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Abstract:

The reader will find in this paper a simple theoretical framework where to analyze the usual issues from environmental and economic growth literature, including the problem of global warming, the corresponding climate change, and the impact on economic system. This paper considers two environmental externalities: the first one is local and gives account of the marginal damage from emissions flow; the second one is aggregate, or global, and relates to the extreme damage which may happen if the accumulated stock of pollutants is on the threshold of a worldwide catastrophe. In this context dominated by market failures, the decentralized equilibrium path is inefficient and unsustainable, while the socially optimal balanced growth path shows a singularity with trajectories truncating and changing of course. With respect to the economy's long-run performance we study how environment matters in different ways and at different stages, and give conditions for sustainability of sustained growth.

Keywords: Environment, Externalities, Global Warming, Climate Change, Catastrophe, Optimal Sustained Growth, Sustainability.

JEL classification: C61, C62, O41, Q5.

1 Introduction

In a seminal work on endogenous growth theory and the environment [Smulders (1999)], we can read that "*Many other models only incorporate a flow variable to represent the environment. Thus ignoring the accumulation of wastes and the irreversibility of environmental damage, these models are not able to examine the possible conflict between short-run and long-run consequences of economic growth on the environment, but they prove to be a useful simplification to examine, for instance, the effects of different environmental tax issues*". The lesson is that the stock of accumulated pollutants has to be explicitly incorporated into models when we study the issue of long-run growth sustainability. In such a case, if pollution increases with economic growth, it may happen that growth ceases when the stock level reaches a certain upper bound. Moreover, long-run sustainability depends not only on the level of emissions but also on the assimilative capacity of the environment. Indeed, as López (1994) points out, the world's capacity to absorb pollution is bounded and, once pollution stock approaches the absolute tolerable limit, economic growth would not become feasible anymore because the economy will be falling down into an extreme situation of catastrophic state.

The previous point leads us thinking that it is of great relevance to ask whether there are limits to growth. Of course, the global economic collapse is more likely that arises in an economy following a sustained long-run growth path if pollution emissions appear positively related to the economic activity as a by-product [Gradus and Smulders (1993), Ligthart and Ploeg (1994), Michel and Rotillon (1995), Mohtadi (1996)]. Modern economies satisfy human needs through the consumption of goods and services, which requires obtaining raw materials from the elements of Earth, the production of intermediate and final goods, transport of passengers and merchandises, as well as the provision of all kinds of services. All these productive activities call for both energy consumption and land use. Energy used in economic activity, fundamentally thermal energy (heat) and electricity (alternating and direct current), is obtained from the chemical reactions of oxidation (combustion), from dynamos activated by steam, wind, or water (electromagnetism), and from the photovoltaic panels (solar). The land used in different productive processes represents a transformation of the soil and subsoil that causes deforestation and forest degradation, and whose quantitative impact determines the ecological footprint. The amount of energy and land required is greater the more widespread is agriculture, forestry, animal farming, grazing, and mining, but also depends on the intensity of industrialization and urbanization processes, which bring with them the residential construction, the building construction for industrial and tertiary use, as well as the corresponding infrastructures.

In all the stages of economic activity in which there are (i) chemical reactions associated with the transformation of materials or the combustion of biomass and fossil fuels, and (ii) changes in the Earth's surface that alter and destroy the ecosystems, we find anthropic waste and emissions. For the most part they are pollutant flows that have negative effects, both local and global, on the environment as well as on the economic system itself. We have in mind radioactive and other material wastes, and emissions of carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), hydrogen sulfide (H_2S), chlorofluorocarbons (CFCs), hydro fluorocarbons (HFCs), per fluorocarbons (PFCs), sulphur hexafluoride (SF_6), nitrogen oxides (NO_x), sulphur oxides (SO_x), and carbon monoxide[1] (CO). All these polluting anthropic emissions are closely

[1] The case of carbon monoxide is paradigmatic since it combines the effects of toxicity, local pollution, and global warming. In origin it is lethal by inhalation, locally it contributes to harmful pollution concentrations of the ambient air (Photochemical Smog) depending on weather conditions, and when it is dispersed in the

related to economic activity (production, transport, and consumption) as a by-product, and all of them are undesirable, inevitable, but dimmable.

In a so featured modern economy which we shall handily describe by means of a model of endogenous growth, if pollution tolerance is bounded sooner or later it will be attained the state of ecological catastrophe, which represents an effective and absolute limit to growth. This problem, however, may be mitigated when it is possible for economic agents to undertake emissions abatement activities or control for the degree of pollution associated with production technologies [Gradus and Smulders (1993), Ligthart and Ploeg (1994), Byrne (1997), Stokey (1998), Andreoni and Levinson (2001), Reis (2001)]. Pollution stocks can be diminished not only by increasing the regenerative capacity or by reducing the level of polluting activities, but also by means of pollution abatement actions. These ones contribute to determining the degree of dirtiness associated with technology as well as the net flow of pollutants to the environment. Consequently, improving environmental quality requires abatement expenditures that leave less resources available for growth-oriented investment activities. Hence, the model used to describe a representative economy in such a context must be able to show a trade-off between production growth and environmental quality. Moreover, in presence of environmental externalities it is expected to find lower rates of growth for output and pollution, when pollution is optimally controlled than when it is competitively managed in a decentralized economy.[2] And even more, the opportunity for pollution abatement generates a mechanism that may act as a limit to growth, although less strong than that which arises from a catastrophic event.

There is a long tradition in environmental literature studying the role of catastrophic thresholds in changing utility and agent's behavior when pollution stock exceeds critical levels [Cropper (1976), Clarke and Reed (1994), Tsur and Zemel (1995; 1996; 1998), Naevdal (2006), Naevdal and Oppenheimer (2007), Tsur and Withagen (2011)]. The goal of these articles is not to study the feasibility of sustainable long-run growth but the efficient management of pollution emissions and natural resources exploitation under the risk of an environmental disaster. However, given their focus on ecological catastrophes that imply substantial reductions in society's levels of consumption and utility, both issues appear necessarily related. And this is what we are to analyze in the present paper.

Aronsson et al. (1998), in a Ramsey-type growth model, integrates the study of catastrophes with the theory of economic growth. The paper introduces uncertainty because of the use of nuclear power to produce the energy needed in production, and it is well known that there is a fundamental risk concerning the possibility that radioactivity may leak out into the environment, either from an accident in the nuclear plant or from improper storage of waste. This is important because, in case of accident, damage would be severe and irreversible. However, as many of the most serious nuclear accidents have shown, e.g., Three Mile Island in 1979, Chernobyl in 1986, and Fukushima in 2011, damages from a nuclear disaster seems to be local rather than global.

Instead, we are interested in global environmental catastrophes like the climate change associated with global warming, a phenomenon attributed to the increase in anthropogenic

atmosphere it has global consequences enhancing the greenhouse effect. The carbon monoxide (CO) reacts with oxygen molecules ($2O_2$) releasing carbon dioxide (CO_2) and tropospheric ozone (O_3).

[2] Things could be different if a pollution externality on the side of production is considered [Gradus and Smulders (1993), Ewijk and Wijnbergen (1995), Mohtadi (1996), Smulders and Gradus (1996)]. In such a case, environmental quality changes production opportunities by affecting the economy's productivity, and an increase in environmental care may boost growth.

greenhouse emissions. Accordingly, the important question for us is whether economic growth and environmental protection are reconcilable. That is, whether optimal sustained growth is compatible with ecological sustainability of the economy as a whole. Moreover, the query on whether environmental concern will eventually limit growth has to be answered looking at two different issues: first, the effects of pollution abatement on the long-run rate of growth; and second, the evolution of the stock of pollutants with respect to the ecologically catastrophic level.

All these issues will be analyzed more accurately in this paper in a simple model of endogenous growth. Given that we are not directly interested on how technological change has been originated, but on conditions under which sustained endogenous growth and ecological sustainability are compatible, our benchmark model will be the traditional Rebelo's (1991) one-sector AK model to which we incorporate variables representing the pollution and environment side. Moreover, this model is suitable for our goal due to its simplicity and easy handling, which allows us to focus on the performance of developed economies and their outcomes in the long term, i.e., the period in which sustainability appears to be a relevant issue.

Welfare depends on consumption but also on the quality of the environment where agents consume. In our model pollution arises from production as a flow that enters the consumer's utility function playing the role of a negative *local externality*. There is an externality because of the existence of numerous households who take into account the local effect that *pollution flow* exerts on their respective utilities, without having any influence on the generating process which mainly depends on firms. Moreover, one central aspect of the analysis below is the explicit consideration of abatement activities, which are costly because they absorb resources reducing investment and consumption possibilities. In this setting, however, households show environmental concern but they do not decide on abatement, whereas firms bear the cost of such an activity but do not receive the corresponding benefits.[3]

Our model considers another externality which arises from the fact that, even if the local negative effect from pollution is not present or has been internalized, households and firms are not aware of the global negative effect from an eventual ecological catastrophe. This is an *aggregate externality* because individual agents decide on the emissions flow but do not control for the accumulated *stock of pollutants*. However, the aggregate stock will eventually have a severe effect on the environment and life on earth, hence on the individual agent's welfare too, at the moment of the economic collapse.[4]

Pollution, which is usually assumed as a pure public bad, has been analyzed in economic models as if it were a monolithic problem without considering its inherent multiplicity of dimensions and effects. However, here we differentiate between local and global effects, and make a distinction between perceived pollution and invisible or diffuse pollution. On the one hand, there are the external effects associated with local pollution. This pollution is easily and clearly recognized by agents and may be corrected by making emerge prices for externalities that capture the marginal harm. The most common instruments vary from the complete allocation of property rights (Coase) allowing for multilateral negotiation, to the environmental taxation (Pigou) setting taxes and subsidies which equate the private and the social marginal

[3] We ignore here any other local externality associated with the flow of pollution, which could play a significant role by affecting the productivity of factors via the health of workers or the quality of inputs.

[4] The global effects on the environment and life on earth may include biodiversity destruction, deforestation and soils erosion, acid rains, global warming and climate change, as well as depletion of the stratospheric ozone layer.

cost of emissions. On the other hand, there are the external effects associated with global pollution, which arises from many small sources but eventually manifests in a huge aggregate effect. This feature, despite the evident simplification, shows a strong parallelism with the real problems of the stratospheric ozone layer depletion and the enhanced greenhouse effect caused by anthropic pollution emissions. Between the different ways of correcting these two problems that represent the most obvious examples of an aggregate externality at work, it has been used the command-and-control approach (emission standards) for the first, while it has been implemented a cap-and-trade scheme (market for tradable emission permits) for the second.

