor not perfectly elastic. The latter property is obtained by assuming that economic agents have heterogeneous idiosyncratic preferences for regions. The aim of Moretti (2011) is to analyze how local shocks propagate to the rest of the economy in the long run, with a focus on the labor market. He discusses the case where agents have different skills, while we keep labor homogeneous. However, regional unemployment disparities are not studied by Moretti.

Contrary to most of the search and matching literature, the spatial heterogeneity is explicit in our framework. In each region, imperfect information and lack of coordination among agents are summarized by a regional-specific matching function with constant returns to scale. Because of these frictions, a realized match between a job seeker and a job vacancy creates a pure economic rent. This rent, also called the surplus of the match, is shared through Nash bargaining. Frictions generate congestion externalities which are not internalized by decentralized agents unless the so-called Hosios (1990) condition is met. This general property in the search-matching literature does not hold in our setting where job seekers living in a given region can choose to search all over the country (with possibly a lower efficiency out of the region of residence). If they do, regions are much more interrelated, with a range of consequences. More specifically, when decentralized agents decide whether to search nationally, they look at their private interest and ignore the consequences of their choices on job creation in all regions. In particular, national job-seekers can get more than one job offer but will only accept one and this generates a loss of resources. In addition, when opening vacancies in a region, firms do not internalize the changes in the matching probability and hence in net output in the rest of the economy. Therefore, as soon as some workers search all over the country, we can show that the Hosios condition is never sufficient to decentralize the (constrained) efficient allocation (which maximizes net output subject to the matching constraints). When regions are symmetric, we also show that efficiency requires that either nobody searches all over the country or all job seekers do.

Next, we develop a numerical exercise for a stylized US economy where regions are initially symmetric and the Hosios condition prevails. Augmenting the effectiveness of search out of the region of residence increases the unemployment rate everywhere because the induced negative effect on vacancy creation outweighs the direct favorable effect on the risk of unemployment. This drop in labor demand is a response to lower

Eurostat (2013).

acceptance rates of job offers when additional job-seekers get more than one offer. A rise in productivity in one region sharply reduces the local unemployment rate and increases it elsewhere. Raising the cost of vacancy creation in a region is detrimental to this region and favorable to the rest of the economy. A change in amenities in a region turns out to have negligible effects on unemployment rates. Finally, the decentralized economy is far from efficient. The efficient unemployment rates amount to 1% while they are equal to 7% in the decentralized economy. For a very wide range of parameters, efficiency requires that nobody searches in the whole country while 65% of the workforce does it in the decentralized economy.

To our knowledge, efficiency has not often been studied in models with imperfect labor market and inter-regional migration. Boadway *et al.* (2004) build a static model with search-matching frictions, immobile workers but mobile firms. They aim at studying policies to restore efficiency when there is an inefficient distribution of firms because of agglomeration effects in matching and production. Within the limit of this paper, we do not look at policy interventions to improve efficiency.

Although a spatial equilibrium model with genuine unemployment has for long been missing, some papers have recently partly filled the gap. Leaving aside the literature where regions are so close that commuting is an alternative to relocation, we think that the literature about regional unemployment differentials can be divided in two groups according to the type of search: either one needs to move before starting to seek a job in the region of residence or one can search all over the country and then move if needed.

In the first case, some papers extend the island model of Lucas and Prescott (1974) whose economy is populated by a large number of segmented perfectly competitive labor markets where only labor is mobile (workers being allowed to visit only one island per period). Lkhagvasuren (2012) adds search-matching frictions as well as match-location specific productivity shocks in an otherwise standard islands model to reproduce the volatility of unemployment rates in the United States. Focusing also on one (small) region out of many, Wrede (2012) studies the relationships between wages, rents, unemployment and the quality of life in a dynamic framework. He assumes a standard search-matching framework and analyzes how regional amenities affect unemployment and the quality of life. Inspired by Ortega (2000)'s international migration model, Fonseca (2003) develops a two-region model. Ignoring the cost of search, she explains the link between regional migration and unemployment rates when a productivity shock arises. The model is then calibrated using Spanish data and simulated. A higher productivity in one region reduces its unemployment rate, a fact that has been widely documented. Building upon Beaudry *et al.* (2012), Beaudry *et al.* (2013) introduce search-matching frictions in spatial equilibrium setting with wage bargaining, free mobility of jobs, a very stylized housing market, and amenities with congestion externalities. We have these features in common. In their paper, with some exogenous probability, the jobless population gets the opportunity to move to another city in order to seek jobs, while we let agents choose between two strategies: regional and national search. Furthermore, Beaudry *et al.* (2013) do not look at efficiency while we do. They develop a thorough empirical analysis while we calibrate and simulate our model.

Second, some recent papers assume that workers can seek a job in the whole country. In a setting with many regions, Amior (2012) studies wages' responses to a housing shock in the presence of skill heterogeneity. He assumes national search in a search-matching framework as well as a random migration cost. Domingues Dos Santos (2011) builds a search-matching dynamic framework with two regions that are each considered as a line. She finds that increasing search effectiveness is beneficial for unemployment rates in both regions. However, she keeps wages exogenous. Using a search-matching dynamic framework with national search and endogenous wages, Antoun (2010) assumes two types of agents who differ in their preference for a region. He finds that a positive productivity shock in one region decreases unemployment locally but raises it in the other region. We extend these models by endogeneizing the choice between regional and national search under wage bargaining.⁵ Contrary to these papers, we also develop a normative analysis by looking at efficiency. However, we keep our framework static while they all assume a dynamic setting.

In the new economic geography literature, Epifani and Gancia (2005) analyze the simultaneous emergence of both agglomeration economies and unemployment rate differentials. For this purpose, they build a dynamic two-sector two-region model with transport costs and search-matching frictions. They assume a congestion effect in the utility which could reflect the housing market. They emphasize the role of migration following a productivity shock, which raises the unemployment rate in the short run but decreases it in the long run. Francis (2009) extends this framework to endogenous job destruction.

Finally, initial microeconomic evaluations about the effectiveness of on-line job search have reached rather pessimistic conclusions. See Kuhn and Mansour (2011) for a recent summary. According to this paper which based on more recent data, Internet searchers' unemployment duration is however shorter. Our paper contributes to this literature as well but from an equilibrium perspective.

The rest of this paper is organized as follows. In Section 2, we describe the model and its equilibrium. In Section 3, we study the equilibrium efficiency. A numerical analysis is conducted in Section 4. Section 5 concludes.

2 The model

This section develops a model with two distant regions. It first proposes a benchmark case under perfect competition on the labor market. Then, it introduces search-matching frictions in the labor market. To start with, it is assumed that a match can only be formed if the job vacancy and the unemployed worker are located in the same region. This standard assumption is relaxed in a last subsection where vacancies and job seekers located in different regions are allowed to meet.

We consider a static model of an economy made of two large regions ($i \in \{1, 2\}$). Each region is a point in space. The distance between the two regions is such that

⁵Molho (2001) develops a job-search framework with exogenous wages with both types of search. We extend his approach by integrating it in a general equilibrium model with endogenous wage.

commuting is ruled out, while inter-regional mobility through migration is allowed at no cost. The aggregate national labor force is made of an exogenous large number N of homogeneous risk-neutral workers. A worker living in region i supplies one unit of labor if the wage is above the value of time if she stays at home, denoted b_i . Firms are free to locate in the region they prefer. They use labor to produce a unique consumption good which is sold in a competitive market⁶ and taken as the *numeraire*.

There is also a perfectly competitive housing market. Each individual consumes one unit of housing.⁷ All the housing stock is owned by absentee landlords. As in Moretti (2011), the housing supply curve in region i is exogenous and linear:

$$r_i = \bar{r}_i + s_i N_i^P \quad s_i \geq 0, \quad i \in \{1, 2\} \quad (1)$$

where r_i is the rent paid for a unit of housing and N_i^P denotes housing demand, i.e. the population living in region i when the housing market clears (we call it the *ex-post* population). The level of parameter s_i depends on geographic and institutional regional specific features. At the limit, when this parameter becomes huge, the housing supply is close to vertical (a fixed supply being the standard assumption in the *Rosen-Roback* framework). When s_i is finite, the housing market creates a congestion effect: One additional resident in a region rises the cost of living of all inhabitants.⁸

Workers have idiosyncratic preference for regions. Agent j gets utility c_{ij} from living in region i . As Moretti (2011), we assume that the relative preference for region 1 over region 2, $c_{1j} - c_{2j}$, is uniformly distributed on a given support $[-v; v]$, $v > 0$. The presence of a distribution of relative preferences implies that the elasticity of inter-regional labor mobility is finite. A higher value of v entails less intense responses to regional differences in, say, the real wage.

The indirect utility V_{ij}^e of an *employed* individual j living in region i is, as in Moretti (2011), assumed to be additive and defined by:

$$V_{ij}^e = w_{ij} + a_i + c_{ij} - r_i \quad (2)$$

where w_{ij} represents the wage earned by agent j in region i and a_i is a measure of exogenous local consumption amenities in region i , such as the climate. These amenities are public goods and are not affected by the number of inhabitants in a region (no

⁶It is easily shown that each regional good market is balanced. See Appendix A for a proof.

⁷We however neglect the amount of land occupied by plants. To introduce firms' demand for land would complicate the model without yielding more insights.

⁸This very simplified way of modeling the housing market is similar in spirit with the approach adopted by Beaudry *et al.* (2013) and, as far as supply is concerned, with Amior (2012). Glaeser (2008) models the use of local inputs in the housing production function. Notowidigdo (2011) takes into account the limited depreciation of housing and studies the impact of positive and negative labor demand shocks. In a recent paper, Karahan and Rhee (2013) analyze the link between labor and housing market outcomes through a directed search model on the housing market. The housing market is often treated in the urban economics literature through bid-rent offers. See Zenou (2009). Note that what we call the housing market could actually represent any source of congestion related to the regional number of inhabitants.

rivalry).⁹ Similarly, the indirect utility V_{ij}^u of an *unemployed* person j residing in region i is:

$$V_{ij}^u = b_i + a_i + c_{ij} - r_i. \quad (3)$$

2.1 The perfectly competitive labor market benchmark

Before introducing search-matching frictions, it is useful to present the equilibrium under perfect competition in the labor market. Throughout the paper, we assume constant returns in the production of the consumption good.¹⁰ Let $y_i > b_i$ be the constant real marginal product of labor in region i . The whole labor force in this region is employed in equilibrium and is paid at the marginal product: $w_{ij} = w_i = y_i$.

Workers are free to locate in the region they prefer. Agent j chooses a dwelling in region 1 if $V_{1j}^e \geq V_{2j}^e$, otherwise she decides to live in region 2. Hence, there exists a value of the relative preference $c_{1j} - c_{2j}$, say x , such that the worker is indifferent between the two regions:

$$x = y_2 - y_1 + a_2 - a_1 + r_1 - r_2$$

with $r_1 = \bar{r}_1 + s_1 N_1^P$ and $r_2 = \bar{r}_2 + s_2 N_2^P$. Individuals whose relative preference for region 1 over region 2 is higher than x locate in region 1, while in the opposite case, they decide to live in 2. Therefore, provided that $-v < x < v$, the *ex-post* population sizes are respectively $N_1^P = \frac{v-x}{2v}N$ and $N_2^P = \frac{v+x}{2v}N$. If the condition is not met, one gets a corner solution.

Substituting N_i^P in the rents (1), the threshold x is explicitly given by

$$x = \frac{2v [y_2 - y_1 + a_2 - a_1 + \bar{r}_1 - \bar{r}_2 + \frac{s_1 - s_2}{2}N]}{2v + (s_1 + s_2)N} \quad (4)$$

which determines the partition of the population and hence of employment if the equilibrium is interior. This holds true under conditions on the parameters immediately derived by substituting (4) into the inequalities : $-v < x < v$.

Condition 1. *A necessary and sufficient condition for an interior equilibrium is that $v > 0$ verifies the two following conditions:*

$$v > y_2 - y_1 + a_2 - a_1 + \bar{r}_1 - \bar{r}_2 - s_2 N \quad (5)$$

$$v > y_1 - y_2 + a_1 - a_2 + \bar{r}_2 - \bar{r}_1 - s_1 N \quad (6)$$

⁹Contrary to what is sometimes done in the literature (see e.g. Wrede (2012) or Brueckner and Neumark (2011)), amenities a_i do not affect the production function.

¹⁰Although very standard in the search-matching literature, this assumption does not account for an empirical regularity according to which firms are more productive in larger cities. In Europe however, there is no clear-cut evidence on the strength of this relationship nor on the mechanisms at work remain not well understood (Combes *et al.*, 2012; Andini *et al.*, 2013). In the US, Beaudry *et al.* (2013) find no evidence of agglomeration effects on productivity (over 10-year periods). So, we think that our assumption is not too strong a simplification, in particular in the US context.

If $y_2 - y_1 + a_2 - a_1 + \bar{r}_1 - \bar{r}_2 > 0$, (5) can be binding if its RHS is positive, condition (6) being then necessarily met since v is positive. In the opposite case, (6) can put a constraint on v , but then not (5).

In this model, the impact of various local shocks is easy to derive. As an example, consider first an increase in productivity in region 1. As the wage rate rises, this region becomes more attractive. So, workers with a lower idiosyncratic relative preference $c_{1j} - c_{2j}$ move to region 1, i.e. x goes down and the population in region 1 goes up. However, as relative rents $r_1 - r_2$ rise as well, the decrease in x is less than proportional:

$$dx/dy_1 = -2v/[2v + (s_1 + s_2)N] > -1. \quad (7)$$

It is easily verified that the higher v , the lower the marginal impact of y_1 on *ex-post* population sizes in absolute value.

