Beyond the Arrow effect: income distribution and multi-quality in a Schumpeterian framework

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Beyond the Arrow effect: income distribution and multi-quality firms in a Schumpeterian framework*

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Abstract

This paper introduces multi-quality firms within a Schumpeterian framework. Featuring non-homothetic preferences and income disparities in an otherwise standard quality-ladder model, I indeed show that the resulting differences in the willingness to pay for quality among consumers generate both positive investments in R&D by industry leaders *and* positive market shares for more than one quality, hence allowing for the emergence of multi-product firms within a vertical innovation framework. This positive investment in R&D by incumbents is obtained with complete equal treatment in the R&D field between the incumbent patentholder and the challengers: in our framework, the incentive for a leader to invest in R&D stems from the possibility for an incumbent having innovated twice in a row to efficiently discriminate between rich and poor consumers displaying differences in their willingness to pay for quality. I hence exemplify a so far overlooked demand-driven rationale for innovation by incumbents. I am then also able to analyze the impact of inequality both on long-term growth and on the allocation of R&D activities between challengers and incumbents. I find that redistributive policies generally lead to an increase in the long-run growth rate, and to variations in the share of the overall R&D expenditures being undertaken by incumbents.

Keywords: Growth, Innovation, Income inequality, Multi-Product firms.

JEL classification: O3, O4, F4.

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1 Introduction

The importance and specificities of multi-product firms (MPFs) have lately been exemplified by a growing body of literature.[1] In particular, because of unique supply and demand linkages, MPFs' product-market decisions such as intra-firm portfolio adjustments or investment in product innovation have been shown to obey to specific incentives (Eckel and Neary, 2010; Dhingra, 2013). Dynamic R&D-driven growth models studying the behavior and impact on aggregate innovation of MPFs have already been provided for the cases where firms are multi-industry (Klette and Kortum, 2004; Akcigit and Kerr, 2010) or multi-varieties (Minniti, 2006). However, the standard quality-ladder framework has so far not been able to account for the existence of "multi-quality" firms, i.e. firms selling more than one quality-differentiated version of the same good.[2] Indeed, the "creative destruction" mechanism at the heart of Schumpeterian models traditionally not only deters leaders from investing in R&D (the well-known "Arrow effect"), but also guarantees the systematic exit of any quality that has moved away from the frontier.[3]

The present paper builds on these considerations, and provides a model accounting for the existence of multi-quality leaders within a dynamic Schumpeterian framework. More precisely, I argue that as long as preferences are non-homothetic, income distribution impacts the strength and scope of the "creative destruction" process: income differences then account for both the survival of more than one quality at the equilibrium *and* for positive investment in R&D by incumbents. The result is the endogenous emergence in a dynamic framework of multi-quality leaders whose product portfolio composition and investment in R&D activities are both influenced by the extent of income disparities.

The intuition behind this result is straightforward, and is related to the well-explored notion of second-degree price discrimination. For a monopolist, serving costumers who do not care much for quality creates negative externalities, since it hinders the captation of costumer surplus from those who have a stronger taste for quality. Mussa and Rosen (1978) have demonstrated that a monopolist confronted to such disparities in consumers' taste for quality optimally chooses to offer lower quality items charged at a lower price to the less enthusiastic consumers, opening the possibility of charging higher prices to more adamant buyers of high quality units. In their microeconomic *static* set-up, the monopolist has by assumption a whole product line at its disposition. In a standard quality-ladder dynamic framework on the other hand, the monopolist only has access to as many qualities as times

[1] Among others, Bernard et al. (2010) estimate that MPFs account for 41% of the total number of US firms as well as for 91% of total output; also, they estimate that the contribution to the US output growth of product mix decisions of MPFs (i.e. product adding and dropping) is greater than the one of firm entry and exit.

[2] As an example of such a firm, one can think of Apple, which commercializes simultaneously the Iphone4 and the Iphone5 on the US market, and even keeps offering the Iphone3 in India.

[3] Mussa and Rosen (1978) study pricing decisions of multi-quality firms, but in a static framework precluding any specific modeling of the R&D process leading to the initial design of the product line. Klette and Kortum (2004) as well as Akcigit and Kerr (2010) feature MPFs in a quality-ladder world; however, multi-product firms are also multi-industry firms in their models, with only one quality being sold within each product line.

he has innovated. I demonstrate that in such a dynamic set-up, internalization of such negative externalities then leads to investment in R&D by incumbent monopolists, and in case of success, to the existence of firms simultaneously offering more than one quality of their product.

I first demonstrate the general nature of the identified price-discrimination mechanism in a partial equilibrium framework. I show that provided there exists differences in the willingness to pay for quality among consumers, the expected value of innovating once more differs between challengers and incumbents: the Arrow effect operating under free entry then becomes compatible with positive investment in R&D by incumbents. I then integrate such a mechanism in a Schumpeterian model by featuring non-homothetic preferences in an otherwise traditional quality-ladder framework, hence allowing for more than one quality to be consumed at the equilibrium in the presence of differences in wealth endowment.[4]

In such a framework, a *challenger* winning the latest innovation race and being the producer of the highest quality needs to decide between two alternatives: capturing the whole market by charging a price sufficiently low to appeal to the poorest households, or selling its product at a higher price only to the wealthiest consumers, at the cost of abandoning the rest of the market to its direct competitor (i.e. the previous quality leader). On the other hand, an *incumbent* winning an innovation race has two successive qualities at its disposal: he can then efficiently discriminate between rich and poor consumers by offering two distinct price/quality bundles, capturing the whole market *and* reaping the maximum surplus from the wealthy consumers at the same time. I then model R&D races in which both incumbents and challengers are participating, and show that without any advantage of any kind in the R&D field and under free entry, the incumbent still invests a strictly positive amount in R&D. Such a behavior directly stems from the existing increment between the profits realized when being a successful challenger and a successful incumbent.

I then move to studying the impact of income distribution on the innovation incentives of both challengers and incumbents, and by extension on long-term growth. I show that redistributive policies leading to a reduction in the wealth gap between rich and poor are beneficial for long-term growth. I am also able to show that income distribution impacts the *allocation* of overall R&D expenditures between challengers and incumbents.

My main contribution is to provide a framework endogenously accounting for the emergence of multi-quality leaders in the presence of income disparities among consumers. Be-

[4]This property is obtained by imposing unit consumption of quality goods in a two-class society, the rest of a consumer's income being spent on standardized goods: within each industry, a given consumer then buys the quality that, *given its price*, offers him the highest utility. By contrast, in the standard quality-ladder models (Segerstrom et al., 1990; Grossman and Helpman, 1991a; Aghion and Howitt, 1992), the quality goods are divisible, and the preferences of the consumers are hence homothetic. Within each sector, the pricing strategy chosen by the current quality leader (whether he charges the unconstrained monopoly price or resorts to limit pricing) then systematically ensures that only the highest price-adjusted quality is consumed at the equilibrium, even when differences in wealth endowments are allowed for: the poorest consumers only consume a lower share of the top quality good.

yond its originality, such a result bears several implications. First, while so far the incentives for innovation by quality leaders have essentially been modeled as stemming from the structure of the R&D process, this paper is the first to provide a *demand-driven* incentive for investment in R&D by incumbents. Second, such a framework makes it possible to investigate the impact of income distribution on both the length of multi-quality firms' product portfolio and the intensity of their innovation activities, hence contributing to the literature analyzing the effects of income inequality on innovation. I particularly show that beyond the effect on the level of long-run growth already identified by the literature (Zweimuller and Brunner, 2005; Foellmi and Zweimuller, 2006), income distribution also impacts the allocation of overall R&D activities between incumbents and challengers.

Relation to literature.

This paper contributes to the literature accounting for innovation by incumbents in quality-ladder models. Segerstrom and Zolnierek (1999) as well as Segerstrom (2007) have obtained positive investment in R&D by the incumbent by assuming that the expertise granted by quality leadership confers R&D cost advantages. Etro (2004, 2008) models sequential patent races with concave R&D costs where the incumbent, acting as a Stackelberg leader, is given the opportunity to make a strategic precommitment to a given level of R&D investment: the quality leader then has an incentive to invest in R&D in order to deter outsiders' entry. Denicolo and Zanchettin (2012) as well as Acemoglu and Cao (2010) provide models where incumbents and challengers participate to two different kinds of R&D races, differing in terms of costs and rewards: leaders invest in R&D to improve their products (incremental innovation), while challengers participate to R&D races in the hope of leapfrogging the existing incumbent (radical innovation). All those models have hence explored various possible incentives for innovation by incumbent stemming from the structure of the R&D process, i.e. from the *supply side*, while our paper on the other hand provides a *demand-based* rationale for leader R&D, stemming from the perspective of more efficient price discrimination in the case of successive successful innovations. All those papers also feature homothetic preferences, hence guaranteeing that only the highest quality will be produced and consumed within each industry, and precluding the emergence of multi-quality leaders as a consequence of positive innovation by incumbent.

A paper more closely related to this work is the one of Aghion et al. (2001), who analyze the influence of product market competition on innovation intensity, developing a framework in which goods of different quality are imperfect substitutes and can therefore coexist in the market. They show that the perspective to lessen the competition pressure (and broaden the market share) provides the incentive for the incumbent to resort to step-by-step innovation in order to improve its own product. They however preclude free entry by exogenously imposing that only two firms are active and invest in R&D, while our paper on the other hand provides a product market-driven incentive that is robust to the free entry condition.

This work also contributes to the small literature studying the R&D investment of multi-product firms in a dynamic, general equilibrium framework. Klette and Kortum (2004) as well as Akcigit and Kerr (2010) have already modeled industry leaders investing in exploration R&D so as to expand their activities in other sectors; however, they do not account for multi-quality firms, i.e. leaders widening their product portfolio within a given industry. Minniti (2006) embeds multi-product firms selling more than one variety of a given good in an endogenous growth model; however, he does so in a horizontal differentiation framework under the assumption of love-for-variety at the individual level.