As we have mentioned above, the economic activity releases an assorted collection of greenhouse gases into the atmosphere. To a certain extent they are removed by sinks but an important part of them are stored in the atmospheric reservoir. This increases the greenhouse effect which results in an additional global warming of the Earth's surface and atmosphere. In consequence, we assume that once the accumulated stock of greenhouse gases in the atmosphere goes beyond a critical level representing the pollution threshold, it may adversely affect ecosystems causing a drastic climate change and serious disruptions to economic and social activity. This is the kind of disaster we have in mind in this paper, because of the sudden appearance and because of the substantial loses it entails. Catastrophes may be gradual if the associated economic adverse effects manifest in a progressive way but, for the sake of simplicity, the catastrophe in this paper is taken as *abrupt*, reducing consumption and utility instantaneously to zero. On the other hand, catastrophes may be reversible when the economy can still recover from a severe damage, even if it means to bear important cleaning costs or waiting for a long period of time. However, our catastrophe is considered as *irreversible* because once the catastrophic event occurs it will be impossible to reverse its effects. We have represented this by truncating permanently the planning horizon. Finally, in this paper we assume that the critical level of pollution is *known* with certainty though the ecological catastrophe remains as an endogenous event, which may be avoided or not depending on agents' decisions about consumption, production, and abatement. This is in sharp contrast with the case of nuclear disasters where uncertainty is a fundamental feature associated with the randomness of an exogenous event occurrence. In the real world there is a lot of small uncertainties associated with the links in the chain that goes from emissions, to greenhouse effect, to global warming, to climate change, and to the adverse economic effects because of the lack of an exact scientific knowledge with respect to the chemical, physical, and economic processes involved in this kind of ecological catastrophe. Nevertheless, successive IPCC reports and other studies have provided enough scientific evidence to increase our level of knowledge and, therefore, the previous uncertainty concerning those processes has been substantially diminished. In particular, there is today a wide consensus about the critical concentration level of greenhouse gases in the atmosphere, which has been established at around 550 ppm CO_2-equivalent [Stern (2007)].

Pollution stock, rather than flow, is connected with the greatest and widest-ranging market failure we can imagine, given that faced to the anthropic waste and emissions problem, individual economic agents (households and firms) are somehow concerned about the local effects of pollution, but they do not consider in any way the global effect for decision making. In this context, some kind of aggregate coordinating action is needed, and the Kyoto protocol provides a basis for correcting such a failure. The Kyoto protocol is in fact an agreement which acts as a global central planner facing the aggregate externality problem. The agreement sets individual emission targets, and then implements some flexibility mechanisms to control the aggregate emissions flow on the basis of three elements: an international market for Emission Permits

Trading and two project-based mechanisms, the Joint Implementation and the Clean Development Mechanism. On the other hand, it puts in place a regulatory system for verification and monitoring of the emissions reduction. In short, the Kyoto protocol may be seen as the necessary cooperating context where the adverse consequences of pollution externalities could be corrected and sustainability guaranteed.

Because of the two environmental externalities the competitive equilibrium does not work well. On the one hand, the equilibrium path is not Pareto-optimal and, on the other hand, this path leads the economy to the state of ecological catastrophe. Consequently, we will study the opportunities for an efficient management of the economy with special attention to the problem of the environmental and economic collapse. Efficiency alone, as traditionally assumed, is not sufficient for sustainability but, as we will show, Pareto optimality is necessary to produce sustainable outcomes. Central to this study is also the problem of *sustainable development*,[5] and consequently we will look for conditions under which environmental quality and growth could go hand in hand. In other words, all along this article we will try to find a concrete answer to the following usual questions: *(i)* Do environmental externalities influence growth? *(ii)* What are conditions for *sustained* balanced growth when environment matters? *(iii)* Which is the effect of environmental concern on the rate of growth? *(iv)* Will pollution controls and abatement reduce growth rates? *(v)* Under what conditions is *sustainability* feasible? *(vi)* Is it possible to get in the long-run sustained and sustainable growth? We want to remark that our goal is not to study the problem of how to implement the efficient solution in a decentralized economy. Nevertheless, the results we obtain show a great parallelism with the policy coordinating framework and the measures put in practice according to the Kyoto protocol recommendations.

The article is organized as follows. Section 2 describes the economy and introduces the assumptions featuring a general equilibrium one-sector endogenous growth model in which pollution is a by-product of economic activity, but it may be reduced by spending a fraction of the aggregate output on abatement. In section 3 we briefly study the decentralized competitive equilibrium without regulation. In section 4 we study the socially optimal solution assuming sufficient conditions for interior solutions. Using the unconstrained trajectories, we characterize the growth path and analyze under what conditions sustained balanced growth is feasible. Subsection 4.2 focuses on ecological sustainability and the non-catastrophic growth process. In section 5 we solve the general dynamic optimization problem allowing for corner solutions, we study necessary conditions for global optimality with respect to the two types of pollution externalities, and show that, if pollution stock approaches the critical level, the central planner has to change the value of the dirtiness index in line with the Kyoto protocol recommendations. We also characterize growth in the aftermath and compare with the previous one. Finally, section 6 summarizes and concludes.

[5] There are two categories of sustainability, which may be neatly identified in the following quotation from Stern (2012): "many ecological economists argue that sustainability requires minimum levels of natural capital or natural resources to be maintained as human made inputs have limited ability to substitute for them in the provision of human welfare. This idea is termed *strong sustainability.* By contrast, many mainstream environmental economists assume that human made inputs can substitute extensively for natural inputs. They argue that sustainability could be achieved as long as sufficient investment is made in human produced capital. This is referred to as *weak sustainability".*

2 The economy

2.1 Production

The model economy is a one sector closed economy. Output is obtained according to an aggregate production function of the AK type where capital is the only factor needed to produce,

$$Y(t) = AK(t). \tag{1}$$

In this model K is an aggregate composite of different sorts of capital which, in a broad sense, includes physical as well as human capital. The efficiency parameter $A > 0$ represents a constant technological level. Hence, there is not exogenous technical progress in the model. For the sake of simplicity, we assume that this production function arises from the direct summation of the individual production functions for many identical firms.

Moreover, pollution plays a central role in this economy. Although pollution emissions may be originated in the processes of consumption and production, we focus on pollution generated by firms. Pollution emissions are a by-product of economic activity, arising as an unintended output[6] when firms produce according to the best available technology represented by equation (1). There is not free disposability of the residuals. Instead, firms are allowed to engage in costly abatement activities to mitigate emissions they produce, and the environment shows a certain absorptive capacity to clean up pollution. All these issues are better captured in a multiple relationship framework which is specified along the next two subsections.

2.2 Pollution and abatement

One feature of this model that is absent from the canonical endogenous growth AK model, is the existence of a stock of pollutants S, which is increased by polluting activities such as production Y, but is reduced by abatement $\mathcal{B}$ as well as by the corresponding natural regeneration at a constant rate $\delta > 0$.[7] Moreover, it is assumed an upper bound for S, called $S^{\max}$, which plays the role of a threshold value for which as soon as the current stock goes beyond it, a catastrophic state is reached in the economy. The critical level $S^{\max}$ is known with certainty, but the occurrence of the catastrophic event is endogenous and depends on agents' economic decisions. Under these assumptions, *sustainable development* will be characterized as a situation where the main economic variables show positive long-run balanced growth while, at the same time, they contribute to generate an accumulated stock of pollutants smaller than (or equal to) the critical value $S^{\max}$. This is equivalent to require that conditions for both weak and strong sustainability be satisfied.

The above-mentioned abatement effort $\mathcal{B}$, which is costly and endogenously decided by agents, will be measured in terms of output $Y(t)$ in such a way that these two variables relate

[6] According to Murty et al. (2012), the process of transformation of inputs into intended output triggers additional reactions in nature which inevitably result in pollution emissions as a by-product.

[7] An alternative to this constant exponential rate of pollution decay, which implies that natural regeneration is a linear function of the pollution stock, is the inverted U-shaped decay function modelized by Tahvonen and Withagen (1996) and Tahvonen and Salo (1996). This one implies that a pollution stock level that is sufficiently high will reduce the rate of natural regeneration to zero, in particular when it reaches the irreversibility threshold. Moreover, Ayong-Le-Kama et al. (2011) considers a two-stage structure for the natural regeneration rate, which is a positive constant or zero depending on whether the stock of pollutants is found below or above the irreversibility threshold. On the other hand, the first of these alternatives assumes that the critical stock level is known with certainty, while for the second one it is unknown.

to each other according to

$$\mathcal{B}(t) = Y(t) - Y_N(t) = (1 - z(t))\,Y(t). \tag{2}$$

Here $z(t)$ represents, as in Stokey (1998) and the opposite to Reis (2001), a measure of the effective dirtiness of the technique used to produce. Obviously, $z(t) = 1 - \frac{\mathcal{B}(t)}{Y(t)} \in [0,1]$ because resources devoted to clean pollution can never exceed the current production. Therefore, any choice for z close to zero or one automatically makes the existing technique less or more polluting respectively. The above expression introduces a definition for the net output[8] as

$$Y_N(t) = z(t)Y(t). \tag{3}$$

The equation governing the motion of S may be written as $\overset{\bullet}{S}(t) = P\left(Y(t), \mathcal{B}(t)\right) - \delta S(t)$, where $P\left(Y(t), \mathcal{B}(t)\right)$ represents the net flow of waste and emissions associated with the endogenously determined levels of production-polluting and abatement activities.[9] This flow is increasing with respect to Y and decreasing with respect to $\mathcal{B}$, i.e., $P_1 > 0$ and $P_2 < 0$. Function $P(.)$ is assumed homogeneous of degree zero, i.e., a proportionally equal increase in both output and abatement leaves net emissions unchanged independently of the population size. Consequently, the emissions function may be rewritten as $P\left(Y(t), \mathcal{B}(t)\right) = G\left(\frac{\mathcal{B}(t)}{Y(t)}\right)$, where we assume strict concavity: $G' < 0$, $\lim\limits_{x \to 0^+} G' < 0$, $-\infty < \lim\limits_{x \to 1^-} G' < 0$, $G'' < 0$, $G(0) = G^M > 0$, and $G(1) = 0$. Actually, $G^M > \delta S^{\max}$ represents an *effective* upper limit for the emissions function, which is high enough to lead the economy, if it prevails, to the state of ecological catastrophe.[10] Now, substituting the previous variable transformations into the differential equation for the motion of the stock of pollutants we get

$$\overset{\bullet}{S}(t) = G\left(1 - z(t)\right) - \delta S(t), \tag{4}$$

where $G_z = -G' > 0$, $0 < \lim\limits_{z \to 0^+} G_z < +\infty$, $\lim\limits_{z \to 1^-} G_z > 0$, and $G_{zz} = G'' < 0$. Taking as reference $z = 1$, which implies that no abatement effort is done and that emissions flow reaches the maximum level G^M, the larger the reduction in z the more effective the reduction in emissions. Or, put in other words, as long as we produce with a cleaner technology the effectiveness, measured in terms of emissions reduction, of any additional pollution abatement that reduces z will be larger.[11]

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[8] In this model, given that gross output is $Y = AK$ and net output is $Y_N = zAK$, it is useful from now on to differentiate between the gross productivity of capital, A, and the net-of-abatement productivity of capital, Az.