As second example, if relative amenities $a_2 - a_1$ go up, region 2 attracts more inhabitants. Wages are unaffected by this change because of constant returns to scale, while relative rents $r_1 - r_2$ decrease. So, as one can verify from (4), the rise in the *ex-post* population size in region 2 is again less than proportional. For a thorough analysis of the competitive benchmark case (under decreasing returns to scale), the reader is referred to Moretti (2011).

2.2 Introducing labor market frictions

To generate a setting where unemployment is an equilibrium phenomenon, we now introduce search-matching frictions (Mortensen and Pissarides (1999); Pissarides (2000)). In each regional labor market, we assume a regional-specific random matching process. Adopting a one-job-one-firm setting, firms decide in which region they open at most one vacancy. The cost κ_i of opening a vacancy is constant, exogenous and region specific.¹¹ If the vacancy is filled, a firm produces $y_i > b_i$ units of the consumption good. So, depending on the origin of the worker, a firm makes a profit $J_{ij} = y_i - w_{ij}$ on a filled position.

2.2.1 The timing of decisions

At the beginning of the unique period, everybody is unemployed, chooses in which region to reside, and decides to search for a job either regionally or nationally (i.e. either one only searches for a job in the region where one lives or one searches in both regions at the same time). The reason why some workers would only search in their region of residence rather than nationally is intuitive. If a worker has a sufficiently strong relative idiosyncratic preference for her region of residence, she will not accept to migrate to take a position. Since, following Decreuse (2008), we assume a small cost of refusing a job offer, this individual will then not take part to the matching process in the other region.

In a second step, firms open vacancies and possibly meet a worker. This worker then accepts or not the job offer. When a match is formed with a job seeker who does not

¹¹Capital is assumed to move freely across regions through vacancy creation. Ignoring credit market imperfections, entrepreneurs have no problem financing their vacancy cost κ_i .

live in the firm's region, this worker relocates. Allowing unemployed workers to relocate at this stage would complicate the exposition without yielding more insights. After the relocation step, employed workers and firms bargain over wages. Fourth, production takes place, the housing and good markets clear.

The moment at which wages are negotiated matters when a relocation of the worker is involved. If this moment occurs before the decision to migrate is taken, through Nash bargaining, the worker will get a partial compensation for the difference in the regional non-wage components of utility ($a_i + c_{ij} - r_i$). To implement this timing, one has to assume that the employer is aware of the idiosyncratic preferences of the worker for both regions. One can doubt that this information is available. So, we prefer the timing indicated above: The bargained wage will then not compensate the worker for the difference in $a_i + c_{ij} - r_i$.¹²

Given this timing, some additional notations have to be introduced. Before the matching process, N_i agents choose to reside in region i (N_i is called the *ex-ante* population in region i). Population in region i is composed of N_i^N agents who search nationally and N_i^R individuals who only search in their region of residence ($N_i = N_i^N + N_i^R$). After the relocation stage, N_i^P workers live in region i . For agents located in region i , the notation $-i$ will designate the other region.

2.2.2 The matching process

Following Molho (2001), Antoun (2010) and Domingues Dos Santos (2011), and in the same spirit as Amior (2012), workers can search for a job in the whole country. We allow for distant search, meaning that search in a region can be conducted while living in the other one. Compared to the alternative (i.e. one needs to move in a region before being able to look for a job in it), this assumption looks to us more in accordance with currently available communication technologies. The search-matching effectiveness of those living in the region where vacancies are open is normalized to one. For residents of the other region, this effectiveness takes an exogenous value α with $0 \leq \alpha \leq 1$. The number of hirings in each region is given by a regional-specific matching function $M_i(\cdot, \cdot)$ with:

$$M_i(\mathcal{V}_i, N_i + \alpha N_{-i}^N) < \min\{\mathcal{V}_i, N_i + \alpha N_{-i}^N\}, \quad i \in \{1, 2\}, \quad (8)$$

where \mathcal{V}_i represents the number of vacancies opened in region i and $N_i + \alpha N_{-i}^N$ is the number of job seekers measured in efficiency units (where N_{-i}^N stands for the national job seekers located in the other region). The upper bound in (8) is needed in a static framework to guarantee that both sides of the regional labor market will be rationed in equilibrium. Following Pissarides (2000) and a large empirical literature, the matching function has constant returns to scale and is increasing and concave in both arguments.

¹²Moreover, if the framework was dynamic, this timing would raise another issue. Under the standard assumption of automatic renegotiation (Pissarides 2000, p. 15), the wage would then be revised after the relocation step and would be chosen exactly as proposed in the timing of events we privilege.

Defining tightness in region i as

$$\theta_i = \frac{\mathcal{V}_i}{N_i + \alpha N_{-i}^N},$$

$m_i(\theta_i)$ designates the probability M_i/\mathcal{V}_i that a vacancy in region i meets a worker, with $0 < m_i(\theta_i) < 1$ by the inequality in (8) and $m'_i(\theta_i) < 0$ because of search-matching congestion externalities. So, unfilled jobs find a partner more easily in a region able to attract more job-seekers. The probability that an unemployed worker living in i meets a firm located in region i is $p_i(\theta_i) \equiv \theta_i m_i(\theta_i)$, with $0 < p_i(\theta_i) < 1$. Job-seekers find a job more easily in a thicker local labor market: $[p_i(\theta_i)]' > 0$.¹³ The probability that an unemployed worker searching nationally and living in $-i$ meets a firm settled in region i is $\alpha p_i(\theta_i)$. In case of national search, for someone living in i , the probability of getting an offer in i and no offer from the other region is $p_i(\theta_i)(1 - \alpha p_{-i}(\theta_{-i}))$. The probability of the opposite event is $\alpha p_{-i}(\theta_{-i})(1 - p_i(\theta_i))$. The probability of getting an offer from each region is $\alpha p_i(\theta_i)p_{-i}(\theta_{-i})$. In this case, the worker accepts the best opportunity for her, i.e. the position that offers her the highest indirect utility level. Finally, this worker living in i faces a probability $(1 - p_i(\theta_i))(1 - \alpha p_{-i}(\theta_{-i}))$ of getting no offer at all and of remaining unemployed.

2.3 A model with regional search only

In this section, we assume, as it is standard in the literature, that an unemployed worker can only search for a vacant position in the region where she has decided to live. So, we impose $\alpha = 0$. Next, we will relax this assumption. We solve the problem by backwards induction. The housing supply is given by equation (1). The housing demand is represented by the population living *ex-post* in the region, N_i^P .

2.3.1 Individual wage negotiation

Individual Nash bargaining takes place *ex-post*, once the cost of opening a vacancy is sunk. So, when a vacancy and a job seeker have met, the wage solves the following maximization:

$$\max_{w_{ij}} (V_{ij}^e - V_{ij}^u)^{\beta_i} (J_{ij} - V_i)^{1-\beta_i} \quad (9)$$

where V_i is the value of an unfilled vacancy and $\beta_i \in [0, 1)$ denotes the bargaining power of a worker in region i . The first-order condition can be rewritten as:

$$w_{ij} = \beta_i y_i + (1 - \beta_i) b_i - \beta_i V_i. \quad (10)$$

Hence, the wage is independent of the location of the unemployed and can therefore be denoted by w_i . It is lower than the marginal product y_i . As $w_i > b_i$, under free-entry, workers always take the position.

¹³As is standard, we assume Inada conditions: $\lim_{\theta \rightarrow 0} m(\theta) = 1$; $\lim_{\theta \rightarrow 0} p(\theta) = 0$; $\lim_{\theta \rightarrow +\infty} m(\theta) = 0$; $\lim_{\theta \rightarrow +\infty} p(\theta) = 1$.

2.3.2 Opening of vacancies

The expected value of a vacant position V_i is equal to $-\kappa_i + m_i(\theta_i)(y_i - w_i)$. Firms open vacancies freely until this value V_i is nil in both regions. Anticipating correctly the outcome of the wage bargain, the free-entry condition becomes:

$$\frac{\kappa_i}{(1 - \beta_i)m_i(\theta_i)} = y_i - b_i, \quad \forall i \in \{1, 2\}. \quad (11)$$

The probability of filling a vacancy $m_i(\theta_i)$ increases with the (*ex-post*) surplus of a match $y_i - b_i$ and decreases with the cost of opening a vacancy κ_i and workers' bargaining power β_i .

2.3.3 Location choice

Agents decide in which region to locate in order to maximize their expected utility. They thus compare the expected utility of living in region 1, $p_1(\theta_1)V_{1j}^e + (1 - p_1(\theta_1))V_{1j}^u$, with the expected utility of living in region 2, $p_2(\theta_2)V_{2j}^e + (1 - p_2(\theta_2))V_{2j}^u$. The worker whose relative preference for region 1 over region 2 is above $b_2 - b_1 + a_2 - a_1 + r_1 - r_2 + p_2(\theta_2)(w_2 - b_2) - p_1(\theta_1)(w_1 - b_1)$ chooses to live in region 1, while if their relative preference is below this threshold, workers reside in region 2. Let us define

$$\Delta_1 = b_2 - b_1 + a_2 - a_1 + \bar{r}_1 - \bar{r}_2 \quad \text{and} \quad \Delta_2 = b_2 - b_1 + a_2 - a_1 + r_1 - r_2.$$

We get the following lemma:

Lemma 1. *Given that agents perfectly anticipate the wage w_i defined in (10), there is a threshold*

$$x = \Delta_2 + p_2(\theta_2)\beta_2(y_2 - b_2) - p_1(\theta_1)\beta_1(y_1 - b_1), \quad (12)$$

assumed to be in $(-v, v)$, such that

- *A job seeker decides to live in region 1 if $c_{1j} - c_{2j} \geq x$;*
- *Else, she decides to reside in region 2.*

When unemployed workers need to move before starting to search for a job, a higher (respectively, lower) time value of being unemployed in region 2 (resp., 1) or higher relative levels of amenities $a_2 - a_1$ as well as lower relative rents $r_2 - r_1$ induce a higher threshold x , meaning that more workers locate in 2. A rise in tightness in region 1 has an unambiguous negative effect on x : as the probability of getting a job in region 1 increases (and therefore the probability of remaining unemployed goes down), more workers decide to locate there. A similar clear-cut conclusion holds if θ_2 increases.

2.3.4 Equilibrium

Let u_i designate the unemployment rate in region i at the end of the matching process.

Definition 1. When $\alpha = 0$, an interior equilibrium is a vector $\{w_i, \theta_i, u_i, N_i, N_i^P\}_{i \in \{1,2\}}$ and a scalar x where w_i is given by (10), in which under free-entry $V_i = 0$, θ_i is fixed by (11), $u_i = 1 - p_i(\theta_i)$, $N_1 = N_1^P = \frac{v-x}{2v}N$, $N_2 = N_2^P = \frac{v+x}{2v}N$, and

$$x = \frac{2v [\Delta_1 + \frac{s_1-s_2}{2}N + p_2(\theta_2)\beta_2(y_2 - b_2) - p_1(\theta_1)\beta_1(y_1 - b_1)]}{2v + (s_1 + s_2)N} \quad (13)$$

assumed to be in $(-v, v)$.

The equilibrium is recursive. Once tightness is fixed in each region by the free-entry condition, the equilibrium value of x is known and population sizes are determined as well. The equilibrium unemployment rate in a region is only affected by the determinants of tightness in this region. By looking at (11), these determinants are regional-specific. So, a change in say the marginal product of labor in a region has no spill-over effect on the equilibrium unemployment rate in the other region.

Condition 2. A necessary and sufficient condition for an interior equilibrium is that $v > 0$ verifies the two following conditions:

$$v > \Delta_1 - s_2N + \beta_2p_2(\theta_2)(y_2 - b_2) - \beta_1p_1(\theta_1)(y_1 - b_1) \quad (14)$$

$$v > -\Delta_1 - s_1N + \beta_1p_1(\theta_1)(y_1 - b_1) - \beta_2p_2(\theta_2)(y_2 - b_2) \quad (15)$$

where equilibrium tightness levels θ_i are a function of parameters only thanks to (11).

A comment similar to the one made after Condition 1 could be replicated here.

2.3.5 Comparison with the frictionless case

Let us consider the same marginal shocks as in the competitive case. When productivity rises in region 1, wages increase less than under perfect competition (see (10)). So, firms' profit $y_i - w_i$ increases and hence more vacancies are created in this region. By totally differentiating (11), it can be shown that

$$\frac{d\theta_1}{\theta_1} = \frac{dy_1}{\eta_1(y_1 - b_1)} > 0 \quad \text{and} \quad \frac{du_1}{dy_1} = -\frac{p_1(\theta_1)}{y_1 - b_1} \frac{1 - \eta_1}{\eta_1} < 0 \quad (16)$$

where $\eta_1 \equiv -\frac{m'(\theta_1)\theta_1}{m(\theta_1)}$ is the elasticity of the probability at which vacancies are filled in region 1 with respect to θ_1 in absolute value. Since the prospect of getting a job in region 1 gets better and w_1 rises, more workers locate in region 1. The total impact on the location of workers is on the one hand less important than in the competitive case because the rise in the wage is less important. On the other hand, an induced effect appears in the frictional case only: A rise in regional productivity improves the chances of finding a job in this region. Under some conditions the net effects can be ranked:

Proposition 1. Under the Hosios condition ($\beta_1 = \eta_1$) or if the workers' bargaining power is inefficiently low ($\beta_1 < \eta_1$), the change in the population sizes induced by a rise in productivity in region 1 is smaller in absolute value in the frictional case with $\alpha = 0$ than in the competitive case. A corresponding proposition can be expressed for a rise in productivity in region 2.

