# Income Inequality, School Choice and the Endogenous Gentrification of US Cities

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#### Abstract

Why in some urban areas do rich and poor households cohabit at the community level while, in others, we observe a sorting by income? To answer this question I develop a two-community general equilibrium framework of school quality, residential choice and tax decision. The model predicts that in highly unequal societies low and high income households choose to live in the same community but segregate by schooling. When inequality is smaller, we observe the typical sorting by income across communities. The effect of inequality on the quality of public schools depends on the relative size of the housing market of each community. When inequality increases, if the housing conditions of the community in which rich and poor households cohabit are affordable, then an inflow of high income middle class households towards this community emerges (gentrification). As a consequence, inequality impacts negatively the quality of the public school because both rich and poor households vote for lower taxation.

Key Words: Gentrification; Housing; Inequality; Segregation; Stratification. JEL Codes: D72, H42, I24, R21.

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#### 1 Introduction

It is well established that a peculiar characteristic of U.S. metropolitan areas is the concentration of poverty close to inner cities. According to the U.S. Census Bureau, 17.6% of the population in the inner cities of all U.S. metropolitan areas had an income below the poverty level in 1999<sup>1</sup>. However, in some urban areas high income households also reside very close to business districts that are located within the central areas of cities. Not surprisingly in some inner cities high income households reside in the same area as low income households. The cohabitation of heterogeneous income groups in the same community, in particular in the same school district, may have important implications for income redistribution, access to high quality education, public policies, political decisions and socio-economic opportunities.

A central issue is to understand the consequences of the emergence of mixed income communities, since the presence of different income groups within the same community may create segregation in terms of access to high quality schooling and, therefore, hinder the upward mobility of poor households. Why in some urban areas, do we observe a sorting by income at community level while, in others, rich and poor households cohabit? How does this difference relate to the quality of public schooling and private enrollment within school districts? Does income inequality have a role in explaining schooling quality, residential choices and community segregation?

The aim of this paper is to develop a general equilibrium model of private/public school choice, political decisions and residential choices, that can answer to these questions. My theoretical structure builds on de la Croix and Doepke (2009). The authors develop a single district economy in which agents perfectly segregate by education. The segregation pattern is driven by the fact

<sup>&</sup>lt;sup>1</sup>In contrast, only 8.4% of the population in the suburbs lived in poverty during the same year. Source: U.S. Census Bureau, 2000 Census, Summary File 3. The data also point out that poverty in inner cities had declined since 1990. See Berube and Frey (2002) for an analysis of poverty rates in the 102 largest U.S. metropolitan areas based on the 2000 Census, and Berube and Kneebone (2006) for a similar study based on the 2000 Census and 2005 ACS.

that parents prefer to enroll their children in private schools when these schools provide a higher education quality than what is provided in the public system<sup>2</sup>. Compared to de la Croix and Doepke's work, I focus on a multi-community economy in which housing market and fiscal policies interact with school and residential location choices, and therefore, with the quality of public education in different communities. In particular, the framework I provide involves an urban area composed of two communities with homogeneous land and fixed boundaries, which can be interpreted as two different school districts. Parents have to decide in which community to live and in which type of school to send their children, choosing between a tax financed public school or a private school financed by tuition fees. The quality of public education in each district is determined by the amount of spending per student financed through property taxes on housing value<sup>3</sup>. Moreover, a voting process takes place in each community in order to determine the local tax rate and, therefore, the public education spending.

I first investigate the existence of an equilibrium in which poor and rich households cohabit in the same community and send their children respectively to public and private school. In contrast, middle income households reside in the other community and choose the local public school. From a political perspective, my model ends up with the result provided in Epple and Romano (1996): if this political equilibrium exists, then a coalition of rich and poor households will be opposed by a coalition of middle income households. This outcome, namely *'the ends against the middle'*, implies that high and low income households vote for low taxation and public school spending, while the middle income households vote for a high level of redistribution.

An important prediction of my theoretical model is related to the link

 $<sup>^{2}</sup>$ The choice between public and private education has already been studied by many authors: Stiglitz (1974), Epple and Romano (1998), Glomm and Ravikumar (1998), in a single district economy, Nechyba (2000), Bears et al. (2001), Martinez-Mora (2006), in a multi-district economy.

<sup>&</sup>lt;sup>3</sup>If we assume a proportional income tax rather than a property value tax the model predicts the same qualitative results.

between inequality and gentrification of inner cities<sup>4</sup>. During recent decades, since the early 70's, U.S. metropolitan areas experienced an increase in income inequality, driven largely by income growth in the top half of the income distribution<sup>5</sup>. Increase in inequality has been accompanied by a change in the population composition of some central areas of cities, caused by a replacement of poor households by high income households of the middle class.

My model is able to replicate the phenomenon of gentrification in urban areas characterized by income mixing and high housing density. In particular, when inner cities are characterized by a large number of housing units with respect to the suburbs, as for instance in the U.S. cities with earlier urban development, the model predicts an endogenous gentrification of the central areas as a consequence of inequality. If the conditions on the housing market in the central district are such that housing prices are affordable, then it represents an economic opportunity for high income middle class households to live in this district. Together with the possibility to opt-out of the public school system and to pay lower taxes, these families may decide to move away from their community of residence and settle in neighborhoods close to the central business district. Consequently, when inequality increases, the model predicts a population reallocation that may provoke a displacement of poor households towards the suburbs.

In my model the effect of inequality on public spending per student is ambiguous and depends on the relative size of the endowment in housing in the two communities and, therefore, on population reallocation across districts. When the size of the housing market in the community in which we observe income mixing is sufficiently small compared to the other community, inequality decreases the tax rate but increases the public spending per pupil. In

 $<sup>^{4}</sup>$ Glass (1964) coined the term gentrification: it denotes the influx of middle class households to cities, displacing the lower class worker residents towards the suburbs. Consequently, average households income and housing prices increase in neighborhoods close to CBD.

<sup>&</sup>lt;sup>5</sup>See Gottschalk (1997), Goldin and Katz (2001), Piketty and Saez (2003), Corocan and Evans (2010). According to the U.S. Census Bureau, inequality as measured by the Gini coefficient increased from 0.415 in 1979 to 0.469 in 2009.

this case high income middle class households do not move toward the income mixing community and we end up with one of the results in de la Croix and Doepke (2009): higher inequality is positively associated with public spending per student. If this condition on the housing market is not satisfied, inequality impacts negatively on the public spending per student.

With respect to the seminal work by Epple and Romano (1996), my framework provides some important differences. On the one hand, I consider a multicommunity model in which a competitive housing market determines housing prices in each community, and where households can move at no cost across communities. On the other hand, while they focus on majority equilibrium, I assume a probabilistic voting mechanism<sup>6</sup>. As a distinct theoretical contribution, I derive analytically the conditions under which an inter-community political equilibrium with income mixing exists and is unique. I find that an income mixing equilibrium is more likely if the mass of households in metropolitan areas is more concentrated in the tails of the income distribution rather than at the average level. If the conditions for this particular type of equilibrium are not satisfied, then the model predicts a perfect stratification across communities according to income. In this case the fully public regime prevails, and the community with lower (higher) quality of public education is populated by households with low (high) income.

Fernandez and Rogerson (1996) and Benabou (1996) develop a multicommunity model in which public education is financed through a proportional income tax determined by majority voting within each community. They find perfect income stratification across communities. Bearse et al. (2001) follow this approach studying the impact of the centralization of funding and vouchers reform and allowing households to opt-out of the public system. They find that funding education through vouchers lowers average public spending on

<sup>&</sup>lt;sup>6</sup>I follow the most recent literature in adopting a probabilistic voting mechanism for the determination of education policies. This political mechanism guarantees the existence of the equilibrium. See de la Croix and Doepke (2009), Dottori and Shen (2009), Arcalean and Schiopu (2010).

education. Epple et al. (1994) characterize the conditions for the existence of an equilibrium in a model with freely mobile households across communities, in which there is a competitive housing market and a majority voting process on local proportional property tax on the amount of housing consumed. Nechyba (1999) shows that in a multi-community model with a fixed stock of heterogeneous houses perfect income stratification is not guaranteed in equilibrium. Along these lines, Martinez-Mora (2006), provides a framework in which in each community there is a competitive housing market, a private school option, and a majority voting process on property tax to finance public education spending. Assuming that an urban area offers lower quality public schooling, the resulting equilibrium will be one of two types: urban trap or urban mixing equilibrium<sup>7</sup>.

The paper is organized as follows: in section 2, I present some stylized facts showing the existence of income mixing communities and the consequences in terms of public education spending, local taxation and private enrollment within metropolitan areas. Section 3 develops the theoretical model. In this section I also derive the analytical condition for the existence of an income mixing equilibrium and for a perfect income stratification equilibrium. Section 4 focuses on the effect of inequality on residential decisions and fiscal policies as well as on the endogenous gentrification pattern of some U.S. cities. Section 5 concludes.

#### 2 Stylized Facts

In this section I analyze some stylized facts characterizing U.S. cities using the 2000 Decennial Census and 2000 Common Core of Data, in order to show that

<sup>&</sup>lt;sup>7</sup>In an urban mixing equilibrium only very rich households enroll their children in private school, while low income and middle income groups perfectly stratify by income across the urban area and the suburbs. In an urban trap equilibrium intermediate income households also opt-out of the public system enrolling their children in private schools, while higher income households prefer to use the local public school of high quality. See Martinez-Mora (2006).

communities with income mixing exist.