This paper is finally also related to the literature examining the relationship between long-term growth and income distribution operating through the demand side. Foellmi and Zweimuller (2006) model a similar two-class society, and demonstrate that a lower level of inequality is systematically detrimental to long-term growth. They however obtain this result in an horizontal innovation framework, where the rewards for innovation are from a different nature than in Schumpeterian models. Zweimuller and Brunner (2005) on the other hand have studied the impact of disparities in purchasing power of households in a quality-ladder framework, showing that a reduction in the level of inequality within the economy is beneficial for innovation intensity and hence for growth. While I rely on their modeling strategy and obtain results similar to theirs concerning the challenger innovation rate, their model however does not feature innovation by incumbent, hence precluding the emergence of multi-quality firms and only capturing part of the effects of the level of inequality on the innovation rate in such a framework.

The rest of the paper is organized as follows. Section 2 illustrates in a simple partial equilibrium framework how differences in the willingness to pay for quality impact the innovation incentives of both challengers and incumbents. Section 3 presents the structure of our general equilibrium model, while section 4 studies its steady state properties. Section 5 then analyzes the effects of the extent of inequality on the innovation intensity. Section 6 concludes.

2 Reconciling the Arrow effect with incumbent's innovation

In order to demonstrate the generality of the mechanism driving the emergence of multi-quality leaders in our model, I first isolate it within a partial equilibrium framework. I hence model R&D races meeting the most standard assumptions of the baseline Schumpeterian growth model (Barro and i Martin (2003), chapter 7; Acemoglu (2008), chapter 14).

More precisely, I consider the R&D investment decisions of firms aiming at entering a final good industry characterized by an array of quality-differentiated products. Each innovation increases the quality by a rung q, with the κ-th innovation being of quality q^{κ}. The successful researcher retaining the exclusive rights over the latest technology obtains a flow of monopoly profits $\pi(\kappa)$.[5] I assume that the probability to innovate $p(\kappa)$ in an

[5]Indeed, whether he needs to resort to limit pricing or can charge the unconstrained monopoly price,

industry where the highest quality currently available is q^κ depends linearly on the total expenditures over R&D $Z(\kappa)$: more precisely, I have $p(\kappa) = \psi(\kappa)Z(\kappa)$, with $\psi(\kappa)$ capturing the effect of the current technology position κ.[6] The expected value of an innovation is then $E[v(\kappa)] = \frac{\pi(\kappa)}{r+p(\kappa)}$, with r being the interest rate over time (I consider the steady state of such an economy, and hence assume r to be constant). I assume that both challengers and incumbents have the *possibility* to invest in R&D, and denote by $Z^c(\kappa)$ and $Z^i(\kappa)$ the respective amounts being invested.

In an industry where the highest quality currently available is q^κ, the standard, free-entry condition for challengers equates the costs incurred when engaging in R&D $Z^c(\kappa)$ and the expected value of innovating $p(\kappa)E[v(\kappa+1)]$:

$$Z^c(\kappa)\left(1 - \psi(\kappa)E[v(\kappa+1)]\right) = 0 \tag{1}$$

On the other hand, the Hamilton-Jacobi-Bellman equation of the incumbent deciding whether to invest in R&D or not is of the form:

$$rv(\kappa) = \max_{Z^i(\kappa)\geq 0} \{\pi(\kappa) - Z^i(\kappa) + \psi(\kappa)Z^i(\kappa)(E[v(\kappa+1)] - E[v(\kappa)]) - \psi(\kappa)Z^c(\kappa)E[v(\kappa)]\}$$

with the first order condition (f.o.c.) being:

$$(\underbrace{-1 + \psi(\kappa)E[v(\kappa+1)]}_{(*)} \underbrace{-\psi(\kappa)E[v(\kappa)]}_{(**)})Z^i(\kappa) = 0$$

The value of the $(*)$ term is null under the free-entry condition (1). The remaining term $(**)$ is negative, and represents the well-known "Arrow effect", capturing the fact that the incumbent would loose its current profits if it innovated a second time. We are hence confronted to the classic result that under free-entry, incumbents do not have any incentive to carry out research in a vertical framework, since they would cannibalize their own market in case of a successful innovation.[7]

This result relies on the "creative destruction" phenomenon at work in quality-ladder models: since a new quality has an objective advantage over all the previous ones, its producer can (and will) exclude all the other competitors from the market. However, the industrial organization literature studying competition and pricing decisions in vertically-differentiated markets has since long shown that quality differentiation *does not* preclude the survival of more than one quality and/or more than one producer. Indeed, provided there exist *differences in the willingness to pay for quality among consumers*, strategic pricing of firms in a situation of natural oligopoly or monopoly will lead to more than one

the successful innovator is systematically able to charge a price that will ensure him a monopoly position (Grossman and Helpman, 1991b; Aghion and Howitt, 1992)

[6] I hence do not impose decreasing returns, neither at the firm nor at the industry level.

[7] As already stated in our literature review, models where incumbents innovate have already been provided (Aghion et al., 2001; Segerstrom, 2007; Etro, 2008; Acemoglu and Cao, 2010). However, they all depart in one way or the other from the standard specification I outlined in my example.

quality being sold and consumed at the equilibrium. Such differences among consumers in the price they are ready to pay for a given quality are generated either by income differences among consumers displaying non-homothetic preferences[8] (Gabszewicz and Thisse, 1980; Shaked and Sutton, 1982), or by exogenously imposed different tastes for quality (Mussa and Rosen, 1978; Glass, 1997). In such a framework, Gabszewicz and Thisse (1980) as well as Shaked and Sutton (1982) have shown that competition among vertically-differentiated firms yields several qualities being sold at different prices at the equilibrium (the total number of qualities is however naturally limited by the existence of marginal production costs increasing along quality). Similarly, Mussa and Rosen (1978) have proved that a monopoly firm having at its disposal a whole product line and being unable to perfectly discriminate among heterogenous consumers[9] offers a whole menu comprising different qualities sold at different prices. To sum it up, "*vertical product differentiation refers to a class of products which cohabit simultaneously on a given market, even though customers agree on a unanimous ranking between them. (...)The survival of a low-quality product then rests on the seller's ability to sell it at a reduced price, (...) specializing in the segment of costumers whose propensity to spend is low, either because they have relatively lower income, or relatively less intensive preferences, than other costumers*" (Gabszewicz and Thisse, 1986).

I hence claim that *provided consumers display differences in their willingness to pay for quality*, the profits realized by a firm having a product line comprising two qualities are *superior* to the profits realized by a firm having the knowledge to produce only one quality level. Indeed, a firm being able to produce and sell two qualities will be able to better *discriminate* among consumers differing in their willingness to pay, capturing the incremental profits generated by charging a higher price to quality-loving consumers, while still offering a lower quality (charged at a lower price) to consumers less prone to value quality. In other words, I claim that in a framework allowing for differences in the willingness to pay to arise, the expected value of being the winner of the next innovation race is higher for the incumbent than for the challenger: $E[v_i(\kappa{+}1)] > E[v_c(\kappa{+}1)]$. Taking into account those different valuations of further innovating, the free-entry condition for challengers then becomes:

$$Z^c(\kappa)\,(1 - \psi(\kappa)E[v_c(\kappa+1)]) = 0 \tag{2}$$

while the HJB equation of the incumbent yields the following f.o.c.:

$$(\underbrace{-1 + \psi(\kappa)E[v_i(\kappa+1)]}_{>0}\underbrace{-\psi(\kappa)E[v_c(\kappa)]}_{(**)})Z^i(\kappa) = 0$$

[8]Indeed, income differences alone do not guarantee differences in the willingness to pay: in the case of homothetic preferences such as the standard quality-augmented CES utility function, the constant elasticity of substitution along income will lead poor and rich individuals to consume the same quality, but in different amounts.

[9]Perfect discrimination means that a monopolist can distinguish among consumers prior to any actual sale, and charge different prices to different consumers for the same good.

The negative cannibalization term (**) is now compensated by a positive term. The Arrow effect is hence a priori not incompatible with investment in R&D by incumbents any more. Indeed, as long as the incumbent has not fully exploited the price discrimination possibilities offered when having more than one quality at one's disposal, the free entry condition will *not* preclude a positive amount being invested in R&D by incumbents.

Having precisely identified the mechanism at work in a partial equilibrium framework, I now present an economy displaying the required feature, i.e. differences in the willingness to pay for quality among consumers. More precisely, I model non-homothetic preferences through unit consumption of the quality good, and incorporate this feature in an otherwise canonical quality-ladder framework displaying income inequality.

3 The model

3.1 Consumers

The economy is populated by a fixed number L of consumers that live infinitely and supply one unit of labor each period, paid at a constant wage w. While all consumers are identical with respect to their preferences and their labor income, they are assumed to differ with respect to asset ownership: more precisely, I assume a two-class society with rich (R) and poor (P) consumers being distinguished by their wealth $\omega_R(t)$ and $\omega_P(t)$.[10]

The share of "poor" consumers within the population is denoted by β. The extent of inequality within the economy is determined by this share, as well as by the repartition between rich and poor of the aggregate stock of assets within the economy $\Omega(t)$. $d \in (0, 1)$ is defined as the ratio of the value of the stock of assets owned by a poor consumer *relative* to the average per-capita wealth: $d = \frac{\omega_P(t)}{\Omega(t)/L}$. The wealth position of the rich can be computed for a given d and β, and we finally have $\omega_P(t) = d\frac{\Omega(t)}{L}$ and $\omega_R(t) = \frac{1-\beta d}{1-\beta}\frac{\Omega(t)}{L}$.

Current income $y_i(t)$ of an individual belonging to the group i ($i = P, R$) is then of the form:

$$y_i(t) = w + r(t)\omega_i(t) \tag{3}$$

with $r(t)$ being the interest rate.

It is important to keep in mind that the existence of such income differences is however not sufficient to generate differences in the willingness to pay of consumers, previously identified as crucial for our result (cf section 2). Indeed, in the case of a standard quality-augmented CES utility function, constant elasticity of substitution along income would guarantee that both poor and rich consumers end up consuming the same quality, but in

[10] All the results presented in the paper pertaining to investment in R&D by incumbents are robust under the alternative specification of inequality being generated through differences in income, i.e. through different endowments in labor efficiency units.

different proportions. The introduction of non-homotheticity (i.e. variation of the composition of the consumption bundle along income) in the preference structure is hence essential, and constitutes the only deviation of the framework presented here from the standard quality-ladder model.