Proof. Differentiating (13) with respect to y_1 and taking into account the adjustment of tightness (16), one can check that in the frictional case

$$\frac{dx}{dy_1} = \frac{-2v}{2v + (s_1 + s_2)N} \frac{\beta_1}{\eta_1} p_1(\theta_1)$$

which has to be compared with (7) in the competitive case. Under the so-called Hosios condition, i.e $\beta_1 = \eta_1$, it is well-known that search-matching externalities are internalized by decentralized agents (see e.g. Pissarides, 2000). Then, as $p_1(\theta_1) < 1$, one gets the announced property. This conclusion is reinforced if $\beta_1 < \eta_1$ so that too few vacancies are created in equilibrium. No conclusion can be drawn if instead $\beta_1 > \eta_1$. \square

Furthermore, by (16), we now have that an increase in the productivity in i yields a lower unemployment rate in the region. As tightness in the other region remains constant, the unemployment rate there is not affected.

The impact of a variation in amenities is the same whether the labor market is frictional or perfectly competitive. For, consumption amenities do not impact the free-entry conditions, which determine the levels of labor market tightness. Therefore, if region 2 becomes relatively more attractive, agents' relative preference $c_{1j} - c_{2j}$ should be higher to locate in region 1 (x increases), but unemployment rates remain constant in both regions.

2.4 Regional and national search

We now let workers search simultaneously in both regions ($0 < \alpha \leq 1$) if they prefer to do so. Appendix B shows that taking search and location decisions simultaneously or choosing first the location and then the searching area is equivalent. Therefore, to ease the exposition, the presentation below opts for the second timing. To present the various stages of the model we move backwards. The housing market is taken into account as in Section 2.3. At the time of individual bargaining in any region i , a worker migrating from $-i$ has already moved in region i and thus has the same fall back position as a worker settled in region i from the start. The generalized Nash bargaining process is therefore (9). The wage is still given by (10) and denoted by w_i .

2.4.1 Acceptance of a job offer

A worker searching in her region only always accepts a job offer, as $V_{ij}^e > V_{ij}^u$ in a free-entry equilibrium. Similarly, a worker searching nationally who only gets a job offer from a firm located in the region where she lives always takes the position. In case this worker only receives a job offer from a firm settled in the other region, she always accepts the job, as she decided to search for a job there (as shown in Appendix B). However, if a worker searching nationally gets two offers, one from each region, she rejects one of them (incurring an arbitrary small cost ε) and accepts the other one. To take this decision, the unemployed worker compares the indirect utility she gets from working in region 1,

V_{1j}^e , with the one from working in region 2, V_{2j}^e .¹⁴ The agent whose relative preference $c_{1j} - c_{2j}$ is above the threshold $w_2 - w_1 + a_2 - a_1 + r_1 - r_2$ chooses to work in region 1 rather than in region 2. So,

Lemma 2. *As agents perfectly anticipate the wage w_i defined by (10), there is a threshold*

$$\hat{x} = \Delta_2 + \beta_2(y_2 - b_2) - \beta_1(y_1 - b_1), \quad (17)$$

assumed to be in $(-v, v)$, such that

- *Whenever a job seeker searching nationally has one job offer from each region, she accepts the job offer in region 2 if $c_{1j} - c_{2j} < \hat{x}$;*
- *Else, she accepts the job offer in region 1.*

A higher relative wage in region 2, $w_2 - w_1$, or higher relative amenities in region 2, $a_2 - a_1$, as well as a higher relative rents in region 1, $r_1 - r_2$, obviously induce more workers to choose to work in region 2 whenever receiving two job offers.

2.4.2 Vacancy creation

Firms open vacancies in region i until the expected gain V_i is nil ($i \in \{1, 2\}$). This condition is now $m_i(\theta_i)\pi_i(y_i - w_i) = \kappa_i$, where π_i is new and designates the conditional probability that the meeting leads to a filled vacancy (see Section 2.4.4 for more details). Combining (10) and the free-entry condition yields

$$\frac{\kappa_i}{(1 - \beta_i)m_i(\theta_i)} = \pi_i(y_i - b_i), \quad \forall i \in \{1, 2\}. \quad (18)$$

The rate at which vacancies are on average filled, $\pi_i m_i(\theta_i)$, varies with the parameters exactly as in paragraph 2.3.2.

2.4.3 Search decision and location choice

As announced above, we here treat these decisions sequentially.

Search decision

Let p_i be a short notation for $p_i(\theta_i)$. An individual j living in region 2 decides to search regionally or nationally by comparing the expected utility in both cases. The expected utility if the agent located in 2 searches nationally is

$$p_2(1 - \alpha p_1)V_{2j}^e + \alpha p_1(1 - p_2)V_{1j}^e + \alpha p_1 p_2(\max\{V_{1j}^e; V_{2j}^e\} - \varepsilon) + (1 - p_2)(1 - \alpha p_1)V_{2j}^u. \quad (19)$$

¹⁴Recall that the wage bargain takes place after migration, if any. So, someone searching in all the country who gets two offers selects one of them, then migrates if the vacant position is in the other region, and finally bargains over the wage. At that moment, the job offer previously rejected cannot be recalled. Hence, the worker's outside option is the value in unemployment where she now lives and the wage is given by (10).

The expected utility of a job seeker living in region 2 and searching for a job in this region only is

$$p_2 V_{2j}^e + (1 - p_2) V_{2j}^u.$$

When the small cost of refusing a job offer ε tends to zero, the individual whose relative preference for region 1 over region 2, $c_{1j} - c_{2j}$, reaches a level above $b_2 - w_1 + a_2 - a_1 + r_1 - r_2$ searches nationally. Otherwise, she decides to look for a job in region 2 only. This is shown in Appendix B (the comparison of cases *e* and *f*).

A similar development is conducted for a worker settled in region 1. The comparison of expected utility between national and regional search becomes:

$$\begin{aligned} p_1(1 - \alpha p_2) V_{1j}^e + \alpha p_2(1 - p_1) V_{2j}^e + \alpha p_1 p_2 (\max \{V_{1j}^e; V_{2j}^e\} - \varepsilon) \\ + (1 - p_1)(1 - \alpha p_2) V_{1j}^u &\stackrel{\geq}{\leq} p_1 V_{1j}^e + (1 - p_1) V_{1j}^u \\ \Leftrightarrow c_{1j} - c_{2j} &\stackrel{\leq}{\geq} w_2 - b_1 + a_2 - a_1 + r_1 - r_2 \end{aligned}$$

when ε tends to zero. A job seeker located in region 1 whose relative preference for region 1 over region 2 is higher than $w_2 - b_1 + a_2 - a_1 + r_1 - r_2$ searches in region 1 only. If agent's relative preference is below this threshold, the worker looks for a job in the whole country (see the comparison of cases *a* and *c* in Appendix B). Under perfect anticipation of bargained wages, the following lemma defines two thresholds and three types of job seekers:

Lemma 3. *When $\alpha > 0$, let*

$$z_1 = \beta_1(b_1 - y_1) + \Delta_2 \tag{20}$$

$$z_2 = \beta_2(y_2 - b_2) + \Delta_2 \tag{21}$$

with $z_1 < z_2$. Given these thresholds assumed to be in $(-v, v)$,

- If $c_{1j} - c_{2j} < z_1$, agent *j* searches in region 2 only;
- If $z_1 \leq c_{1j} - c_{2j} \leq z_2$, agent *j* searches nationally;
- If $c_{1j} - c_{2j} > z_2$, agent *j* searches in region 1 only.

The shares of these three groups in the total population are respectively $\frac{v+z_1}{2v}$, $\frac{z_2-z_1}{2v}$ and $\frac{v-z_2}{2v}$. Remembering (17), it is easily seen that $z_1 \leq \hat{x} \leq z_2$.

By comparing their expected utility in case of regional and national search, unemployed workers turn out to compare the utility levels when they are actually employed in the other region and when they remain unemployed in their region of residence. These utility levels are not in expected terms and thus search decisions are independent of probabilities to get a job offer. Therefore, the number of workers who search nationally is independent of search effectiveness α . A rise in Δ_2 shifts z_1 and z_2 upwards, while keeping $z_2 - z_1$ unchanged. Hence, more unemployed workers search in region 2 only

and less do so in 1 only, while the share of the population searching nationally remains constant. If the value of time b_i increases or the productivity y_i decreases (which yields a drop in w_i), workers prefer this region relatively more to search for a job there (the number of regional job seekers in $-i$ goes up).

Location choice

As an unemployed worker who decides to look for a job regionally only locates in her region of search, we have to compare the expected utility of an agent j who searches nationally while being located in region 1 or in region 2. These expected utility levels are respectively

$$p_1(1 - \alpha p_2)V_{1j}^e + \alpha p_2(1 - p_1)V_{2j}^e + \alpha p_1 p_2(\max\{V_{1j}^e; V_{2j}^e\} - \varepsilon) + (1 - p_1)(1 - \alpha p_2)V_{1j}^u \quad (22)$$

and (19), as shown in Appendix B.

Lemma 4. *Let*

$$x = \Delta_2 + \frac{1 - \alpha}{1 - \alpha p_1 - \alpha p_2 + \alpha p_1 p_2} (p_2 \beta_2 (y_2 - b_2) - p_1 \beta_1 (y_1 - b_1)), \quad (23)$$

with $0 \leq \frac{1 - \alpha}{1 - \alpha p_1 - \alpha p_2 + \alpha p_1 p_2} < 1$ and, by (20) and (21), $z_1 \leq x \leq z_2$.

- If $c_{1j} - c_{2j} < x$, then agent j locates in region 2;
- Else, worker j settles in region 1.

The share of the population living *ex-ante* in region 2 (respectively, 1) is then $\frac{v+x}{2v}$ (respectively $\frac{v-x}{2v}$).

Compared to (12) when α was assumed to be nil, increasing the differential in expected rents $p_2 \beta_2 (y_2 - b_2) - p_1 \beta_1 (y_1 - b_1)$ has now a less positive effect on the number of people choosing to locate in region 2 since there is the opportunity of searching nationwide wherever one lives. At the limit, if search is equally efficient wherever one looks for a job ($\alpha = 1$), this differential does not affect the location choice any more. When $\alpha > 0$, an increase in relative amenities in region 2, $a_2 - a_1$, or a decrease in relative rents in region 2, $r_2 - r_1$, as well as a rise (respectively, a drop) in the value of home production in region 2 (resp., 1) still induce more workers to locate in 2 *ex-ante*. However, an increase in tightness in region 1 has several effects. First, if one lives in region 1, the increase in the probability of being employed in this region equals the decrease in the probability of being unemployed. As the individual stays in the same region, the net gain is proportional to $w_1 - b_1$. Second, if one lives in region 2, the increase in the probability of being employed in region 1 equals the decrease in the probability of staying unemployed in region 2. This effect is proportional to $V_{1j}^e - V_{2j}^u$. Third, the decline in the probability of being employed in 2 is the same wherever one lives. So, this effect cancels out. The first and the second effects push the difference in idiosyncratic preference of the indifferent agent, x , in opposite directions so that the net effect is ambiguous. This conclusion also holds if θ_2 increases. So, a first major difference with the case where people only search in their region of residence is that a rise in the number of vacancies in a region has no clear-cut impact on the location choice any more.

2.4.4 Summary of the acceptance, search and location decisions

Equations (20), (21) and (23) provide the threshold values for the search and location decisions. Since $z_1 \leq x \leq z_2$,

Lemma 5. *Given (20), (21) and (23), search and location decisions verify the following conditions:*

- If $c_{1j} - c_{2j} < z_1$, agent j locates in region 2 and searches there only;
- If $z_1 \leq c_{1j} - c_{2j} < x$, agent j settles in region 2 and searches in the whole country;
- If $x \leq c_{1j} - c_{2j} \leq z_2$, agent j locates in region 1 and looks for a job nationally;
- If $c_{1j} - c_{2j} > z_2$, agent j settles in region 1 and looks for a job in region 1 only.

Figure 1 illustrates this partition of the total population if $-v < z_1, z_2 < v$.

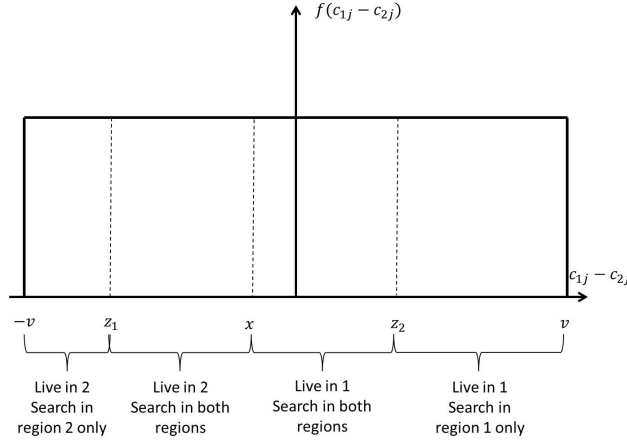


Figure 1: The partition of the population in case of an interior solution.

Comparing threshold values x and \hat{x} , one cannot rank them since x varies with the levels of tightness. When region are fully symmetric however, these 2 thresholds are equal to zero.