Figure 1 describes the relationship between per capita income and distance of residence from the central business district (CBD) in the county where the 6 biggest U.S. cities in 1999 are situated<sup>8</sup>. The 2000 Decennial Census dataset provides the per capita income of each Census tract composing the county. I calculate the distance of each tract from the CBD using the coordinates of the latitude and longitude provided in the dataset. The location of the CBD is taken from the Census's geographic reference manual list 1982 that identifies the CBD Census tracts for all metropolitan areas<sup>9</sup>. Since the selected counties have different dimensions, I restrict the analysis to a distance of 15 miles from CBD<sup>10</sup>.

Firstly we can infer evidence from Figure 1 that in some cities (New York, Chicago and Philadelphia) high income households reside very close to the CBD, i.e. within a radius of 5 miles, while in other cities (Los Angeles, Houston and Phoenix) the area close to CBD is mostly populated by very low income households. A second observable stylized fact is the existence of a U-shaped relationship between per capita income and distance from CBD in Chicago and Philadelphia and a monotonic increasing relationship in Los Angeles, Houston and Phoenix within a radius of 10 miles<sup>11</sup>.

 $<sup>^{8}</sup>$ These 6 cities are the county boards of the respective county and the only U.S. cities with more than 1.3 million habitants. The rank has not changed in 2010.

<sup>&</sup>lt;sup>9</sup>For cities with more than one census tract in the CBD, I choose the tract with the higher per capita income as the tract of reference. Choosing the most populated tract does not change the qualitative results.

<sup>&</sup>lt;sup>10</sup>Among these counties New York County is the most densely populated but it is the smallest in terms of size with a maximum distance of 9 miles from the CBD, while in Los Angeles this distance is around 60 miles.

<sup>&</sup>lt;sup>11</sup>The same evidence is provided in Glaeser et al. (2008) showing that this U-shaped pattern emerges in most of the oldest U.S. cities rather than in the newest cities. The U-shaped relationship could also be observable in the New York metropolitan area, although the evidence at county level suggests a monotonic decreasing relationship. New York County has in fact a dimension of 23 square miles. Moreover, if we plot the relationship between median housing value or rent and distance from the CBD, the trends are very similar to those observed in Figure 1. The cities characterized by a coexistence of very high and very low income households in the area within a radius of 10 miles from the CBD exhibit the same U-shape observed in Figure 1. In the same way, the inner cities in which very poor people live close to the CBD display a monotonic increasing relationship between housing value and distance from the CBD within counties.



Figure 1: Income and Distance from CBD

Despite these differences in the composition of population in areas close to the CBD, i.e. U-shaped versus monotonic, a common characteristic of all these cities is the coexistence of both very high and very low income households in a radius of 10 miles of distance from the tract defining the CBD. In particular, in U.S. counties, we observe a cohabitation of heterogeneous income groups within central school districts, while suburban school districts appear to be more homogeneous in terms of income distribution<sup>12</sup>.

Figure 2 maps the income deciles in 1999 in Cook and Maricopa Counties<sup>13</sup>, in which the cities of Chicago and Phoenix are respectively situated (2.a and

<sup>&</sup>lt;sup>12</sup>In this paper we consider the school district to be the unified school district, which includes elementary and secondary educational levels. Geographically, school districts are not necessarily completely contained in a county, since they are local governments with powers similar to that of a county.

<sup>&</sup>lt;sup>13</sup>As an example I concentrate on one U-shaped (Chicago) and one Monotonic city (Phoenix). The results can be extended to all the cities in Figure 1 and other American cities. Maps are elaborated with ArcGIS using the 2000 Decennial Census data and the Topologically Integrated Geographic Encoding and Referencing system, TIGER.



Figure 2: Per Capita Income Distribution within Counties and Cities

(a) Cook County



(b) Chicago City



(c) Maricopa County



(d) Phoenix City

2.c). The maps clearly show the existence of income mixing areas in central school districts, while the suburban school districts are characterized by a less heterogeneous income distribution within their boundaries. Moreover, it should be noted that in the central school districts (2.b and 2.d) the presence of high income households is much stronger in Chicago than in Phoenix. In the city of Phoenix, for instance, only a minority of rich households lives in the central district, while the majority lives in suburban districts. Even if in both counties we observe a cohabitation of different income groups within central school districts, households in Maricopa County tend to be more stratified by income across communities, and some of those are populated only by households belonging the the same income decile.

These differences in population composition within school districts may have important consequences in terms of schooling segregation between children of families of different social status living in the same school district. In districts in which the cohabitation of heterogeneous income groups is strong, as for instance in central school districts, households will choose for their offspring a different type of school, segregating their children by school type, presumably more than in districts in which the income is homogeneously distributed.

Consider for instance areas close to the CBD within central school districts in which different income groups live<sup>14</sup>. From Figure 1 we know that these CBD areas are populated by very rich households, in U-shaped cities, or by very poor households, in monotonic cities. Table 1 aggregates the data respectively for two U-shaped cities, Chicago and Philadelphia, and for three monotonic cities, Los Angeles, Houston and Phoenix, plotted in Figure 1. Not surprisingly the summary statistics for these different types of cities vary broadly in terms of

<sup>&</sup>lt;sup>14</sup>We refer the the CBD area as all the census tracts within a distance of maximum 7.5 miles from the CBD. We choose a distance of 7.5 miles in order to create two different areas contained in the central school district, the CBD area and the vicinity, that can be populated by very different income groups. The qualitative results do not change if we analyze the summary statistics for a distance of 5 miles or 10 miles from CBD. As before we limit the reference area to 15 from CBD.

poverty rate, public spending per pupil, enrollment rate in private school and local taxation.

	City type	Mean	St dev	Min	Max
Expenditure per student (\$)	U-shaped Monotonic	$7292 \\ 5910$	$3127 \\ 1911$	$\begin{array}{c} 4610\\ 3886 \end{array}$	$\frac{19357}{14638}$
Local revenue per student(\$)	U-shaped Monotonic	$5635 \\ 3239$	2104 1806	$2264 \\ 1253$	12948 7982
Share of poor in $\%$	U-shaped Monotonic	$26.74 \\ 31.42$	$16.37 \\ 15.03$	$\begin{array}{c} 1.4 \\ 1.3 \end{array}$	$92.7 \\ 79.8$
Private Enrollment in $\%$	U-shaped Monotonic	$21.37 \\ 9.24$	$20.28 \\ 28.48$	$\begin{array}{c} 1 \\ 0.3 \end{array}$	$\begin{array}{c} 100 \\ 89.5 \end{array}$

 Table 1: CBD area in U-Shaped and Monotonic Cities: Summary Statistics

Source: 2000 Common Core of Data and 2000 Decennial Census. Number of observations: 1703 Census tracts in Philadelphia County and Cook County and 3268 Census tracts in Maricopa County, Los Angeles County and Harris County.

The concentration of very high or very low income households close to the CBD area might help to explain these differences. In U-shaped cities, in fact, high income households live very close to the CBD areas and choose private schools for their offspring<sup>15</sup>. In monotonic cities, indeed, areas located close to the CBD are mostly populated by low income households. Not surprisingly, in the central areas of monotonic cities we observe a lower enrollment rate in private school, local revenue per student and expenditure in public school, compared to the same area in U-shaped cities. As a matter of fact, the co-habitation of different income groups within the same community or district may create disparities in terms of access to education and generate schooling segregation between pupils of families of different social status.

The theoretical literature has mainly focused on studying the conditions under which perfect income stratification among communities emerges. In this paper I mainly focus on the case in which cities are characterized by income mixing and schooling segregation within the same school district.

 $<sup>^{15}</sup>$ Evidence suggests that almost two-thirds of students in private schools are from families with incomes greater than 50,000 in 1997/1998; likewise, only 8% of private school students are from families with incomes below 20,000. See Reardon and Yun, 2003.

#### 3 The Model

I analyze a general equilibrium model of two communities in which housing market and fiscal policies may interact with the quality of public education, school choices and residential decisions. The economy is populated by a continuum of households of measure one. Each household consists of one adult and one school-aged child. Adults are differentiated by their income endowment x, where x can be thought as the wage that an adult can obtain in the labor market. I focus on a uniform distribution of income over the interval  $[\mu - \sigma; \mu + \sigma]$  for positive  $\mu > \sigma > 0^{16}$ .

In the model an economy consists of a set of public school districts with fixed boundaries within a county. For simplicity I assume that the county is made up of two communities or school districts<sup>17</sup>. Households' preferences can be represented by a utility function U(h, z). More precisely, households have identical preferences defined on the quality of education of their children, z, and on the private housing consumption, h.

Education can be provided by public and private schools that are mutually exclusive and use the same technology to produce educational services. Public schools follow a residence-based admission policy: households living in a community use the local public school. Each community assesses an *ad valorem* tax on the value of housing to finance the public education system. The tax rate,  $\tau$ , and therefore the amount of public spending on education, are determined by a political vote of residents of the community. I assume that communities impose a proportional property tax on the value of houses rather than an income tax to be consistent with the observation that in the U.S.

<sup>&</sup>lt;sup>16</sup>As in de la Croix and Doepke (2009), the uniform distribution, with mean  $\mu$  and standard deviation  $\sigma$ , is chosen for simplicity. Accordingly, the associated density function is given by f(x) = 0 for  $x < \mu - \sigma$  and  $x > \mu + \sigma$  and  $f(x) = \frac{\mu}{2\sigma}$  for  $\mu - \sigma \le x \le \mu + \sigma$ . In a recent paper Arcalean and Schiopu (2010) assume that income is distributed according to a Pareto distribution rather than a uniform distribution in order to give a more flexible parametrization of the income distribution.