The economy features a continuum of sectors indexed by s, which varies along the unit interval: $s \in [0,1]$. Within each sector, two types of final goods are available: a homogenous commodity and a differentiated good. Homogenous goods are produced with labor alone, and require a unit labor input of $1/w$. They are competitively priced and their price hence serves as the numeraire. Regarding the differentiated goods, it is assumed that at any date t, a sequence of qualities $q_j(s,t)$, $j = 0, -1, -2, ...$ exist and can be produced within each industry s, with $q_0(s,t)$ being the best quality available, $q_{-1}(s,t)$ the second-best, etc. Two successive quality levels differ by a fixed factor $k > 1$: $q_j(s,t) = k.q_{j-1}(s,t)$.

Consumers have additively separable preferences over their consumption of all goods, with the instantaneous utility function $\mathcal{U}_i(t)$ of a type i consumer being of the form:

$$\mathcal{U}_i(t) = \int_0^1 u_i(s,t)ds = \int_0^1 \ln c_i(s,t) + \ln q_{ij}(s,t)ds \tag{4}$$

By assumption, consumers value only **one unit** of each differentiated good.[11] For each industry s at each period t, an individual belonging to group i hence chooses to consume the quality level $q_j(s,t)$ that offers him the highest utility, considering its price $p(s,t,q_j(s,t))$. I denote this quality $q_{ij}(s,t)$, and the index of consumed qualities over industries $Q_i(t) = \int_0^1 q_{ij}(s,t)ds$. He then spends the rest of his income over the consumption of $C_i(t)$ units of homogenous commodities, with $C_i(t) = \int_0^1 c_i(s,t)ds$.

At time τ, the intertemporal decision problem of a type i consumer is to maximize:

$$\int_\tau^\infty \left(\int_0^1 \ln c_i(s,t) + \ln q_{ij}(s,t)ds \right) e^{-\rho(t-\tau)}dt$$
$$\text{s.t.} \quad \omega_i(\tau) + \int_\tau^\infty w e^{-r(t)(t-\tau)}dt \geq \int_\tau^\infty C_i(t)e^{-r(t)(t-\tau)}dt + \int_\tau^\infty P(t,Q_i(t))e^{-r(t)(t-\tau)}dt$$

with $P(t,Q_i(t)) = \int_0^1 p(s,t,q_{ij}(s,t))ds$ and ρ being the rate of time preference. Given an expected time path for both the interest rate $r(t)$ and the relation between quality and price $P(t,Q_i(t))$, it is then possible to determine the optimal time path of $C_i(t)$ (i.e. the consumption devoted to standardized commodities) and of $Q_i(t)$ (i.e. the chosen quality for each quality-differentiated good) for a consumer of type i.

Separability of utility (both over time and across goods) guarantees that for any given foreseen time path $P(t,Q_i(t))$ of expenditures devoted to the continuum of quality goods

[11] Unit consumption of the quality-differentiated goods ensures the non-homotheticity of the preference structure in this model. This particular way to model non-homotheticity is the most classic in qualitative choice models featuring strategic pricing of firms (Gabszewicz and Thisse, 1980; Shaked and Sutton, 1982). One could also have obtained differences in the willingness to pay by imposing exogenously different tastes for quality (Glass, 1997).

that does not exhaust life-time resources, the optimal path of consumption expenditures on homogenous commodities has to fulfill the standard first-order condition of such a maximization problem:

$$\frac{\dot{C}_i(t)}{C_i(t)} = r(t) - \rho \tag{5}$$

The optimal time path of $Q_i(t)$, on the other hand, cannot be characterized by a differential equation, since the quality choices are discrete. It is possible to notice however that within each industry s, the choice of the quality $q_{ij}(s,t)$ being consumed by a type i individual depends on the pricing decisions $p(s,t,q_{ij}(s,t))$ made by profit-maximizing firms. I hence set aside the discrete quality choices on the part of consumers until having defined the market and price structure for each of the quality sectors.

The focus of this article is on the balanced growth path properties of such a model. In such a product-innovation model (i.e. precluding any productivity improvement) fixing the wage level w and imposing unit consumption of the quality goods, the steady state is characterized by constant levels of wealth Ω and individual consumption of the standardized commodities C_i.[12] As as result we have that $r = \rho$ at the steady-state, and the time subscripts are dropped for the rest of the model exposition.

3.2 Market structure and pricing

The market for quality goods is non-competitive. Labor is the only input, with constant unit labor requirement $a < 1$.[13]

The quality goods being characterized by unit consumption and fixed quality increments, firms use prices as strategic variables. Firms know the shares of groups P and R in the population, the respective incomes y_R and y_P as well as the preferences of the consumers, but *cannot distinguish individuals by income.* Firms within each sector only consider their strategic interactions with other firms of the same sector, and do not internalize the impact of their pricing decisions on consumption allocation across sectors. Hence, for the sake of notational simplicity, the industry indices are momentarily dropped.

In order to describe the strategic decisions operated by firms within a given industry, it is necessary to define the "threshold" price $p^T_{i,\{j,j-m\}}$ for which a consumer belonging to group i is indifferent between quality j and quality $j-m$. Determining such a threshold price amounts to solving the following equality:

$$\ln(y_i - p^T_{i,\{j,j-m\}}) + \ln q_j = \ln(y_i - p_{j-m}) + \ln q_{j-m}$$

[12] As I will discuss later on, consumers still become better-off over time due to the quality improvement of the differentiated good and the resulting long-run growth of individual utility.

[13] Given the model assumes unit consumption of the quality goods, a necessarily has to be inferior to 1.

Considering the fact that $q_j = k^m q_{j-m}$, solving for $p^T_{i,\{j,j-m\}}$ in the above equality yields:

$$p^T_{i,\{j,j-m\}} = y_i \left(\frac{k^m - 1}{k^m} \right) + \frac{p_{j-m}}{k^m} \tag{6}$$

The price $p^T_{i,\{j,j-m\}}$ is the maximum price that the firm selling the quality j in industry s can charge to a type i consumer in order to have a positive market share, when facing the firm selling quality $j - m$. As one can see, this threshold price positively depends on the income y_i of consumer i, as well as on the price charged by the competitor p_{j-m}.

Having defined this threshold price, and along with Zweimuller and Brunner (2005), it is possible to establish that:

Lemma 1: *Within each industry $s \in [0, 1]$, if $p_j \geq wa$ holds for the price of some quality q_j, $j = -1, -2, ...$, then for the producer of any higher quality q_{j+m}, $1 \leq m \leq -j$, there exists a price $p_{j+m} > wa$, such that any consumer prefers quality q_{j+m} to q_j.*

Proof: For a given group of consumers i, $p^T_{i,\{j+m,j\}} = y_i \left(\frac{k^m-1}{k^m}\right) + \frac{p_j}{k^m}$ is a weighted average of y_i and p_j. Given the fact that only prices being below their income are taken into account by consumers i, we have that $p_j < y_i$, and we can hence conclude that $p^T_{i,\{j+m,j\}} > p_j$. Hence, it is always possible for the producer of the quality $j + m$ to set a price $p_{j+m} > p_j \geq wa$ such that $p_{j+m} \leq p^T_{i,\{j+m,j\}}$, i.e. such that quality q_{j+m} is preferred to quality q_j by the consumers of group i. This ends the proof. □

Hence, within each industry s, if we take for granted that a producer never sells its quality at a price below the unit production cost wa, it is always possible for the producer of the highest quality to drive all of its competitors out of the market while still making strictly positive profits. Along this result, any firm entering the market with a new highest quality q_0 has to consider the following trade-off concerning the pricing of its product: setting the highest possible price for any given group of costumers, vs. lowering its price in order to capture a further group of consumers. It is then possible to show that in an economy characterized by two distinct groups of consumers (R and P), we have:

Lemma 2: *Within each sector $s \in [0, 1]$, we have that at equilibrium,*
(1) The highest quality is produced,
(2) At most the two highest qualities q_0 and q_{-1} are actually produced,
(3) The equilibrium price p_{-1} fulfills $wa \leq p_{-1} \leq p^T_{P,\{-1,-2\}}$, with $p^T_{P,\{-1,-2\}}$ denoting the maximum price the producer of the q_{-1} quality can set in order to deter the producer of the q_{-2} quality from entry.

The proof is made in Zweimuller and Brunner (2005). The intuition is that since there are only two distinct groups of consumers, at most two distinct qualities can be sold, and at least one is always consumed, since it is assumed every individual buys one unit of the quality good. By Lemma 1, higher qualities drive out lower ones, hence the two qualities being still possibly active are q_0 and q_{-1}. At equilibrium, no firm can make a loss, hence the price p_j being charged for any quality q_j active on the market is necessarily superior or equal to the production cost wa. Finally, $p_{-1} \leq p^T_{P,\{-1,-2\}}$ follows from the fact that otherwise the producer of quality q_{-2} could enter the market.

As it can be seen from lemma 2, two different situations are possible for the equilibrium market structure and associated prices within each industry $s \in [0, 1]$: either only the top quality good q_0 is sold to both groups of consumers (groups P and R), *or* the top quality good is sold only to the rich consumers (group R) while the second best quality good is sold to the poor consumers (group P). Lemma 1 shows that the decision regarding the market structure belongs to the producer of the highest quality q_0, considering that he is always able to set a price that will drive its competitors out. This decision depends on two factors: (i) the extent of inequality within the economy, and (ii) whether the winner of the latest innovation race (who is also the producer of the highest quality good) is a former incumbent or challenger.