2.4.5 Acceptance probability and vacancy creation

A detailed explanation is provided in Appendix C. Consider a vacant position in region 1. The mass of job seekers searching for a job in 1 is $\frac{v-x+\alpha(x-z_1)}{2v}N$ in efficiency units. Conditional on meeting one of these unemployed workers, all those whose relative preference $c_{1j} - c_{2j}$ lies above \hat{x} accept for sure an offer from region 1. For those between z_1 and \hat{x} , this is only the case if they get no offer from region 2. So, conditional on a

contact between a vacancy in region 1 and a job seeker, the acceptance probability is (with a corresponding expression for π_2):

$$\pi_1 = 1 - \frac{\alpha p_2(\hat{x} - z_1)}{v - x + \alpha(x - z_1)} \quad (24)$$

$$\pi_2 = 1 - \frac{\alpha p_1(z_2 - \hat{x})}{v + x + \alpha(z_2 - x)} \quad (25)$$

It is easily checked that the higher \hat{x} , the more workers accept job offers in region 2 and so the lower the acceptance rate in 1. The higher the number of workers searching in region 2 only (an increasing function of z_1), the higher the acceptance rate in 1. Finally, an increase in the probability of getting a job offer in region 2 decreases the acceptance rate in region 1. Similarly, π_2 increases with \hat{x} and decreases with z_2 and p_1 . The impact of search-matching effectiveness α should be stressed: a higher α leads to a lower conditional acceptance probability, as the probability of getting two job offers increases.

Combining (18) with (24)-(25) leads to the following free-entry conditions:

$$\frac{\kappa_1}{(1 - \beta_1)m_1(\theta_1)} = \frac{v - x + \alpha(x - z_1) - \alpha p_2(\hat{x} - z_1)}{v - x + \alpha(x - z_1)}(y_1 - b_1) \quad (26)$$

$$\frac{\kappa_2}{(1 - \beta_2)m_2(\theta_2)} = \frac{v + x + \alpha(z_2 - x) - \alpha p_1(z_2 - \hat{x})}{v + x + \alpha(z_2 - x)}(y_2 - b_2) \quad (27)$$

Through the endogenous acceptance rate, vacancy creation in any region is now affected by characteristics of the other region, namely parameters (like the productivity and the value of time) and tightness. This is a second major difference with the case where people only search in their region of residence.

2.4.6 Populations' definitions and unemployment rates

In Subsection 2.2.1, we defined the *ex-ante* populations N_i , split into N_i^N workers searching in all the country and N_i^R workers searching in their region of residence only, as well as the *ex-post* populations N_i^P . Since $z_1 \leq x \leq z_2$, an interior solution is such that:

$$N_1^R = \frac{(v - z_2)}{2v}N \quad N_1^N = \frac{(z_2 - x)}{2v}N \quad (28)$$

$$N_2^R = \frac{(z_1 + v)}{2v}N \quad N_2^N = \frac{(x - z_1)}{2v}N \quad (29)$$

with $N_1 = N_1^N + N_1^R = \frac{v-x}{2v}N$, $N_2 = N_2^N + N_2^R = \frac{v+x}{2v}N$, $N_1 + N_2 = N$, and

$$N_1^P = \frac{(v - x) - \alpha p_2(1 - p_1)(z_2 - x) + \alpha p_1(1 - p_2)(x - z_1) + \alpha p_1 p_2(x - \hat{x})}{2v}N \quad (30)$$

$$N_2^P = \frac{(v + x) + \alpha p_2(1 - p_1)(z_2 - x) - \alpha p_1(1 - p_2)(x - z_1) - \alpha p_1 p_2(x - \hat{x})}{2v}N \quad (31)$$

with $N_1^P + N_2^P = N$. *Ex-post*, the number of inhabitants in, say, region 1 is the sum of 4 terms. The first term represents the population living *ex-ante* in region 1. The second

term corresponds to the workers who were living *ex-ante* in 1 and who leave region 1 as they only get a position in region 2. The third term is composed of the agents who lived *ex-ante* in region 2 and who move as they only get an offer from region 1. Finally, the fourth term represents the number of workers who got 2 offers. This term is positive whenever $x > \hat{x}$, meaning that some more workers living in 2 *ex-ante* accept a position in region 1. The partial effect of a rise in tightness in a region is to increase the size of the *ex-post* population in this region and to decrease it in the other one. A rise in the threshold x reduces N_1^P and augments N_2^P . Since the impact of tightness on x is ambiguous, the total effect of a rise in tightness on the regional *ex-post* population sizes is ambiguous as well. This third difference with respect to the case where $\alpha = 0$ is a consequence of the first one. A rise in any of the thresholds z_1 , z_2 or \hat{x} lowers N_1^P and augments N_2^P .

The number of (*ex-post*) unemployed workers in, say, region 1 is composed of the agents living *ex-ante* in 1 who did not get a job offer in region 1, $(1 - p_1)\frac{v-x}{2v}N$, to which we subtract the workers who did not get an offer from region 1 but well from region 2 ($\alpha p_2(1 - p_1)\frac{z_2-x}{2v}N$). The unemployment rates which are the ratio of the number of (*ex-post*) unemployed workers over the (*ex-post*) population, can be written as

$$u_1 = \frac{(1 - p_1)(v - x - \alpha p_2(z_2 - x))}{v - x - \alpha p_2(1 - p_1)(z_2 - x) + \alpha p_1(1 - p_2)(x - z_1) + \alpha p_1 p_2(x - \hat{x})} \quad (32)$$

$$u_2 = \frac{(1 - p_2)(v + x - \alpha p_1(x - z_1))}{v + x + \alpha p_2(1 - p_1)(z_2 - x) - \alpha p_1(1 - p_2)(x - z_1) - \alpha p_1 p_2(x - \hat{x})} \quad (33)$$

Lemma 6. *As in the case where $\alpha = 0$, the unemployment rate u_i decreases with tightness in region i , θ_i . The following partial effects are new. Tightness in the other region, θ_{-i} , and the threshold x affect u_i , but the sign of these effects is ambiguous. The unemployment rate in region i decreases with z_2 and it increases with z_1 . The unemployment rate in region 1 increases with \hat{x} , while the opposite holds for the unemployment rate in region 2. Finally, when region are symmetric, a rise in search effectiveness α implies a lower regional unemployment rate.*

These properties are easily derived by differentiating (32) and (33). An increase in region i labor market tightness boosts the probability that a worker living in region i finds a job and it rises the probability that a worker located in the other region gets a position in region i (which increases the labor force living in region i). Consequently, the unemployment rate in region i goes down. A rise in tightness in the other region $-i$ has an ambiguous impact on the unemployment rate in region i . The probability of leaving region i increases. Both the number of unemployed workers and the size of the labor force go down, leading to an ambiguous impact on u_i . Unemployment rates also vary in an ambiguous way with the threshold x . As x goes up, the number N_1 of agents living *ex-ante* in region 1 shrinks while N_2 increases. The levels of regional unemployment, hence the numerators of (32) and (33), change in the same way. The *ex-post* population sizes, which are the denominator of (32) and (33), vary in the same directions: N_1^P decreases and N_2^P increases. Hence, we do not get clear-cut conclusions.

Some partial effects have however a clear sign. More workers searching for a job in their region of residence only (i.e. an increase in z_1 or a decrease in z_2) rises the unemployment rate in both regions. More workers searching all over the country therefore reduces the unemployment rates in both regions.

In a standard Mortensen-Pissarides setting (where geographical heterogeneities are concealed in an aggregate matching function), the size of the labor force does not affect the equilibrium unemployment rate (as eventually the number of vacancies rises proportionately, leaving the equilibrium level of tightness unaffected). This equilibrium property is not different here (N plays no role in (32)-(33)). However, if $\alpha > 0$, the equilibrium unemployment rates are affected by the partition of the population between the two regions and between the two statuses of national versus regional job seekers.

2.4.7 Equilibrium

Definition 2. When $0 < \alpha \leq 1$, an interior equilibrium is a vector $\{x, \hat{x}, z_1, z_2\}$ assumed to be in $(-v, v)$ and a vector $\{w_i, \theta_i, u_i, N_i^N, N_i^R, N_i^P\}_{i \in \{1,2\}}$, solving (10), in which under free-entry $V_i = 0$, (17), (18), (20), (21), (23), (24), (25), (28), (29), (30), (31), (32) and (33) with:

$$r_1 = \bar{r}_1 + \frac{s_1}{2v} N (v - x - \alpha p_2(1 - p_1)(z_2 - x) + \alpha p_1(1 - p_2)(x - z_1) + \alpha p_1 p_2(x - \hat{x}))$$

$$r_2 = \bar{r}_2 + \frac{s_2}{2v} N (v + x + \alpha p_2(1 - p_1)(z_2 - x) - \alpha p_1(1 - p_2)(x - z_1) - \alpha p_1 p_2(x - \hat{x}))$$

We now consider conditions for an interior equilibrium. As we already know that $z_1 \leq \hat{x}, x \leq z_2$, we need to guarantee that $-v < z_1 < z_2 < v$:

Condition 3. Sufficient conditions for an interior solution are

$$v > \beta_1(y_1 - b_1) - \Delta_1 + s_2 N \quad (34)$$

$$v > \beta_2(y_2 - b_2) + \Delta_1 + s_1 N \quad (35)$$

Proof See Appendix D.1.

When $\alpha \neq 0$, the existence and uniqueness of the equilibrium can only be shown analytically when regions are fully symmetric (see Appendix D.2 for a proof).

3 Efficiency

This section studies the efficiency of the *laissez-faire*¹⁵ decentralized equilibria introduced in the previous section. We first derive the optimal allocation when workers can only search in the region where they live ($\alpha = 0$) and compare it with the decentralized equilibrium derived in Section 2.3. It will turn out that the decentralized equilibrium is efficient when the Hosios condition is satisfied. In a second stage, we derive the optimal allocation when $\alpha > 0$ and analyze the differences between this allocation and

¹⁵This expression is added since there is no public intervention in Section 2.

the decentralized equilibrium characterized in Section 2.4. The Hosios condition is then not sufficient to guarantee efficiency of the decentralized equilibrium.

3.1 The case where $\alpha = 0$

The central planner's chooses the levels of tightness and the threshold x to maximize net output subject to the same matching frictions as decentralized agents. Net output is the sum of output produced, home production, amenities and agents' idiosyncratic preferences, net of vacancy costs. We also add the total surplus created by the housing market.¹⁶ We write this problem as¹⁷:

$$\begin{aligned} \max_{\theta_1, \theta_2, x} & y_1 L_1 + y_2 L_2 + b_1(N_1^P - L_1) + b_2(N_2^P - L_2) + a_1 N_1^P + a_2 N_2^P - \kappa_1 \mathcal{V}_1 \\ & - \kappa_2 \mathcal{V}_2 + \frac{N}{2v} \left[\int_{-v}^x c_{2j} dj \right] + \frac{N}{2v} \left[\int_x^v c_{1j} dj \right] - (\bar{r}_1 + \frac{s_1 N_1^P}{2}) N_1^P - (\bar{r}_2 + \frac{s_2 N_2^P}{2}) N_2^P \end{aligned} \quad (36)$$

where

$$\begin{aligned} \mathcal{V}_1 &= \theta_1 \frac{N}{2v} (v - x) & L_1 &= \frac{N}{2v} p_1 (v - x) & N_1^P &= \frac{N}{2v} (v - x) \\ \mathcal{V}_2 &= \theta_2 \frac{N}{2v} (v + x) & L_2 &= \frac{N}{2v} p_2 (v + x) & N_2^P &= \frac{N}{2v} (v + x) \end{aligned}$$

and $-v \leq x \leq v$. The first-order conditions writes:

$$\frac{\kappa_1}{(1 - \eta_1)m_1(\theta_1)} = y_1 - b_1 \quad (37)$$

$$\frac{\kappa_2}{(1 - \eta_2)m_2(\theta_2)} = y_2 - b_2 \quad (38)$$

$$x = \Delta_2 + p_2(y_2 - b_2 - \frac{\kappa_2}{m_2(\theta_2)}) - p_1(y_1 - b_1 - \frac{\kappa_1}{m_1(\theta_1)}) + \mu_1 - \mu_2 \quad (39)$$

$$\mu_1(v - x) = 0 \text{ and } \mu_2(v + x) = 0 \quad (40)$$

where $\eta_i = \frac{-\theta_i m'_i(\theta_i)}{m_i(\theta_i)}$, μ_1 is the Lagrangian multiplier associated to the constraint $-v \leq x$ and μ_2 , the Lagrangian multiplier associated to the constraint $x \leq v$.

Imagine that the Hosios condition is satisfied (i.e. $\beta_i = \eta_i$) and the solution is interior (i.e. $\mu_1 = \mu_2 = 0$). Then, as in a standard Mortensen-Pissarides setting, the equilibrium levels of tightness, and hence the unemployment rates, are chosen optimally (compare equations (11) and (37)-(38)). Furthermore, the partition of the population is optimal. By (37) and (38), the optimality condition (39) can be rewritten as:

$$x = \Delta_2 + p_2 \eta_2 (y_2 - b_2) - p_1 \eta_1 (y_1 - b_1)$$

¹⁶Because the demand for housing is vertical, the consumer surplus and hence the total surplus on each housing market is infinite. It can however be shown that the total surplus measured at the level of the country is made of an infinite constant minus the costs of production of housing. This constant term does not matter for the optimal allocation.