<sup>&</sup>lt;sup>17</sup>In section 4 I will interpret the two communities as the central school district and the suburban school districts of a county.

property taxes support most of the funding that local governments provide for education<sup>18</sup>.

A household can move across districts at no direct cost and chooses the community in which to reside. To keep the analysis simple, I assume that each community has a fixed amount of homogeneous land from which housing is produced through the same constant returns to scale production function. Land, and therefore houses, are owned by a competitive *absentee landlord* to whom households have to pay the rent at the market price. Alternatively, I may suppose that households are owners and buy land at the market price<sup>19</sup>.

#### 3.1 Timing of the Events

Households have to make three explicit decisions: they have to choose in which community to live, they have to decide whether to educate their children either in public or private schools and they have to vote for the level of public funding for education. Parents have perfect foresight regarding the outcome of the political vote and, therefore, the fiscal policy adopted by the government of the community in which they decide to live. The timing of the events can be described by the following two-stage process. In the first stage, each adult simultaneously settles in a community, assuming that housing prices endogenously adjust to equate demand and supply for housing, and decides to which type of school, free of charge public school or tuition fee private education, he/she sends his/her child. In the second stage the adult residents of each community vote on property tax rate and public school expenditure. All households have to pay taxes even if they decide to opt-out of the public system. The outcome of the voting process determines the quality of public

<sup>&</sup>lt;sup>18</sup>In reality local governments determine the level of spending in education through a combination of property and income taxes. In this paper I concentrate only on housing value taxation in order to avoid an additional source of taxation and complicate the analysis. This assumption is standard in the literature that incorporates the housing market. See for instance the multi-community models developed by Epple and Zelenitz (1981), Epple, Filimon and Romer (1984), Epple and Romer (1991).

<sup>&</sup>lt;sup>19</sup>See Hansen and Kessler (2001) for a discussion on absentee landlord assumption.

education in the two school districts.

This timing structure can be justified by observing that public education policy can frequently be adjusted through a yearly budget vote, while residential decisions depend on predetermined factors and cannot be easily adjusted. The same argument can be found in de la Croix and Doepke (2009) in a model with endogenous fertility. Similarly, they observe that the choice between public and private education entails substantial switching costs, especially when education segregation is linked to residential segregation.

#### **3.2** The Structure of the Economy

I consider an economy composed of two communities that may differ in the amount of land contained in their fixed boundaries<sup>20</sup>. The structure I have in mind is represented by a county composed by two different communities. Each community contains a set of public schools that provide identical quality of education and could be thought of as school districts<sup>21</sup>.

Communities are politically independent but economically integrated. The assumption of political independence between communities implies that each local government can choose fiscal policies autonomously. Economic integration excludes barriers to migration or trade exchange, and allows households to be perfectly mobile among communities at no direct  $\cos^{22}$ . Moreover, we exclude peer group effects so that the quality of public education in each school district is only determined by the amount of spending per pupil financed through property taxes on housing values<sup>23</sup>.

In my framework parents have perfect foresight over the outcome of the political process and, consequently, over the policies adopted by the local gov-

 $<sup>^{20}</sup>$ Notice that considering an economy composed of two communities with the same amount of land represents a particular case of the structure analyzed in this paper.

<sup>&</sup>lt;sup>21</sup>Alternatively, we may consider that there is only one public school per district. As already underlined by Epple and Sieg (1999) and Nechyba (2000), this specification may imply a mix of community qualities and permit us to observe the empirically important possibility of the coexistence of rich and poor households within a single school district.

 $<sup>^{22}</sup>$  The same argument can be found in Hansen and Kessler (2001).

 $<sup>^{23}\</sup>mathrm{See}$  Benabou (1993) for a model with peer group effects.

ernment of each community. Put differently, the expectations about school quality will be realized. Taking fiscal policies as given, households have to choose where to reside and to which type of school to send their children. As in Bearse et al. (2001) households face four residential/school choices: (*i*) district 1 and public school, (*ii*) district 1 and private school, (*iii*) district 2 and public school, and (*iv*) district 2 and private school<sup>24</sup>.

The stylized facts presented in Section 2 underline the existence of income mixing communities within U.S. counties in which low and high income households cohabit at community level, in particular within unified central school districts. The theoretical literature has focused on studying the conditions under which perfect income stratification among communities emerges. By contrast, the aim of this paper is to concentrate on the conditions that justify the existence of income mixing communities in which schooling segregation between children of families belonging to heterogeneous income groups emerges.

For this reason, I restrict my analysis to the specific case in which the quality of public schools is such that  $\tilde{x}_1 < \tilde{x}(q_1) < \tilde{x}_2$  and  $\tilde{x}_2 \leq \tilde{x}(q_2)$  where  $\tilde{x}_1$  and  $\tilde{x}_2$  are the income thresholds that leave households indifferent to locate in one community or in the other,  $q_1$  and  $q_2$  the quality of public school in the two communities, and  $\tilde{x}(q_1)$  and  $\tilde{x}(q_2)$  the income thresholds that determine the households' decision to opt-out of the public school system <sup>25</sup>. If this political equilibrium exists, it will be characterized by a population distribution in which households with income  $\in [\mu - \sigma, \tilde{x}_1]U[\tilde{x}_2, \mu + \sigma]$  settle in community 1 while households with income  $\in [\tilde{x}_1, \tilde{x}_2[$  in community 2. In particular, under the assumption that  $\tilde{x}_1 < \tilde{x}(q_1) < \tilde{x}_2$  and  $\tilde{x}_2 \leq \tilde{x}(q_2)$ , we can observe that communities are not perfectly stratified by income<sup>26</sup> :

<sup>&</sup>lt;sup>24</sup>The peculiarity of my model with respect to the work of Bearse et al. (2001) is that we consider a housing market, property tax rate rather than income tax, but also that the tax rate is endogenously determined and depends on how the segregating forces, housing markets and local government spending, operate in communities that offer different qualities of public schooling.

<sup>&</sup>lt;sup>25</sup>Later on, I will provide a condition on the parameters such that the expected quality of public education in community 1 and 2 guarantees that  $\tilde{x}_1 < \tilde{x}(q_1) < \tilde{x}_2$  and  $\tilde{x}_2 \leq \tilde{x}(q_2)$ .

<sup>&</sup>lt;sup>26</sup>Only households choosing public school perfectly stratify by income. Bearse et al. (2001)

(i) All households with income  $x \leq \tilde{x}_1$  live in community 1 and send their children to a public school of quality  $q_1$ .

(*ii*) All households with income  $\tilde{x}_1 < x < \tilde{x}_2$  live in community 2 and send their children to a public school of quality  $q_2$ .

(*iii*) All households with income  $x \ge \tilde{x}_2$  live in community 1 and send their children to private school.

Figure 3 describes graphically the households' distribution in this economy, in which rich and poor households cohabit in community 1 and send their children to different types of school.

Figure 3: The Economy Structure



This type of equilibrium has first been studied by the pioneer work by Epple and Romano (1996) in a multi-community model with private options, showing that the likely majority voting equilibrium is characterized by two opposing coalitions of voters: the first composed of the middle income class who vote for high level of taxation and public education spending; the second composed of low and high income households who prefer a lower level of redistribution<sup>27</sup>.

find the same conclusion assuming that the tax revenue in region 1 is always lower than in region 2.

<sup>&</sup>lt;sup>27</sup>In my framework, redistribution is interpreted as public education spending. Corocan

Empirical evidence suggests the existence of this equilibrium, namely 'The ends against the middle'.

#### 3.3 Households' Problem

Households' preferences can be represented by a utility function U(h, z). Adults have identical preferences and care about the quality of their children's education, z, and housing consumption h. Parents may choose to educate their offspring either in public school, z = q, where q denotes the schooling quality, or in private school, z = e, where e represents education spending in the private market. In addition to property taxes on housing value, in the latter case parents have to pay the tuition fees, which cover the full cost of private education. Public and private schools use the same technology to produce educational services<sup>28</sup>. A household can consume either public or private school services, but not both. Together with the assumption that public schools are financed through local property taxes, the possibility to opt-out of the public system by choosing private education is a good approximation of the U.S. school system. For simplicity I adopt a logarithmic utility function.

The problem of the representative adult agent can be written as follows:

$$\begin{cases} \max_{h,e} U(h,z) = ln[h] + ln[max\{q,e\}] \\ s.t. \quad ph(1+\tau) = x - e \end{cases}$$
(1)

where p is the net of tax housing price and is determined in the competitive housing market of each community. Substituting the budget constraint into the objective function, I can rewrite the utility of the representative household as follow:

$$u[q, e, x, \tau, p] = ln \left[ \frac{x - e}{(1 + \tau)p} \right] + ln[max\{q, e\}]$$

$$\tag{2}$$

and Evans (2010) observe that the coalition of rich and poor households votes for low education spending because rich parents prefer that their children opt-out to public education system, while poor parents prefer a greater level of consumption rather than redistribution.

<sup>&</sup>lt;sup>28</sup>For exposition reasons quality units are normalized such that the price per unit of private schooling is equal to 1.

