

The social economic impact of AIDS: Accounting for intergenerational transmission, productivity and fertility

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The social economic impact of AIDS: Accounting for intergenerational transmission, productivity and fertility *

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Abstract

In this paper we develop a model that aims to investigate the economic and demographic impacts of three effects of the HIV-AIDS epidemic in developing countries. The direct effect of the HIV epidemic is that it hits the inherited characteristics of young adults. The two indirect effects, resulting from the first, are the reduction in productivity of adults and the transmission of the disease to their offsprings. We allow these different effects to act either separately or together, and we investigate the marginal efficiency of health expenditures on the survival probability of individuals and demographics. The direct effect of the HIV virus is that it leads adults to increase their own health expenditure and to decrease that of their children. On the contrary, the transmission effect of the HIV virus leads parents to spend more on their children than on their own. We show that the reduction in productivity of young adults decreases health expenditures for themselves and their children. Furthermore, we find that the productivity effect dominates by large the two others. Moreover, when adults decide to have fewer children because of HIV, we show that the ratio of low to high skilled workers increases. This demographic impact impoverishes the economy in the short and medium run.

Keywords: orphans, epidemic, transmission, productivity shock, survival rate

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1 Introduction

The AIDS epidemic is one of the most destructive health crises of modern times, ravaging families and communities throughout the world. This pandemic has massive demographic effects and as proved by significant empirical evidence based on the examination of the effects of AIDS crisis on the level and growth of income.

The disease is responsible for willing many young adults and there by leaving vast number of children across Africa or Asia without one or both parents. The loss of a parent because of AIDS has not only serious consequences on a child's access to basic necessities such as shelter, food, clothing, health and education but can also affect its emotional and psychological state (see Case, Pakson and Ableidinger (2004), Gertler et al (2003)). More precisely children whose parents are living with AIDS often experience negative changes in their lives not only long before orphaned but also after the death of their parents.

Usually parental presence provides emotional support and transmits them values that are important for the evolution of a child's human capital (see Gertler et al (2003), Bell and Gersbach (2008)). These values are lost when a parent is infected by AIDS because he is no longer able to spend time as he could before his sickness. Furthermore, after the death of their parents, orphans who live with foster families might be discriminated against by their foster parents who favor of their biological children (see Case, Pakson and Ableidinger (2004)). Consequently, it is possible that orphans may miss out on school enrollment, have their schooling interrupted or perform poorly in school (see Case, Pakson and Ableidinger (2004)).

In this paper, we develop a model which captures three effects of an AIDS epidemic. More precisely, the HIV virus hits the inherited characteristics of young adults and at the same time this shock implies two indirect effects. The two indirect effects are the reduction in productivity of the adults and the transmission of the disease to their offspring. Allowing those different effects to act either separately or together and investigating the marginal efficiency of the health expenditures on the survival probability of an individual and on demographics in our simulations we find a tremendous economic and demographic impact mainly in the short and in the medium run.

Our model is related to Boucekkine and Laffargue (2010) which investigate the short, medium and long-term distributional impact of orphans on the whole economy. It is a three period OLG model consisting of children, young junior adults and senior adults. A young junior adult has an exogenous number of infants and he is wholly altruistic concerning their survival and what their position in the future generation would be. He decides about the investment on his health, his education and also the health and education expenditures of his children. The probabilities of survival of a child and a junior adult are dependent on their amount of health and education spending and their inherited characteristics, their own human capital. These decisions determine the survival probabilities of their children and themselves because they depend mainly on health spending.

It is noteworthy to mention that except our paper and Boucekkine and Laffargue (2010) that examine the effects of epidemic in short and in medium run the rest of the literature focus on the effects of AIDS in the long run (see Cuddington et al (1994), Kambou et al (1992) and Arndt and Lewis (2001) Bloom and Mahal (1997) Bonnel (2000), Bell, Devarajan and Gersbach (2003) and Papageorgiou and Stoytcheva (2005)).