[9] An alternative way of representing the flow P(.) may be found in Pommeret and Schubert (2009) where pollution emissions are assumed linear in K, and abatement is represented by technological progress that lowers the coefficient. They assume an exogenous process which consists in a once-and-for-all switching to a new technology of abatement, but keeping endogenous the choice of the date to implement the adoption.

[10] Tahvonen and Salo (1996) conceives this upper level as the emissions level for which residents decide to move to other locations. On the other hand, we can interpret these emissions as a maximum level beyond which production starts to be delocalized by transferring abroad the polluting technology.

[11] There is not empirical evidence about this assumption and, moreover, even if we assume convexity, given the nature of the model, the results below do not change. Instead, the concavity assumption allows for a huge simplification of all subsequent analysis concerning sufficient conditions for optimality and the convergence of integrals.

2.3 Investment

According to the aggregate resources constraint, net output may be devoted to consumption or capital accumulation. For the sake of simplicity we do not consider capital depreciation. Hence, net investment equals gross investment and the capital stock is governed by the differential equation

$$C(t) + \dot{K}(t) = Y(t) - \mathcal{B}(t). \tag{5}$$

This equation also reflects the cost of the abatement activity in a very simple way: one additional unit of abatement effort is automatically 'transformed' into a lower unit of output available for consumption or capital accumulation. This particular 'one-to-one' transformation contributes to simplify our analysis.

2.4 Preferences

The economy is populated by many identical and infinitely lived agents. Population, denoted by N, is assumed to be growing at a constant rate $0 < n < A$. The initial population $N(t_0)$ is normalized to one. Individual preferences are assumed to be represented by a twice continuously differentiable instantaneous utility function $V\left(c\left(t\right), P\left(t\right)\right)$, which depends positively on the current per capita consumption c and negatively on the polluting emissions flow P [Gradus and Smulders (1993), Ligthart and Ploeg (1994), Selden and Song (1995), Reis (2001), Eriksson and Zehaie (2005), Pommeret and Schubert (2009)]. Under this assumption, households do not take care for the stock of pollutants S accumulated in the environment and the atmosphere, but only for the current net flow of polluting waste and emissions P. This is because the local stock effect of pollution is assumed short-lived and the abatement activity, which reduces emissions and facilitates regeneration, makes it negligible.[12] On the other hand, the assumed role for S and P in our model reflects the combined hypothesis of either discontinuous and continuous dependence of damages on pollution (the abrupt damage of a catastrophe *via* the stock S, and the marginal damage of pollution *via* the flow P); Hence, a non-zero marginal disutility shows that adverse effects occur even at pollution levels below the critical threshold.

As we have previously shown, emissions depend positively on production and negatively on pollution abatement, two variables that appear related to each other according to (2). Given the characterization of the emissions function, P is an increasing monotonous transformation of z. Therefore, the instantaneous utility function may be written as $U\left(c(t), z(t)\right)$ with $U_c > 0$ and $U_z < 0$, where the two ordinal utility functions V and U represent the same preference

[12] This also implies that we ignore any global stock effect in the representation of households' preferences, but this is because of the nature of the aggregate externality considered here. Following Weitzman (2007) in p. 713, the damage associated with the global stock effect is not modeled entering the instantaneous utility function as a direct argument, but as changing parameters of the intertemporal utility function. Nevertheless, an important stream of literature considers that welfare depends on the stock of pollution rather than on the current flow [Huang and Cai (1994), Mohtadi (1996), Tahvonen and Salo (1996), Byrne (1997), Kelly (2003)]. However, if the flow of pollution is increasing with production, then capital accumulation that increases future output also increases future flows of pollution. Hence, we find a general consensus in the literature [Gradus and Smulders (1993), Smulders and Gradus (1996), Aghion and Howitt (1998), Reis (2001), Eriksson and Zehaie (2005)] according to which, if we consider the stock of pollution as an argument in the utility function, we will obtain the same fundamental long-run results but at the cost of a more complex analysis. Only transitional dynamics would change significantly if pollution stock is directly involved in preferences. In any case, our interest here is in the long-run dynamics.

ordering. Moreover, we assume decreasing marginal utilities: $U_{cc} < 0$ and $U_{zz} < 0$, as well as strict concavity with respect to both arguments taken together, $U_{cc}U_{zz} - (U_{cz})^2 > 0$.

The structure of the model allows for the existence of a long-run balanced growth path, defined as an allocation in which consumption per capita grows at a constant rate and the dirtiness index is constant. To ensure that such a path may exist in this model we assume that the particular instantaneous utility function is multiplicatively separable and of the CIES form[13] [King, Plosser and Rebelo (1988), Bovenberg and Smulders (1995; 1996), Smulders and Gradus (1996), Ladrón de Guevara et al. (1999)]

$$U\left(c(t), z(t)\right) = \frac{c(t)^{1-\Phi}}{1-\Phi}\left(1 - z(t)\right)^{\alpha(1-\Phi)}. \tag{6}$$

In this function, the parameter α $\left(\equiv \frac{U_{1-z}}{U_c}\frac{1-z}{c} = -\frac{U_z}{U_c}\frac{1-z}{c}\right)$ represents the relative weight of environmental care in utility and is assumed to be positive and lower than one, $0 < \alpha < 1$. Moreover, the inverse of the constant intertemporal elasticity of substitution[14] is allowed to take values above or below unity, $0 < \Phi \lesseqgtr 1$. The previous utility function fulfills all the above mentioned assumptions concerning first and second derivatives. The strict concavity assumption requires as sufficient condition that the determinant of the Hessian matrix be positive, which implies the additional parameter constraint $\Phi > \frac{\alpha}{1+\alpha}$.[15]

[13] Bovenberg and Smulders (1995) also considers additional restrictions on ecological relationships and technology. With respect to the first, we remind that hereafter we are going to study conditions for ecologically non-catastrophic states. With respect to the second, we have to recall that our one sector and one accumulable factor model builds upon a linear production function which summarizes the whole set of technological requirements postulated by these authors.

[14] From now on, intertemporal elasticity of substitution (IES) will refer to the *conventional* IES coefficient given by $\frac{-Uc}{c \cdot U_{cc}} = \frac{1}{\Phi}$, because the utility function (6) shows a *long-run* IES coefficient given by $\frac{1}{\Phi - \alpha(1-\Phi)}$.

[15] Environmental literature has long dealt with the sign of the second order cross derivative of the instantaneous utility function [Michel and Rotillon (1995), Mohtadi (1996)]. From $U\left(c(t), \frac{\mathcal{B}(t)}{Y(t)}\right)$ we know that utility indirectly depends on the environmental quality, which increases with abatement effort scaled by economy's dimension. The latter can be measured by output as well as by capital stock, given the linear form of the production function. In the particular case of equation (6) we get $U_{c2} = -U_{cz} = \alpha(1-\Phi)c^{-\Phi}\left(1-z\right)^{\alpha(1-\Phi)-1}$, which is negative (positive) as long as Φ is greater (smaller) than one. That is, as long as the intertemporal elasticity of substitution is smaller (greater) than one. Empirical evidence seems to corroborate the case of a low intertemporal elasticity of substitution and, hence, $U_{c2} < 0$. This implies that the marginal utility of consumption decreases as the environmental quality increases. Namely, consumption and pollution emissions are complements in terms of preferences. However, the model works exactly the same in the opposite case in which $U_{c2} > 0$.

3 The competitive decentralized solution

First of all, we will consider the structure of this economy from the point of view of the non-regulated competitive equilibrium. In this economy, assuming that there are no depreciation charges, each competitive firm faces the following stationary optimization problem, given the absence of adjustment costs and any other intertemporal element in its present value maximization problem,

$$\begin{aligned} \max_{\{K_i,z_i\}} \Pi_i &= Y_i - rK_i - \mathcal{B}_i \\ \text{s.t.} \quad 0 &\leqslant z_i \leqslant 1, \\ Y_i &= AK_i, \\ \mathcal{B}_i &= (1-z_i)\,Y_i, \\ K_i &> 0. \end{aligned} \tag{7}$$

The first order conditions are

$$r = Az_i, \tag{8}$$

$$0 = (1-z_i)\,AK_i. \tag{9}$$

From (9), because of the slackness condition, we observe that it is optimal for the individual firm to choose $z_i = 1$, and then by (8) we get $r = A$. These results imply zero quasi-rents at the equilibrium. But this also means that individual firms have no incentive to allocate resources to pollution abatement because of the externality originated in the conflict between the private nature of the cost of this activity and the social nature of its benefits, which are beyond the firm's control. Consequently, in the competitive equilibrium we will observe that production is undertaken by firms with the most polluting of the available production techniques.

Instead, households preferences are sensitive to the pollution emissions flow, and so it has been represented in their utility functions. They show a clear preference for reducing z below unity according to the assumption $U_z < 0\ \forall z$. Hence, we don't need to explicitly solve the optimization problem for households to see that, given the properties of the utility function, they will never choose such an extreme value for z. These two opposite and incompatible results for firms and households is the cause of a fundamental market mismatch between abatement demand and supply, which reflects the consequences of the above-mentioned local externality. In other words, there is a market failure associated with the absence of incentives for individual agents to internalize the negative external effect and, consequently, the decentralized equilibrium path is not Pareto optimal.

On the other hand, we need to consider the evolution of the aggregate stock of pollutants S because it is crucial from the point of view of the long-run sustainability of the economy's performance. In a decentralized economy S is not individually controlled by firms or households because both take as given the level of this stock at the moment of their decision making. Likewise, its evolution, although determined endogenously, is taken as exogenous. In the competitive equilibrium the firm's choice of the dirtiest technique, $z = 1$, has dreadful implications for the sustainability of outcomes. Consider the equation governing the motion of the aggregate pollution stock under such an extreme value: $\overset{\bullet}{S}(t) = G^M - \delta S(t)$, which finds the solution $S(t) = \frac{G^M}{\delta} - \left(\frac{G^M}{\delta} - S_0\right) e^{-\delta(t-t_0)}$. This means that S monotonically increases converging to the value $\frac{G^M}{\delta} > S^{\max}$, which implies that soon or later but in finite time the economy will inevitably reach the state of ecological catastrophe. In fact, the time elapsed until the competitive economy falls into the catastrophic state, $T_d^c = t_d^c - t_0$, may be calculated from the condition $S\left(t_d^c\right) = S^{\max}$. The result we get is

$$T_d^c = \ln\left(\frac{G^M - \delta S_0}{G^M - \delta S^{\max}}\right)^{\frac{1}{\delta}}, \tag{10}$$

a function that depends on the involved parameters according to the following signs of their partial derivatives $T_d^c = T_d\left(\overset{-}{G^M}, \overset{-}{S_0}, \overset{+}{S^{\max}}, \overset{+/-}{\delta}\right)$. The undetermined sign comes from the ambiguous impact of δ on $S\left(t\right)$, which is the consequence of two opposite effects. An increase (decrease) in δ, on the one hand reduces (increases) the limit value of S but, on the other it reduces (increases) the initial gap and increases (reduces) the speed at which this gap vanishes.