More precisely, each industry $s \in [0, 1]$ fluctuates between two states over time, with its position being determined by the identity of the winner of the last innovation race. The two possible states (SC) and (SI) can be characterized in the following way:

- **"Successful Challenger" (SC) state**: a challenger is the winner of the last R&D race, i.e. the new quality leader is *different* from the former quality leader. The new quality leader then only has the highest quality q_0 at its disposal.[14] One or two qualities can then be sold on the market, *depending on the pricing strategy chosen by the new quality leader* (which will itself depend on the wealth distribution in the economy). The market structure in this state can then either be a *monopoly* (only quality q_0 is sold), with the new quality leader charging a price that enables him to capture the whole market, or a *duopoly* (both qualities q_0 and q_{-1} are sold), with the new quality leader charging a higher price and serving only the upper part of the market, leaving the lower part to the second-best quality producer.

- **"Successful Incumbent" (SI) state**: the former quality leader, still carrying out R&D, is the winner of the last R&D race, and hence has two successive qualities at its disposal. According to lemma 2, the market structure is then necessarily a *monopoly*. However, as it will be demonstrated below, the quality leader will then offer two different quality/price bundles in order to *discriminate* between the groups P and R of the population (Mussa and Rosen, 1978), and hence both qualities q_0 and q_{-1} are sold.

[14] Indeed, I assume that being the inventor of a quality is necessary to be able to produce it; that is, a successful challenger knows how to produce the quality q_0, but cannot undertake the production of the now second-best quality q_{-1}, for which the former leader keeps a comparative advantage.

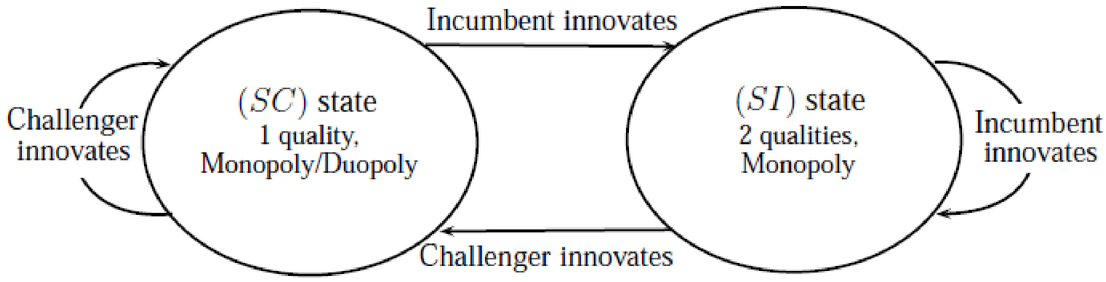

Figure 1: Two possible states

Figure 1 illustrates the fluctuations between the two possible states over time. I now describe in more details the possible market structures, prices and associated profits in the two existing states.

3.2.1 Prices and profits in the (SC) state

As already stated, the market structure in the (SC) state is either a monopoly or a duopoly, depending on the level of inequality within the economy.

Case 1: Monopoly price regime in the (SC) state

It corresponds to the case where the wealth structure makes it optimal for the quality leader to set a price enabling him to sell the unique quality he has at its disposal to the entire market, driving the former quality leader out of the market.

$p^T_{i,\{0,-1\}}$ is the maximum price the producer of quality q_0 can set in order to capture the consumers of the group i for a given price p_{-1} of quality q_{-1}. It is first possible to notice that setting a price that captures the consumers belonging to the group P automatically ensures that the rich consumers will consume the highest quality q_0 too, since $p^T_{i,\{0,-1\}}$ is increasing along y_i. Hence, the optimal price chosen by a quality leader willing to capture the whole market is $p^T_{P,\{0,-1\}}$, given that the producer of quality q_{-1} engages in marginal cost pricing (i.e. $q_{-1} = wa$). I denote by p_P the price being then charged by the quality leader, with the associated profits π_M:

$$p_P = y_P \frac{k-1}{k} + \frac{wa}{k} \tag{7}$$

$$\pi_M = L(p_P - wa) \tag{8}$$

Case 2: Duopoly price regime in the (SC) state

It corresponds to the case where the wealth structure makes it optimal for the new quality leader to set a price capturing only the upper part of the market, abandoning the lower part to the producer of the second-best quality. The two highest qualities q_0 and q_{-1} are then sold at the equilibrium, being produced by two different firms.

Zweimuller and Brunner (2005) have defined a possible equilibrium in that case, under the condition on the punishment strategies of the infinitely repeated pricing game that no firm is punished if it changes its price without affecting the other firm's profit (Proof: cf Zweimuller and Brunner (2005), p. 242). At this equilibrium, the new quality leader optimally chooses to charge the highest possible price enabling him to capture the group of rich consumers $p^T_{R,\{0,-1\}}$, given the expected strategy of the producer of the second-best quality. The former quality leader charges the highest possible price enabling him to capture the poor group of consumers $p^T_{P,\{-1,-2\}}$, given that the producer of quality q_{-2} engages in marginal cost pricing.[15]

I call p_R the price being charged by the new quality leader for the highest quality, while p_P is the price charged by the follower for the second-best quality. I also define the associated profits π_L for the quality leader and π_F for the producer of the second-best quality:

$$p_R = y_R \frac{k-1}{k} + y_P \frac{k-1}{k^2} + \frac{wa}{k^2} \tag{9}$$

$$\pi_F = \beta(p_P - wa), \qquad \pi_L = (1-\beta)(p_R - wa) \tag{10}$$

Selection of the equilibrium price regime.

Having described the prices and profits for both possible market structures, I still need to define under which parametric conditions on wealth distribution each price regime occurs. It can be however be seen from the expressions of π_M, π_L and π_F that they depend on the endogenous equilibrium value of overall wealth Ω. I will hence comment the parametric conditions governing the occurence of each regime once I have fully defined the steady state equilibrium of our economy (section 5.3).

3.2.2 Prices and profits in the (SI) state

Two qualities are systematically sold in the (SI) state. Indeed, a leader having at its disposal two successive qualities and facing two groups of consumers having different levels of income will always find it optimal to offer two distinct price-quality bundles in order to maximize its profit (Mussa and Rosen, 1978). The market structure is then a **monopoly**. The price charged by the monopolist for its second-best quality will be the maximal price enabling him to capture the poor group of consumers $p^T_{P,\{-1,-2\}}$, given that the producer of quality q_{-2} engages in marginal cost pricing. The price charged for the highest quality will then be the maximal price $p^T_{R,\{0,-1\}}$, given $p^T_{P,\{-1,-2\}}$. Those are respectively the prices p_P and p_R already previously defined. The profits π_{SI} of such a discriminating monopolist

[15] It is important to insist once more on the fact that the strategy chosen in this case by the producer of quality q_{-1} is only made possible because of the decision of the new quality leader to charge a higher price, capturing only the upper part of the market: had the new leader found optimal to charge $p^T_{P,\{0,-1\}}$ instead of $p^T_{R,\{0,-1\}}$ for quality q_0, the former leader would have been driven out of the market and we would be back to case 1 (monopoly price regime).

are then simply of the form:

$$\pi_{SI} = \beta L(p_P - wa) + (1-\beta)L(p_R - wa) = \pi_F + \pi_L \tag{11}$$

Having defined the possible market structure, prices and profits in every possible state, I can now move to the description of the R&D process, which is the engine of growth in our model.

3.3 R&D sector

Firms carry out R&D in order to discover the next quality level. Two types of firms have the possibility to engage in R&D races: the current quality leader (incumbent), and followers (challengers). I assume free entry, with every firm having access to the same R&D technology. Innovations are random, and occur for a given firm i according to a Poisson process of hazard rate ϕ_i. Labor is the only input, and I assume constant returns to R&D at the firm level: in order to have an immediate probability of innovating of ϕ_i, a firm needs to hire $F\phi_i$ labor units, F being a positive constant inversely related to the efficiency of the R&D technology.[16]

I define v_C as the value of a challenger firm, v_{SC} as the expected present value of a quality leader having innovated once, and v_{SI} as the expected present value of a quality leader having innovated twice. Free entry and constant returns to scale imply that R&D challengers have no market value, whatever state the economy finds itself in: $v_C = 0$. Free entry of challengers in the successive R&D races also yields the traditional equality constraint between expected profits of innovating for the first time $\phi_C v_{SC}$ and engaged costs $\phi_C wF$ (free entry condition):

$$v_{SC} \quad = \quad wF \tag{12}$$

The incumbent on the other hand participates to the race while having already innovated at least once, and hence being the current producer of the leading quality in case (SC)/of the two highest qualities in case (SI).
In industries being currently in the (SC) state, the incumbent faces the following Hamilton-Jacobi-Bellman equation:

$$\rho v_{SC} \quad = \quad \max_{\phi_{I,SC} \geq 0} \{\pi_M - wF\phi_{I,SC} + \phi_{I,SC}(v_{SI} - v_{SC}) + \phi_C(v_F - v_{SC})\} \tag{13}$$

The incumbent in the (SC) state earns the profits π_{SC} (the precise form of π_{SC} depending on the equilibrium price regime and corresponding market structure in the (SC) state), and incurs the R&D costs $wF\phi_{I,SC}$. With instantaneous probability $\phi_{I,SC}$, the leader innovates once more, the industry jumps to the state (SI), and the value of the leader

[16]The condition of constant marginal costs of R&D can however be loosened, and our results are robust to the introduction of decreasing returns to R&D investments, both at the industry and at the firm level.

(now producing and selling two distinct qualities) climbs to v_{SI}.[17] However, with overall instantaneous probability ϕ_C, some R&D challenger innovates, and the quality leader falls back to being a follower: its value drops to v_F (again, the precise form of v_F depends on the market structure in the (SC) case). The industry then remains in the state (SC), and only one quality is produced.