Our analysis consists of two main parts studying both the economic and demographic impact of an AIDS epidemic though a theoretical framework and simulations. The first part is the economic impact of the disease concerning the decisions of young adults about their own and their children's health expenditures. We allow for 3 different effects of the

epidemiological shock. The first one is the decrease in the human capital of junior adults from the epidemic based on the proposition found in the literature that an epidemic hits mainly young adults (see for instance Boucekkine and Laffargue (2010), Corringan et al (2004), Boucekkine, Desbordes and Latzer (2009) and Guinness and Alban (2000)). The other mechanisms are indirect. We suppose that during the epidemic period there is a decrease of productivity and that parents can transmit their epidemiological shock to their children. Our results show that the parents decrease their investment because of lower income and also because they are totally altruistic and they care for their children. On the other hand, the reduction of productivity decreases their children's health expenditures and the transmission of epidemic leads the parents to increase it and to decrease theirs.

The second effect concerns the demographic impact due to the spread of the disease. In particular, we observe that there are two main mechanisms like in Boucekkine and Laffargue (2010). First, children of wealthy parents have a higher survival probability than those that have poor parents. This mechanism leads to an increase of the proportion of young adults with high human capital in medium term. The second mechanism is defined by the increase in the number children who become orphans. Orphans have a lower probability to obtain a high level of human capital. Thus, including the above shocks, we notice that the first mechanism (the survival probability of wealthy people is higher than of the poor) dominates in the period of epidemic and the second (the increase in the number of orphans) for the period after the epidemical shock in productivity takes place. On the other hand, the first mechanism dominates all generations in the medium term for the diffusion of the shock.

Also, we incorporate the decision of adults for fertility as a consequence of AIDS. There is a conflict concerning its effect on fertility. In particular, Young (2005), Young (2007) and Boucekkine, Desbordes and Latzer (2009) find that the HIV is lowering fertility in Sub-Saharan Africa. In contrast, Kalemli-Ozcan (2006), applying a Solow model, shows a negative relationship between AIDS and schooling and a positive relationship between AIDS and total fertility rates that lower per capita growth and welfare for the African generations. Our case is based on the line of Young (2007) and Boucekkine, Desbordes and Latzer (2009) and we show that the proportion of young adults of low human capital increases and impoverishes the economy in the short and medium run.

Finally, our model like Boucekkine and Laffargue (2010) concludes that epidemics, modeled as one period exogenous shocks to adults' survival probability and later on children's survival probability through transmission, have no impact on the long term output distribution.

Furthermore, there are many papers investigating the effects of AIDS using general equilibrium models. For instance, Bell, Devarajan and Gersback (2003) develop a two parent model giving an important role to the number of orphans because of AIDS. They distinguish different categories of orphans and they bring out useful conclusions about the distributional effects of epidemics. They argue that AIDS destroys human capital by killing mostly young people and it also compromises the mechanism through which knowledge and abilities are transmitted through generations showing as a result a negative effect of this epidemic on long run growth. Their diffusion is not affected by health and education expenditures.

The most important contribution of our paper is based on the construction of a different mechanism of transmission: an epidemic can spread from young adults to their infants. It is true that parents can transmit their values and knowledge to their offspring but they might also transmit their disease to them. One of the ways of transmitting HIV virus is from mothers to children through breast feeding. This way of transmission has even worst results because the children are not only orphans in case that their parents die but also they are infected and can be refused by other families and society in general.

Using a OLG framework, Corrigan, Glomm and Mendez (2005) show that the growth effects of AIDS are enormous in the long run. They argue that children receive a different level of education depending on whether their parents are healthy or infected. Dividing in two categories the adults healthy and infected, they investigate how medical expenditures can enhance the labor productivity of AIDS infected workers. This framework inspired us to investigate the consequences of reduction of productivity not only concerning the young adults and their children's health investments but also its impact on demographics. As pointed out earlier, none of the previous papers investigate the effects of an epidemic on the marginal efficiency of health and education expenditures. Furthermore so far the literature focused on long term effects.

As referred to previously, another important aspect of our paper is that it captures the channel of inequality in the short, medium and long run. The first theoretical approach that relates health expenditures, mortality and inequality across generation was proposed by Chackraborty and Das (2005). The authors focus their analysis on the fact that poor parents invest less in their own health and have a high probability of dying. Hence, they save less and thereby leaving lower bequests to their children if they die. Also they argue that parents care for the health of their children but in the formulation of the survival probability of children they do not take into account their health expenditures.

The rest of paper is organized as follows: In section 2, we present the benchmark model, section 3 develops an extension of the benchmark model including all the different epidemiological situations and section 4 provides simulations in the benchmark and extended model and introduces an exercise on fertility. Section 5 concludes.