Parents preferring public education will choose e = 0. Let us define as  $u^{q}[q, 0, x, \tau, p]$  and  $u^{e}[0, e, x, \tau, p]$  the utility of a household respectively choosing public or private school for his/her child. The problem can be written as:

$$\begin{cases} \max_{h} u^{q}[q, 0, x, \tau, p] & \text{if public education} \\ \max_{h, e} u^{e}[0, e, x, \tau, p] & \text{if private education} \end{cases}$$
(3)

The solutions to this problem are given by:

$$\begin{cases} e = 0, \qquad h^q = \frac{x}{(1+\tau)p} & \text{if public education} \\ e = \frac{x}{2}, \qquad h^e = \frac{x}{[2(1+\tau)]p} & \text{if private education} \end{cases}$$
(4)

with  $h^q$  ( $h^e$ ) is the housing demand under public (private) education choice<sup>29</sup>. As we can expect, an increase in property value tax rate or in net of tax housing prices will reduce the consumption of housing.

It is convenient to represent consumer preferences in the space (x, p). Substituting the optimal households' choices into the maximization problem allows us to derive the indirect utility functions of adults choosing public,  $V^{q}[x, q, p]$ , or private,  $V^{e}[x, p]$  education for their children:

$$\begin{cases} V^{q}[x,q,p] = ln\left[\frac{x}{(1+\tau)p}\right] + ln[q] \\ V^{e}[x,p] = ln\left[\frac{x}{[2(1+\tau)]p}\right] + ln\left[\frac{x}{2}\right] \end{cases}$$
(5)

Comparing indirect utility functions we observe that a household will optout of the public education system if and only if  $V^e[x,p] > V^q[x,q,p]$ . The boundary above which parents prefer private education for their offspring is defined by the threshold:

$$\tilde{x}(q) = 4q \tag{6}$$

with q, the expected quality of public school.

 $<sup>^{29}{\</sup>rm Given}$  the utility function chosen, it is clear that, when education is available, half of income goes to each spending: education and housing.

Households with an income level x higher than the threshold  $\tilde{x}(q)$  will strictly prefer to pay for private education. The expected quality of public education will determine the position of this threshold in the income distribution and the share of children participating in the public school system. As already suggested in de la Croix and Doepke (2009) we can note that education quality is a normal good because parents with higher income demand more of it.

#### 3.4 The Political Mechanism

The analysis here follows the general structure of the most recent literature by assuming that the tax rate and the quality of public education are determined via probabilistic voting in which each individual carries the same political weight in the political process<sup>30</sup>.

The social welfare functions maximized in the two communities by the political mechanism are respectively given by:

$$W_{1}[\tau_{1},q_{1}] = \int_{\mu-\sigma}^{\tilde{x}_{1}} u[q_{1},0,x,\tau_{1},p_{1}]f(x)dx + \int_{\tilde{x}_{2}}^{\mu+\sigma} u[0,e_{1},x,\tau_{1},p_{1}]f(x)dx \quad (7)$$

$$W_1[\tau_2, q_2] = \int_{\tilde{x}_1}^{x_2} u[q_2, 0, x, \tau_2, p_2] f(x) dx$$
(8)

Welfare maximization is constrained to the local government budget rule of the community, that is:

$$\tau_1 \int_{\mu-\sigma}^{\tilde{x}_1} p_1 h_1^q f(x) dx + \tau_1 \int_{\tilde{x}_2}^{\mu+\sigma} p_1 h_1^e f(x) dx = \int_{\mu-\sigma}^{\tilde{x}_1} q_1 f(x) dx \tag{9}$$

$$\tau_2 \int_{\tilde{x}_1}^{\tilde{x}_2} p_2 h_2^q f(x) dx = \int_{\tilde{x}_1}^{\tilde{x}_2} q_2 f(x) dx \tag{10}$$

The left-hand side of these two constraints represents total revenues from the taxation on housing values. The right-hand sides give the amount of total spending in public education. Replacing households' housing demands (4) in

 $<sup>^{30}\</sup>mathrm{See}$  for instance de la Croix and Doepke (2009), Dottori and Sheng (2010), Arcalean and Schiopu (2010).

the balanced budget rules (9) and (10), allows us to express the property tax rates as an increasing function of the quality of public education:

$$\tau_1[q_1] = \frac{4q_1(\tilde{x}_1 - \mu + \sigma)}{2\tilde{x}_1^2 - \tilde{x}_2^2 - 2q_1(\tilde{x}_1 - \mu + \sigma) + 6\mu\sigma - \mu^2}$$
(11)

$$\tau_2[q_2] = \frac{2q_2}{(\tilde{x}_1 + \tilde{x}_2) - 2q_2} \tag{12}$$

Notice that housing prices do not directly influence the policies voted by adult residents<sup>31</sup>. Moreover, the timing of the events imposes that residential and educational choices are predetermined when the voting process occurs simultaneously in the two communities. Maximizing the welfare functions (7) and (8) with respect to the corresponding local budget constraint (11) and (12), taking the first order conditions for a maximum and solving for education quality, allows us to define the voting outcomes:

$$q_1^* = \frac{2\tilde{x}_1^2 - \tilde{x}_2^2 - (\mu^2 + \sigma^2) + 6\mu\sigma}{8\tilde{x}_1 - 4(\tilde{x}_2 + \mu - 3\sigma)}$$
(13)

$$q_2^* = \frac{(\tilde{x}_1 + \tilde{x}_2)}{4} \tag{14}$$

Equations (13) and (14) describe the education policies voted and therefore adopted in community 1 and community  $2^{32}$ . Looking at the corresponding optimal tax rates we observe that  $\tau_1^* = \frac{(\tilde{x}_1 - \mu + \sigma)}{\tilde{x}_1 - \tilde{x}_2 + 2\sigma}$  and  $\tau_2^* = 1$ .

#### 3.5 Housing Market

In each community a local housing market in which prices are determined competitively exists. Each community has a fixed amount of homogeneous land from which housing stock is produced through the same constant return

 $<sup>^{31}</sup>$ Given the logarithmic utility function, the FOCs do not depend on housing prices but depend on income thresholds. In terms of timing this means that housing prices are determined only after the voting process

<sup>&</sup>lt;sup>32</sup>Given the timing structure of the model, parents have perfect foresight regarding the outcome of the political vote. Solving the problem backwards, we need to derive the residential decisions in order to discuss the economic interpretation of these results.

to scale production function. Communities may differ only in the amount of land contained within their boundaries. Following Epple et al. (1984) and Epple and Romer (1991), I assume the existence of an Absentee Landlord who resides outside the economy and owns the land<sup>33</sup>.

The aggregate housing demand in each community is obtained by integrating the household's housing demand over the income interval:

$$h_1^d(x,\tau_1,p_1) = \int_{\mu-\sigma}^{\tilde{x}_1} h_1^q f(x) dx + \int_{\tilde{x}_2}^{\mu+\sigma} h_1^e f(x) dx$$
(15)

$$h_2^d(x,\tau_2,p_2) = \int_{\tilde{x}_1}^{\tilde{x}_2} h_2^q f(x) dx$$
(16)

In equilibrium the aggregate community housing demands equal the community housing supply,  $h_i^d(x, \tau_i, p_i) = k_i$ , where  $i = \{1, 2\}$ . For simplicity we assume that  $k_i$  represents the total housing units in each community. Housing prices will be given by:

$$p_1(\tau_1) = \frac{2\tilde{x}_1^2 - \tilde{x}_2^2 - (\mu^2 + \sigma^2) + 6\mu\sigma}{4k_1(1 + \tau_1^*)}$$
(17)

$$p_2(\tau_2) = \frac{\tilde{x}_2^2 - \tilde{x}_1^2}{2k_2(1 + \tau_2^*)};$$
(18)

Since agents have perfect foresight, they take into account their expectation concerning the outcome of the voting process when they formulate their housing demand. Substituting the equilibrium tax rates into (17) and (18) I can derive the housing prices in the two communities as a function of the income thresholds and parameters:

$$p_1^* = \frac{(2\tilde{x}_1^2 - \tilde{x}_2^2 - (\mu^2 + \sigma^2) + 6\mu\sigma)(\tilde{x}_2 - \tilde{x}_1 - 2\sigma)}{4k_1(\tilde{x}_2 - 2\tilde{x}_1 + \mu - 3\sigma)}$$
(19)

$$p_2^* = \frac{\tilde{x}_2^2 - \tilde{x}_1^2}{4k_2} \tag{20}$$

<sup>&</sup>lt;sup>33</sup>The assumption of a fixed amount of homogeneous land, and therefore constant housing supply, is clearly an unrealistic assumption. However, in a static model with exogenous fertility it seems reasonable to consider the housing supply as perfectly inelastic.

Given  $\tilde{x}_i$  housing prices are decreasing in the amount of land supplied by the Absentee Landlord: the larger the supply, the lower the corresponding housing price. Now I have all the elements to study the existence and the properties of the equilibrium.

#### 3.6 Residential Choices

Households have to choose in which community to live. When taking this decision they forecast public policies that in equilibrium are consistent with the realized policies. Rational households may react by moving to the other community, when the local government of the community where they reside implements unattractive fiscal policies. Given economic integration across communities, each household is free to move from one district to the other at no direct cost. As a consequence of these households' choices, housing prices adjust endogenously. In equilibrium no household has an incentive to move, since the residential choice is the decision that maximizes the family's expected utility:

$$u[q_1^*, 0, \tilde{x}_1, t_1^*, p_1^*] = u[q_2^*, 0, \tilde{x}_1, t_2^*, p_2^*]$$
(21)

$$u[q_2^*, 0, \tilde{x}_2, t_2^*, p_2^*] = u[0, e^*[\tilde{x}_2], \tilde{x}_2, t_1^*, p_1^*]$$
(22)

Equation (21) states that in equilibrium a household with income  $\tilde{x}_1$  is indifferent between living in community 1 and sending his/her child to public school of quality  $q_1^*$ , and living in the community 2 and send his/her child to public school of quality  $q_2^*$ . Similarly, equation (22) implies that a household with income  $\tilde{x}_2$  is indifferent between residing in district 1 and opting out of the public school system, and living in the other district and sending his/her child to the local public school<sup>34</sup>.