¹⁷In expression $\int_{-v}^x c_{2j} dj$ and $\int_x^v c_{1j} dj$, there is a slight abuse of notation since v and x are values for the difference $c_{1j} - c_{2j}$. This notation is equivalent to assuming a bijective relationship between the identifier of workers, j , and their relative preference for region 1, $c_{1j} - c_{2j}$.

which is equivalent to the corresponding condition in the decentralized equilibrium (12) when the Hosios condition is satisfied. In sum,

Proposition 2. *If $\alpha = 0$, in the laissez-faire economy, the sizes of the workforce and the decentralized equilibrium unemployment rates are efficient if the Hosios condition is met in both regions.*

3.2 The case where $\alpha > 0$

The constrained central planner chooses tightness in both labor markets and allocates the population between the two regions and the two statuses of job search (national versus regional) by fixing the four thresholds $\{\hat{x}, x, z_1, z_2\}$. If part of the workforce searches in both regions, the probabilities of receiving two offers is computed as in the decentralized economy. The planner's problem consists in maximizing net output¹⁸

$$\begin{aligned}
& \max_{\theta_1, \theta_2, z_1, z_2, x, \hat{x}} y_1 L_1 + y_2 L_2 + b_1(N_1^P - L_1) + b_2(N_2^P - L_2) + a_1 N_1^P + a_2 N_2^P \\
& + \frac{N}{2v} \left[\int_{-v}^x c_{2j} dj \right] + \frac{N}{2v} \left[\int_x^v c_{1j} dj \right] + \frac{N}{2v} \alpha p_1 p_2 \left[\int_{\hat{x}}^x (c_{1j} - c_{2j}) dj \right] \\
& + \frac{N}{2v} \alpha p_1 (1 - p_2) \left[\int_{z_1}^x (c_{1j} - c_{2j}) dj \right] - \frac{N}{2v} \alpha p_2 (1 - p_1) \left[\int_x^{z_2} (c_{1j} - c_{2j}) dj \right] \\
& - \kappa_1 \mathcal{V}_1 - \kappa_2 \mathcal{V}_2 - (\bar{r}_1 + \frac{s_1 N_1^P}{2}) N_1^P - (\bar{r}_2 + \frac{s_2 N_2^P}{2}) N_2^P \\
& \text{subject to the following constraints:} \\
& \mathcal{V}_1 = \theta_1 \frac{N}{2v} [v - x + \alpha(x - z_1)], \quad \mathcal{V}_2 = \theta_2 \frac{N}{2v} [v + x + \alpha(z_2 - x)] \\
& L_1 = \frac{N}{2v} [p_1(v - x) + \alpha p_1(1 - p_2)(x - z_1) + \alpha p_1 p_2(x - \hat{x})] \\
& L_2 = \frac{N}{2v} [p_2(v + x) + \alpha p_2(1 - p_1)(z_2 - x) - \alpha p_1 p_2(x - \hat{x})] \\
& N_1^P = \frac{N}{2v} [v - x - \alpha p_2(1 - p_1)(z_2 - x) + \alpha p_1(1 - p_2)(x - z_1) + \alpha p_1 p_2(x - \hat{x})] \\
& N_2^P = \frac{N}{2v} [v + x + \alpha p_2(1 - p_1)(z_2 - x) - \alpha p_1(1 - p_2)(x - z_1) - \alpha p_1 p_2(x - \hat{x})] \\
& -v \leq z_1 \leq x \leq z_2 \leq v, \quad z_1 \leq \hat{x} \leq z_2
\end{aligned}$$

An efficient allocation is a vector $\{\theta_1, \theta_2, z_1, z_2, \hat{x}, x\}$ solving the following first-order conditions:

$$\begin{aligned}
& \frac{\kappa_1}{(1 - \eta_1)m_1(\theta_1)} = \pi_1(y_1 - b_1) - \frac{\alpha p_2(z_2 - \hat{x})}{v - x + \alpha(x - z_1)}(y_2 - b_2) \\
& - \frac{\alpha(1 - p_2)(x - z_1)}{v - x + \alpha(x - z_1)} \left(\Delta_2 - \frac{x + z_1}{2} \right) - \frac{\alpha p_2(z_2 - \hat{x})}{v - x + \alpha(x - z_1)} \left(\Delta_2 - \frac{z_2 + \hat{x}}{2} \right) \quad (41)
\end{aligned}$$

¹⁸With the same slight abuse of notations as in the previous subsection.

$$\begin{aligned} \frac{\kappa_2}{(1-\eta_2)m_2(\theta_2)} &= \pi_2(y_2 - b_2) - \frac{\alpha p_1(\hat{x} - z_1)}{v + x + \alpha(z_2 - x)}(y_1 - b_1) \\ &+ \frac{\alpha(1-p_1)(z_2 - x)}{v + x + \alpha(z_2 - x)} \left(\Delta_2 - \frac{z_2 + x}{2} \right) + \frac{\alpha p_1(\hat{x} - z_1)}{v + x + \alpha(z_2 - x)} \left(\Delta_2 - \frac{\hat{x} + z_1}{2} \right) \end{aligned} \quad (42)$$

$$\hat{x} = \Delta_2 + y_2 - b_2 - (y_1 - b_1) + \frac{\mu_5 - \mu_6}{(N/2v)\alpha p_1 p_2} \quad (43)$$

$$z_1 = b_1 - y_1 + \Delta_2 + \frac{\kappa_1}{m_1(\theta_1)} \frac{1}{1-p_2} + \frac{\mu_1 - \mu_2 - \mu_5}{(N/2v)\alpha p_1(1-p_2)} \quad (44)$$

$$z_2 = y_2 - b_2 + \Delta_2 - \frac{\kappa_2}{m_2(\theta_2)} \frac{1}{1-p_1} + \frac{\mu_3 - \mu_4 + \mu_6}{(N/2v)\alpha p_2(1-p_1)} \quad (45)$$

$$\begin{aligned} x = \Delta_2 + & \frac{(1-\alpha) \left[p_2(y_2 - b_2 - \frac{\kappa_2}{m_2(\theta_2)}) - p_1(y_1 - b_1 - \frac{\kappa_1}{m_1(\theta_1)}) \right]}{1 - \alpha p_1 - \alpha p_2 + \alpha p_1 p_2} \\ & + \frac{\mu_2 - \mu_3}{(N/2v)(1 - \alpha p_1 - \alpha p_2 + \alpha p_1 p_2)} \end{aligned} \quad (46)$$

$$\begin{array}{lll} \mu_1(v + z_1) = 0 & \mu_2(x - z_1) = 0 & \mu_3(z_2 - x) = 0 \\ \mu_4(v - z_2) = 0 & \mu_5(\hat{x} - z_1) = 0 & \mu_6(z_2 - \hat{x}) = 0 \end{array}$$

in which π_1 (resp., π_2) verifies (24) (resp., (25)). To compare the decentralized equilibrium and the optimal allocation, let us assume that both are interior solutions. A comparison of the above optimality conditions and of a decentralized equilibrium in Subsection 2.4.7 shows that the *laissez-faire* economy has no reason to be efficient. When choosing the number of vacancies in each region, the planner takes into account the impacts of a rise in the number of vacancies on the value of leisure, the amenities, the rents and the idiosyncratic preference when a worker has to migrate to take a job offer (see the second line of (41) and (42)).¹⁹ compensation cannot appear if $\alpha = 0$. For then, workers never migrate to take a position. These workers thus enjoy the same level of amenities, rents and idiosyncratic preferences whether they are employed or unemployed. This compensation is not present in the decentralized equilibrium. It can be checked that allowing to bargain before taking the decision to migrate does not eliminate these sources of inefficiencies.

The decentralized choice of tightness is also inefficient because the planner recognizes that an additional vacancy in region i reduces the chances of a match between residents of region i and vacancies in region $-i$, while decentralized agents do not. Imagine that the acceptance probabilities π_i were the same at the optimum and in the decentralized

¹⁹In both equations, the first part of the second line measures the change in the share of workers seeking a job in region i who live in the other region and get a position in region i . This change multiplies an expression that recognizes that these workers do not enjoy leisure, amenities, rents and their idiosyncratic preference in region $-i$ since they move to region i . The last part of the second line is also the product of two terms. The first one quantifies the change in the share of workers searching a job in region i , who only get an offer in region $-i$ or get two offers, and who migrate to region $-i$. The second term captures that these workers do not enjoy leisure, amenities, rents and their idiosyncratic preference in region i since they move to region $-i$.

economy. By subtracting the nonnegative term $[\alpha p_2(z_2 - \hat{x})(y_2 - b_2)]/[v - x + \alpha(x - z_1)]$ from $\pi_1(y_1 - b_1)$ in (41), the planner internalizes an induced effect on employment in region 2 and this pushes optimal tightness downwards. The same holds true in (42).

Considering now the partition of regional workforces between the two statuses of job search, we need to compare (44) and (20) or (45) and (21). Two effects are at work. First, because the planner considers the net gain in output, $y_i - b_i$, while the decentralized workers only consider the share $\beta_i(y_i - b_i)$ that accrues to them, the decentralized value of z_i are lower than the efficient ones. Second, a marginal rise in, say, z_1 reduces the size of the workforce living in region 2 and searching also in region 1. This lowers expected net output by an amount proportional to $(y_1 - b_1)p_1(1 - p_2)$ but reduces the cost of vacancy creation by an amount proportional to $\kappa_1\theta_1$. After division by $p_1(1 - p_2)$, the loss in expected output becomes $y_1 - b_1$ while the gain becomes the product of the expected cost of opening a vacancy in region 1, $\kappa_1/m_1(\theta_1)$, and of $1/(1 - p_2)$. This gain is not taken into account by decentralized decisions (see (20) and (21)) but well by the planner (see the expressions $\kappa_i/[(1 - p_{-i})m_i(\theta_i)]$ in (44) and (45)). So, through this second effect, too many workers search for a job nationally.

Turning to the choice of residence, a comparison of (46) and (23) indicates one source of inefficiency. When fixing x , the planner compares the expected increase in net output in each region, $p_i(y_i - b_i - \kappa_i/m_i(\theta_i))$, while the decentralized job seekers compares the expected net increase in income $p_i\beta_i(y_i - b_i)$. Contrary to the case where $\alpha = 0$, the Hosios condition does not reconcile the two perspectives since in any case (41) and (42) do not yield an equality between the expected cost of opening a vacancy and $(1 - \eta_i)(y_i - b_i)$ when $\alpha > 0$. It should be mentioned that this source of inefficiency in the choice of x would disappear if $\alpha = 1$, i.e. in the limit case where job seekers are equally effective in the matching process wherever they search. In sum,

Proposition 3. *If $\alpha > 0$, even if the Hosios condition is met in both regions, the decentralized laissez-faire equilibrium unemployment rates, regional partition of the workforce and numbers of job seekers searching in the whole country are inefficient. In general, one cannot rank the optimum and the equilibrium.*

If we assume two symmetric regions, the efficient allocation is symmetric as well. Hence, the optimal x and \hat{x} are set to zero. We can denote the thresholds $z_2 = -z_1 = z \in [0, v]$ and the probability of being recruited $p_1 = p_2 = p$. The planner's objective function becomes the net gain of firms' production

$$2((y - b)L - \kappa\theta V) = 2[(y - b)p(v + \alpha z(1 - p)) - \kappa\theta(v + \alpha z)] \quad (47)$$

plus a constant term. It is easily seen that this objective function is linear in z , with a coefficient of proportionality equal to $2\alpha((y - b)p(1 - p) - \kappa\theta)$. When this coefficient is positive, z takes the highest possible value (i.e. v). When it is negative, $z = 0$. Whatever z , tightness is determined by a simplified version of (41)-(42), namely:

$$\frac{\kappa}{(1 - \eta)m(\theta)} = \frac{v + \alpha(1 - 2p)z}{v + \alpha z}(y - b)$$

Proposition 4. *When regions are symmetric, the threshold values z_1 and z_2 are equal in absolute value. The optimum is a corner solution: Either everybody searches nationally or nobody.*

4 Numerical exercise

To get clear-cut results regarding various local shocks and to compare the decentralized and the optimal economy, we calibrate and simulate the model. Our calibration uses mainly parameters found in the literature to reproduce the US unemployment rate during the period 2005-2011.

4.1 Calibration

We assume symmetric regions. We normalize regional productivity levels y_i and the total size of the population N to 1. We assume Cobb-Douglas matching functions $M_i = \bar{h} \mathcal{V}_i^{0.5} (N_i + \alpha N_{-i}^N)^{0.5}$ with $\bar{h} = 0.7$.²⁰ To satisfy the Hosios condition, workers' bargaining power β_i are set to 0.5. The expected cost of opening a vacancy is derived from Pissarides (2009). In his dynamic framework, it amounts to 0.43, which means that firms expect to pay 43% of their monthly output per vacancy. To transpose this value to our static framework, we need to multiply 0.43 by the sum of the interest rate (0.004) and the separation rate (0.036). So, the expected cost of opening a vacancy ($\kappa_i / [\pi_i m_i(\theta_i)]$) equals 0.0172. To match the free-entry conditions (18), we set the home production value to 0.9656. Under free-entry, the wage (10) equals 0.98. The difference between the wage and home production may look small. However, as Pissarides (2009) explains, the permanent income of employed workers is only marginally above the permanent income of unemployed workers, even if the difference in a dynamic framework between current wage and current home production is quite large. We assume that, at equilibrium, 35% of the regional workforce searches in the region of residence only. Using the above calibration, we get a value for the z_2 threshold of 0.0172. By the definition of N_i^R , this allows to parametrize v to 0.03. We take $s_i = 0.22$ for the slope of the housing supply, which corresponds to the mean of the significant elasticities in Green *et al.* (2005). We then assume that $\alpha = 0.12$, which means that workers searching out of their region of residence are 8 times less efficient than in their own region. Finally, we calibrate the cost of opening a vacancy κ_i to match an average unemployment rate in the US in the period 2005-2011, namely 7%. So, $\kappa_i = 0.01$. The value of the parameters are summarized in Table 1 and the equilibrium values are provided in Table 2, both in Appendix E.

4.2 Simulation

We now consider a range of shocks whose values are displayed in Table 1. The simulated equilibria after these shocks are presented in Table 2. The relative changes compared to the baseline equilibrium are available in Table 3.

²⁰In a static setting, the Cobb-Douglas specification does not guarantee that the hiring rate tends to 1 when θ_i becomes sufficiently big. In the simulations we take care of this difficulty.