2 Benchmark model

We consider a discrete time, perfect foresight dynamic model of a small open economy. Our benchmark model is a simplification of the model of Boucekkine and Laffargue (2010). People live for three periods, successively as children, junior adults and senior adults. We will examine the choices of a junior adult in the given period t . In a second paragraph we will describe the temporary equilibrium of the model in this period. Also, we assume the existence of only a single good, health care. This good must be interpreted in a broader sense, like the investment in human capital. Junior adults make all the decisions concerning their own and their children's health expenditures.

A junior adult enters in the period t with an endowment in human capital h . As it is pointed out earlier, the health care or the investment of human capital is considered as a single good in the economy. This good is produced by firms which use the human capital as the only input under the assumption of constant returns to scale. Human capital endowment, h can be considered as a storage good. We impose the similar assumption than Boucekkine and Laffargue (2010) that the productivity of human capital is equal to 1. Furthermore, every agent decides an amount of savings (the health care is the storage good) s and in his investment in health l for the period t under the budget constraint

$$h = s + l. \quad (1)$$

Spending in health has an effect on the lifetime of the junior adult. We define the probability of senior adult being alive in period $t + 1$ as $\pi(l)$. Each junior adult has an exogenous number n of children at the end of period t . Senior adults do not receive any wage. Also, the agent invests e_{+1} in the health of his children. The probability for each of them to be alive in the period $t + 2$ will depend on this investment. The children do not take any decisions. If the agent is alive in period $t + 1$ and takes care of his children this probability will be $\lambda(e_{+1})$. If he is dead, these children will be orphans and the probability becomes $c\lambda(e_{+1})$ with $0 < c_1 < c < 1$. So the budget constraint of the agent in the period $t + 1$ is

$$s = ne_{+1}. \quad (2)$$

We suppose that the amount of investment spent by the junior adult for the health of his children will be the same if he stays alive or she dies in the beginning of period $t + 1$. According to this assumption the intertemporal budget constraint of the junior adult is

$$h = ne_{+1} + l. \quad (3)$$

The utility of the junior adult in period t is

$$U = n\lambda(e_{+1})\{\pi(l)vh + (1 - \pi(l))vch\}. \quad (4)$$

The utility of our model is similar with Boucekkine and Laffargue (2010) with the main difference that we consider that everybody has the same human capital h . In obedience to this utility, the junior adult is considered wholly altruistic. His utility only depends on the expected human capital accumulated by his children who will reach the adult age. This specification coincides with the evolutionary biology (see Galor and Moav (2002) and 2005). Consistent with this the parents maximize the probability of survival and the quality of their children.

Another noteworthy feature is that vh represents the satisfaction a child brings to his parent when he reaches the adult age with human capital h . When it dies the satisfaction is 0. In

order to simplify more the utility function, we impose $r_1 = vh$ and $r_2 = vch$. Furthermore, we set $r = \frac{r_1}{r_2} - 1$ which is $r = \frac{1}{c} - 1$.

Consequently, the utility function becomes after removing the multiplicative term,

$$U = \lambda(e_{+1})\{\pi(l)r + 1\}. \quad (5)$$

where r is the satisfaction premium brought by children when at least one parent survives, the utility for parents to stay alive. In this case, the survival probability of each child is higher by the factor $1/c$.

Finally, the junior adult must solve in period t the program:

$$Max_{l, e_{+1}} \lambda(e_{+1})\{\pi(l)r + 1\} \quad (6)$$

$$h = ne_{+1} + l \quad (7)$$

$$l, e_{+1} > 0. \quad (8)$$

Furthermore, we determine more precise the specifications of the survival functions:

$$\lambda(e_{+1}) = \min \left\{ (Ae_{+1} + A')^{1-\alpha} / (1-\alpha), 1 \right\} \quad (9)$$

$$\pi(l) = \min \left\{ (Bl + B')^{1-\beta} / (1-\beta), 1 \right\} \quad (10)$$

$$\text{with } 0 < \beta, \alpha < 1, A, B, B' > 0, 0 \leq A' < (1-\alpha)^{1/(1-\alpha)}, B' < (1-\beta)^{1/(1-\beta)}. \quad (11)$$

We suppose that we are always in the intervals where both functions are strictly increasing. The intuition behind this is based on empirical results like Deaton (2003) who noticed that health spending, health state and longevity of an individual are increasing and concave functions of his income.