Solving the system composed of equations (21) and (22) allows us to deter-

 $<sup>^{34}</sup>$  Without loss of generality, we can observe that households with income  $\tilde{x}_1$  and  $\tilde{x}_2$  reside in community 1.

mine the income thresholds as a function of parameters:  $\tilde{x}_1 = f(k_1, k_2, \mu, \sigma)$ and  $\tilde{x}_2 = g(k_1, k_2, \mu, \sigma)^{35}$ . According to the assumption that income is distributed uniformly over the interval  $[\mu - \sigma; \mu + \sigma]$ , I can derive the share of households in each community and in each type of school. Let us define by  $\Psi_{P,i}$  and  $\Psi_{R,i}$  the fraction of children participating in public (P) and private (R) schools respectively, in community  $i = \{1, 2\}$ .

$$\begin{cases}
\Psi_{P,1} = \frac{\tilde{x}_1 - (\mu - \sigma)}{2\sigma} \\
\Psi_{P,2} = \frac{\tilde{x}_2 - \tilde{x}_1}{2\sigma} \\
\Psi_{R,1} = \frac{(\mu + \sigma) - \tilde{x}_2}{2\sigma} \\
\Psi_{R,2} = 0
\end{cases}$$
(23)

Given perfect foresight and exogenous fertility we always have that the number of households with income  $x \leq \tilde{x}_1$  is equal to  $\Psi_{P,1}$ , with income  $\tilde{x}_1 < x < \tilde{x}_2$  is equal to  $\Psi_{P,2}$ , and with income  $x \geq \tilde{x}_2$  is equal to  $\Psi_{R,1}$ .

Interestingly enough, at given  $\tilde{x}_i$ , the population density in the mixed income community,  $\frac{\Psi_{P,1}+\Psi_{R,1}}{k_1} = \frac{\tilde{x}_1-\tilde{x}_2+2\sigma}{2k_1\sigma}$ , is positively correlated with the standard deviation of the income distribution<sup>36</sup>.

**Definition (Intercommunity Equilibrium)** An equilibrium is a partition of households across communities and schools, a pair of income thresholds  $(\tilde{x}_1; \tilde{x}_2)$  a vector of policies  $(\tau_1^*, \tau_2^*, q_1^*, q_2^*)$  and housing prices  $(p_1, p_2)$  such that: (i) households maximize utility;

- (*ii*) the housing markets clear;
- *(iii)* the regional budgets are balanced;

*(iv)* public education spending and property taxes are decided by simultaneous voting in both regions;

(v) no agent wishes to move.

 $<sup>^{35}\</sup>mathrm{See}$  Appendix A for the analytical expression of these thresholds.

<sup>&</sup>lt;sup>36</sup>More precisely, the societies characterized by a concentration of income in the tails of the distribution, rather than at the average level, exhibit a higher ratio between population and housing in mixed income communities.

Epple et al. (1984) give a similar definition of intercommunity equilibrium. In a static model of residential location it is required that no agent has an incentive to move, in the sense that moving from one district to another cannot increase the household's utility. Figure 4 gives a representation of the equilibrium in the space  $(x_1, x_2)^{37}$ . The blue (red) curve is the set of  $x_1$  and  $x_2$  such that  $u[q_1^*, 0, \tilde{x}_1, t_1^*, p_1^*] = u[q_2^*, 0, \tilde{x}_1, t_2^*, p_2^*]$   $(u[q_2^*, 0, \tilde{x}_2, t_2^*, p_2^*] = u[0, e^*[x_2], \tilde{x}_2, t_1^*, p_1^*])$ , respectively. The intersection between the two curves determines the equilibrium values  $\tilde{x}_1$  and  $\tilde{x}_2$ .



Assumption 1  $\sigma > \bar{\sigma}$ , with  $\bar{\sigma} = \frac{(k_1+k_2)\mu}{k_1+3k_2}$ .

**Proposition 1** Under Assumption 1, there exists a unique and interior equilibrium pair of income thresholds  $(\tilde{x}_1; \tilde{x}_2) \in [\mu - \sigma, \mu + \sigma]$  such that the system composed of equations (21) and (22) is satisfied and the expected quality of public education is such that  $\tilde{x}_1 < \tilde{x}(q_1) < \tilde{x}_2$  and  $\tilde{x}_2 \leq \tilde{x}(q_2)$ .

<sup>&</sup>lt;sup>37</sup>As an example, I set the following parameters' values:  $k_1 = k_2 = 1$ ,  $\mu = 4$ ,  $\sigma = 2.5$ , implying  $\bar{\sigma}$ 

#### **Proof** See Appendix A

Proposition 1 states that if the standard deviation of the income distribution is sufficiently large, then the corresponding unique equilibrium is characterized by a distribution of households across communities and types of school in which high income parents send their children to private schools, while low and middle income parents enroll their offspring in public schools of different quality. In particular, all households with income  $x < \tilde{x}_2$  perfectly stratify across communities: poor households reside in community 1 while middle class households live in community 2. Moreover, in community 1 poor and rich households cohabit and segregate themselves by schooling<sup>38</sup>. To sum up, if this type of equilibrium exists, an income mixing community emerges. Therefore, (i) households in community 2 will never choose private education for their children: no private school is provided in this community; (ii) school quality in community 2 is higher than in community 1,  $q_1 < q_2$ ; (iii) property tax rate in community 1 is lower than in community 2,  $\tau_1 < \tau_2$ ; (iv) both districts are occupied.

The reason behind these statements is quite intuitive: rich families would be better off by choosing private education since education is a normal good. Consequently, they prefer a low level of taxation because they do not use the public school for their children. Poor families, by contrast, cannot opt-out of the public system but prefer not to pay high taxes in order to consume more housing. High and low income households cohabit in the same community and vote for a low level of taxation. The fact that both districts must be occupied implies that taxation and, therefore, quality of public education, will be higher in community 2. Middle income households, in fact, are not able to enroll their children in private school but they are more demanding in terms of school quality with respect to low income households. As education is a normal good, they vote for a higher level of taxation and redistribution, and

<sup>&</sup>lt;sup>38</sup>From a political point of view, as already observed in Epple and Romer (1996), in this equilibrium a coalition of middle income will be opposed by a coalition of rich and poor households.

settle in community 2. Unlike the literature in which the political outcome generates a perfect income stratification across communities, my theoretical model is able to replicate the empirical evidence of the urban income mixing shown in Section 2.

Interestingly enough, the threshold  $\bar{\sigma} = \frac{(k_1+k_2)\mu}{k_1+3k_2}$  is positive correlated with the mean,  $\mu$ , of the income distribution. *Ceteris paribus*, this means that economies with a high income mean are less likely to be characterized by income mixing communities<sup>39</sup>. The threshold  $\bar{\sigma}$  is also positively correlated with the relative dimension of communities 1 and 2,  $\frac{k_1}{k_2}$ . The more housing units in community 1, the greater  $\sigma$  should be in order to guarantee the existence of the equilibrium<sup>40</sup>. When the standard deviation of the income distribution is sufficiently low, i.e.  $\sigma \leq \bar{\sigma}$ , households perfectly stratify by income. In this case the fully public regime prevails. The richest group decides to reside in the community in which the high quality public school is provided. The rest of the population lives in the other community enrolling their offspring in the local public school of lower quality.

**Proposition 2** If  $\sigma \to \overline{\sigma}$ , then  $\tilde{x}_2 \to \mu + \sigma$  and  $\Psi_{R,1} \to 0$ . The resulting equilibrium is characterized by a fully public regime and perfect income stratification across communities.

#### **Proof** See Appendix B

De la Croix and Doepke (2009) find a similar result in a model with one community, income taxes, no housing market and endogenous fertility: when inequality is low, all parents use the public school. The key message of my stylized model is that in more unequal societies,  $\sigma > \bar{\sigma}$ , we may observe the coexistence of different income groups in an urban mixing community characterized by schooling segregation. The quality of public school in this

<sup>&</sup>lt;sup>39</sup>In other words, in economies with high income mean the standard deviation that guarantees income mixing communities must be higher.

<sup>&</sup>lt;sup>40</sup>Dividing both the denominator and the numerator of  $\bar{\sigma}$  by  $k_2$ , it is easy to prove that  $\frac{\partial \bar{\sigma}}{\partial (k_1/k_2)} > 0.$ 

community will be low and the opportunities in terms of social improvement for poor families less likely.

### 4 Inequality, Spending on Public Schools and Gentrification of Inner Cities

In this section I study the impact of a mean preserving spread on the equilibrium vector of fiscal policies, housing prices, spending on public schools and residential choices. I proxy inequality by the standard deviation of the income distribution<sup>41</sup>. Independently from the model parameters, a mean preserving spread leads to higher private school enrollment, lower participation in public schools and higher housing prices in both communities. An increase in inequality spreads the tails of the income distribution and generates an effect on income thresholds that modifies the residential choices via the housing market and voting process. However, the effect on school quality in the income mixed community is ambiguous and depends on the housing market in the economy.