In the (SI) state, the incumbent faces the following Hamilton-Jacobi-Bellman equation:

$$\rho v_{SI} = \max_{\phi_{I,SI}\geq 0}\{\pi_{SI} - wF\phi_{I,SI} + \phi_{I,SI}(v_{SI} - v_{SI}) + \phi_C(v_F - v_{SI})\} \tag{14}$$

The incumbent in the (SI) state earns the profits π_{SI} of a monopolist being able to discriminate between rich and poor consumers by offering two distinct price/quantity bundles. He incurs the R&D costs $wF\phi_{I,SI}$. With instantaneous probability $\phi_{I,SI}$, the incumbent innovates once more, in which case its value remains v_{SI}, since we have established with Lemma 2 that at most two successive quantities are sold at equilibrium. Hence, the incumbent will still be the producer of the two qualities being sold, *but* he will drive himself out of the market for the former quality q_{-1}, that has become quality q_{-2} with the latest quality jump. The industry then remains in state (SI). With instantaneous probability ϕ_C,[18] some R&D follower innovates, and the quality leader then falls back to being an R&D challenger: its value falls to v_F. The industry then jumps to the state (SC), and only the new highest quality is sold by the latest successful innovator.

In both states, the incumbent firm chooses its R&D effort so as to maximize the right-hand side of its Bellman equation. (13) and (14) then yield the following first order conditions:

$$v_{SI} - v_{SC} = wF \tag{15}$$
$$-wF = 0 \quad \Rightarrow \quad \phi_{I,SI} = 0 \tag{16}$$

Hence, it is possible to obtain a relation between the R&D costs and the incremental value that would result from innovating in both states. Given that the incremental value of a further innovation for an incumbent in the (SI) state is null in an economy with only two distinct population groups, it immediately follows that the optimal investment in R&D in that state is zero.[19] From then on, I hence refer to the investment in R&D of the incumbent firm in the (SC) state as simply ϕ_I.

[17] Accordingly to the crucial condition identified in Section 2 in order to generate innovation by incumbent, this expected value of innovating for a second time v_{SI} is different from the expected value of innovating for the first time v_{SC}.

[18] The challengers invest the same amount in the R&D sector ϕ_C in both states (SC) and (SI), since they face the same expected reward v_{SC} in both cases: a successful innovation by a challenger indeed always brings the industry back to state (SC).

[19] I believe it would be possible to generalize this model to more than two groups of population, or a continuum of quality valuations as in Mussa and Rosen (1978). Intuitively, the incumbent would then keep investing in R&D beyond the second innovation in a row.

Using the optimality constraints (15) and (16) in (13) and (14), the following expressions for the expected values v_{SC} and v_{SI} are obtained:

$$v_{SC} = \frac{\pi_{SC} + \phi_C v_F}{\rho + \phi_C} \quad (17)$$

$$v_{SI} = \frac{\pi_{SI} + \phi_C v_F}{\rho + \phi_C} \quad (18)$$

I am now left to detail the possible values taken by v_F and π_{SC}, which depend on whether the equilibrium market structure in the (SC) state (depending on the income distribution within our economy) is a monopoly (Case 1) or a duopoly (Case 2).

- *Case 1 - Monopoly price regime in the* (SC) *state.* We then have $\pi_{SC} = \pi_M$, and the value v_F of a firm that has been leap-frogged by a challenger is null: indeed, the new leader charges a price that captures the whole market, leaving no room for the producer of the second-best quality. The previous leader then falls back to a challenger status, and we have $v_F = v_C = 0$. Using (12), (15), (17) and (18), it is possible to obtain the two following equalities between incurred costs and expected profits when the equilibrium market structure in the (SC) state is a monopoly:

$$wF = \frac{\pi_M}{\rho + \phi_C} \quad (19)$$

$$2wF = \frac{\pi_{SI}}{\rho + \phi_C} \quad (20)$$

- *Case 2 - Duopoly price regime in the* (SC) *state.* We then have $\pi_{SC} = \pi_L$, and the value v_F of the previous leader (now producer of the second-best quality) is strictly positive, since in Case 2 the new leader has optimally chosen to charge a price capturing only the upper part of the market. This "follower" faces the following Hamilton-Jabobi-Bellman equation:

$$\rho v_F = \max_{\phi_F \geq 0} \{\pi_F - wF\phi_F + \phi_F(v_{SC} - v_F) + (\phi_C + \phi_I)(v_C - v_F)\} \quad (21)$$

  The follower sells the second-best quality to the lower part of the market, earning the profits π_F. He incurs the R&D costs $wF\phi_F$. With instantaneous probability ϕ_F, he is successful in innovating once more, and its value jumps back to v_{SC}. With instantaneous probability $\phi_C + \phi_I$, either some R&D follower or the current quality leader innovates, and the follower is definitively driven out of the market: its value falls to $v_C = 0$. Solving for an interior solution to this maximization problem yields the condition $v_{SC} - v_F = wF$, which, combined with condition (12), would imply $v_F = 0$. This is however not the case, since the follower's profits π_F when the market structure in the (SC) state is a duopoly are strictly positive. We then necessarily have $\phi_F = 0$. Plugging this value back into (21), we obtain that $v_F = \frac{\pi_F}{\rho + \phi_C + \phi_I}$. Using (12), (15), (17) and (18), the two following equalities between incurred costs and expected profits are obtained when the equilibrium market structure in the (SC)

state is a duopoly:

$$wF = \frac{\pi_L + \phi_C \left(\frac{\pi_F}{r+\phi_C+\phi_I}\right)}{\rho + \phi_C} \tag{22}$$

$$2wF = \frac{\pi_{SI} + \phi_C \left(\frac{\pi_F}{r+\phi_C+\phi_I}\right)}{\rho + \phi_C} \tag{23}$$

4 Steady state equilibrium

4.1 Labor market equilibrium

I denote by μ_{SC} the fraction of all industries for which the latest innovator is a challenger (i.e. being in the (SC) state), and by μ_{SI} the fraction of all industries for which the last innovation race has been won by an incumbent (i.e. being in the (SI) state). Those measures sum to one: $\mu_{SC} + \mu_{SI} = 1$. Additionally, since the focus of this article is the analysis of the steady state, the flows in must equal the flows out of each state, so that each fraction remains constant: the further condition $\phi_C \mu_{SI} = \phi_I \mu_{SC}$ then holds. Indeed, for each industry being in the (SC) state, the probability to exit this state is equal to the probability ϕ_I of an incumbent innovating; for each industry being in the (SI) state, the probability to enter the (SC) state corresponds to the probability ϕ_C of a challenger innovating. Combining the two conditions, I finally obtain the following values for μ_{SC} and μ_{SI}:

$$\mu_{SC} = \frac{\phi_C}{\phi_I + \phi_C}, \qquad \mu_{SI} = \frac{\phi_I}{\phi_C + \phi_I}$$

In the industries being in the (SC) state, both incumbents and challenger invest in research activities, with $F(\phi_I + \phi_C)$ being the total number of people hired in the R&D sector. On the other hand, in industries being in the (SI) state, only the challengers carry out research, and $F\phi_C$ units of labor are devoted to R&D activities. Since I imposed unit consumption of the quality good and the same marginal cost of production regardless of the quality level, aL units of labor are devoted to the production of the quality good in each industry. Finally, $L/w(\beta(y_P - p_P) + (1-\beta)(y_R - p_{SC}))$ are the units of labor devoted to the production of the standardized good in the industries being in the (SC) state (where the price p^R_{SC} being charged to the rich consumers depends on the market structure in the (SC) state), while $L/w(\beta(y_P - p_P) + (1-\beta)(y_R - p_R))$ units are devoted to it in the industries being in the (SI) state.

The following equation then describes the equilibrium on the labor market:

$$\begin{aligned} L &= \frac{\phi_C}{\phi_I + \phi_C}\left[F(\phi_I + \phi_C) + aL + (L/w)(\beta(y_P - p_P) + (1-\beta)(y_R - p^R_{SC}))\right] \\ &+ \frac{\phi_I}{\phi_I + \phi_C}\left[F\phi_C + aL + (L/w)(\beta(y_P - p_P) + (1-\beta)(y_R - p_R))\right] \end{aligned} \tag{24}$$

It will prove convenient to express (24) in terms of profit flows. Multiplying both sides by w, replacing y_P and y_R by their respective values, splitting waL into $\beta waL + (1-\beta)waL$ and rearranging terms, we finally get:

- *Case 1 - Monopoly price regime in the* (SC) *state.* We have $p_{SC}^R = p_P$, and obtain the following labor equilibrium equation:

$$wF\phi_C + \frac{\phi_C}{\phi_I + \phi_C}(wF\phi_I - \pi_M) + \rho\Omega - \frac{\phi_I}{\phi_I + \phi_C}\pi_{SI} = 0 \tag{25}$$

- *Case 2 - Duopoly price regime in the* (SC) *state.* We have $p_{SC}^R = p_R$, and obtain the following labor equilibrium equation:

$$wF\phi_C + \frac{\phi_C}{\phi_I + \phi_C}wF\phi_I + \rho\Omega - \pi_{SI} = 0 \tag{26}$$

4.2 Steady state analysis

The endogenous variables at the steady state are the profits accruing to the active producers in the two possible states, the overall wealth Ω, as well as the R&D investment decisions of the incumbent ϕ_I and the challengers ϕ_C. Since the number of active producers as well as the shape of their profits depends on whether we have a monopoly or a duopoly market structure in the (SC) state, two cases have to be distinguished in the definition of the steady state:

- *Case 1 - Monopoly price regime in the* (SC) *state.* The 5 equations defining the economy steady state are the profits realized by the monopolist in both possible states (8) and (11), the equality constraints between the incurred R&D costs and the expected value of an innovation in both states (19) and (20), as well as the labor market equilibrium condition (25).

- *Case 2 - Duopoly price regime in the* (SC) *state.* The 5 equations defining the economy steady state are the profits realized by the active producers in both possible states (10) and (11), the equality constraints between the incurred R&D costs and the expected value of an innovation in both states (22) and (23), as well as the labor market equilibrium condition (26).