In short, the presence of multilateral and diffuse local and aggregate pollution externalities in a decentralized competitive economy leads to insufficient abatement and excessive pollution, a result that is clearly inefficient. This market failure calls for some sort of public intervention that adjusts the incentive system, allowing prices to capture the true social costs and benefits of abatement and pollution. Moreover, without any corrective environmental policy the environment will be damaged up to the level of irreversible catastrophe, and sustained growth, if there exists, will not be sustainable.

4 The optimal interior solution under sufficient conditions for sustainability

Now, we will focus on the socially optimal solution for the model economy described in section 2.[16] As we have shown, there are two externalities connected with the environmental problem: one of local nature and the other global. The first one conforms to the traditional marginalist view according to which an externality is present whenever the welfare (utility or profits) of some agents (mainly households) depends on variables whose values are chosen at the margin by others (notably firms). Instead, the second one works like an extreme aggregate externality because individual decisions on pollution emissions do have uncontrolled effects on the aggregate stock of pollutants, which in turn will have an abrupt and drastic effect on the agent's welfare from the exact moment of the ecological catastrophe and the corresponding economic collapse.[17]

The socially optimal solution simultaneously internalizes both externalities. First, by considering the true social costs and benefits from pollution abatement and, second, by taking into account the global negative effect from an eventual ecological catastrophe associated with the accumulated stock of pollutants. In this setting, the central planner looks at the social welfare, which is defined as

$$W = \int_{t_0}^{+\infty} \frac{c^{1-\Phi}}{1-\Phi} (1-z)^{\alpha(1-\Phi)} e^{-(\rho-n)(t-t_0)} dt$$

if $S(\tau) \leqslant S^{\max} \quad \forall \tau$, or

$$W^c = \int_{t_0}^{t^c} \frac{c^{1-\Phi}}{1-\Phi} (1-z)^{\alpha(1-\Phi)} e^{-(\rho-n)(t-t_0)} dt \tag{11}$$

if $S(\tau) > S^{\max} \quad \forall \tau > t^c$, where $t^c < +\infty$ represents the period in which occurs the ecological and economic catastrophe.

Assuming that the central planner is intended for maximizing social welfare, the way he can reach this target implies to maximize W subject to the constraint $S(t) \leqslant S^{\max}$. However, in this section we shall study interior solutions alone. Therefore, although the broad dynamic optimization problem should be formulated introducing as an explicit constraint the no-catastrophe condition, which implies that the central planner takes care *ex-ante* of trajectories leading to catastrophic states and optimally decides to avoid them by choosing the controls appropriately, we leave such a general procedure for a next section. For now, we specify the optimization problem without this state constraint that applies throughout the planing period, but still consider the dynamic constraint which takes into account the evolution of the aggregate stock of pollutants in the environment.[18] Accordingly, we first obtain the *unconstrained*

[16] The matter of how to replicate the efficient path in a competitive economy by means of an optimal environmental policy, is beyond the scope of this paper. However, the reader will find in Mohtadi (1996), Smulders and Gradus (1996), as well as in Rubio and Aznar (2002) a discussion on the way the government can implement emission standards, pollution fees, and public abatement, which allow the competitive economy to generate efficient outcomes in an endogenous growth model with an AK technology. These papers include a local pollution externality in the model but they ignore the complementary aggregate externality.

[17] We assume here that the negative effect on welfare is instantaneous rather than progressive. Although the latter would be more realistic given the parallelism established with the economic consequences of the climate change induced by global warming, our assumption simplifies analysis without substantially changing the nature of the problem.

[18] We do that in line with the recommendation of Chiang (1992): "*Although we cannot in general expect the unconstrained solution to work, it is not a bad idea to try anyway. Should that solution turn out to satisfy the*

optimal trajectories and then we check whether they are ecologically sustainable or not. In this way, we get sufficient conditions on parameters that ensure the sustainability of such trajectories. Moreover, a similar decision is made here with respect to the static control constraints $0 \leqslant z(t) \leqslant 1$. At this stage we also ignore these constraints but, later on, we will check them along the solution trajectories. This will lead us to identify two parameter conditions which make the control variable bounded satisfying the above constraints.

Hence, assuming interior solutions for controls and states with respect to their static constraints, the planner's problem consists in choosing the sequence $\{c(t), z(t), t \geq t_0\}$ which, for a given positive social rate of discount $\rho > n$, solve the optimization problem

$$\max_{\{K,S,c,z\}} \int_{t_0}^{+\infty} \frac{c^{1-\Phi}}{1-\Phi} (1-z)^{\alpha(1-\Phi)} e^{-(\rho-n)(t-t_0)} dt$$

$$s.t. \qquad (1)\text{-}(5), \tag{12}$$

for $k(t_0) = k_0 > 0$ and $s(t_0) = s_0 > 0$ given.

From now on we will use lowercase letters to represent variables in per capita terms.[19] The current value Hamiltonian is

$$\underset{\{c,z,q,k,\mu,s\}}{H^c} = \frac{c^{1-\Phi}(1-z)^{\alpha(1-\Phi)}}{1-\Phi} + q(Akz - c - nk) + \mu(g(1-z) - (\delta+n)s), \tag{13}$$

where q and μ are the co-states for k and s, respectively, and represent their corresponding shadow prices. The first order necessary conditions are

$$q = c^{-\Phi}(1-z)^{\alpha(1-\Phi)}, \tag{14}$$

$$q + \mu \frac{g_z}{Ak} = \frac{\alpha c^{1-\Phi}(1-z)^{\alpha(1-\Phi)}}{Ak(1-z)}. \tag{15}$$

As we have seen, gross product may be allocated to consumption, investment, or abatement. On the margin, according to (14) goods must be equally valuable if they are consumed or accumulated as new physical capital. Namely, the marginal utility of consumption today must be equal to the marginal shadow value of physical capital (consumption tomorrow). According to (15) at equilibrium the implicit price of a more dirty technique, that is, the value measured in units of utility of the marginal net product from a more dirty technique (qAk) plus the shadow value of the marginal emission associated with such a more dirty technique (μg_z), must be equal to the marginal utility of a cleaner one. Namely, the entire valuation of a marginal reduction in resources devoted to abatement, which contributes to increase consumption (present or future) as well as the stock of pollutants, must be equal to the marginal utility of those resources

constraint, then the problem would be solved. Even if not, useful clues will usually emerge regarding the nature of the true solution". Nevertheless, the reader will find below in section 5 the complete formulation and a detailed study of the optimal control problem, including every relevant constraint.

[19] Recall that $G(1-z)$ is a strictly concave function representing the aggregate emissions flow, which depends positively on the dirtiness index z. Hence, the function $g(1-z)$, which represents the per capita emissions flow, preserves the characterization attributed to the aggregate function: $g' < 0$, $\lim_{x\to 0^+} g' < 0$, $-\infty < \lim_{x\to 1^-} g' < 0$, $g'' < 0$, $G^M \geqslant g(0) = \frac{G^M}{N(t)} = G^M e^{-n(t-t_0)} > 0$, and $g(1) = 0$. Moreover, $g_z = -g' > 0$, $0 < \lim_{z\to 0^+} g_z < +\infty$, $\lim_{z\to 1^-} g_z > 0$, and $g_{zz} = g'' < 0$.

when devoted to abatement, which contribute to increase environmental quality. Moreover, the dynamic conditions for quantities and shadow prices are

$$\overset{\bullet}{k} = Akz - c - nk, \tag{16}$$

$$\overset{\bullet}{q} = \rho q - Azq, \tag{17}$$

$$\overset{\bullet}{s} = g(1-z) - (\delta + n) s, \tag{18}$$

$$\overset{\bullet}{\mu} = (\rho + \delta)\mu, \tag{19}$$

together with initial conditions k_0 and s_0 and the transversality conditions

$$\lim_{t \to +\infty} e^{-(\rho - n)(t - t_0)} qk = 0, \tag{20}$$

$$\lim_{t \to +\infty} e^{-(\rho - n)(t - t_0)} \mu s = 0. \tag{21}$$

We can start by studying the block of equations related to the stock of pollutants and its shadow price. So, we first solve equation (19) getting as solution $\mu(t) = \mu(t_0) e^{(\rho+\delta)(t-t_0)}$. Then, we integrate (18) and obtain the expression $s(t) = s_0 e^{-(\delta+n)(t-t_0)} + \int_{t_0}^{t} g(1 - z(\tau)) e^{-(\delta+n)(t-\tau)} d\tau$. Finally, if we substitute both expressions into the transversality condition (21) it results that $\lim_{t \to +\infty} \mu(t_0) \left\{ s_0 + \int_{t_0}^{t} g(1 - z(\tau)) e^{(\delta+n)(\tau - t_0)} d\tau \right\} = 0$. It is easy to check that this condition holds if, and only if, $\mu(t_0) = 0$ because the integral on the r.h.s. cannot be negative. Consequently, the above-mentioned solution to (19) determines that, $\forall t \geq t_0$,

$$\mu(t) = 0. \tag{22}$$

This result implies that even if the central planner internalizes all the pollution-based externalities, he optimally assigns zero value to the social shadow price of the accumulated stock of pollutants. This is because the optimization problem (12) has been solved under the assumption that any solution is an interior solution. In other words, it has been assumed that the static constraint involving the stock of pollutants $S(t) \leqslant S^{\max}$ is satisfied at any time by every optimal solution. This also means that the central planner behaves disregarding the potential damage of a drastic ecological catastrophe on welfare. Obviously, the optimal solution may not be in accordance with such an assumption but, for the moment in this section, we follow the strategy of thinking that it conforms, and focus on finding conditions which guarantee that the assumption holds.