**Proposition 3** The effect of inequality on the quality of public school  $q_1^*$  and on the income threshold  $\tilde{x}_2$  depends on the relative size of the endowment in housing:

(i) If k<sub>1</sub>/k<sub>2</sub> ≥ 1, then ∂q<sub>1</sub>/∂σ < 0 and ∂x̃<sub>2</sub>/∂σ < 0;</li>
(ii) If k<sub>1</sub>/k<sub>2</sub> < 1, then there exists a unique k̄ > 0 such that:
(ii.i) if k̄ > k<sub>2</sub> > k<sub>1</sub>, then the same results as (i) hold;
(ii.ii) if k<sub>2</sub> > k̄ > k<sub>1</sub>, then ∂q<sub>1</sub>/∂σ > 0, ∂x̃<sub>2</sub>/∂σ > 0.
Independently from k<sub>i</sub> the sign of the other derivatives does not change: ∂x̃<sub>1</sub>/∂σ < 0, ∂r<sub>1</sub>/∂σ < 0, ∂q<sub>1</sub>/∂σ < 0, ∂p<sub>1</sub>/∂σ < 0, ∂p<sub>2</sub>/∂σ > 0.

**Proof** See Appendix C

 $<sup>^{41}</sup>$ Inequality as measured by the Gini index has increased during the period 1990-2000 in most U.S. counties: from 0.408 to 0.456 in Cook County, from 0.391 to 0.431 in Maricopa County, from 0.433 to 0.490 in Los Angeles County, from 0.427 to 0.468 in Harris County, from 0.409 to 0.451 in Philadelphia County, from 0.421 to 0.480 in Suffolk County.

When inequality increases, if housing units in community 1 are sufficiently numerous compared to units in community 2, (cases (i) and (ii.i) in Proposition 3), both income thresholds move to the left: households with income  $\tilde{x}_1$  strictly prefer to reside in district 2 and send their children to the local public school of quality  $q_2$ , while households with income  $\tilde{x}_2$  strictly prefer to reside in district 1, opt-out of the public system and enroll their children in private school (see Figure 5). When the relative size of the housing market is such that the housing conditions in community 1 are attractive for the families of the well-off middle class, a mean preserving spread will generate an inflow of high income middle class households towards this community and, consequently, a lower tax rate and public school quality. The share of population in community 1 increases and a large fraction of poor and rich households will reside in this community enrolling respectively their children in the low quality public school and in the private school, segregating themselves by schooling. Due to population reallocation, an increase in inequality impacts negatively on the quality of public schools in community 1, because high income middle class households migrate from community 2 and vote for low taxation. At the same time, the quality of public school in community 2 is negative correlated with inequality. The fact that high income households migrate to community 1 reduces the tax base, given a constant tax rate, so that the spending in public education decreases. Even if the share of population is decreasing in community 2, the public spending per student will be lower after an increase in inequality.

By contrast, when inequality increases but the housing supply in community 2 is sufficiently large compared to community 1 (case (*ii.ii*) in Proposition 3), the income thresholds  $\tilde{x}_2$  move to the right, and households with this income level now strictly prefer to live in community 2 and send their children to the local public school of quality  $q_2$ . Since housing prices and, consequently, the tax base are high in community 1, high middle class households prefer to live in the other community. A mean preserving spread makes community 1





less attractive for richer households of middle income class, because housing conditions are not attractive. However, even if the tax rate goes down due to an increase in the share of rich households, the share of high income households in this community will be relatively smaller compared to the previous cases; as a consequence, the tax rate decreases less than in the previous case. Due to this population reallocation as well as to a small housing supply that implies high tax base and housing prices, an increase in per student spending in public education emerges as a consequence of inequality (see Figure 6). Also in this case, inequality impacts negatively on the quality of public school in community 2. Given the constant tax rate in this community, the shift in threshold  $\tilde{x_1}$  dominates the shift in threshold  $\tilde{x_2}$ , that is, the increasing share of low income households is greater than the increasing share of high income households, so that the public spending per student decreases as a consequence of a mean preserving spread.





#### 4.1 Residential Reallocation in Chicago and Phoenix

The stylized facts presented in section 2 show the existence of mixed income communities in US cities, in particular in areas close to the CBD or within central school districts. A natural interpretation of the model is to consider community 1 as the central school district with income mixing, and community 2 as the suburban district with more homogeneous income distribution. When the central school district is characterized by an advanced housing development, as for instance when  $\frac{k_1}{k_2} \geq 1$  my model predicts the endogenous gentrification of the central school district within cities as a consequence of inequality, that is an inflow of high income middle class households towards the city center, and the subsequent displacement of low income households towards the suburb<sup>42</sup>.

Figure 7 maps the housing density for two cities with different housing characteristics within the county in which they are situated. In the central

<sup>&</sup>lt;sup>42</sup>This phenomenon is typical of cities that experienced an earlier urban development. Brueckner and Rosenthal (2009) develop a model predicting that eventual redevelopment of aging dwellings in inner cities creates a young downtown housing stock that attracts high income households, leading to gentrification.







district of Chicago we can clearly observe a housing density and development that are much more advanced compared to the central district in the city of Phoenix<sup>43</sup>. The case of Chicago represents the alternatives (i) and (ii.i) in Proposition 3, while the case of Phoenix the alternative (ii.ii).

In Chicago central school district the housing supply in the central city is relatively large compared to the suburb. In this case, my model predicts that high income middle class households are attracted by the consumption opportunities related to the low housing prices and a low level of taxation, so that they move towards the CBD, as we can see in Figure 8. An influx of high income households towards the central school district emerges from 1990 to 2000. As a consequence of this residential choice, the median housing value increases in the areas close to the CBD. As discussed previously, this process, namely

<sup>&</sup>lt;sup>43</sup>For space reasons I consider only the case of one city with earlier urban development within the central school district, Chicago, and one city with recent urban development of the inner city, Phoenix. The same exercise can be done with other U.S. cities.



Figure 8: Per Capita Income and Median Housing Rent in Cook County

(a) Per capita income 1990



(b) Per capita income 2000



(c) Median housing value 1990



(d) Median housing value in 2000



Figure 9: Per Capita Income and Median Housing Rent in Maricopa County

(a) Per capita income 1990



(b) Per capita income 2000



(c) Median housing value 1990



(d) Median housing value in 2000

the gentrification of the inner cities, impacts negatively on publicly funded education, and consequently on human capital accumulation and economic opportunity for upward mobility of low income households, since the per pupil spending on public education goes down as a consequence of the population reallocation across communities and of housing market adjustments.

Figure 9 describes the variation in per capita income and median housing value in Maricopa County and in particular in the city of Phoenix. As shown in Figure 7 in this county the housing density in the central school district is similar and even smaller compared to the suburban school districts. In this case we do not observe a shift of high income households towards the central school district. In other words we have no gentrification of the inner city as observed in Chicago during the same period of time, as suggested by the theoretical model. Consequently, in this scenario the model predicts the result provided by de la Croix and Doepke (2009): an increase in per student public school spending when inequality increases.

#### 5 Conclusion

In this paper I developed a general equilibrium model which shows mixed income communities within cities rather than a perfect income stratification across communities. With respect to the previous literature, my model includes simultaneously: public versus private education choice, multi-community structure with independent local government, competitive housing market within each community, property tax rate on housing value rather than income tax, a probabilistic voting mechanism and residential choices. My framework can be used to study the relationship between income distribution and the population composition of cities. One of the main predictions of the theoretical model is that in highly unequal economies, households segregate by schooling within communities, while they stratify by income across communities in less unequal economies.

The effect of inequality on the quality of public education depends on the relative size of the endowment of housing. The housing market is crucial in my framework, since its characteristics affect the housing residential choice and may imply an endogenous gentrification of the cities. The interpretation of the theoretical model as inner city Vs suburb can be used to replicate the stylized facts related to inflow or not of high income households towards the city center in U.S. cities. If cities are characterized by a high concentration of housing units within their central school district then high income middle class households move from the suburban district to the central district. In this case the effect of inequality on the redistribution and quality of public school will be negative. By contrast, if the relative size of housing endowment within the central area is sufficiently small compared to the suburb, as in U.S. cities with recent urban development, then we do not observe a migration of high income households towards inner cities. In this case the quality of public school is positive correlated to inequality. A natural extension of this theoretical paper is to test these conclusions empirically as well as the link between segregation and inequality.

## Appendices

#### A Proof of Proposition 1

Substituting (4), (11), (12), (13), (14), (19) and (20) into the system composed of equations (21) and (22) and solving simultaneously at the equilibrium, allow us to determine the income thresholds that leave indifferent households between living in community 1 or in community 2:

$$\tilde{x}_1 = \frac{\alpha + 2\beta}{2k_2(k_1 + k_2)}$$
$$\tilde{x}_2 = \frac{\alpha + \beta}{k_2(2k_1 + k_2)}$$

with  $\alpha = \sqrt{(k_1 + k_2)^2 (k_1^2(\mu - 3\sigma)^2 + 32k_1 - k_2\sigma^2 + 16k_2^2\sigma^2)} + k_1^2(\mu - 3\sigma) + 3k_1k_2(\mu - 3\sigma)$ , and  $\beta = k_2^2\mu - 3k_2^2\sigma$ . After some algebraical manipulation, we find that the solution of the system is an interior,  $\mu + \sigma > \tilde{x}_2 > \tilde{x}_1 > \mu - \sigma > 0$ , if and only if  $\mu > \sigma > \frac{(k_1 + k_2)\mu}{k_1 + 3k_2} = \bar{\sigma}$ . Given restrictions on parameter values,  $\mu > \sigma > 0, k_1 > 0$  and  $k_2 > 0$ , it is easy to show that  $\tilde{x}_2 > \tilde{x}_1$  because  $\frac{k_2}{2k_1} > 0 > \frac{\beta}{\alpha}$ . Notice also that when  $\sigma > \frac{(k_1 + k_2)\mu}{k_1 + 3k_2} = \bar{\sigma}$  then  $\mu - 3\sigma < 0$ . The equilibrium condition reduces to  $\frac{\mu}{2} < \sigma < \mu$  if we consider equal housing units in the two communities, that is  $k_1 = k_2 = \bar{k}$ .