4.2.1 Rising search effectiveness in the other region

Consider a 25% rise in the effectiveness of job search in the other region (α goes up from 0.12 to 0.15). Recalling (20) and (21), this does not modify workers' decisions regarding the searching area. Because of the symmetry assumption, the *ex-ante* and the *ex-post* partitions of workers across regions do not vary either. Rising α implies that more workers get two job offers and refuse one of them. This yields a lower conditional acceptance rate in each region. Firms need to face a higher probability of meeting a worker because of free-entry and therefore the levels of tightness go down. These results can be shown analytically.

Regarding the unemployment rates, two effects are simultaneously at work. First, as more workers get two offers, less remain unemployed. Second, as tightness drops, agents face a lower probability of meeting a firm. Under our calibration, this second effect dominates. Therefore, an increase in the search effectiveness α leads to *higher* unemployment rates (from 7% to 8 %).

4.2.2 Rising labor productivity in region 1

Starting from symmetric regions, an increase in productivity of 0.1% in, say, region 1, augments the (*ex-post*) surplus created by a match in the region. To restore the free-entry condition (18), the probability of filling a vacancy in region 1 has to decrease (through an increase in tightness and/or a decrease in the conditional acceptance rate). In region 2, as the surplus of a match stays constant, the filling probability in region 2 remains unchanged. To better understand the mechanisms at work, we first assume that rents remain constant and simulate the model. We then relax this assumption.

Without taking adjustments in rents into account (so that Δ_2 is fixed), workers get a higher wage in region 1 and are therefore more numerous to locate there (x decreases). For the same reason, there are less workers searching regionally in 2 (z_1 goes down) while the number of regional job seekers in 1 remains unchanged. Furthermore, as the wage increases in region 1 whereas it stays constant in region 2, more workers accept the position in 1 when they get a job offer from each region. As a result, \hat{x} goes down. This affects the conditional acceptance probabilities: The probability increases in region 1 while it decreases in region 2. Therefore, tightness goes down in region 2 while it goes up in region 1. This leads to a higher *ex-post* population in region 1 and a decrease in the *ex-post* population in region 2. Furthermore, the unemployment rate goes down in region 1 but goes up in region 2.

However, rents are not constant in our model. Because of the increase in *ex-post* population in region 1, relative rents $r_1 - r_2$ increase. Although not a general feature, this rise is, under our calibration, bigger than the one in wages, leading eventually to changes in the equilibrium values that go in the opposite direction compared to the description above (see Tables 2 and 3). More specifically, wages go up by 0.0005 in 1 while they remain constant in 2. Relative rents $r_1 - r_2$ increase by 0.0009. This leads to more regional job seekers in region 2 (a rise of 4.4%) and to less regional job seekers in region 1 (their number decreases by 9.8%). Because relative rents increase more than

relative wages, workers with two offers in hands are more willing to accept a job offer in 2 and \hat{x} goes up. However, we still have that more workers locate *ex-ante* in region 1 (x goes down). This is due to the increase in tightness in region 1. Even if national job seekers who get two job offers need a higher relative preference for region 1 to accept the position, the conditional probability of acceptance π_1 is higher than before.²¹ Conversely, the conditional probability of accepting a position in 2 goes down, as well as tightness θ_2 . Consequently, an increase in productivity in a region can lead to regional unemployment rate disparities. The unemployment rate in region 1 goes down by 2.5 percentage points, while it rises in region 2 by 0.4 percentage points. Given the smallness of the regional-specific productivity shock, the magnitude of these impacts should be emphasized. By the static nature of the model, these effects should be interpreted as long run impacts.

4.2.3 Rising the cost of opening a vacancy in region 1

Creating vacancies in region 1 becomes more costly (by, say, 1%) while the (*ex-post*) surplus of a match remains constant. So, firms have less incentives to open vacancies in this region and the probability of filling a vacancy in region 1 increases. The latter is the product of the probability of meeting an applicant m_1 and the conditional acceptance rate π_1 . At this stage, nothing changes in region 2.

We first switch off the housing market and simulate the model. As wages remain constant in both regions and rents are kept fixed, the search and acceptance decisions are not modified (\hat{x} , the z_i 's and hence the π_i 's stay at this stage constant). Therefore, the above rise in the probability of filling a vacancy in region 1 is achieved by a decrease in θ_1 . This rises the conditional acceptance probability in region 2 (recall (25)). To keep the free-entry condition, tightness in region 2 has to increase. By the same reasoning, a rise in tightness in region 2 leads to a decrease in the conditional acceptance rate in region 1 which reinforces the decline in θ_1 . As a result, workers need a higher relative preference for region 1 to locate there, and x goes up. So, the *ex-ante* population increases in region 2. This also lead to a larger *ex-post* population in region 2.

Switching on the housing market, the rise in *ex-post* population in region 2 diminishes the rent gap $r_1 - r_2$. This leads to new search and acceptance decisions. Eventually, a rise in κ_1 by 1% leads to a decrease in relative rents in region 1 by 0.0002. This induces new search decisions: The number of regional job seekers in 1 increases by 1.6% (it goes down by the same amount in 2), i.e. both z_i 's shrink. Workers need a lower relative preference for region 1 to accept a position there (\hat{x} , z_1 and z_2 decrease by the same amount, keeping $z_2 - \hat{x}$ and $\hat{x} - z_1$ and therefore p_i 's (at this stage) unchanged). For the reasons explained earlier, the probability of meeting a worker increases by almost 1% in region 1, yielding a decrease by 0.1% of the conditional acceptance rate in region 2. This implies a similar drop in the probability of meeting a worker in region 2, which yields a slight decrease in the conditional acceptance probability in region 1.

²¹This is mainly due to the impact on tightness levels: as the probability of getting an offer from region 2 decreases but the share of workers that are possibly refusing a job offer from region 1 $\frac{\hat{x}-z_1}{2v}$ remains constant, the conditional acceptance probability in 1 rises.

As a result, following a 1% increase in the cost of opening a vacancy in region 1, the unemployment rate rises by 0.9 percentage point in this region while it remains almost unchanged in region 2.

4.2.4 Rising relative amenities $a_2 - a_1$

Let us increase $a_2 - a_1$ from zero to 0.001. Amenities do not impact the wage-setting process, and therefore, the firm-worker matching probabilities initially stay constant. We first freeze the housing market. When region 2 becomes more attractive, there are more regional job seekers there, less regional job seekers in region 1 (both z_i 's increase), more inhabitants in region 2 *ex-ante* (x rises) and someone who holds two job offers needs a higher relative preference for region 1 to refuse a job offer in 2 (\hat{x} increases). Still, we have that the number of national job seekers remain constant (the difference $z_2 - z_1$ but also $\hat{x} - z_1$, $z_2 - \hat{x}$, as well as, at this stage, $x - z_1$ and $z_2 - x$ stay constant). The increase in x leads to a higher conditional acceptance rate in region 1 and a lower one in region 2. Because of free-entry, the meeting probability for a firm goes down in region 1 but rises in region 2, reinforcing the initial impact on acceptance probabilities but mitigating the variation of x . Overall, this leads to a rise in *ex-post* population in region 2 and to a decrease in the unemployment rate in region 2.

Taking the adjustment in rents into account, the increasing *ex-post* population in 2 puts a downward pressure on relative rents in region 1 ($r_1 - r_2$ decreases by 0.0009). Under the current calibration, this partially but not totally offsets the effects described above. The number of regional job seekers in 2 goes up by 1.07% (it increases by the same amount in region 1). The *ex-ante* population in region 1 decreases by 0.07%. This is due to an increase in tightness in region 2 by 0.06%, leading to a decrease in the conditional acceptance rate of 0.03%. Overall, unemployment rate goes down by 0.02 percentage point in region 2 (to 6.98%), while it rises up to 7.02% in region 1. These adjustments are negligible compared to those observed above for a change in productivity of the same magnitude.

4.2.5 The efficient allocation and the efficiency gap

We finally compare the *laissez-faire* decentralized equilibrium and the efficient allocation under the Hosios condition.²² The central planner opts for levels of tightness which are higher than in the decentralized case, so that the efficient unemployment rate is much lower than in the decentralized economy (1 versus 7%). From Proposition 4, we know that the efficient value of z equals either 0 or v . Simulating the model, we find that, while 65% of the population searches nationally in the decentralized economy, it is optimal that everyone searches regionally only ($z = 0$). The conclusion that workers should only

²²With symmetric regions, the efficient allocation is symmetric as well. Hence, the optimal x and \hat{x} are set to zero. To compute the optimal values of $-z_1 = z_2 = z \geq 0$ and of $p_1 = p_2 = p \geq 0$ (from which the corresponding value of θ is deducted), we discretize z in $[0; 0.03]$ and p in $[0, 1]$ (allowing each to take 9000 values), then we evaluate the social objective of the planner for each of the 9000×9000 values. Finally, we select the global optimum.

search regionally appears to be very robust to changes in the parameters. Varying search effectiveness in the other region α from 0.01 to 0.98 does not modify our conclusion. It is only optimal that everybody searches nationally ($z = v$) when $\alpha \in (0.98; 1]$. Let superscript c designates the calibrated values in Table 1. With $\alpha^c = 0.12$, $z = 0$ is still optimal when we successively consider the following changes in the other parameters: $\kappa \in [0.1 * \kappa^c, 1.5 * \kappa^c]$, $b \in [0.1 * b^c, 1 * b^c]$, $v \in [v^c, 30 * v^c]$, $y \in [y^c, 1.0025 * y^c]$ ²³ and $\bar{h} \in [0.7 * \bar{h}^c, 1.5 * \bar{h}^c]$.

Net output levels at the social optimum and at the decentralized equilibrium differ only by the net gain of firms' production (47). We compute an "efficiency gap" as this difference in net output divided by the decentralized value of (47). Figure 2 draws the evolution of this efficiency gap with α . As already mentioned, the efficiency gap is positive whenever α is positive. The gap is at first increasing with α . It reaches a maximum value of 18% when $\alpha = 0.76$ and slightly decreases above this value. When $\alpha > 0.98$, the central planner chooses $z = v$ and the difference grows again. In sum, our calibrated economy which satisfies the Hosios condition appears to be very far from efficient.

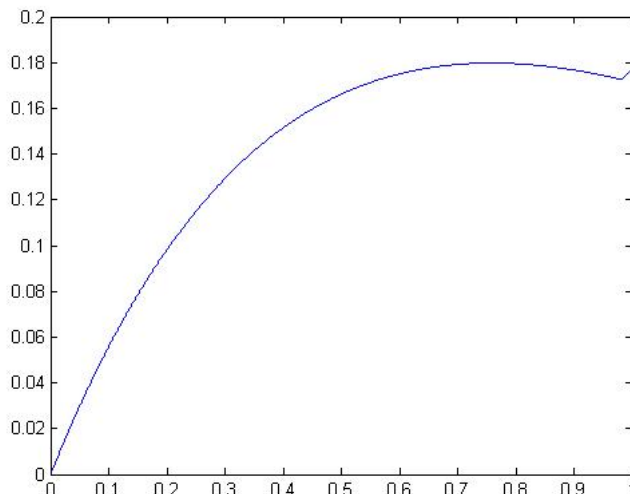


Figure 2: Evolution of the efficiency gap with α

5 Conclusion

This paper studies equilibrium unemployment in a two-region static economy where homogeneous workers and jobs are free to move and the housing market clears. Since

²³Above this upper-bound, the decentralized unemployment rates become negative. The efficient unemployment rate is already nil when $y = 1.0005$.

individuals have idiosyncratic and heterogeneous relative preferences for regions, geographical mobility is finite. Moreover, housing supply is not fixed. We develop a tractable search-matching equilibrium in which searching for a job in another region is possible without first migrating there. Current communication technologies motivate this assumption. The paper compares this environment with the perfectly competitive case and with the frictional case where job seekers can only search for jobs in the region where they actually live. We show that letting the unemployed workers search for jobs all over the country substantially changes the mechanisms at work. Search-matching externalities are amplified by the opportunity of searching in a region where one does not live and by the fact that some workers can simultaneously receive a job offer from each region. Hence, some vacant positions remain unfilled, which leads to a waste of resources.

We study the efficiency of the *laissez-faire* decentralized economy. In standard search-matching models with wage bargaining, the equilibrium is efficient if the Hosios condition is verified. In our model, the Hosios condition is not sufficient to guarantee efficiency. Workers and firms take decisions without internalizing the effect of their choices on net output in both regions.

The model is calibrated for the US and two symmetric regions. We adopt a conservative assumption according to which searching out of one's region of residence is 8 times less efficient than regional search. Simulations show that increasing matching effectiveness in the other region by 25% rises the unemployment rate all over the country by 1 percentage point. Moreover, small asymmetric shocks on regional productivity and on the cost of job creation generate substantial disparities in unemployment rates. It also shows that the optimal allocation is a corner solution where no one searches all over the country (this result is very robust to changes in parameters) and the regional unemployment rates are 6 percentage points lower than in the decentralized equilibrium.

This paper does not claim to have evaluated the general equilibrium impact of the Internet on the matching process. It has only focused on the implications of searching before possibly moving to another region under the standard assumptions of constant returns to scale in the matching process and in production. Beaudry *et al.* (2013) find no significant effects of agglomeration forces on productivity in the US. So, we feel confident that the latter assumption is not too strong a simplification. The waste of resources when a vacancy remains unfilled would be reduced if firms received several applications (see e.g. Blanchard and Diamond (1994), where job-seekers however only send one application). The conclusion about the inefficiency of national search would be affected by the presence of migration costs since the workforce incurs the latter only if job-search turns out to be successful. Extending the model to a dynamic setting seems natural. However, we think that this extension would require some simplifying assumptions. Finally, since the *laissez-faire* decentralized economy turns out to be inefficient, the role of public intervention is an interesting avenue for further research.