Proposition 1 (Existence and uniqueness of a steady state equilibrium):

- (1) *Monopoly price regime in the* (SC) *state: under the parametric condition* $F < \frac{(1-a)L(k+\beta-1)}{\rho(k-d(2k+\beta-1))} < 2F$*, the system formed by equations* (8)*,* (11)*,* (19)*,* (20) *and* (25) *has a unique solution in* $(\pi_M, \pi_{SI}, \phi_C, \phi_I, \Omega)$*, all strictly positive.*

- (2) *Duopoly price regime in the* (SC) *state: under certain parametric conditions (cf. Appendix A), the system formed by equations* (10)*,* (11)*,* (22)*,* (23) *and* (26) *has a unique solution in* $(\pi_L, \pi_S I, \phi_C, \phi_I, \Omega)$*, all strictly positive.*

Proof: **(1) Monopoly price regime in the (SC) state.** (8) and (11) define π_M and π_{SI} are linear functions of Ω. Substituting for π_M and π_{SI} in (19) and (20), it is hence possible to obtain a linear system of 2 equations with 2 unknowns, jointly determining the values of Ω and ϕ_C. Substituting for the obtained values, (25) uniquely determines the value of ϕ_I. The following analytical solutions are obtained:

$$\phi_C = \frac{(1-d)(1-a)(k-1)L}{F(k-d(2k+\beta-1))} - \rho \tag{27}$$

$$\Omega = \frac{(1-a)wL(k+\beta-1)}{\rho(k-d(2k+\beta-1))} \tag{28}$$

$$\pi_M = \left(\frac{k-1}{k}\right)(1-a)wL + d\left(\frac{k-1}{k}\right)\rho\Omega \tag{29}$$

$$\pi_{SI} = \left(\frac{k-1}{k^2}\right)(1-a)(k+1-\beta)wL + \left(\frac{k-1}{k^2}\right)(k+(1-\beta)d)\rho\Omega \tag{30}$$

$$\phi_I = \frac{\phi_C(\Omega - wF)}{2wF-\Omega} = \psi(\phi_C, \Omega) \tag{31}$$

It is then necessary to impose the following condition on Ω for ϕ_I to be positive (which is also sufficient to ensure $\phi_C > 0$):

$$wF < \Omega < 2wF \quad \Leftrightarrow \quad F < \frac{(1-a)L(k+\beta-1)}{\rho(k-d(2k+\beta-1))} < 2F$$

This ends the proof.

(2) Duopoly price regime in the (SC) state. cf Appendix A.

I have hence demonstrated that in an economy where disparities in purchasing power exist, incumbents have an incentive to keep investing in R&D beyond their first successful innovation, because of the positive increment in profits that exists when innovating for a second time. The result is the **endogenous emergence of multi-quality leaders in a dynamic quality-ladder model**. This result can be commented in the light of the microeconomic literature analyzing price discrimination by a monopolist having at its disposal a product range including different quality levels. In such a context, Mussa and Rosen (1978) have demonstrated that serving costumers who place smaller valuations on quality creates **negative externalities** for the monopolist, preventing him from capturing the maximum costumer surplus from those who have a stronger taste for quality. In their static framework, the multi-quality monopolist then internalizes the existing negative externalities by inducing less enthusiastic consumers to buy lower quality items charged at a lower price, opening the possibility of charging higher prices to more adamant buyers of high quality units. In our dynamic model with endogenous innovation, the monopolist only has access to as many qualities as R&D races he has won: the negative externalities stemming from having to serve two distinct groups of consumers having different quality valuation is then internalized by expanding the line of product towards *higher* (and not lower) qualities, i.e. through R&D investment.

Another implication of this result is that in the case where there exist wealth disparities within an economy, positive investment in R&D by quality leaders is obtained with complete equal treatment in the R&D field between the incumbent patentholder and the challengers, as well as without any concavity in the R&D cost function. I am indeed modeling constant returns to R&D investments, and not allowing for any R&D cost advantage of the incumbent over the followers (Segerstrom and Zolnierek, 1999; Segerstrom, 2007) or any sequentiality in the patent races (Etro, 2008). This model hence exemplifies the existence of so far overlooked incentives for innovation by incumbent stemming from the demand structure rather than from the supply side (i.e. R&D sector characteristics and R&D capabilities of challenger and incumbent firms).

I have demonstrated in a dynamic, general equilibrium framework that the existence of differences in the willingness to pay for quality among consumers is sufficient to account for the existence of multi-quality leaders, since income disparities generate (1) the survival of more than one quality at the equilibrium, and (2) positive investment in R&D activities by incumbents. Such a model makes it possible to study the specific innovation incentives of vertically differentiated multi-product firms. In particular, I believe I am able to extend and deepen the study of the impact of income inequality on long term economic growth. This will be the aim of the next section. Since the impact of income distribution on growth depends on the price regime in the (SC) state, I however first comment the parametric conditions governing the choice of the equilibrium price regime when a challenger has won the latest innovation race.

4.3 Selection of the equilibrium price regime in the (SC) state

As noted in section 3.1, the selection of the (deterministic) equilibrium market structure in the (SC) state is up to the winner of the latest innovation race (i.e. the quality winner), who chooses the optimal price regime considering the distribution of wealth in the economy. More formally, the decision will be taken by the new leader comparing the expected profits in both cases, assuming the latter anticipates correctly the overall wealth as well as the R&D investment rates in both cases. The condition for the leader to choose a monopoly rather than a duopoly market structure is hence of the form:

$$\frac{\pi_M(\Omega^M)}{\rho+\phi_C^M}+\phi_I^M\frac{\pi_{SI}(\Omega^M)}{\rho+\phi_C^M}>\frac{\pi_L(\Omega^D)+\phi_C^D\frac{\pi_F(\Omega^D)}{\rho+\phi_C^D+\phi_I^D}}{\rho+\phi_C^D}+\phi_I^D\frac{\pi_{SI}(\Omega^D)+\phi_C^D\frac{\pi_F(\Omega^D)}{\rho+\phi_C^D+\phi_I^D}}{\rho+\phi_C^D}$$

with the supercript M (respectively D) describing the value taken by the endogenous variables in the case of a monopoly (resp. duopoly) price regime in the (SC) state. Although this condition might seem complex, constant returns to R&D effort at the firm level enable us to simplify this expression using (17) and (18). Indeed, whatever the equilibrium price regime in the (SC) state, expected incremental values $\phi_C^i v_{SC}$ and $\phi_I^i(v_{SI}-v_{SC})$ of a first and a second innovation have to be equal to the incurred costs, i.e. $\phi_C^i wF$ and $\phi_I^i wF$. The above condition then simplifies to $wF + 2wF\phi_I^M > wF + 2wF\phi_I^D$, i.e. $\phi_I^M > \phi_I^D$. Determining the equilibrium market structure in the (SC) state hence amounts to comparing the investment by incumbent in both possible price regimes. Such a condition implies that the leader systematically chooses the market structure ensuring him the greatest probability of reaching the status of "discriminating" monopolist, i.e. of endogenizing the negative externalities stemming from wealth inequality. It is important to keep in mind that this however does not amount to choosing the case displaying the highest long-term growth rate, since part of the overall R&D effort is carried out by challengers, and this conditions gives no information on the respective size of ϕ_C^M and ϕ_C^D.

5 Distribution of income and long-term growth

I hence provide a framework that exemplifies the impact of income distribution (through the resulting differences in willingness to pay for quality among consumers) on the emergence and the behavior of multi-quality leaders. Such a model can then also be used to deepen our understanding of the existing interactions between income distribution and long-run growth operating through the demand market (Zweimuller, 2000; Foellmi and Zweimuller, 2006; Zweimuller and Brunner, 2005). Indeed, Foellmi and Zweimuller (2006) have shown that a mean-preserving spread of income could increase the long-run rate of growth in a *horizontal* differentiation context. More closely related to our model, Zweimuller and Brunner (2005) have shown that a rising level of inequality systematically decreases the long-term growth in a quality-ladder framework. However, as already previously commented, they preclude any investment by incumbents, hence only capturing part of the influence of income distribution on innovation incentives. I provide a more complete picture, making it possible to not only make predictions on the overall growth rate, but also to comment on the allocation of the R&D activities among different possible actors (i.e. incumbents and challengers).

I first note that in this model, consumers become better off due to the successive improvements of the quality consumption good, and hence the economy growth rate is linked to the innovation intensity of both challengers and leaders. More precisely, we have that the long-run growth rate γ can be expressed in the following way: $\gamma = (\ln k)\phi_C(1 + \frac{\phi_I}{\phi_I + \phi_C})$. In the following analysis, I then consider two types of variations in the extent of wealth disparities and their impact on R&D investment and growth: (a) an increase in d for a given β (i.e. a mean-preserving redistribution from the rich to the poor consumers), and (b) an increase in β for a given d (i.e. an increasing concentration of wealth among a small group of people).

I obtain analytical results in the case we have a monopoly price regime in the (SC) state:

Proposition 2 (Wealth distribution and long-term growth):
When the equilibrium market structure is a monopoly in the (SC) state, we have the following comparative statics for varying values of β and d.