Being  $\tilde{x}_1$  the income threshold that leave indifferent to live in community 1 or in community 2 when choosing public school (21), all households with income  $x < \tilde{x}_1$  will choose to live in community 1. Households with income  $x > \tilde{x}_1$  have to choose whether to reside in community 2 or in community 1 and enroll their children in private school (22). This choice depends on the income threshold  $\tilde{x}_2$ : all households with income  $x > \tilde{x}_2$  will choose to live in community 1 and opt-out of the public school system. All households with income  $\tilde{x}_1 < x < \tilde{x}_2$  will reside in community 2. In equilibrium these migration conditions are simultaneously satisfied and no agent has an incentive to move because utilities are maximized and housing markets clear. If Assumption 1 is satisfied and equations (21) and (22) are increasing and continuous in  $x \in [\mu - \sigma, \mu + \sigma]$ , then we have to show:

- i) if  $x < \tilde{x}_1$  then  $u[q_1^*, 0, x, t_1^*, p_1^*] > u[q_2^*, 0, x, t_2^*, p_2^*]$
- ii) if  $\tilde{x}_2 > x > \tilde{x}_1$  then  $u[q_1^*, 0, x, t_1^*, p_1^*] < u[q_2^*, 0, x, t_2^*, p_2^*]$  and  $u[q_2^*, 0, x, t_2^*, p_2^*] > u[0, e^*[x], x, t_1^*, p_1^*]$
- iii) if  $x > \tilde{x}_2$  then  $u[q_2^*, 0, x, t_2^*, p_2^*] < u[0, e^*[x], x, t_1^*, p_1^*]$

In order to prove (i) it is sufficient to show that when  $x < \tilde{x}_1$  the utility level evaluated in x when choosing public school and living in community 1 is strictly greater than the utility level evaluated in  $\tilde{x}_1$ . This condition is verified when  $\log\left(-\frac{4k_1x}{\mu^2 - 6\mu\sigma + \sigma^2 - 2x^2 + \tilde{x}_2^2}\right) - \log\left(-\frac{4k_1\tilde{x}_1}{\mu^2 - 6\mu\sigma + \sigma^2 - 2\tilde{x}_1^2 + \tilde{x}_2^2}\right) + \log\left(-\frac{\mu^2 - 6\mu\sigma + \sigma^2 - 2\tilde{x}_1^2 + \tilde{x}_2^2}{8x - 4(\mu - 3\sigma + \tilde{x}_2)}\right) - \log\left(-\frac{\mu^2 - 6\mu\sigma + \sigma^2 - 2\tilde{x}_1^2 + \tilde{x}_2^2}{8x_1 - 4(\mu - 3\sigma + \tilde{x}_2)}\right) > 0.$  Taking the exponential function, given the optimal income threshold  $\tilde{x}_2$  and given the condition for the existence of the interior equilibrium  $\sigma > \bar{\sigma}$ , we observe that  $\frac{x(8\tilde{x}_1-4(\mu-3\sigma+x_2))}{\tilde{x}_1(8x-4(\mu-3\sigma+\tilde{x}_2))} > 1 \text{ when } x < \tilde{x}_1 \text{ since } \tilde{x}_2 < 3\sigma - \mu \text{ in equilibrium. To prove}$ (ii) we have to show that the utility level evaluated in x when choosing public school and living in community 2 is larger than the utility evaluated in  $\tilde{x}_2$  and in  $\tilde{x}_1$ . The condition is verified when  $\log\left(\frac{2k_2x}{x^2-\tilde{x}_1^2}\right) - \log\left(\frac{2k_2\tilde{x}_2}{\tilde{x}_2^2-\tilde{x}_1^2}\right) + \log(x+\tilde{x}_1) - \log\left(\frac{2k_2\tilde{x}_2}{\tilde{x}_1^2-\tilde{x}_1^2-\tilde{x}_1^2}\right) + \log(x+\tilde{x}_1) - \log\left(\frac{2k_2\tilde{x}_2}{\tilde{x}_1^2-\tilde{x}_1^2 \log(\tilde{x}_1 + \tilde{x}_2) > 0$ . Taking the exponential we observe that the above condition holds whenever  $\frac{x(\tilde{x}_2-\tilde{x}_1)}{(x-\tilde{x}_1)\tilde{x}_2} > 1$ , that is, when  $x < \tilde{x}_2$ . Using (i), we also have that  $x > \tilde{x}_1$ . (iii) can be proved showing that the utility when choosing private in region 1 evaluated in x is strictly greater than the utility evaluated in  $\tilde{x}_2$  when  $x > \tilde{x}_2$ . This condition is verified when  $\log\left(-\frac{2k_1x}{\mu^2 - 6\mu\sigma + \sigma^2 + x^2 - 2\tilde{x}_1^2}\right) - \frac{k_1x}{\mu^2 - 6\mu\sigma + \sigma^2 + x^2 - 2\tilde{x}_1^2}$  $\log\left(-\frac{2k_1\tilde{x}_2}{\mu^2 - 6\mu\sigma + \sigma^2 - 2\tilde{x}_1^2 + \tilde{x}_2^2}\right) + \log(x) - \log(\tilde{x}_2) > 0.$  Taking the exponential function, the condition reduces to  $\frac{x^2(\mu^2-6\mu\sigma+\sigma^2-2\tilde{x}_1^2+\tilde{x}_2^2)}{\tilde{x}_2^2(\mu^2-6\mu\sigma+\sigma^2+x^2-2\tilde{x}_1^2)} > 1$ . Using optimal  $\tilde{x}_1$  and given Assumption 1, i.e. the condition for the existence of the interior equilibrium  $\sigma > \bar{\sigma}$ , we observe that the latter condition is always verified when  $x > \tilde{x}_2.$ 

#### **B** Proof of Proposition 2

Equation  $\tilde{x}_2$  is continuous in  $\sigma$ . The limit of  $\tilde{x}_2$  when  $\sigma \to \bar{\sigma} = \frac{(k_1+k_2)\mu}{k_1+3k_2}$  is equal to  $-\frac{2k_1\mu(k_1^2+3k_1k_2+k_2^2)+2(k_1+3k_2)}{k_2(2k_1+k_2)(k_1+3k_2)}\sqrt{\frac{\mu^2(k_1+k_2)^2(k_1^2+4k_1k_2+2k_2^2)^2}{(k_1+3k_2)^2}}$ . Given the parameters, this limit belongs to the domain  $[\mu - \sigma; \mu + \sigma]$  if and only if  $\sigma > \bar{\sigma}$ . Moreover, the limit of  $\Psi_{R,1}$  when  $\sigma \to \bar{\sigma}$  is zero since  $\tilde{x}_2 \to \mu + \sigma$ . When  $\sigma < \bar{\sigma}$  than  $\tilde{x}_2(\sigma) > \mu + \sigma$  that is excluded being outside of the domain of the income distribution.

#### C Proof of Proposition 3

First, derive the sign of the derivatives of the residential thresholds  $\tilde{x}_1$  and  $\tilde{x}_2$  with respect to  $\sigma$ . Using the income thresholds in appendix A we can show that  $\frac{\partial \tilde{x}_1}{\partial \sigma} < 0 \ \forall k_i > 0$ :

$$\frac{\partial \tilde{x}_1}{\partial \sigma} = -\frac{a + \frac{b}{\sqrt{c}}}{d}$$

where  $a = 3k_1^2 + 9k_1k_2 + 6k_2^2$ ,  $b = (k_1 + k_2)^2 (3k_1^2(\mu - 3\sigma) - 32k_1k_2\sigma - 16k_2^2\sigma)$ ,  $c = (k_1 + k_2)^2 (k_1^2(\mu - 3\sigma)^2 + 32k_1k_2\sigma^2 + 16k_2^2\sigma^2)$  and  $d = 2k_2(k_1 + k_2)$ . Being the denominator positive  $\forall k_i > 0$ , we have to study the sign of  $a + \frac{b}{\sqrt{c}}$ . Given that in equilibrium  $\mu > \sigma > \frac{(k_1 + k_2)\mu}{k_1 + 3k_2} > 0$ , we can observe that a > 0and  $\frac{b}{\sqrt{c}} < 0$ . After some algebraical manipulations it is easy to show that  $a\sqrt{c} - b = 3 (k_1^2 + 3k_1k_2 + 2k_2^2) \sqrt{(k_1^2(\mu - 3\sigma)^2 + 32k_1k_2\sigma^2 + 16k_2^2\sigma^2)} + (k_1 + k_2)^2 (3k_1^2(3\sigma - \mu) + 32k_1k_2\sigma + 16k_2^2\sigma)$ 

that is always positive. Consequently,  $\frac{\partial \tilde{x}_1}{\partial \sigma} < 0$ .