In this case (22), (14), and (15) give the tangency condition

$$z = 1 - \frac{\alpha}{A} \frac{c}{k}, \tag{23}$$

as well as the two control functions

$$c = c(k, q) = \left(\frac{\alpha}{A}\right)^{\frac{\alpha(1-\Phi)}{\Phi - \alpha(1-\Phi)}} q^{\frac{-1}{\Phi - \alpha(1-\Phi)}} k^{\frac{-\alpha(1-\Phi)}{\Phi - \alpha(1-\Phi)}}, \tag{24}$$

$$z = z(k, q) = 1 - \left(\frac{\alpha}{A}\right)^{\frac{\Phi}{\Phi - \alpha(1-\Phi)}} q^{\frac{-1}{\Phi - \alpha(1-\Phi)}} k^{\frac{-\Phi}{\Phi - \alpha(1-\Phi)}}. \tag{25}$$

Substituting (24) and (25) into (16) and (17), we get the dynamic system

$$\overset{\bullet}{k} = (A-n)\,k - A^{\frac{-\alpha(1-\Phi)}{\Phi-\alpha(1-\Phi)}}\left(\alpha^{\frac{\Phi}{\Phi-\alpha(1-\Phi)}} + \alpha^{\frac{\alpha(1-\Phi)}{\Phi-\alpha(1-\Phi)}}\right) q^{\frac{-1}{\Phi-\alpha(1-\Phi)}}\,k^{\frac{-\alpha(1-\Phi)}{\Phi-\alpha(1-\Phi)}}, \tag{26}$$

$$\overset{\bullet}{q} = (\rho - A)q + A^{\frac{-\alpha(1-\Phi)}{\Phi-\alpha(1-\Phi)}}\,\alpha^{\frac{\Phi}{\Phi-\alpha(1-\Phi)}}\,q^{\frac{-(1-\Phi+\alpha(1-\Phi))}{\Phi-\alpha(1-\Phi)}}\,k^{\frac{-\Phi}{\Phi-\alpha(1-\Phi)}}, \tag{27}$$

with the initial condition $k(t_0) = k_0$ and the transversality condition (20). These two differential equations conform a nonlinear dynamic system, which has a particular structure that makes it susceptible of being solved in closed form. Applying the method developed in Ruiz-Tamarit and Ventura-Marco (2011), particularly *Propositions 1* and *2*, we conclude that it does exist a unique optimal solution trajectory for $k(t)$ and $q(t)$ with the closed form representation

$$k^*(t) = k_0 \exp\left\{\frac{A-\rho-\alpha(\rho-n)}{\Phi-\alpha(1-\Phi)}(t-t_0)\right\}, \tag{28}$$

$$q^*(t) = q(t_0) \exp\left\{-\Phi\frac{A-\rho-\alpha(\rho-n)}{\Phi-\alpha(1-\Phi)}(t-t_0)\right\}, \tag{29}$$

$$q(t_0)^{\frac{1}{\Phi-\alpha(1-\Phi)}}\,k_0^{\frac{\Phi}{\Phi-\alpha(1-\Phi)}} = \frac{\Phi-\alpha(1-\Phi)}{\rho-A+\Phi(A-n)}\left(\frac{\alpha}{A}\right)^{\frac{\alpha(1-\Phi)}{\Phi-\alpha(1-\Phi)}}. \tag{30}$$

Given the initial capital stock, k_0, equation (30), which arises directly from the transversality condition, gives the initial value for the shadow price, $q(t_0)$. Once the two initial values are known, equations (28) and (29) determine unequivocally the complete trajectories for these two variables. For any $q(t_0)$ other than the one given by (30) the economy places on an explosive trajectory which does not satisfy optimality conditions, in particular the transversality condition (20). Moreover, given $b_x \equiv \alpha^{\frac{\alpha(1-\Phi)}{\Phi-\alpha(1-\Phi)}} > 0$ the transversality condition holds if, and only if, $a_x \equiv \frac{\rho-A+\Phi(A-n)}{\Phi-\alpha(1-\Phi)} > 0$. This parameter constraint must be satisfied for any positive intertemporal elasticity of substitution, i.e., $0 < \Phi \gtrless 1$, what is not obvious. However, the strict concavity assumption on the utility function imposes the additional parameter constraint

$$\Phi > \alpha(1-\Phi). \tag{31}$$

Hence, the transversality condition (20) holds if, and only if,

$$\rho > A(1-\Phi) + \Phi n. \tag{32}$$

4.1 Characterizing the *sustained* balanced growth path

Given (28) and the production function in per capita terms that arises from (1), we obtain

$$y^*(t) = Ak_0 \exp\left\{\frac{A-\rho-\alpha(\rho-n)}{\Phi-\alpha(1-\Phi)}(t-t_0)\right\}, \tag{33}$$

$$\gamma_y^*(t) = \gamma_k^*(t) = \gamma^* = \frac{A-\rho-\alpha(\rho-n)}{\Phi-\alpha(1-\Phi)}. \tag{34}$$

The growth rates of per capita capital stock and output are equal to each other and constant over time along their respective optimal solution trajectories. Using the control functions for consumption and the degree of dirtiness associated with the technique, as given in (24) and (25), we get the optimal solution trajectories for these two variables

$$c^*(t) = \frac{\rho - A + \Phi(A-n)}{\Phi - \alpha(1-\Phi)} k_0 \exp\left\{ \frac{A - \rho - \alpha(\rho - n)}{\Phi - \alpha(1-\Phi)}(t - t_0) \right\}, \tag{35}$$

$$\frac{c^*(t)}{k^*(t)} = \left(\frac{c}{k}\right)^* = \frac{\rho - A + \Phi(A-n)}{\Phi - \alpha(1-\Phi)}, \tag{36}$$

$$\gamma_c^*(t) = -\frac{\gamma_q^*(t)}{\Phi} = \gamma^*, \tag{37}$$

$$z^*(t) = z^* = \frac{A\Phi - \alpha\rho + \alpha\Phi n}{A(\Phi - \alpha(1-\Phi))}, \tag{38}$$

$$\gamma_z^* = 0. \tag{39}$$

The dirtiness index is expected to be bounded, i.e., $0 \leqslant z^* \leqslant 1$. However, for this to be ensured we need additional parameter constraints. In particular,

$$\Phi(A-n) + n(\Phi - \alpha(1-\Phi)) \geqslant \alpha(\rho - n), \tag{40}$$

$$\Phi(A-n) \geqslant A - \rho, \tag{41}$$

where it is easily checked that (41) encompasses (32). Therefore, the transversality condition leads to $z^* < 1$.

These results completely characterize the socially optimal solution when society, or the central planner, solves the market failure inherent to the local pollution externality. This solution endogenizes the full costs and benefits associated with emissions flow and abatement but, as explained above, it doesn't take care of the possible damage of an ecological and economic catastrophe caused by the global environmental externality. Along the optimal solution variables k, c, and y grow at the same constant rate; the ratio consumption to capital stock is constant and positive; and the dirtiness index remains fixed forever at a constant value between zero and one. Therefore, the model does not predict transitional dynamics and all the endogenous variables conform a balanced growth path from the beginning.[20]

If we examine a little more into the previous results, we find that there could be either positive or negative growth, as well as stationarity. Given (34) and the strict concavity assumption on the utility function, a positive rate of growth $\gamma^* > 0$ arises when $A - \rho > \alpha(\rho - n)$. This condition is compatible with the parameter constraints corresponding to the transversality condition as well as with the lower and upper bounds for z^*, giving

$$\Phi(A-n) + n(\Phi - \alpha(1-\Phi)) > \Phi(A-n) > A - \rho > \alpha(\rho - n) > 0. \tag{42}$$

[20] The rationale for a constant rate of growth is that the social rate of return to capital is constant. In this model

$$r^* \equiv \frac{\partial Y_N}{\partial K} = Az^* = \frac{A\Phi - \alpha\rho + \alpha\Phi n}{\Phi - \alpha(1-\Phi)},$$

the real return to capital is endogenously determined by preferences and technology parameters. The previous expression also shows that only in the absence of environmental concern, $\alpha = 0$, the interest rate is equal to A, as in the canonical model. Otherwise, it is lower because of the transversality condition (32).

Moreover, the above expression represents in this model the Fisher-Keynes-Ramsey equation for the interest rate because it is equivalent to $r^* = \rho + \Phi\gamma^*$.

On the other hand, stationarity $\gamma^* = 0$ arises when $A - \rho = \alpha(\rho - n)$, which combined with the remaining parameter constraints leads to

$$\Phi\left(A-n\right)+n\left(\Phi-\alpha(1-\Phi)\right)>\Phi\left(A-n\right)>A-\rho=\alpha\left(\rho-n\right)>0. \tag{43}$$

In case of stationarity all variables conform a steady state, which is 'chosen' among a multiplicity by the predetermined initial value of the per capita capital stock.

Finally, although less economically relevant, we could find negative growth, $\gamma^* < 0$, when $A - \rho < \alpha(\rho - n)$. This is the only case which gives a chance for $A \leqslant \rho$.[21]

The absence of transitional dynamics that makes the short-run dynamics identical to the long-run ones, leads us to undertake the comparative statics analysis for the socially optimal rate of growth and the dirtiness index. The parameter dependences for these two endogenous variables may be summarized as follows

$$\gamma^* = \gamma\left(\overset{+}{A}, \overset{-}{\rho}, \overset{-}{\Phi}, \overset{-}{\alpha}, \overset{+}{n}\right), \tag{44}$$

$$z^* = z\left(\overset{+/\pm}{A}, \overset{-}{\rho}, \overset{-}{\Phi}, \overset{-}{\alpha}, \overset{+}{n}\right). \tag{45}$$

For the rate of growth, the signs associated with A, ρ, and Φ are the usual in the canonical AK model: the larger the capital productivity and the higher the patience of agents, the greater the rate of growth. A newer but very intuitive result is found here: the higher the weight of environmental care in utility the smaller the rate of growth. That is, for higher values of α that imply a higher marginal utility of abatement and a lower rate of return on capital, the central planner optimally decides to devote more resources to abatement and less to capital accumulation and, hence, to growth. An striking result arises in this model associated with the incontestable positive relationship between the rate of growth and the population growth rate. This result, however, is absolutely dependent on the presence of environmental concern in the model: only in the case in which $\alpha > 0$ we observe that a higher population growth rate leads the central planner to divert resources from abatement and consumption towards capital accumulation. This investment flow is strong enough to compensate for the new capital requirements due to a greater population (capital dilution), and sufficient to breed a greater rate of growth. Moreover, this positive effect is stronger as higher is the weight of environmental care in the utility function.[22]

The dirtiness index, in turn, depends positively on the productivity parameter when the intertemporal elasticity of substitution is greater than one, but the sign of this relationship cannot be analytically decided for values of the elasticity lower than one. Moreover, if the constraint for positive growth along the balanced path holds, the higher the level of patience of agents the higher the value of the dirtiness index. This happens because when consumers are highly patient the central planner optimally decides to reallocate resources towards capital accumulation, which enhance growth. This happens so intensively that even diverts some of the resources previously devoted to pollution abatement, which leads to produce with a more dirty technique. Because of this crowding out effect, the higher the weight of environmental care in the utility function the smaller the dirtiness index. Finally, the greater the population

[21] More technical details about these cases, as well as a complete geometric characterization, may be found in Ruiz-Tamarit and Ventura-Marco (2011).