Also, in our model we consider the survival functions such that

$$\lambda(0) = A'^{1-\alpha} / (1-\alpha) > 0 \quad (12)$$

and

$$\pi(0) = B'^{1-\beta} / (1-\beta) > 0. \quad (13)$$

This idea comes in contrast to Chakraborty and Das (2005) who assume that the survival probability falls to zero with zero investment and it concides with Boucekkine and Laffargue (2010) who consider that $\lambda(0)$ and $\pi(0)$ can be interpreted as the inherent health situations which are unrelated with the health investments (see Finlay 2005). Contrary to Boucekkine and Laffargue (2010), we will however not set the intrinsic physical characteristics of children (A') equal to zero considering the fact that the main contribution of our theoretical model is to study the consequence of the transmission of an epidemic shock from parents to children. Hence, in our extension model, we relate A' to B' .

We denote the efficiency of junior adults' health expenditures as the derivative of their probability of survival with respect to health expenditures.

$$\frac{\partial \pi(l)}{\partial l} = B(Bl + B')^{-\beta}. \quad (14)$$

Moreover, we have that :

$$\frac{\partial^2 \pi(l)}{(\partial l \partial B)} = [(1-\beta)Bl + B'](Bl + B')^{-1-\beta} > 0, \quad \frac{\partial^2 \pi(l)}{(\partial l \partial B')} = -\beta B(Bl + B')^{-1-\beta} < 0, \quad (15)$$

observing these derivatives, we can note that the marginal efficiency of investment does not respond in the same way to shocks on B and B' . The efficiency of health expenditures falls down with an epidemic lowering B and increases with an epidemic lowering B' . This efficiency decreases for a composite epidemic which decreases by the same proportion the values of parameters B and B' . The same can be inferred when the shocks affect infant mortality.

Here, we focus on the case where an epidemic can be defined as a drop in the inherent health of parents. This is compatible with W-shaped age profile of mortality observed in the majority of epidemics like AIDS and Spanish flu. The demographic impact of such epidemic is stronger on junior adults than on children and old ages. In particular, it affects the working ages. More precisely, it will affect the young junior adults in this framework. After the explanations and the survival functions given above, the optimization program becomes

$$\text{Max}(Ae_{+1} + A')^{1-\alpha} [r(Bl + B')^{1-\beta} / (1-\beta) + 1] / (1-\alpha), \quad (16)$$

$$h = ne_{+1} + l \quad (17)$$

$$l, e_{+1} > 0, (Ae_{+1} + A') \leq (1-\alpha)^{1/(1-\alpha)}, Bl + B' \leq (1-\beta)^{1/(1-\beta)}. \quad (18)$$

We make the following assumptions.

Assumption1. The parameters of the model must satisfy the constraints

$$Bh + \frac{A'}{A}nB + B' \leq \left(1 + \frac{1-\alpha}{1-\beta}\right) \left(1 + \frac{1}{r}\right) (1-\beta)^{1/1-\beta} \quad (19)$$

$$\frac{Bn}{A} (1-\alpha)^{1/1-\alpha} \geq n \frac{BA}{A'} + Bh \quad (20)$$

$$Bh + \frac{A'}{A}nB + B' > \frac{1-\alpha}{r} B'^\beta + \frac{1-\alpha}{1-\beta} B' \quad (21)$$

$$(Bh + B')^{-\beta} \left(\frac{A'}{A}nB\right) < \frac{1-\alpha}{r} \quad (22)$$

Lemma 2.1 Program has a unique solution defined by the two equations

$$\frac{Bh + \frac{A'}{A}nB + B'}{Bl + B'} - \frac{1-\alpha}{r(Bl + B')^{1-\beta}} = 1 + \frac{1-\alpha}{1-\beta} \quad (23)$$

$$\frac{h-l}{n} = e_{+1}. \quad (24)$$

Proof See Boucekkine and Laffargue (2007).

Since the benchmark model is a simplification of Boucekkine and Laffargue (2007) the following lemmas are valid.

Lemma 2.2 Lemma 2. (a) A junior adult endowed with high human capital invests more in his health than a junior adult endowed with low human capital (b) The investment of a junior adult in his own health increases with the utility for parents of being alive. (c) If A' is zero, the investment of a junior adult in his own health is independent of the number of children otherwise it increases when the later number goes up.