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Appendices

A Good’s market clearing

In case of perfectly competitive labor markets, the level of output produced regionally is the product of the constant marginal product and the employment level $y_i L_i$. As wages are equal to productivity, the consumption side can be expressed as $w_i L_i = y_i L_i$, where L_i equals the size of the resident workforce N_i^P .

In case of imperfect labor markets, regional good markets also always clear. For, the production in a region corresponds to the sum of the output of the firms and the home production of the unemployed, $y_i L_i + b_i(N_i^P - L_i)$. The regional aggregate consumption is the sum of employed workers’ consumption, the unemployed workers’ consumption, the entrepreneurs’ profit and the cost of vacancy creation $\kappa_i \mathcal{V}_i$:

$$w_i L_i + b_i(N_i^P - L_i) + (y_i - w_i)L_i - \kappa_i \mathcal{V}_i + \kappa_i \mathcal{V}_i = y_i L_i + b_i(N_i^P - L_i).$$

B Search and location decisions are taken simultaneously

The following table summarizes the different cases an agent faces:

Where to live	Where to search		
	Region 1	Region 2	Region 1 and region 2
Region 1	case a	case b	case c
Region 2	case d	case e	case f

We will proceed by first computing the expected utility of an individual in each case. By doing so, we will be able to drop cases b and d. In a second step, we will define 3 thresholds out of the 6 that could be computed from the 4 remaining cases. By ranking these thresholds and comparing the expected utility levels, we will be able to rank the expected utility levels and show that these thresholds are those we get if we assume that agents take the location and search decisions sequentially.

1. Expected utility in each case

Case a: The utility if the individual lives in 1 and searches in 1 only

$$p_1 V_{1j}^e + (1 - p_1) V_{1j}^u$$

Case b: The utility if the individual lives in 1 and searches in 2 only

$$\alpha p_2 V_{2j}^e + (1 - \alpha p_2) V_{1j}^u$$

Case c: The utility if the individual lives in 1 and searches in both regions

$$p_1(1 - \alpha p_2) V_{1j}^e + \alpha p_2(1 - p_1) V_{2j}^e + (1 - p_1)(1 - \alpha p_2) V_{1j}^u + \alpha p_1 p_2 \max \{ V_{1j}^e; V_{2j}^e - \varepsilon \}$$

case d: The utility if the individual lives in 2 and searches in 1 only

$$\alpha p_1 V_{1j}^e + (1 - \alpha p_1) V_{2j}^u$$

Case e: The utility if the individual lives in 2 and searches in 2 only

$$p_2 V_{2j}^e + (1 - p_2) V_{2j}^u$$

Case f: The utility if the individual lives in 2 and searches in both regions

$$p_2(1 - \alpha p_1) V_{2j}^e + \alpha p_1(1 - p_2) V_{1j}^e + (1 - p_2)(1 - \alpha p_1) V_{2j}^u + \alpha p_1 p_2 \max \{ V_{1j}^e - \varepsilon; V_{2j}^e \}$$

We assume that the cost ε of refusing a job offer tends to zero.

2. Case b is dominated by case c if

$$\begin{aligned} & \alpha p_2 V_{2j}^e + (1 - \alpha p_2) V_{1j}^u \\ & < p_1(1 - \alpha p_2) V_{1j}^e + \alpha p_2(1 - p_1) V_{2j}^e + (1 - p_1)(1 - \alpha p_2) V_{1j}^u + \alpha p_1 p_2 \max \{ V_{1j}^e; V_{2j}^e \} \\ & \Leftrightarrow 0 < p_1(V_{1j}^e - V_{1j}^u) + \alpha p_1 p_2 [\max \{ V_{1j}^e; V_{2j}^e \} - V_{1j}^e - V_{2j}^e + V_{1j}^u] \end{aligned}$$

Two subcases should be considered:

- If $\max \{V_{1j}^e; V_{2j}^e\} = V_{2j}^e$, then the comparison becomes:

$$\begin{aligned} 0 &< p_1(V_{1j}^e - V_{1j}^u) + \alpha p_1 p_2 (V_{1j}^u - V_{1j}^e) \\ 0 &< (1 - \alpha p_2) p_1 (V_{1j}^e - V_{1j}^u) \end{aligned}$$

This always holds. Similarly,

- If $\max \{V_{1j}^e; V_{2j}^e\} = V_{1j}^e$, then the comparison becomes:

$$\begin{aligned} 0 &< p_1(V_{1j}^e - V_{1j}^u) + \alpha p_1 p_2 (V_{1j}^u - V_{2j}^e) \\ 0 &< (1 - \alpha p_2) p_1 (V_{1j}^e - V_{1j}^u) + \alpha p_1 p_2 (V_{1j}^e - V_{2j}^e) \end{aligned}$$

This always holds since $V_{1j}^e \geq V_{2j}^e$. As a result, *case b* will never be optimal for agent j .

3. *Case d* is dominated by *case f* if

$$\begin{aligned} &\alpha p_1 V_{1j}^e + (1 - \alpha p_1) V_{2j}^u \\ &< p_2(1 - \alpha p_1) V_{2j}^e + \alpha p_1(1 - p_2) V_{1j}^e + (1 - p_2)(1 - \alpha p_1) V_{2j}^u + \alpha p_1 p_2 \max \{V_{1j}^e; V_{2j}^e\} \\ &\Leftrightarrow 0 < p_2(V_{2j}^e - V_{2j}^u) + \alpha p_1 p_2 [\max \{V_{1j}^e; V_{2j}^e\} - V_{1j}^e - V_{2j}^e + V_{2j}^u] \end{aligned}$$

Two subcases should be considered:

- If $\max \{V_{1j}^e; V_{2j}^e\} = V_{1j}^e$, then the comparison becomes:

$$\begin{aligned} 0 &< p_2(V_{2j}^e - V_{2j}^u) + \alpha p_1 p_2 (V_{2j}^u - V_{2j}^e) \\ 0 &< (1 - \alpha p_1) p_2 (V_{2j}^e - V_{2j}^u) \end{aligned}$$

This always holds. Similarly,

- If $\max \{V_{1j}^e; V_{2j}^e\} = V_{2j}^e$, then the comparison becomes:

$$\begin{aligned} 0 &< p_2(V_{2j}^e - V_{2j}^u) + \alpha p_1 p_2 (V_{2j}^u - V_{1j}^e) \\ 0 &< (1 - \alpha p_1) p_2 (V_{2j}^e - V_{2j}^u) + \alpha p_1 p_2 (V_{2j}^e - V_{1j}^e) \end{aligned}$$

This always holds since $V_{2j}^e \geq V_{1j}^e$. As a result, *case d* will never be optimal for agent j .

4. Defining the thresholds

With the 4 remaining cases, we define 3 threshold values and show that this is sufficient to get a dominant case for each value of $c_{1j} - c_{2j}$:

Definition of the threshold between *case c* and *case f*

$$\begin{aligned}
& p_1(1 - \alpha p_2)V_{1j}^e + \alpha p_2(1 - p_1)V_{2j}^e + (1 - p_1)(1 - \alpha p_2)V_{1j}^u + \alpha p_1 p_2 \max \{V_{1j}^e; V_{2j}^e\} \\
&= p_2(1 - \alpha p_1)V_{2j}^e + \alpha p_1(1 - p_2)V_{1j}^e + (1 - p_2)(1 - \alpha p_1)V_{2j}^u + \alpha p_1 p_2 \max \{V_{1j}^e; V_{2j}^e\} \\
&\Leftrightarrow (1 - \alpha)p_1 V_{1j}^e - (1 - \alpha)p_2 V_{2j}^e + (1 - p_1)(1 - \alpha p_2)V_{1j}^u - (1 - p_2)(1 - \alpha p_1)V_{2j}^u = 0 \\
&\quad \Leftrightarrow (1 - \alpha)p_1(V_{1j}^e - V_{1j}^u) - (1 - \alpha)p_2(V_{2j}^e - V_{2j}^u) \\
&\quad \quad + (V_{1j}^u - V_{2j}^u)(1 - \alpha p_1 - \alpha p_2 + \alpha p_1 p_2) = 0
\end{aligned}$$

Using the definitions of utilities, we get equation (23):

$$x = a_2 - a_1 + b_2 - b_1 + r_1 - r_2 + (1 - \alpha) \frac{p_2(w_2 - b_2) - p_1(w_1 - b_1)}{1 - \alpha p_1 - \alpha p_2 + \alpha p_1 p_2}$$

Definition of the threshold between *case e* and *case f*

$$\begin{aligned}
& p_2 V_{2j}^e + (1 - p_2)V_{2j}^u = p_2(1 - \alpha p_1)V_{2j}^e + \alpha p_1(1 - p_2)V_{1j}^e \\
& \quad + (1 - p_2)(1 - \alpha p_1)V_{2j}^u + \alpha p_1 p_2 \max \{V_{1j}^e; V_{2j}^e\} \\
& \Leftrightarrow \alpha p_1(V_{1j}^e - V_{2j}^u) + \alpha p_1 p_2 [\max \{V_{1j}^e; V_{2j}^e\} - V_{1j}^e - V_{2j}^e + V_{2j}^u] = 0
\end{aligned}$$

Two subcases should be considered:

- If $\max \{V_{1j}^e; V_{2j}^e\} = V_{2j}^e$,

$$\begin{aligned}
& (1 - p_2)(V_{1j}^e - V_{2j}^u) = 0 \\
& \Leftrightarrow V_{1j}^e = V_{2j}^u \text{ as } p_2 < 1 \\
& \Leftrightarrow z_1 = b_1 - w_1 + b_2 - b_1 + a_2 - a_1 + r_1 - r_2
\end{aligned}$$

- If $\max \{V_{1j}^e; V_{2j}^e\} = V_{1j}^e$,

$$\begin{aligned}
& (1 - p_2)(V_{1j}^e - V_{2j}^u) + p_2(V_{1j}^e - V_{2j}^e) = 0 \\
& \Leftrightarrow \tilde{z}_1 = b_1 - w_1 + b_2 - b_1 + a_2 - a_1 + r_1 - r_2 + p_2(w_2 - b_2)
\end{aligned}$$

However, the assumption of the subcases, $V_{1j}^e \geq V_{2j}^e$, implies that in this case relative preference are such that:

$$c_{1j} - c_{2j} \geq b_1 - w_1 + b_2 - b_1 + a_2 - a_1 + r_1 - r_2 + w_2 - b_2 > \tilde{z}_1$$

which leads a contradiction as we assume in the definition of \tilde{z}_1 that $V_{1j}^e \geq V_{2j}^e$. Therefore, the only possible threshold value between *case e* and *case f* is z_1 .

Definition of the threshold between *case a* and *case c*

$$\begin{aligned}
p_1 V_{1j}^e + (1 - p_1) V_{1j}^u &= p_1 (1 - \alpha p_2) V_{1j}^e + \alpha p_2 (1 - p_1) V_{2j}^e \\
&\quad + (1 - p_1) (1 - \alpha p_2) V_{1j}^u + \alpha p_1 p_2 \max \{ V_{1j}^e; V_{2j}^e \} \\
&\Leftrightarrow \alpha p_2 (V_{2j}^e - V_{1j}^u) + \alpha p_1 p_2 [\max \{ V_{1j}^e; V_{2j}^e \} - V_{1j}^e - V_{2j}^e + V_{1j}^u] = 0
\end{aligned}$$

Two subcases should be considered:

- If $\max \{ V_{1j}^e; V_{2j}^e \} = V_{1j}^e$,

$$\begin{aligned}
(1 - p_1) (V_{2j}^e - V_{1j}^u) &= 0 \\
&\Leftrightarrow V_{2j}^e = V_{1j}^u \text{ as } p_1 < 1 \\
&\Leftrightarrow z_2 = w_2 - b_2 + b_2 - b_1 + a_2 - a_1 + r_1 - r_2
\end{aligned}$$

- If $\max \{ V_{1j}^e; V_{2j}^e \} = V_{2j}^e$,

$$\begin{aligned}
(1 - p_1) (V_{2j}^e - V_{1j}^u) + p_1 (V_{2j}^e - V_{1j}^e) &= 0 \\
&\Leftrightarrow \tilde{z}_2 = w_2 - b_2 + b_2 - b_1 + a_2 - a_1 + r_1 - r_2 - p_1 (w_1 - b_1)
\end{aligned}$$

However, the assumption of the subcase, $V_{2j}^e \geq V_{1j}^e$, implies that in this case relative preference are such that:

$$c_{1j} - c_{2j} \leq w_2 - b_2 + b_2 - b_1 + a_2 - a_1 + r_1 - r_2 + b_1 - w_1 < \tilde{z}_1$$

which leads to a contradiction as we assume in the definition of \tilde{z}_2 that $V_{2j}^e \geq V_{1j}^e$. Therefore, the only possible threshold value between *case a* and *case c* is z_2 .

5. Ranking the thresholds

It is easily seen that $z_1 \leq x \leq z_2$.

6. Dominant strategies

- the individual whose relative preference is x is indifferent between living in 1 and searching in both regions (*case c*) and living in 2 and searching in both regions (*case f*);
- the individual whose relative preference is z_1 is indifferent between living in 2 and searching in 2 only (*case e*) and living in 2 and searching in both regions (*case f*);
- the individual whose relative preference is z_2 is indifferent between living in 1 and searching in 1 only (*case a*) and living in 1 and searching in both (*case c*);

We conclude from Figure 3 that the three threshold values we chose at first are sufficient to get a dominant strategy for each value of the relative preference $c_{1j} - c_{2j}$. These values are equivalent to those obtained when location and search decisions are taken sequentially.

	z_1	x	z_2
f dominates c	f dominates c	c dominates f	c dominates f
e dominates f	f dominates e	f dominates e	f dominates e
c dominates a	c dominates a	c dominates a	a dominates c
$e > f > c > a$	$f > c > a$ and $f > e$	$c > f > e$ and $c > a$	$a > c > f > e$
e gives the highest utility	f gives the highest utility	c gives the highest utility	a gives the highest utility

Figure 3: Dominant strategies

C Conditional acceptance rates

In this appendix, we show formulas (24) and (25). In the first part, we focus on the conditional acceptance rate when the vacancy is located in region 1. We then turn to the opposite case.