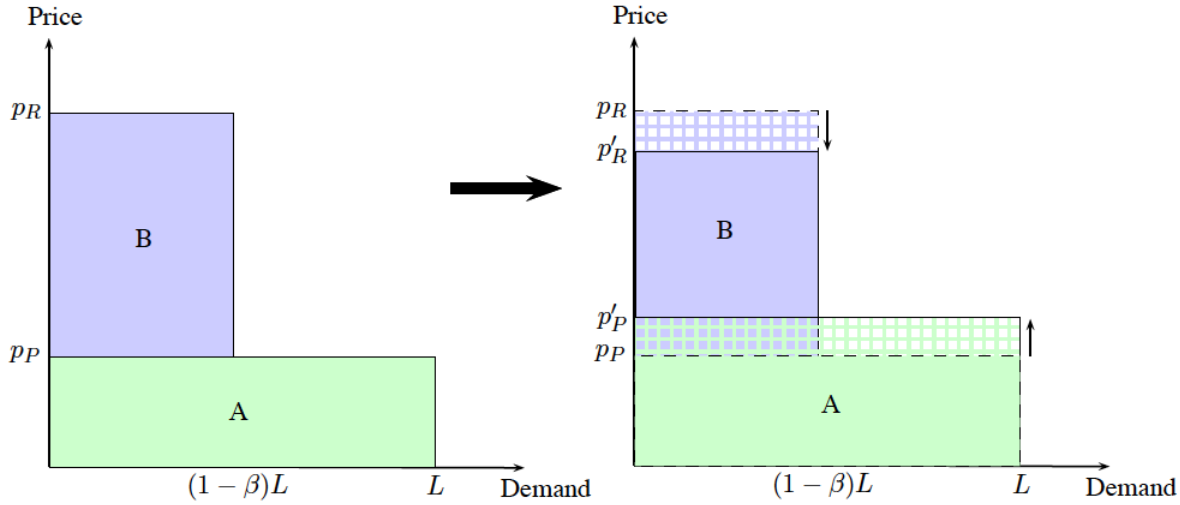

Figure 2: Effects of a shock on d (for Ω constant)

- *(a) Effect of an increase in the relative wealth of poor consumers d: the challengers' innovation rate ϕ_C, the incumbent's innovation rate ϕ_I, the overall wealth Ω and the long-run growth rate γ increase along d.*
- *(b) Effect of a mean-preserving increase in the population share of the poor β: the challengers' innovation rate ϕ_C, the incumbent's innovation rate ϕ_I, the overall wealth Ω and the long-run growth rate γ increase along β.*

Proof: Full analytical expressions for the comparative statics concerning ϕ_C and Ω can be obtained from the expressions (27) and (28):

$$\begin{aligned}
\frac{\partial \phi_C}{\partial d} &= \frac{(1-a)(k-1)L(k-1+\beta)}{F(k-d(2k-1+\beta))^2} > 0 \\
\frac{\partial \Omega}{\partial d} &= \frac{(1-a)wL(k-1+\beta)(2k-1+\beta)}{\rho(k-d(2k-1+\beta))^2} > 0 \\
\frac{\partial \phi_C}{\partial \beta} &= \frac{(1-a)(1-d)d(k-1)L}{F(k-d(2k-1+\beta))^2} > 0 \\
\frac{\partial \Omega}{\partial \beta} &= \frac{(1-a)(1-d)kwL}{\rho(k-d(2k-1+\beta))^2} > 0
\end{aligned}$$

And since we have $\phi_I = \psi(\phi_C, \Omega)$ with $\psi'_\Omega > 0$ and $\psi'_{\phi_C} > 0$, an increase in Ω and ϕ_C (whether it be following an increase in d or in β) necessarily entails an increase in ϕ_I. Also, substituting for the value of ϕ_I as a function of ϕ_C and Ω, the long-run growth rate expression simplifies to: $\gamma = \frac{\Omega \phi_C}{wF} = \xi(\phi_C, \Omega)$. Hence, an increase in both Ω and ϕ_C necessarily increases long-term growth. This ends the proof.□

(a) Let us first comment the effects of an increase in d when we have a monopoly price regime in the (SC) state. I first note that such a rise in the ratio of the wealth of a poor consumer *relative* to the average per-capita wealth leads to a *decrease* in the level of inequality. A simple intuition for the positive variation of ϕ_C in the case of an increase in d can then be found by considering the variations in expected gains of successfully innovating for the first time. For a given level of wealth Ω, an increase in d has a *positive price effect* on the profits of a successful challenger, since he can charge a higher price and still capture the whole market (the critical income in the (SC) state is the income of poor households): considering Figure 2, area A increases.

The intuition for the variation of ϕ_I following a shock on d can be found considering the labor equilibrium equation for a fixed, given value of overall wealth Ω. The increase in ϕ_C we have identified following an increase in d can only be obtained by reallocating part of the labor force from the production of the standardized good to R&D activities (indeed, the unit consumption assumption pins down the amount of labor devoted to the production of the quality good to a fixed amount). More precisely, since the price p_P has increased, both the poor and the rich will devote less of their overall income to the consumption of the standardized good in the (SC) state. In the (SI) state on the other hand, the decrease in p_R implies that while the poor have decreased their consumption of the standardized good, the rich have been able to increase theirs. It hence means that the extent of the labor force reallocation to R&D activities following a shock on d is stronger for industries in the (SC) state than for those in the (SI) state. Since incumbents only invest in R&D activities in industries being in the (SC) state, it immediately follows that ϕ_I increases following a positive variation of d.

Finally, the increase in overall wealth Ω helps us reconciling this increase in ϕ_I with the shrinking area B of Figure 2. Indeed, for a given level of wealth Ω, the expected increment in profits when innovating for a second time has decreased following a rise in d. However, the increase in Ω more than offsets the redistribution from the rich to the poor, and p_R ends up increasing as well following an increase in d, yielding a positive variation of the increment of innovating a second time following a rise in d.

(b) I now move to commenting the effects of an increase in β when we have a monopoly price regime in the (SC) state.

I first note that a mean-preserving rise in the share of the population being poor β corresponds to a higher concentration of wealth among a small group of people. Indeed, it implies an increase in the *relative* income of a rich consumer ($\frac{\partial d_R}{\partial \beta} = \frac{1-d}{(1-\beta)^2} > 0$): there are more poor with the same income, and fewer rich with more income. The straightforward variation in that case is the resulting increase in ϕ_I: indeed, for a given level of overall wealth Ω, such a concentration of wealth among rich consumers yields an increase in the increment of expected profits following a second successful innovation (p_R increases, and p_P remains at the same level). In Figure 3, area B increases.

The intuition concerning the variation of ϕ_C can then be found considering the labor market equilibrium condition. For a fixed level of wealth Ω, such an increase in p_R (and no variation in p_P, since the income of the poor $d\frac{\Omega}{L}$ has not been impacted) necessarily results in a decrease in the overall amount of income being devoted to the consumption

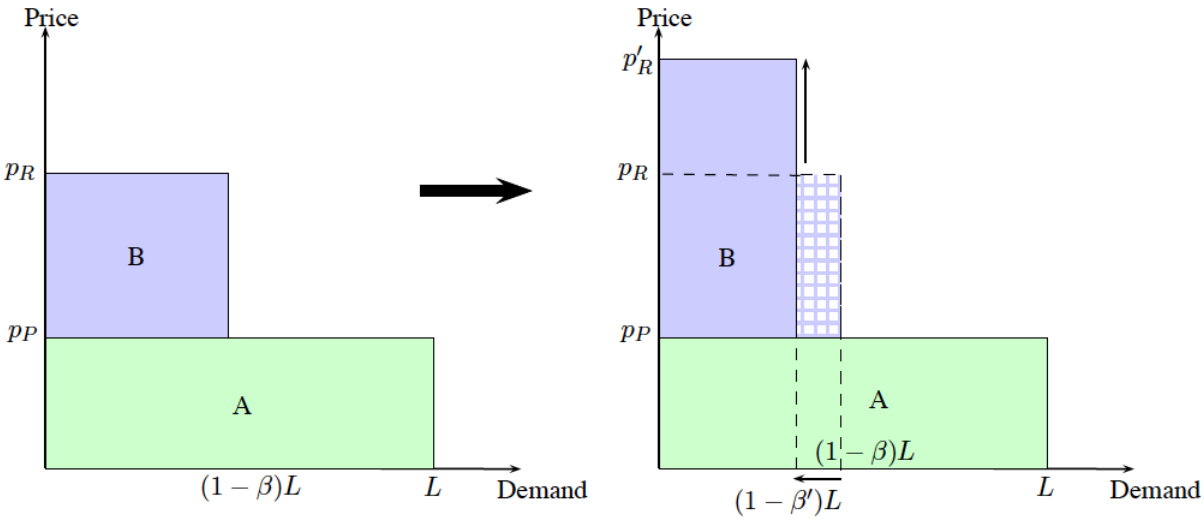

Figure 3: Effects of a shock on β (for Ω constant)

of the standardized good. Since in the (SI) state only challengers carry out research, this reallocation of labor towards R&D necessarily yields an increase in ϕ_C.

Finally, the increase in overall wealth Ω helps us reconciling this increase in ϕ_C with the constant area A of Figure 3. Indeed, this increase in Ω finally yields a positive variation of the increment of innovating a first time following a rise in β, hence justifying the increase in R&D investment of challengers.

In the case of a duopoly price regime in the (SC) state, the absence of explicit analytical expression of the different endogenous variables (cf. Appendix A) leads us to resort to numerical simulations. I carry out a sensitivity analysis along a wide array of values for parameters F, r, a and k, and the following numerical regularities emerge:

- *Numerical finding 1*: In Case 2, an increase in d decreases ϕ_I and Ω but increases ϕ_C, leading to an increase of long-term growth under a wide array of parametric cases.

- *Numerical finding 2*: In Case 2, an increase in β generates a decrease of ϕ_I, an increase of ϕ_C and non-monotonous variations of Ω under a wide array of parametric cases. The effect on long-run growth is ambiguous.

Several conclusions can be derived from the results presented in this section. First, redistributive policies leading to a reduction in the wealth gap between rich and poor (i.e. an increase in d) are systematically beneficial for long-term growth. Indeed, whether we have a monopoly or a duopoly market structure in the (SC) state, our analytical results and our simulations both show that an increase in d (corresponding to a decrease in the level of wealth inequality) leads to an increase in the long-run growth rate of the economy. This result is in line with the conclusions of Zweimuller and Brunner (2005), who have shown in a similar set-up that innovation by challengers systematically increase following an increase in d. Beyond this impact on long-term growth, I am however also able to further characterize the evolution of the allocation of overall R&D expenditures between challengers and incumbents. In the case we have a monopoly in the (SC) state, I analytically demonstrate that a greater fraction of overall R&D will be carried out by incumbents in the case of a more equal economy. If on the other hand we have a duopoly in the (SI) state, redistributive policies will lead to challengers increasing their share of the overall R&D expenditures.

Second, a mean-preserving higher concentration of wealth among a small group of people (i.e. an increase in β) is found on the other hand to have a positive impact on long-run growth in the case we have a monopoly in the (SC) state. Such a result is essentially driven by the increased incentive to price-discriminate when facing even wealthier consumers on the right tail of the income distribution, and had not been exemplified by Zweimuller and Brunner (2005).