Lemma 2.3 *A junior adult endowed with high human capital invests more in the health of his children than a junior adult endowed with low human capital. (b) The investment of a junior adult in the health of his children decreases with the utility for parents of being alive. (c) if $A' = 0$, the total investment of a junior adult in the health of his children is independent of the number of his children, otherwise it increases when the latter number goes up.*

Lemma 2.4 *(a). The investment of a junior adult in his own health increases in case of epidemic (when the parameter B' decreases).*

Proof See Boucekkine and Laffargue (2007).

In the next section, the extension of our benchmark model is presented before analyzing the demographics associated to the latter.

3 Extension Model

3.1 Model

This section extends the benchmark model. The human capital now can take two values h^- and h^+ with $0 < h^- < h^+$. Further, a child staying alive with alive parents has a probability p of obtaining a human capital of h^+ and a probability $1-p$ of obtaining a human capital of h^- . Similarly, an orphan which succeeds in staying alive has a probability q of achieving high level of human capital and $1-q$ of reaching the low level of human capital. We suppose that

$$0 \leq q < p \leq 1.$$

Given the assumptions, the utility function of a young junior adult takes the following form in period t :

$$U = n\lambda(e_{+1})\{\pi(l)v[p(h^+ - h^-) + h^-] + (1 - \pi(l))vc[q(h^+ - h^-) + h^-]\}. \quad (25)$$

We keep the same assumptions than in the benchmark model and we are following the same simplifications than Boucekkine and Laffargue (2010) which are

$$r_1 = v[p(h^+ - h^-) + h^-], r_2 = vc[q(h^+ - h^-) + h^-] \Rightarrow r = \frac{r_1}{r_2} - 1. \quad (26)$$

Under the assumption now r becomes an increasing function of the inequality of earnings

$$\frac{(h^+ - h^-)}{h^-} \quad (27)$$

are expected the next period. Also, in the following comparative statics we will assume that r and h can change independently. Therefore, in this model we allow for an epidemic shock to have 3 different effects. The first effect acts like in benchmark model i.e. a decrease in the survival probability of young junior adults. The other two effects are indirect. We consider that the epidemic affects also the productivity of workers. Corrigan, Glomm and Mendez (2005) discriminate the healthy and infected individuals, whose productivity decreases during the epidemic period. Furthermore, we impose that the shock of epidemic does not only affect the young junior adults but also their children through a transmission mechanism. Consequently, we relax our assumption of a productivity of human capital being equal to 1 and we consider that the epidemic has an impact on it. Furthermore, every agent sets saving (the health care is storage good) s and his investment in health l for the period t under the budget constraint

$$\Psi h = s + l. \quad (28)$$

The junior adult's program is

$$Max_{l, e_{+1}} \lambda(e_{+1})\{\pi(l)r + 1\} \quad (29)$$

$$\Psi h = ne_{+1} + l \quad (30)$$

$$l, e_{+1} > 0. \quad (31)$$

Where Ψ defines the productivity of infected workers between zero and one during the epidemic. Again, we determine more precisely, the specifications of the survival functions:

$$\lambda(e_{+1}) = \min \left\{ (Ae_{+1} + A')^{1-\alpha} / (1 - \alpha), 1 \right\} \quad (32)$$

$$\pi(l) = \min \left\{ (Bl + B')^{1-\beta} / (1-\beta), 1 \right\} \quad (33)$$

$$\text{with } 0 < \beta, \alpha < 1, A, B, B' > 0, 0 \leq A' < (1-\alpha)^{1/(1-\alpha)}, B' < (1-\beta)^{1/(1-\beta)}. \quad (34)$$

We define a generational trasmission of the shock of epidemics from the parent to child as Bell and Gerspach(2008) did. We impose that A' is influenced by B' . More intuitively, if the parameter B' decreases at the period t , we assume that a fraction of the children of the adults of generation t will be affected. We express this relation as :

$$dA' = \varphi_t dB'. \quad (35)$$

Where φ_t is between 0 and 1. The assumptions are then transformed:

Assumption1. The parameters of the model must satisfy the constraints

$$B\Psi h^+ + \frac{A'}{A}nB + B' \leq \left(1 + \frac{1-\alpha}{1-\beta}\right) \left(1 + \frac{1}{r}\right) (1-\beta)^{1/1-\beta} \quad (36)$$

$$\frac{Bn}{A} (1-\alpha)^{1/1-\alpha} \geq n \frac{BA}{A'} + B\Psi h^+ \quad (37)$$

$$B\Psi h^- + \frac{A'}{A}nB + B' > \frac{1-\alpha}{r} B'^\beta + \frac{1-\alpha}{1-\beta} B' \quad (38)$$

$$(Bh^- + B')^{-\beta} \left(\frac{A'}{A}nB\right) < \frac{1-\alpha}{r} \quad (39)$$

Now, we can establish the following lemmas.