C.1 Conditional acceptance rate in region 1

A vacant position located in region 1 faces $\frac{v-x+\alpha(x-z_1)}{2v}N$ possible workers in efficiency units. These workers always accept a job offer from the firm if their relative preference for region 1 over region 2, $c_{1j} - c_{2j}$, is higher than \hat{x} . If their relative preference is below \hat{x} , job seekers only accept the offer if they have not received one from a firm located in region 2.

We thus compute the conditional acceptance rate as :

$$\frac{2v}{v-x+\alpha(x-z_1)} \{P(c_{1j} - c_{2j} \geq \hat{x})1 + P(c_{1j} - c_{2j} < \hat{x}) P(\text{no offer from 2})\}$$

There are two sub-cases: Whether x is lower or greater than \hat{x} .

Whenever $x < \hat{x}$,

Relative preference	Proba to have this preference	Proba to accept a position in 1
$v > c_{1j} - c_{2j} > \hat{x}$	$\frac{v-\hat{x}}{2v}$	1
$\hat{x} > c_{1j} - c_{2j} > x$	$\frac{\hat{x}-x}{2v}$	$1 - \alpha p_2$
$x > c_{1j} - c_{2j} > z_1$	$\frac{\alpha(x-z_1)}{2v}$	$1 - p_2$

The conditional acceptance rate is thus:

$$\frac{2v}{v-x+\alpha(x-z_1)} \left\{ \frac{v-\hat{x}}{2v} + \frac{\hat{x}-x}{2v}(1 - \alpha p_2) + \alpha \frac{x-z_1}{2v}(1 - p_2) \right\}$$

which leads to equation (24).

Whenever $x > \hat{x}$,

Relative preference	Proba to have this preference	Proba to accept a position in 1
$v > c_{1j} - c_{2j} > x$	$\frac{v-x}{2v}$	1
$x > c_{1j} - c_{2j} > \hat{x}$	$\frac{\alpha(x-\hat{x})}{2v}$	1
$\hat{x} > c_{1j} - c_{2j} > z_1$	$\frac{\alpha(\hat{x}-z_1)}{2v}$	$1 - p_2$

The condition acceptance rate is thus:

$$\frac{2v}{v-x+\alpha(x-z_1)} \left\{ \frac{v-x}{2v} + \frac{\alpha(x-\hat{x})}{2v} 1 + \alpha \frac{\hat{x}-z_1}{2v} (1-p_2) \right\}$$

which leads to equation (24) as well.

C.2 Conditional acceptance rate in region 2

A vacant position located in region 2 faces $\frac{v+x+\alpha(z_2-x)}{2v}N$ possible workers in efficiency units. These workers always accept a job offer from the firm if their relative preference for region 1 over region 2, $c_{1j} - c_{2j}$, is lower than \hat{x} . If their relative preference is higher \hat{x} , job seekers only accept the offer if they have not received one from a firm located in region 1.

We thus compute the conditional acceptance rate as :

$$\frac{2v}{v+x+\alpha(z_2-x)} \{P(c_{1j} - c_{2j} < \hat{x})1 + P(c_{1j} - c_{2j} \geq \hat{x}) P(\text{no offer from 1})\}$$

Here again there are two sub-cases: Whether x is lower or greater than \hat{x} .

Whenever $x < \hat{x}$,

Relative preference	Proba to have this preference	Proba to accept a position in 2
$-v < c_{1j} - c_{2j} < x$	$\frac{v+x}{2v}$	1
$x < c_{1j} - c_{2j} < \hat{x}$	$\frac{\alpha(\hat{x}-x)}{2v}$	1
$\hat{x} < c_{1j} - c_{2j} < z_2$	$\frac{\alpha(z_2-\hat{x})}{2v}$	$1 - p_1$

The condition acceptance rate is thus:

$$\frac{2v}{v+x+\alpha(z_2-x)} \left\{ \frac{v+x}{2v} + \frac{\alpha(\hat{x}-x)}{2v} 1 + \alpha \frac{z_2-\hat{x}}{2v} (1-p_1) \right\}$$

which leads to equation (25).

Whenever $x > \hat{x}$,

Relative preference	Proba to have this preference	Proba to accept a position in 2
$-v < c_{1j} - c_{2j} < \hat{x}$	$\frac{v+\hat{x}}{2v}$	1
$\hat{x} < c_{1j} - c_{2j} < x$	$\frac{x-\hat{x}}{2v}$	$1 - \alpha p_1$
$x < c_{1j} - c_{2j} < z_2$	$\frac{\alpha(z_2-x)}{2v}$	$1 - p_1$

The condition acceptance rate is thus:

$$\frac{2v}{v+x+\alpha(z_2-x)} \left\{ \frac{v+\hat{x}}{2v} + \frac{x-\hat{x}}{2v}(1-\alpha p_1) + \alpha \frac{z_2-x}{2v}(1-p_1) \right\}$$

which leads again to equation (25).

D Existence of an equilibrium

D.1 Conditions for an interior solution

To get conditions on the existence of an interior solution, one needs to determine the extremum values of the z_1 and z_2 thresholds. The value of z_1 and z_2 depends on parameters as well as on endogenous relative rents $r_1 - r_2$, which can be rewritten as:

$$\begin{aligned} \bar{r}_1 - \bar{r}_2 + s_1 N - (s_1 + s_2) \{ \alpha p_2 (1 - p_1) F(z_2) + \alpha p_1 (1 - p_2) F(z_1) \\ + \alpha p_1 p_2 F(\hat{x}) + (1 - \alpha p_1 - \alpha p_2 + \alpha p_1 p_2) F(x) \} N \end{aligned}$$

where $F(\zeta) = \frac{v+\zeta}{2v}$ represents the cumulative density function of the uniform distribution. From this way of writing $r_1 - r_2$, it can be seen that the latter is minimum when $F(z_1) = F(\hat{x}) = F(x) = F(z_2) = 1$, in which case the difference in rents equals $\bar{r}_1 - \bar{r}_2 - s_2 N$. When $F(z_1) = F(\hat{x}) = F(x) = F(z_2) = 0$, $r_1 - r_2$ is instead maximal and equal to $\bar{r}_1 - \bar{r}_2 + s_1 N$.

We are sure that $z_1 = \beta_1(b_1 - y_1) + b_2 - b_1 + a_2 - a_1 + r_1 - r_2 > -v$ if

$$v > \beta_1(y_1 - b_1) + a_1 - a_2 + b_1 - b_2 + \bar{r}_2 - \bar{r}_1 + s_2 N$$

Similarly, we are sure that $z_2 = \beta_2(y_2 - b_2) + b_2 - b_1 + a_2 - a_1 + r_1 - r_2 < v$ if

$$v > \beta_2(y_2 - b_2) + a_2 - a_1 + b_2 - b_1 + \bar{r}_1 - \bar{r}_2 + s_1 N.$$

D.2 Existence of a symmetric equilibrium when regions are symmetric

When both regions have the same exogenous characteristics, a free-entry symmetric equilibrium is characterized by the following conditions:

$$\hat{x} = x = 0$$

$$z_2 = \beta(y - b) = -z_1$$

$$\pi m(\theta) = \frac{\kappa}{(1 - \beta)(y - b)} \quad (48)$$

$$\pi = 1 - \frac{\alpha p(\theta) z_2}{v + \alpha z_2} \quad (49)$$

$$u = \frac{(1 - p)(v - \alpha p(\theta) z_2)}{v}$$

$$N_1^P = N_2^P = 1/2$$

and (28) and (29). This system of equations can be solved recursively. First, the z_1 and z_2 thresholds are functions of parameters only. Second, combining equations (48) and (49) leads to the following implicit relationship in equilibrium tightness:

$$\frac{v + \alpha(1 - p(\theta))\beta(y - b)}{v + \alpha\beta(y - b)} = \frac{1}{m(\theta)} \frac{\kappa}{(1 - \beta)(y - b)}$$

The left-hand side is a negative function of tightness, while the right-hand side depends positively on tightness. So, there is at most one equilibrium. To show the existence of the equilibrium, consider the limit of each side of the last equality when θ tends to 0:

$$\begin{aligned} \lim_{\theta \rightarrow 0} \frac{v + \alpha(1 - p(\theta))\beta(y - b)}{v + \alpha\beta(y - b)} &= 1 \\ \lim_{\theta \rightarrow 0} \frac{1}{m(\theta)} \frac{\kappa}{(1 - \beta)(y - b)} &= 0, \text{ by the Inada conditions.} \end{aligned}$$

So, a unique symmetric equilibrium tightness exists. The other endogenous variables are then determined uniquely as well.

E Numerical exercise

E.1 The values of the parameters

Parameters	Baseline values	shocks
α	0.12	0.15
β_i	0.5	
η_i	β_i	
κ_1	0.0085	0.0086
κ_2	0.0085	
b_i	0.9656	
y_1	1	1.001
y_2	1	
\bar{h}	0.7	
v	0.0265	
$a_2 - a_1$	0	0.001
s_i	0.22	
N	1	

Table 1: Parameters values used for the calibration

E.2 Summary of the simulation

Table 2 reports the baseline equilibrium (first column), the steady-state values after the shocks, and the optimal allocation (last column). Table 3 reports the differences in

percentage points with respect to the baseline equilibrium. When this value was nil, we instead report the absolute variation. Table 3 first presents relative differences due to the shocks. The last column compares the optimum to the decentralized equilibrium.

variables	baseline equilibrium	search eff shock (α)	productivity shock	vacancy cost shock	amenities shock	optimal allocation
\hat{x}	0.0000	0.0000	0.0004	-0.0002	0.0001	0.0000
π_1	0.9331	0.9191	0.9335	0.9331	0.9328	1.0000
π_2	0.9331	0.9191	0.9292	0.9338	0.9334	1.0000
θ_1	1.7445	1.6925	1.8490	1.7100	1.7435	2.0037
θ_2	1.7445	1.6925	1.7298	1.7470	1.7455	2.0037
z_1	-0.0172	-0.0172	-0.0168	-0.0174	-0.0171	0.0000
z_2	0.0172	0.0172	0.0181	0.0170	0.0173	0.0000
$r_1 - r_2$	0.0000	0.0000	0.0009	-0.0002	-0.0009	0.0000
x	0.0000	0.0000	-0.0001	0.0000	0.0001	0.0000
u_1	0.0700	0.0814	0.0445	0.0786	0.0702	0.0091
u_2	0.0700	0.0814	0.0736	0.0694	0.0698	0.0091
N_1^P	0.5000	0.5000	0.5021	0.4997	0.4980	0.5000
N_2^P	0.5000	0.5000	0.4979	0.5003	0.5020	0.5000
m_1	0.5300	0.5381	0.5148	0.5353	0.5301	0.4945
m_2	0.5300	0.5381	0.5322	0.5296	0.5298	0.4945
p_1	0.9246	0.9107	0.9518	0.9154	0.9243	0.9909
p_2	0.9246	0.9107	0.9207	0.9252	0.9248	0.9909
N_1^N	0.3250	0.3250	0.3441	0.3218	0.3248	0.0000
N_2^N	0.3250	0.3250	0.3153	0.3282	0.3252	0.0000
N_1^R	0.1750	0.1750	0.1579	0.1779	0.1731	0.5000
N_2^R	0.1750	0.1750	0.1826	0.1721	0.1769	0.5000

Table 2: Steady state values of the decentralized equilibrium (using the different calibrations) and the optimal allocation

variables	search eff. shock (α)	productivity shock	vacancy cost shock	amenities shock	optimal allocation
\hat{x}	0	0.00040	-0.00015	0.00010	0
π_1	-1.50%	0.04%	0.00%	-0.03%	7.17%
π_2	-1.50%	-0.42%	0.07%	0.03%	7.17%
θ_1	-2.98%	5.99%	-1.98%	-0.06%	14.86%
θ_2	-2.98%	-0.84%	0.14%	0.06%	14.86%
z_1	0.00%	-2.35%	0.88%	-0.58%	-100.00%
z_2	0.00%	5.25%	-0.88%	0.58%	-100.00%
$r_1 - r_2$	0	0.00090	-0.00015	-0.00090	0
x	0	-0.00011	0.00002	0.00011	0
u_1	16.26%	-36.41%	12.26%	0.32%	-86.96%
u_2	16.26%	5.15%	-0.87%	-0.33%	-86.96%
N_1^P	0.00%	0.41%	-0.07%	-0.41%	0.00%
N_2^P	0.00%	-0.41%	0.07%	0.41%	0.00%
m_1	1.52%	-2.87%	1.00%	0.03%	-6.69%
m_2	1.52%	0.42%	-0.07%	-0.03%	-6.69%
p_1	-1.50%	2.95%	-0.99%	-0.03%	7.17%
p_2	-1.50%	-0.42%	0.07%	0.03%	7.17%
N_1^N	0.00%	5.88%	-0.98%	-0.05%	-100.00%
N_2^N	0.00%	-2.98%	0.98%	0.05%	-100.00%
N_1^R	0.00%	-9.76%	1.63%	-1.07%	185.71%
N_2^R	0.00%	4.36%	-1.63%	1.07%	185.71%

Table 3: Relative variation to the baseline steady state

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