The sign of the derivative  $\frac{\partial \tilde{x}_2}{\partial \sigma}$  is not monotonic in  $\sigma$  and depends on the relative size of the housing endowment in the two communities,  $k_1$  and  $k_2$ . Assume for simplicity  $k_1 = 1$  and  $k_1 > k_2$ . The derivative:

$$\frac{\partial \tilde{x}_2}{\partial \sigma} = -\frac{e + \frac{b}{\sqrt{c}}}{f}$$

where  $e = 3 + 9k_2 + 3k_2^2$  and  $f = k_2(2 + k_2)$ .

Being f > 0, we have to study the sign of  $e\sqrt{c} - b$ . If  $k_2 < 1$  we can notice that

$$e\sqrt{c} - b = 3(k_2^2 + 3k_2 + 1)\sqrt{(k_2 + 1)^2((16k_2^2 + 32k_2 + 9)\sigma^2 + \mu^2 - 6\mu\sigma)} + (k_2 + 1)^4((16k_2^2 + 32k_2 + 9)\sigma - 3\mu) > 0$$

Consequently, when  $k_1 > k_2$  we observe  $\frac{\partial \tilde{x}_2}{\partial \sigma} < 0$ .

When  $k_2 > k_1$  the algebra becomes very complicated. We can make the proof using the limits. The limit of  $\frac{\partial \tilde{x}_2}{\partial \sigma}$  when  $k_2 \to 0$  is equal to  $-\infty$  and the limit of  $\frac{\partial \tilde{x}_2}{\partial \sigma}$  when  $k_2 \to \infty$  is equal to  $\frac{4\sqrt{\sigma^2}}{\sigma} - 3$ . Knowing that this derivative is continuous and increasing in  $k_2$  within the domain, it must cross the x-axis only one time, when  $k_2 = \bar{k}$ .

The sign of the derivative  $\frac{\partial q_1}{\partial \sigma}$  reflects the behavior of  $\frac{\partial \tilde{x}_2}{\partial \sigma}$ . In particular, we can observe that if  $\frac{\partial \tilde{x}_2}{\partial \sigma} > 0$  then  $\frac{\partial q_1}{\partial \sigma} > 0$  and viceversa. The derivative  $\frac{\partial q_1}{\partial \sigma}$  is equal to

$$\frac{-3\mu\sqrt{(k_2+1)^2((16k_2(k_2+2)+9)\sigma^2+\mu^2-6\mu\sigma)}-3(k_2(k_2+3)+1)\mu^2+18(k_2(k_2+3)+1)\mu\sigma}}{4k_2(k_2+2)((16k_2(k_2+2)+9)\sigma^2+\mu^2-6\mu\sigma)}$$

$$+\frac{(16k_2(k_2+2)+9)\sigma\Big(\sqrt{(k_2+1)^2((16k_2(k_2+2)+9)\sigma^2+\mu^2-6\mu\sigma)}-3(k_2(k_2+3)+1)\sigma\Big)}{4k_2(k_2+2)((16k_2(k_2+2)+9)\sigma^2+\mu^2-6\mu\sigma)}$$

Assume, as for the study of the sign of  $\frac{\partial \tilde{x}_2}{\partial \sigma}$ , that  $k_1 = 1$ . The limit of  $\frac{\partial q_1}{\partial \sigma}$ when  $k_2 \to 0$  is equal to  $-\infty$  and the limit of  $\frac{\partial q_1}{\partial \sigma}$  when  $k_2 \to \infty$  is equal to  $\frac{\sqrt{\sigma^2}}{\sigma} - \frac{3}{4}$ . Knowing that this derivative is continuous and increasing in  $k_2$  within the domain, it must cross the x-axis only one time, when  $k_2 = \bar{k}$ . Numerically,  $\bar{k}$  is the threshold for both  $\frac{\partial \tilde{x}_2}{\partial \sigma} = 0$  and  $\frac{\partial q_1}{\partial \sigma} = 0$ , and this is true for all possible parameter values. Set for instance  $\mu = 3$  and  $\sigma = 1$ . We get  $\frac{\partial \tilde{x}_2}{\partial \sigma} = 0$  iff  $k_2 = 1.65181$ . At the same time we get  $\frac{\partial q_1}{\partial \sigma} = 0$  iff  $k_2 = 1.65181$ . Set for instance  $\mu = 3.5$  and  $\sigma = 2$ . Both derivatives are zero iff  $k_2 = 1.23508$ . If  $k_2$ is smaller than this threshold value, then the sign of the derivatives  $\frac{\partial \tilde{x}_2}{\partial \sigma}$  and  $\frac{\partial q_1}{\partial \sigma}$  are negative, otherwise they are non-negative.

Let us now study the derivative of  $\frac{\partial \tau_1}{\partial \sigma} = 0$ :  $-\frac{\alpha \left(\sqrt{\beta} + k_1^2(\mu - 3\sigma) + k_1 k_2(\mu - 3\sigma)\right)}{k_2 \sqrt{\gamma} \left(\sqrt{\gamma} + k_1^2(\mu - 11\sigma) + k_1 k_2(\mu - 15\sigma) - 4k_2^2 \sigma\right)^2} = 0$ , with  $\alpha = 8k_1^2 \mu (k_1 + k_2)^2 (2k_1 + k_2)$ ,  $\beta = (k_1 + k_2)^2 (k_1^2 (\mu - 3\sigma)^2 + 32k_1 k_2 \sigma^2 + 16k_2^2 \sigma^2)$ 

and  $\gamma = (k_1 + k_2)^2 (k_1^2(\mu - 3\sigma)^2 + 32k_1k_2\sigma^2 + 16k_2^2\sigma^2)$ . As the denominator is always positive, in order to prove that the tax rate in community 1 is negative correlated with the standard deviation of the income distribution it is sufficient to show that  $\sqrt{\beta} > k_1^2(\mu - 3\sigma) + k_1k_2(\mu - 3\sigma)$ . In equilibrium  $\sigma > \bar{\sigma}$  so that  $\mu - 3\sigma < 0$ . Consequently, given  $\alpha > 0$ ,  $\beta > 0$  and  $\gamma > 0$  we always have  $\frac{\partial \tau_1}{\partial \sigma} < 0$ .

To study the sign of the derivative  $\frac{\partial q_2}{\partial \sigma} = \frac{\frac{\partial \tilde{x}_1}{\partial \sigma} + \frac{\partial \tilde{x}_2}{\partial \sigma}}{4}$ , we have first to derive the intensity of a variation in  $\sigma$  on the thresholds when  $\sigma$  increases. In other words we want to show if  $\left|\frac{\partial \tilde{x}_1}{\partial \sigma}\right| > < \left|\frac{\partial \tilde{x}_2}{\partial \sigma}\right|$ . Using the previous simplification we can observe that the effect of  $\sigma$  on threshold  $\tilde{x}_1$  is always greater than the effect on threshold  $\tilde{x}_2$ . After simple algebraical manipulation, and knowing that a > e and d > f we can observe that  $\left|\frac{\partial \tilde{x}_1}{\partial \sigma}\right| - \left|\frac{\partial \tilde{x}_2}{\partial \sigma}\right|$ :

$$\frac{b}{\sqrt{c}}(f-d) + af - de = \frac{k_2^2(k_1+k_2)^4 \left(-3k_1^2(\mu-3\sigma)+32k_1k_2\sigma+16k_2^2\sigma\right)}{\sqrt{(k_1+k_2)^2 \left(k_1^2(\mu-3\sigma)^2+32k_1k_2\sigma^2+16k_2^2\sigma^2\right)}} + 3k_1k_2^2(k_1+k_2) > 0$$

Being  $\left|\frac{\partial \tilde{x}_1}{\partial \sigma}\right| > \left|\frac{\partial \tilde{x}_2}{\partial \sigma}\right|$  we always observe  $\frac{\partial q_2}{\partial \sigma} < 0$  also when the thresholds move in opposite directions.

Now we have to show that inequality is positive correlated with housing prices in both communities. From equation (20) we know that  $p_2 = \frac{\tilde{x}_2^2 - \tilde{x}_1^2}{4k_2}$ . The derivative  $\frac{\partial p_2}{\partial \sigma} = \frac{\tilde{x}_2 \frac{\partial \tilde{x}_2}{\partial \sigma} - \tilde{x}_1 \frac{\partial \tilde{x}_1}{\partial \sigma}}{2k_2}$ . Knowing that  $\frac{\partial \tilde{x}_1}{\partial \sigma} < 0 \,\forall k_i$  and that  $\frac{\partial \tilde{x}_2}{\partial \sigma} > 0$  when  $\bar{k} > k_2 > k_1$ , it follows directly that  $\tilde{x}_2 \frac{\partial \tilde{x}_2}{\partial \sigma} - \tilde{x}_1 \frac{\partial \tilde{x}_1}{\partial \sigma} > 0$  and consequently  $\frac{\partial p_2}{\partial \sigma} > 0$ . When  $\frac{\partial \tilde{x}_2}{\partial \sigma} < 0$  and using the optimal value for  $x_i$  the expression becomes too complicated to be studied manually. With the help of Mathematica we can observe that  $\tilde{x}_2 \frac{\partial \tilde{x}_2}{\partial \sigma} > \tilde{x}_1 \frac{\partial \tilde{x}_1}{\partial \sigma}, \,\forall \sigma \geq \bar{\sigma}$ . The same argument can be used to prove that  $\frac{\partial p_1^*}{\partial \sigma} > 0, \,\forall \sigma \geq \bar{\sigma}$ .

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