Lemma 3.1 Program has a unique solution defined by the two equations

$$\frac{B\Psi h + \frac{A'}{A}nB + B'}{Bl + B'} - \frac{1-\alpha}{r(Bl + B')^{1-\beta}} = 1 + \frac{1-\alpha}{1-\beta} \quad (40)$$

$$\frac{\Psi h - l}{n} = e_{+1}. \quad (41)$$

Proof Eq. (41) is the constraint of program. We use this constraint to eliminate e_{+1} from the objective function. This function is concave in l . Equation 41 is the first order conditions of the transformed objective function. Let us define the function

$$y(l) = \frac{\frac{A'}{A}nB + B' + B\Psi h}{Bl + B'} - \frac{1-\alpha}{r(Bl + B')^{1-\beta}}. \quad (42)$$

We have

$$y(0) = \frac{A'}{A}nB + 1 + \frac{B}{B'}\Psi h - \frac{1-\alpha}{rB'^{1-\beta}} > 1 + \frac{1-\alpha}{1-\beta} \quad (43)$$

and because of inequality (38)

$$y(h) = 1 + \frac{\frac{A'}{A}nB - Bh(1-\Psi)}{Bh + B'} - \frac{1-\alpha}{r(Bh + B')^{1-\beta}} < 1 < 1 + \frac{1-\alpha}{1-\beta}. \quad (44)$$

$y(l)$ has a unique minimum which is negative, for

$$r(Bh + B')^{1-\beta} = \frac{r \frac{A'}{A} nB + B' + B\Psi h}{(1-\alpha)(1-\beta)}. \quad (45)$$

Hence Eq.(40) determines a unique value for l , which is positive and smaller than h . We have to investigate that this solution satisfies

$$Bl + B' \leq (1-\beta)^{\frac{1}{1-\beta}}.$$

This is equivalent to

$$y\left[\frac{(1-\beta)^{\frac{1}{1-\beta}}}{B} - B'\right] \leq 1 + \frac{(1-\alpha)}{1-\beta},$$

which results from inequality (36). We also check that

$$Ae_{+1} = \frac{A(\Psi h - l)}{n} \leq (1-\alpha)^{\frac{1}{1-\alpha}}$$

or

$$l \geq \Psi h - \frac{(1-\alpha)^{\frac{1}{1-\alpha}}}{A} n.$$

This condition is satisfied because of inequality (37).

Lemma 3.2 • *The probability of survival of a junior adult decreases relatively less for those endowed with a high human capital than those endowed with a low human capital.*

$$\frac{d\pi(l)}{\pi(l)} = \frac{(1-\beta)(Bl + B')^{-\beta} dB'}{\frac{(B \frac{d\Psi}{dB'} h + 1 + n \frac{dA'}{dB'} \frac{1}{A})}{(B\Psi h + B' + n \frac{A'}{A})(Bl + B')^{-\beta} - \frac{(1-\alpha)(1-\beta)}{r}}} \quad (46)$$

- *The probability of survival of his children decreases by*

$$\frac{d\lambda(e_{+1})}{\lambda(e_{+1})} = \frac{(1-\alpha)}{B(\Psi h - l)} \left[1 - \frac{(1-\beta)(Bl + B')^{-\beta}}{\frac{(B \frac{d\Psi}{dB'} h + 1 + n \frac{dA'}{dB'} \frac{1}{A})}{(B\Psi h + B' + n \frac{A'}{A})(Bl + B')^{-\beta} - \frac{(1-\alpha)(1-\beta)}{r}}} \right] dB' \quad (47)$$

- *The probability of survival of a child decreases relatively less if the parents are endowed with high human capital than if his parents are endowed with a low human capital.*