[22] A similar result, although based on a different explanation, may be found in Bartolini and Bonatti (2003).

growth rate the higher the dirtiness associated with the effective production technique, because for higher population growth rates the central planner decides to divert more resources from abatement effort.[23]

These two variables are closely related to each other. Actually, we can make to appear explicitly the relationship between them by using the first order conditions (16) and (23). If we take the first one and divide by k, and then substitute for the ratio $\frac{c}{k}$ from the second, we get for any $\alpha > 0$

$$\gamma = -\left(\frac{A+\alpha n}{\alpha}\right) + \left(\frac{A+\alpha A}{\alpha}\right) z, \tag{46}$$

from which we can deduce the following pairs of reference: $(z_1, \gamma_1) = \left(0, -\frac{A+\alpha n}{\alpha}\right)$ and $(z_2, \gamma_2) = \left(\frac{A+\alpha n}{A+\alpha A}, 0\right)$. The positive relationship found between γ and z suggests that tighter pollution controls and increased abatement, which reduce the dirtiness index, will have negative effects on the optimal rate of growth. This fact reflects the previous crowding out result according to which greener preferences associated with a shift in preferences towards more environmental concern, i.e., a rise in α, affects negatively both the dirtiness index and the rate of growth.

4.2 Sufficient conditions for *sustainability* of the *sustained* path

One major problem considered in models of endogenous growth is the sustainability of the long-run socially optimal and competitive balanced growth paths. This problem has been largely studied in environmental literature where several definitions of sustainability have been proposed [Pezzey (1997), Chichilnisky (1997)]. The most usual concepts of sustainability rely on the feasibility to substitute between natural and human made capital as well as on the intertemporal evolution of consumption and utility. In growth literature sustainability has been conceived as a situation in which utility and consumption follow a non-declining trajectory. In general, sustainability requires that the needs of the present are satisfied without compromising the needs of the future. This suggests a related problem associated with the valuation of the well-being of present and future generations, and points to the equity condition according to which we must avoid the underestimation of future utilities. In our model, the previous requirements for sustainability are always met because the socially optimal choices guarantee a monotonically increasing profile for both consumption and utility.

However, even if conditions for a positive long-run rate of growth are satisfied, there is still a fundamental trade-off between growth and environmental quality that must be carefully analyzed. Here, it is important to recall that environmental quality, which extreme counterpart is the absence of pollution, increases with abatement but diminishes with production. This trade-off results from agent decisions and involves the level of abatement expenditures; hence, the value of the dirtiness index. Consequently, sustainability may also be inspected by looking at the positive relationship between the dirtiness index and the rate of growth that is shown in (46), and which represents in other words the above-mentioned trade-off. According to this equation the more clean the used technology is, the lower the rate of growth. In particular, under the assumption that people show environmental awareness, $\alpha > 0$, our model predicts a lower or, at most, an equal rate of growth relative to the rate of growth arising from the canonical model where no environmental concern does exist, $\alpha = 0$. Moreover, it has been shown that γ^*

[23] The results concerning the population growth rate are consistent with propositions discussed and tested in Cropper and Griffiths (1994). In that paper, the environment is not a factor that restrains productivity as population expands, but a good which quality is degraded by a growing population.

may be zero provided that $\alpha = \frac{A-\rho}{\rho-n} > 0$, which is in accordance with the premise that more environmental concern comes in detriment of growth. But this is an exceptional case and, for any other value of α smaller than the previous one, we are still allowed to conclude that positive sustained growth and environment preservation are compatible in the long-run.

In any case, the main question we shall consider here is whether the results corresponding to the unique optimal solution studied in previous subsections are compatible with the general condition of no environmental and economic catastrophe. This means that we have to check whether the current level of the pollution stock associated with the positive balanced growth path is always lower than, or equal to, the critical level. This exercise is compulsory because the central planner's optimization problem has been solved above under the assumption that the static constraint $S(t) \leqslant S^{\max}$ always holds. However, this is not necessarily the case in the present context and we need to focus on the study of the dynamics of $S(t)$, looking for the sufficient conditions which guarantee that along the socially optimal path, the catastrophic state that reduces consumption and utility instantaneously to zero will never be reached.

The dynamics of the stock of pollutants is governed by $\dot{S}(t) = G(1 - z(t)) - \delta S(t)$, with $0 < S_0 \leqslant S^{\max}$, $G_z > 0$, $G_{zz} < 0$, $G(1) = 0$, and $G(0) = G^M > 0$. Solving backward for $S(t)$ we get

$$S(t) = S_0 e^{-\delta(t-t_0)} + \int_{t_0}^{t} G(1 - z(\tau)) e^{-\delta(t-\tau)} d\tau. \tag{47}$$

The first term on the r.h.s. is a finite value that approaches zero as $(t - t_0)$ tends to $+\infty$, and the integral of the second term is a convergent one as long as the function $G(.)$ grows at most at a positive exponential rate lower than δ. In fact, the assumed more restrictive condition $G_{zz} < 0$ suffices to guarantee the convergence of the integral, given the presence of an exponential discount term. Furthermore, from the AK nature of the model and the corresponding absence of transitional dynamics we know that $z(t)$ takes optimally a constant value z. Thus, the above expression for $S(t)$ may be simplified to

$$S(t) = \left(S_0 - \frac{G(1-z)}{\delta}\right) e^{-\delta(t-t_0)} + \frac{G(1-z)}{\delta}. \tag{48}$$

Given S_0, the stock of pollutants converges monotonically to a constant finite value, which is determined by: *i*) the net emissions corresponding to the dirtiness index z, and *ii*) the exogenous constant rate of natural regeneration δ. That is,

$$\lim_{t \to +\infty} S(t) \equiv S_\infty = \frac{G(1-z)}{\delta} \tag{49}$$

This result, however, does not suffices to prevent the environmental catastrophe. In the model, such a situation may easily arise for high values of z given that $G_z > 0$ and, hence, S_∞ depends positively on z.

On the one hand, the value of z could be chosen at a level $\overline{z}$ for which emissions flow is exactly balanced out by the regeneration corresponding to the natural capacity of the environment to absorb pollution. In such a case, a steady state $\dot{S}(t) = 0$ arises from the beginning[24] and then

[24] Sometimes, the condition for stationarity has been taken as a condition for sustainability of the balanced growth path [Chevé (2000)]. This one may be considered as a very strong, near the conservationists, position where the stock of pollutants and other environmental variables remain constant while the rest of economic variables are still allowed to grow at a constant positive rate. Our analysis shows stationarity as a substitute for sustainability only in the particular case that the ecological upper limit has been reached.

$\forall t \; S(t) = \bar{S}_\infty = S_0 = \frac{G(1-\bar{z})}{\delta}$. This means that

$$\bar{z} = 1 - G^{-1}(\delta S_0). \tag{50}$$

Consequently, if $z < \bar{z}$ then $S(t)$ monotonically decreases below S_0 converging to a certain $S_\infty < S_0$, while if $z > \bar{z}$ then $S(t)$ monotonically increases converging to $S_\infty > S_0$.

On the other hand, the no-catastrophe condition requires that $S(t) \leqslant S^{\max}$ for every $t \geq t_0$ and, in particular, that $S^*(t) \leqslant S^{\max}$, being $S^*(t)$ the socially optimal path for the stock of pollutants. This one emerges from the optimal balanced growth path we are operating, and is determined by substituting the value z^* given in (38) into (48). Because of monotonicity, the no-catastrophe condition holds when $S^*_\infty \leqslant S^{\max}$, where $S^*_\infty = \frac{G(1-z^*)}{\delta}$ is the limiting value for the optimal stock.

Finally, considering that $z^{\max}$ is the constant value of z that eventually makes the stock of pollutants to catch up with the critical level $S^{\max}$, it must satisfy

$$z^{\max} = 1 - G^{-1}(\delta S^{\max}). \tag{51}$$

Then, our search for sufficient conditions for sustainability leads to the constraint

$$\frac{\alpha}{A} \; \frac{\rho - A(1-\Phi) - \Phi n}{\Phi - \alpha(1-\Phi)} \geqslant G^{-1}(\delta S^{\max}). \tag{52}$$

This parameter relationship, if it holds, suffices to make the balanced growth path analyzed above in this section compatible with the requirement of a non-catastrophic state in the economy. It establishes the margins for sustainable and sustained optimal growth. In fact, condition (52) corresponds to $z^* \leqslant z^{\max}$, which represents a more intuitive version of the sufficient condition for sustainability. In this condition it is involved $z^{\max}$, which depends (positively) on the ecologically determined parameters $S^{\max}$ and δ, as well as z^* , which is endogenously decided and depends on preference and technological parameters. In short, (52) establishes the association between ecological and economic determinants that allows for the sustained socially optimal growth to be also ecologically sustainable in the long-run.

5 Necessary conditions for sustainability of the optimal solution

In this section we study optimality as much as sustainability of the long-run balanced growth path from a comprehensive perspective. This is done by simultaneously considering in the formulation of the dynamic optimization problem all the constraints which are relevant for the economy and the environment; i.e. the non-negativity constraint, the control-variable constraint and, specially, the state-variable constraint. As we have previously shown, there are two kind of environmental externalities with differentiated effects on welfare. The problem we develop here endogenizes both, the local and the aggregate externalities, assuming that society, by means of the central planner, maximizes social welfare taking into account the true social costs and benefits from pollution emissions and abatement, but also the global negative effect associated with the potential damage on welfare of an ecological and economic catastrophe. Moreover, we shall describe the socially optimal behavior by studying the paths for the model's variables when it is allowed to reach the upper bound of the ecologically sustainable growth. Namely, the behavior of the economy when the aggregate stock of pollutants in the environment $S(t)$ may catch up with, or even to potentially exceed, the critical level $S^{\max}$, not eventually as a limit but after a finite time span.

This analytical context shows a great parallelism with the real problem concerning the global warming of the Earth's surface and atmosphere caused by the anthropogenically enhanced greenhouse effect. It is well-known that the greenhouse gases accumulated in the atmosphere are reaching, if not already done so, the critical level which will lead to a catastrophic climate change. In this situation, it is of great interest to find out a satisfactory answer for the questions: (i) how can we manage efficiently the present state of affairs characterized by an immoderate use of fossil fuels and other damaging activities which entail an excessive pollution flow, and (ii) what must we do, if we can do something, to guarantee a positive long-run rate of growth still compatible with the environment preservation. One possible, though imperfect, answer is the Kyoto protocol, which is an international agreement that provides the necessary framework for a cooperative solution. It may be seen as playing the role of a global central planner which faces up to the global warming problem. Its implementation entails setting individual pollution targets with the aim of achieving a reduction of the aggregate emissions flow. But, what tells us our model in this regard?