I hence contribute to the analysis of the influence of wealth disparities on long-run growth operating through the demand side. My results partially confirm the predictions obtained by Zweimuller and Brunner (2005) in a similar quality-ladder framework: a reduction in the level of inequality through an increase in the relative income of the poor d leads to an increase in long-run growth. Furthermore, by being able to differentiate the impact of variations in the level of inequality on the incumbent's and the challengers' investment in R&D, I exemplify a so far overlooked influence of wealth distribution on the allocation of R&D spending between the leader and the challengers.

6 Conclusion

In this paper I provided two major contributions to the analysis of the impact of inequality on long-term growth operating through the demand side. I first show that disparities in purchasing power justify investment in R&D by both leaders and challengers, providing a demand-driven rationale for innovation by incumbents. By introducing non-homothetic preferences in an otherwise standard quality-model, I show that the perspective to discriminate efficiently between consumers differing in their willingness to pay for quality is sufficient for the industry leader to overcome the Arrow (1962) effect and keep investing in R&D. The strictly positive innovation rate of the incumbent is here obtained with constant returns to R&D efforts and without any advantage of the incumbent in the R&D field (supply side), by allowing for income inequality to generate different quality valuation of poor and rich consumers (demand side). Second, I then study the impact of redistributive policies on long-run growth, and obtain a negative relationship between inequality and growth. Finally, I show that the level of inequality impacts not only the long-term growth rate, but also the allocation of the R&D effort between challengers and leaders.

Some lines of further work can be quickly sketched. An obvious extension to this model would be to treat the more general case of more than two types of consumers, in order for the incumbent to keep investing in R&D after the second successful race.[20] A

[20]Indeed, as already pointed out, the null investment in R&D by the incumbent in the (SI) state solely

model such as this one can also be extended to a two-country framework, in order to contribute to the developing literature studying the determinants and impact of vertical, intra-industrial trade (Fajgelbaum et al., 2011). Indeed, while the impact on growth of inter-industrial quality trade has already been extensively studied (product life cycle), I believe the framework presented in this paper would be a good starting point for the elaboration of a dynamic model of intra-industrial quality trade (quality life cycle).

stems from the fact that we have only two distinct groups of consumers: once having offered two distinct price-quality bundles, the incumbent does not have any incentive to keep carrying out R&D.

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Appendix A

We first notice using (10) and (11) that π_L, π_F and π_{SI} can be re-expressed as $\pi_L = A_l + B_l\Omega$, $\pi_F = A_f + B_f\Omega$ and $\pi_{SI} = A_f + A_l + (B_l + B_f)\Omega$, with:

$$
\begin{aligned}
A_l &= \left(\frac{k-1}{k^2}\right)(1-a)wL(k-1)(1-\beta), \quad B_l = \left(\frac{k-1}{k^2}\right)\rho(k - d(\beta(k+1)-1)) \\
A_f &= \beta\left(\frac{k-1}{k}\right)(1-a)wL, \quad B_f = \beta\left(\frac{k-1}{k}\right)d\rho
\end{aligned}
$$

We also note that $\rho > B_l$ and $\rho > B_f$.

The 3 sufficient parametric conditions under which there exists a unique positive equilibrium are the following:

$$
\begin{aligned}
2wF\rho > A_f - A_l &> 0 \text{ (i)} \\
B_f - B_l &> 0 \text{ (ii)} \\
A_l - wF\rho &> 0 \text{ (iii)}
\end{aligned}
$$

We now proceed to demonstrating the existence and uniqueness of the equilibrium under those conditions. Replacing wF with its value as expressed in (23) into equation (22), it is possible to obtain the following expression for $\phi_C v_F$:

$$
\phi_C\left(\frac{\pi_F}{\rho + \phi_C + \phi_I}\right) = \pi_{SI} - 2\pi_L \tag{32}
$$

The existence of a positive steady state equilibrium implies that all the elements of the LHS of (32) are positive. The RHS then also has to be positive, which is ensured under the conditions (i) and (ii).

Substituting for $\phi_C v_F$ into (23), it is then possible to express ϕ_C as a function of Ω: $\phi_C = \frac{\pi_{SI}-\pi_L}{wF} - \rho > 0$ under condition (i) and (iii). Substituting for the obtained value of ϕ_C into equations (26) and (32), we obtain two implicit functions $\phi_I = \psi_R(\Omega)$ and $\phi_I = \psi_L(\Omega)$. ψ_R and ψ_L are implicitly defined by writing (26) and (32) respectively as $R(\phi_I, \Omega) = 0$ and $L(\phi_I, \Omega) = 0$ with:

$$
\begin{aligned}
R(.) &= A_f\left(\frac{A_l}{wF} - \rho\right) + ()\,\Omega + (B_l - B_f)\Omega\phi_I + (A_l - A_f)\phi_I + \left(\frac{B_f B_l}{wF}\right)\Omega^2 \\
L(.) &= -\left(\frac{A_f}{wF} - \rho\right)(A_l + \rho wF) + \left(\frac{B_f}{wF}(A_f - A_l - 2wF\rho) + \left(\frac{A_f}{wF} - \rho\right)(\rho - B_d)\right)\Omega \\
&\quad + (B_f + \rho - B_l)\Omega\phi_I + (A_f - A_l - 2wF\rho)\phi_I + \frac{B_f}{wF}(\rho - B_l)\Omega^2
\end{aligned}
$$

We first consider the intercept of the two curves RR and LL (respectively representing the two functions ψ_R and ψ_L in the (ϕ_I, Ω) plane) with the vertical axis. We have $\psi_R(0) = \frac{A_f(\frac{A_l}{wF} - \rho)}{A_f - A_l} > 0$ under conditions (i) and (iii). On the other hand, we have $\psi_L(0) = \frac{-\left(\frac{A_f}{wF} - \rho\right)(A_l + \rho wF)}{B_f + \rho - B_l} < 0$ under condition (iii). We then move to considering the slopes of RR and LL. Using implicit differentiation, we have $\frac{\partial\psi_L}{\partial\Omega} = -\frac{\partial L/\partial\Omega}{\partial L/\partial\phi_I}$ and $\frac{\partial\psi_R}{\partial\Omega} = -\frac{\partial R/\partial\Omega}{\partial R/\partial\phi_I}$. More precisely, we have:

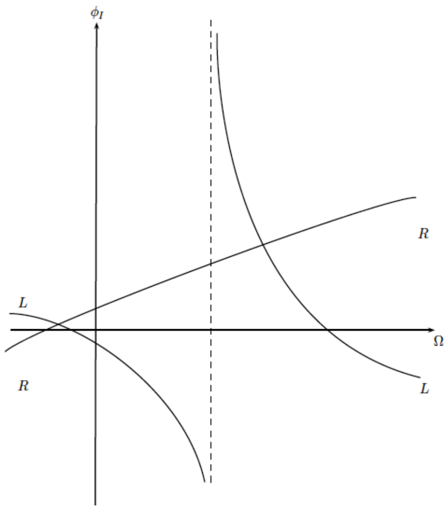

Figure 4: Case 2 steady state equilibrium

$$\begin{aligned}
\frac{\partial R}{\partial \Omega} &= (2\frac{B_f B_l}{wF})\Omega + (B_l - B_f)\phi_I + B_f(\frac{A_l}{wF} - \rho) + \frac{B_l A_f}{wF} > 0 \text{ under condition (iii)} \\
\frac{\partial R}{\partial \phi_I} &= (B_l - B_f)\Omega + A_l - A_f < 0 \text{ under conditions (i) and (ii)} \\
\frac{\partial L}{\partial \Omega} &= 2\frac{B_f}{wF}(\rho - B_l)\Omega + \underbrace{(B_f + \rho - B_l)\phi_I}_{(*)} + \frac{B_f}{wF}\underbrace{(A_f - A_l - 2wF\rho)}_{(**)} + (\frac{A_f}{wF} - \rho)(\rho - B_d) \\
\frac{\partial L}{\partial \phi_I} &= (B_f + \rho - B_l)\Omega + \underbrace{A_f - A_l - 2wF\rho}_{(**)}
\end{aligned}$$

Under conditions (i)-(iii), we unambiguously have that $\frac{\partial \psi_R}{\partial \Omega} > 0$; the curve RR is hence monotonously increasing (cf Figure 4). The shape of curve LL can be analyzed considering the explicit value of ϕ_I obtained when solving for $L(\phi_I, \Omega) = 0$:

$$\phi_I = \psi_L(\Omega) = \frac{-\left(\frac{A_f}{wF} - \rho\right)(A_l + \rho wF) + \left(\frac{B_f}{wF}(A_f - A_l - 2wF\rho) + \left(\frac{A_f}{wF} - \rho\right)(\rho - B_d)\right)\Omega + \frac{B_f}{wF}(\rho - B_l)\Omega^2}{(B_l - \rho - B_f) + 2wF\rho + A_l - A_f}$$

For small values of Ω, ϕ_I is negative (remember that $\psi_L(0) < 0$). We hence have that the term (*) in $\frac{\partial L}{\partial \Omega}$ is negative. Since the term (**) is also negative under condition (i), it guarantees that both $\frac{\partial L}{\partial \Omega} < 0$ and $\frac{\partial L}{\partial \phi_I} < 0$. We hence have $\frac{\partial \psi_L}{\partial \Omega} < 0$ for small values of Ω. As Ω increases, ϕ_I actually becomes more and more negative, with $\phi_I \to -\infty$ as $\Omega \to \Omega_A$ with $\Omega_A = \frac{2wF + A_l - A_f}{\rho + B_f - B_l}$; we hence have $\frac{\partial \psi_L}{\partial \Omega} < 0$ for any $\Omega < \Omega_A$. For $\Omega > \Omega_A$, high values of ϕ_I as well as greater values of Ω ensure both $\frac{\partial L}{\partial \Omega} > 0$ and $\frac{\partial L}{\partial \Omega} > 0$, and we hence still have $\frac{\partial \psi_L}{\partial \Omega} < 0$. The curve LL is hence monotonously decreasing, with an asymptote at $\Omega = \Omega_A$ (cf Figure 4). RR and LL hence necessarily intersect only once, yielding a unique positive equilibrium with (Ω, ϕ_I) strictly positive. This ends the proof. □

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