Let us start by considering the more general optimal control problem involving either non-negativity constraints, pure control variable constraints, and pure state-space constraints, addressed to our endogenous growth model with environmental concern and awareness. In particular, we are now to introduce the constraints

$$\begin{aligned} &\text{(i)} \quad z(t) \geqslant 0, \\ &\text{(ii)} \quad 1 - z(t) \geqslant 0, \\ &\text{(iii)} \quad S(t) \leqslant S^{\max}, \end{aligned} \tag{53}$$

which add to the usual dynamic and boundary constraints for K and S. The third kind of constraint in (53) consists of one constraint in which no control variables are present. This constraint places a restriction on the state space, delimitating the permissible area for the accumulated stock of pollutants. Consequently, writing the current stock in per capita terms we get $S^{\max} - e^{nt}s(t) \geqslant 0$. However, given that $e^{nt}s(t)$ is not allowed to exceed $S^{\max}$, when the constraint is binding, i.e. $S(t) = S^{\max}$, we impose the new condition

$$\frac{d\left(e^{nt}s(t)\right)}{dt} = e^{nt}\left(g\left(1 - z(t)\right) - \delta s(t)\right) \leqslant 0 \quad \left(\text{whenever } e^{nt}s(t) = S^{\max}\right), \tag{54}$$

where we have made use of the equation representing the motion of the stock of pollutants in per capita terms, i.e. $\overset{\bullet}{s}(t) = g\left(1 - z(t)\right) - (\delta + n)\, s(t)$.

Hence, the planner's problem consists in choosing the sequence $\{c(t), z(t), t \geq t_0\}$ which, for a given positive social rate of discount $\rho > n$, solve the optimization problem

$$\max_{\{K,S,c,z\}} \int_{t_0}^{+\infty} \frac{c^{1-\Phi}}{1-\Phi} (1-z)^{\alpha(1-\Phi)} e^{-(\rho-n)(t-t_0)} dt$$

$$s.t. \qquad (1)\text{-}(5),\ (53),\ (54), \tag{55}$$

$$\text{for } k(t_0) = k_0 > 0 \text{ and } s(t_0) = s_0 > 0 \text{ given.}$$

Then, using lowercase letters to represent variables in per capita terms, the general dynamic optimization problem may be written as in the following current value Hamiltonian

$$\underset{\{c,z,q,k,\mu,s,\eta,\theta\}}{H^c} = \frac{c^{1-\Phi}(1-z)^{\alpha(1-\Phi)}}{1-\Phi} + q\left(Akz - c - nk\right) + \mu\left(g(1-z) - (\delta+n)\, s\right) +$$

$$+\eta(1-z) - \theta e^{nt}\left(g(1-z) - \delta s\right). \tag{56}$$

Here, q and μ are the co-states for k and s respectively, and η and θ are Lagrangian multipliers associated with the control-variable constraint and the state-variable constraint respectively. Both η and θ are dynamic multipliers because their corresponding constraints must be satisfied at every period t. Given that the control inequality constraint is linear, the first order necessary conditions arising from Pontryagin's principle and Kuhn-Tucker theorem are

$$q = c^{-\Phi}(1-z)^{\alpha(1-\Phi)}, \tag{57}$$

$$qAk + \mu g_z - \eta - \theta e^{nt} g_z - \frac{\alpha c^{1-\Phi}(1-z)^{\alpha(1-\Phi)}}{1-z} \leqslant 0,$$

$$z \geqslant 0, \quad z\left(qAk + \mu g_z - \eta - \theta e^{nt} g_z - \tfrac{\alpha c^{1-\Phi}(1-z)^{\alpha(1-\Phi)}}{1-z}\right) = 0, \tag{58}$$

$$\overset{\bullet}{k} = Akz - c - nk, \tag{59}$$

$$\overset{\bullet}{q} = \rho q - Azq, \tag{60}$$

$$\overset{\bullet}{s} = g(1-z) - (\delta+n)\, s, \tag{61}$$

$$\overset{\bullet}{\mu} = (\rho+\delta)\,\mu - \delta\theta e^{nt}, \tag{62}$$

$$1 - z \geqslant 0, \quad \eta \geqslant 0, \quad \eta(1-z) = 0, \tag{63}$$

$$g(1-z) - \delta s \leqslant 0, \quad \theta \geqslant 0, \quad \theta\left(g(1-z) - \delta s\right) = 0. \tag{64}$$

To make clear that (62), (63), and (64) only apply when $e^{nt}s(t) = S^{\max}$ we append the complementary-slackness condition and the restriction on the way θ changes over time

$$S \leqslant S^{\max}, \quad \theta (S - S^{\max}) = 0, \quad \dot{\theta} \leqslant 0 \quad (= 0 \text{ when } S < S^{\max}). \tag{65}$$

Finally, we also need the initial conditions k_0 and s_0, and the transversality conditions

$$\lim_{t \to +\infty} e^{-(\rho - n)(t - t_0)} qk = 0, \tag{66}$$

$$\lim_{t \to +\infty} e^{-(\rho - n)(t - t_0)} \mu s = 0. \tag{67}$$

These necessary conditions are also sufficient for a maximum because the Hamiltonian function satisfies the required concavity conditions. Moreover, it is trivial to prove that when the static constraints (53) of the previous optimization problem are nonbinding, the above first order conditions reduce to the ones studied in section 4, and we only get Pareto optimal interior solutions.[25] However, if anyone of such constraints changes its status from nonbinding to binding, then all the first order conditions (57)-(67) become fully operative and Pareto optimal corner solutions are also feasible.

According to this, and given that initially we have $S_0 \leqslant S(t) < S^{\max}$, at t_0 and during all the time until $S(t) = S^{\max}$ the socially optimal path that corresponds to the solution of equations (57)-(67) is the same as the one studied in section 4. In particular, the planner will choose the constant value $0 < z^* = \frac{A\Phi - \alpha\rho + \alpha\Phi n}{A(\Phi - \alpha(1-\Phi))} < 1$ for the dirtiness index. Inside the above interval, we are now to consider the more interesting case in which $z^* > z^{\max}$, and then we shall find $S^*(t^c) = S^{\max}$ for some $+\infty > t^c > t_0$. This implies that the absolute limit to growth, i.e. the ecological and economic catastrophe, appears as a binding constraint in finite time. From substituting the binding state constraint and z^* into (48), we show that

$$t^c = t_0 + \ln \left(\frac{G(1 - z^*) - \delta S_0}{G(1 - z^*) - \delta S^{\max}} \right)^{\frac{1}{\delta}}, \tag{68}$$

which implies that the time elapsed until the economy falls into the catastrophe, $T^c = t^c - t_0$, may be written as a function that depends on the involved variable and parameters according to the following signs of the partial derivatives

$$T^c = T\left(\overset{-}{z^*}, \ \overset{-}{S_0}, \ \overset{+}{S^{\max}}, \ \overset{+/-}{\delta} \right). \tag{69}$$

In what follows we pay special attention to the case in which the pure state-space constraint changes its status from nonbinding to binding, and we study the first order conditions at that moment and thereafter.

Given that we will get $S^*(t^c) = S^{\max}$ for some $+\infty > t^c > t_0$, using condition (65) we first deduce $\dot{\theta}(t^c) < 0$. Then, given that θ is not allowed to be negative, we necessarily conclude that $\theta(t^c) > 0$ and, from (64), also that $g(1 - z(t^c)) = \delta s(t^c)$. In aggregate terms we have

[25] This is because in such a case $\dot{\theta} = 0$ and $\theta = 0$, but also $\mu = 0$ and $\eta = 0$.

$G(1-z(t^c)) = \delta S^*(t^c) = \delta S^{\max}$. Consequently, at t^c the dirtiness index, a variable which admits jumps, will take the value

$$0 < z(t^c) \equiv z^c = 1 - G^{-1}(\delta S^{\max}) = z^{\max} < z^* < 1, \tag{70}$$

irrespective of its value in previous periods. This means that at t^c the central planner may optimally decide a discrete and instantaneous change in the dirtiness index from any $z^* > z^{\max}$ to $z^c = z^{\max}$. It is obvious that $z^c = 0$, which implies $g(1) = 0 = \delta s(t^c)$ and $S(t^c) = 0$, is not an admissible option because in the present context $S(t^c) = S^{\max} > 0$. Moreover, if $z^c = 1$ then $g(0) = \frac{G^M}{N(t^c)} = \delta s(t^c)$, which implies a contradiction because it means $S(t^c) = \frac{G^M}{\delta} > S^{\max}$. Therefore, from (63) we get $\eta(t^c) = 0$. When the central planner runs the economy trying to avert the catastrophic event, he must keep constant $S(t) = S^*(t^c) = S^{\max} \quad \forall t \geqslant t^c$, and then the dirtiness index will remain stuck to the value $0 < z(t) = z^c = z^{\max} < 1 \quad \forall t \geqslant t^c$. Hence, $\dot{\theta}(t) < 0$, $\theta(t) > 0$, and $\eta(t) = 0 \quad \forall t \geqslant t^c$.

On the other hand, from (61) and (64) the dynamic equation for s becomes $\dot{s}(t) = -ns(t)$, from which we get $s(t) = s(t^c)\, e^{-n(t-t^c)} \quad \forall t \geqslant t^c$. Moreover, the general solution to the dynamic equation (62) is $\mu(t) = \mu(t^c)\, e^{(\rho+\delta)(t-t^c)} - \int_{t^c}^{t} \delta\theta(\tau)\, e^{n\tau} e^{(\rho+\delta)(t-\tau)} d\tau \quad \forall t \geqslant t^c$. The path for the shadow price μ couples to such of the stock s, which is explained by population growth alone, and the transversality condition (67) holds if, and only if, $\mu(t^c) = \int_{t^c}^{+\infty} \delta\theta(\tau)\, e^{n\tau} e^{-(\rho+\delta)(\tau-t^c)} d\tau$. The previous integral is bounded because the integrand converges to zero, given that θ is positive but decreasing and $\rho > n$. Therefore, now we find that $\forall t \geqslant t^c$,

$$\mu(t) = \int_{t}^{+\infty} \delta\theta(\tau)\, e^{n\tau} e^{-(\rho+\delta)(\tau-t)} d\tau > 0. \tag{71}$$

As expected, $\mu(t) \neq 0 \quad \forall t \geqslant t^c$ because in addition to the fact that the central planner internalizes all the pollution-based externalities, here the pure state-space constraint is binding and, in the corresponding corner solution, he optimally assigns a positive value to the social shadow price of the accumulated stock of pollutants.

Given all the preceding results, the remaining first order conditions describe the dynamics for q, c, and $k \quad \forall t \geqslant t^c$.[26] Specifically, such dynamics must emerge from equations

$$q = (1-z^c)^{\alpha(1-\Phi)}\, c^{-\Phi}, \tag{72}$$

$$\dot{c} = \left(\frac{Az^c - \rho}{\Phi}\right) c, \tag{73}$$

$$\dot{k} = (Az^c - n)\, k - c, \tag{74}$$

with the initial condition $k(t^c)$ and the transversality condition

$$\lim_{t\to+\infty} e^{-(\rho-n)(t-t^c)} c^{-\Phi} k = 0. \tag{75}$$

The dynamic system (72)-(75), except for the differences in coefficients, shows an equivalent structure to the standard AK model of endogenous growth with no environmental concern. The particular solution $\forall t \geqslant t^c$, which is also socially optimal, is

$$k^c(t) = k(t^c) \exp\left\{\frac{Az^c - \rho}{\Phi}(t - t^c)\right\}, \tag{76}$$

[26] It has to be remarked that in the present context equation (58), or its adapted version for $\forall t \geqslant t^c$, is no longer relevant because the choice of the dirtiness index does not require a marginal condition.

$$\frac{c^c(t)}{k^c(t)} = \frac{\rho - Az^c + \Phi(Az^c - n)}{\Phi}, \tag{77}$$

$$q^c(t) = q(t^c)\exp\left\{-\left(Az^c - \rho\right)(t - t^c)\right\}, \tag{78}$$

$$q(t^c) = \left(\frac{\Phi\left(1 - z^c\right)^{\frac{\alpha(1-\Phi)}{\Phi}}}{\rho - Az^c + \Phi(Az^c - n)}\right)^{\Phi} \frac{1}{k\left(t^c\right)^{\Phi}}, \tag{79}$$

$$\rho > Az^c(1 - \Phi) + \Phi n, \tag{80}$$

$$y^c(t) = Ak\left(t^c\right)\exp\left\{\frac{Az^c - \rho}{\Phi}(t - t^c)\right\}, \tag{81}$$

$$\gamma_y^c(t) = \gamma_c^c(t) = \gamma_k^c(t) = \gamma^c = \frac{Az^c - \rho}{\Phi}. \tag{82}$$

These results completely characterize the economic system after period t^c, just on the limits below the catastrophe, with an accumulated stock of pollutants equal to $S^{\max}$ and the dirtiness index fixed at the level $z^c = z^{\max}$, which only depends on ecologically determined parameters. Moreover, variables k, c, and y grow at a common constant rate, which is positive under the assumption of a constant return to capital greater than the social discount rate, $r^c \equiv Az^c > \rho$. It is interesting to remark that this new rate of growth as well as the new dirtiness index do not depend on the weight of environmental care in utility, α, or the population growth rate, n. Finally, the ratio consumption to capital stock is constant, and positive according to the transversality condition. Once again, the model does not show transitional dynamics beyond t^c and all the endogenous variables conform a unique Pareto optimal balanced growth path.

When we consider the whole trajectories starting from t_0, the optimal balanced growth path appears truncated at t^c. The control variable z experiences a change of scale by jumping down from the constant z^* to the constant $z^{\max}$. That is,

$$\frac{A\Phi - \alpha\rho + \alpha\Phi n}{A(\Phi - \alpha(1 - \Phi))} > z^c. \tag{83}$$

The stock S does not change the scale but it changes drastically the slope, it stops growing and remains constant forever at the level $S^{\max}$. The stock k, which does not change the scale, changes its slope as a consequence of the rate of growth that changes from γ^* to γ^c. That is, because of (83) the constant rate of growth experiences a change of scale by jumping down,

$$\gamma^* = \frac{A - \rho - \alpha(\rho - n)}{\Phi - \alpha(1 - \Phi)} > \frac{Az^c - \rho}{\Phi} = \gamma^c. \tag{84}$$

Instead, the control c changes both the scale and the slope. It changes the slope because of (84), and it changes the scale because

$$\frac{\rho - A + \Phi(A - n)}{\Phi - \alpha(1 - \Phi)} > \frac{\rho - Az^c + \Phi(Az^c - n)}{\Phi}. \tag{85}$$

Inequality (85) also means that the constant ratio consumption to capital stock, which is a non predetermined variable, experiences a change of scale by jumping down.

If we pay attention to the implications of the above results for environmental policy, we find that a central planner managing efficiently the economy, which specifically includes trying to avoid the ecological and economic catastrophe, at period t^c of the optimal balanced growth path he must introduce an immediate change in the controlled index of dirtiness by adjusting both the abatement effort and the emissions flow in the whole economy.

The necessity of an abrupt change in the dirtiness index is even more incontestable in the case of a competitive decentralized economy. In this case market failures lead the economy to choose $z = 1$, which implies that agents use the most polluting of the available techniques, but also that society is condemned to the catastrophe. From (10) and (68) and given that $G^M > G(1 - z^*)$ we get

$$1 < \frac{G^M - \delta S_0}{G^M - \delta S^{\max}} < \frac{G(1 - z^*) - \delta S_0}{G(1 - z^*) - \delta S^{\max}}, \tag{86}$$

which means that the suboptimal competitive economy reaches the catastrophic state faster than the efficiently managed economy, i.e. $T_d^c < T^c$, even though the latter have optimally chosen a $z^* > z^{\max}$.

Therefore, the above results claim for public regulation. Taking the environmental problem as it is faced up by a non-regulated and decentralized economy, we identify different opportunities for government interventions. First, an institutional one that involves the government correcting the local externality associated with pollution emissions and abatement. This may be done by setting the usual Pigouvian taxes and subsidies that make the competitive economy to work efficiently. Second, the government may develop an allocative function, which implies the direct participation by means of a public provision of abatement, greater than the one decided in the decentralized economy. Moreover, given the threat of a global environmental catastrophe, if the optimal choice for endogenous variables is such that leads the stock of pollutants to catch up with the critical level, then the government must abruptly impose cleaner technologies by fixing emission standards and increasing abatement suddenly. These actions will reduce pollution and stabilize the stock of pollutants, a condition that is required to avoid the ecological catastrophe.

It is important to highlight that the tighter standards and higher abatement efforts that allow to reduce net emissions until the sustainable level, also impose a cost on society in terms of lower consumption and welfare. However, this is the only way to preserve sustained growth and, hence, increasing consumption along the optimal path. Recall that if the catastrophe materializes then consumption and utility will be drastically reduced to zero.

On the other hand, the government may implement indirect environmental policies such as information or awareness campaigns [Chevé (2000)]. These ones influence social preferences for environmental conservation, the environmental willingness to pay and, hence, the demand for environmental quality. In other words, participation can make people more environmentally conscious and may prevent that the environment be felt as an obstacle to growth [Bimonte (2001)]. Finally, the government may put into action population controls and other development encouraging measures that accelerate the demographic transition. This is important because of the impact of population growth on the environmental quality as well as on the long-run rate of growth.

According to the thoughts from the Stern review on the economics of climate change [Stern (2007)] and the results from this section, the emissions control and reduction recommended in the Kyoto protocol seems to be a suitable policy if we want to avoid the ecological and economic catastrophe. The evidence about the enhanced greenhouse effect and the global warming trend tells us that the critical level for the accumulated greenhouse gases in the atmosphere is near to be reached. Consequently, only coordinated actions involving governmental policies that stabilize and reduce the aggregate stock of such greenhouse-effect pollutants may be effective at this stage. Moreover, the Kyoto protocol also includes other accompanying policies, which taken together may help to guarantee global efficiency and positive long-run growth. In short, we think that the proposals of this international treaty are essential to enable economic development going forward in a sustainable manner.

6 Conclusions

In this paper, we identify two types of pollution externalities. First, a local externality associated with the emissions flow and the disparity between private and social costs and benefits of abatement activities. Second, an aggregate externality which comes from the fact that even if people are prone to take into account the local negative effect, they are not aware of the global negative effect from an eventual catastrophe associated with the accumulated stock of pollutants. The local negative effect represents a continuous process which marginal damage has been modeled entering the instantaneous utility function as a direct argument. Instead, the global negative effect is discontinuous and its drastic damage on welfare has been hypothesized as changing parameters of the intertemporal utility function. This change, which implies that consumption and utility are abruptly reduced to zero, happens when the accumulated stock of pollutants reaches and exceeds a certain threshold or critical level. To tackle this problem, which shows a strong parallelism with the real problem of the global warming of the Earth's surface and the atmosphere, caused by the anthropogenically enhanced greenhouse effect, we use a general equilibrium one-sector endogenous growth model in which pollution is a by-product of economic activity, but it may be reduced by spending a fraction of the aggregate output on abatement. For the sake of simplicity we adopt a production function of the AK type where the sole factor needed to produce is taken in a broad sense including both physical and human capital.

First of all, under multilateral and diffuse local and global pollution externalities, we find that the decentralized equilibrium path is not Pareto optimal nor environmentally sustainable. This is because, although abatement activities are endogenously decided, there is a market failure creating the wrong incentives that lead to insufficient abatement and excessive pollution. Such a result is clearly inefficient and claims for public regulation.

Second, we study the socially optimal equilibrium which simultaneously internalizes both externalities. In this optimization problem, the central planner considers the true social costs and benefits from pollution abatement, and also takes into account the global negative effect from an eventual catastrophe. We find that the balanced growth path shows a singularity at a point where trajectories truncate. Hence, there are two periods clearly differentiated: previous and after the concentration level of greenhouse gases in the atmosphere catches up with the critical level, which may be set at around 550 ppm CO_2-equivalent. Initially, the positive rate of growth as well as the dirtiness index depend negatively on the weight of environmental care in utility and positively on the population growth rate. We find also a trade-off between environmental quality and growth, which results from agent decisions because increased abatement effort crowds out resources from capital accumulation and growth. Thereafter, a lower optimal dirtiness index is the unique compatible with the no catastrophe condition, but this one only depends on ecologically determined parameters. Moreover, the new rate of growth is still positive, but now it does not depend on the weight of environmental care in utility or the population growth rate.

We conclude that both the suboptimal competitive economy and the efficiently managed economy claim for a change of course in abatement effort and the emissions flow if the accumulated stock of pollutants in the environment has to be stabilized below the catastrophic level. However, this action is required with different degrees of urgency, given that the decentralized economy evolves towards the catastrophe faster than the centralized economy. On the other hand, public intervention to prevent that environment be damaged up to the irreversible level entails a cost for society in terms of lower consumption and welfare. As we have seen, consumption must experience a change of scale and slope. However, this is the only way of preserving sustained growth and, hence, an increasing consumption along the optimal path. Alternatively, the sustained growth is no longer sustainable because consumption and utility will be drastically reduced to zero. In other words, there is still a trade-off between environmental quality and growth according to which sustainability can be guaranteed at the cost of a lower positive long-run rate of growth.

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