Proof We deduce from equation (40)

$$\frac{dl}{dB'} = \frac{1}{\frac{(B \frac{d\Psi}{dB'} h + 1 + n \frac{dA'}{dB'} \frac{1}{A})}{\frac{(B\Psi h + B' + n \frac{A'}{A})B}{Bl + B'} - \frac{(1-\alpha)(1-\beta)B}{r(Bl + B')^{1-\beta}}}} - \frac{1}{B} \quad (48)$$

$$\frac{Bdl}{dB'} = \frac{1}{\frac{(B \frac{d\Psi}{dB'} h + 1 + n \frac{dA'}{dB'} \frac{1}{A})}{\frac{(B\Psi h + B' + n \frac{A'}{A})}{Bl + B'} - \frac{(1-\alpha)(1-\beta)}{r(Bl + B')^{1-\beta}}}} - 1 \quad (49)$$

We notice that if in equation (49) we consider $A' = 0$ and $\Psi=1$ as Boucekkine and Laffargue (2010) impose, we arrive to the same results. Also, we know that

$$\frac{d\pi(l)}{dl} = \frac{\partial\pi(l)}{\partial B'} + \frac{\partial\pi(l)}{\partial l} \frac{dl}{dB'} = (Bl + B')^{-\beta} (1 + B \frac{dl}{dB'}) \quad (50)$$

$$\frac{d\pi(l)}{dB'} = \frac{(Bl + B')^{-\beta}}{\frac{(B \frac{d\Psi}{dB'} h + 1 + n \frac{dA'}{dB'} \frac{1}{A})}{\frac{(B\Psi h + B' + n \frac{A'}{A})}{Bl + B'} - \frac{(1-\alpha)(1-\beta)}{r(Bl + B')^{1-\beta}}}} \quad (51)$$

$$\frac{d\pi(l)}{dB'} / \pi(l) = \frac{(1-\beta)(Bl + B')^{-\beta}}{\frac{(B \frac{d\Psi}{dB'} h + 1 + n \frac{dA'}{dB'} \frac{1}{A})}{(B\Psi h + B' + n \frac{A'}{A})(Bl + B')^{-\beta} - \frac{(1-\alpha)(1-\beta)}{r}}} \quad (52)$$

which is a decreasing function of l and so of h . Thus

$$\frac{d\pi(l^-)}{\pi(l^-)} < \frac{d\pi(l^+)}{\pi(l^+)}$$

Then we substitute Eq. (40) in this expression. We deduce from Eq.(9) and (42)

$$\frac{d\lambda(e_{+1})}{\lambda(e_{+1})} = (1-\alpha) \frac{e_{+1}}{e_{+1}} = -(1-\alpha) \frac{dl}{\Psi h - l} \quad (53)$$

We substitute in the right side of this equation the expression of dl given above and get Eq.(46). Remind that if

$$\Psi h - l = ne_{+1} \quad (54)$$

increases with h , then the factor of dB' in Eq. (47) is a decreasing function of h (see Lemma 2.2 and 2.3). Thus,

$$\frac{d\lambda(e_{+1}^-)}{\lambda(e_{+1}^-)} < \frac{d\lambda(e_{+1}^+)}{\lambda(e_{+1}^+)}.$$

Boucekkine and Laffargue (2010) show that when B' decreases, the parents will increase their health expenditures and reduce those of their children. Now, having the indirect effects, reduction of productivity and transmission of disease to children, we expect a variation in our results. This comes from the intuition that the infected parents will be less productive workers and consequently, will obtain lower income than before. Thus, they will spend less for their own and moreover for their children's health expenditures. Also, since we assume that the parents are wholly altruistic, they will decrease their health investment as the transmission of shock increases and will increase that of their children. Having this variation in the decision of parents concerning health expenditures, we can not conclude which effect dominates and we need to carry out simulations (see section Quantitative analysis).

3.2 Demographic variables

Before applying simulations to the model, the demographic variables need to be investigated. The population alive in period t contains N^{2+} and N^{2-} junior adults with human capital endowments equal to h^+ and h^- respectively. It also contains N^{3+} and N^{3-} senior adults. Moreover, it includes N^{1+} and N^{1-} children who have parents with respective human capital

h^+ and h^- and N^{101+} , N^{101-} are number of orphans with respectively bequests. The parents will be the senior adults according to the equations

$$N^{1+} = nN^{3+} \text{ and } N^{1-} = nN^{3-}. \quad (55)$$

The populations $N^{101+}, N^{2+}, N^{3+}, N^{1+}, N^{101-}, N^{2-}, N^{3-}, N^{1-}$ are determined for the period t . The number of senior adults endowed with a given amount of human capital which will be alive in period $t + 1$ is equal to the number of junior adults of the same human capital who are alive in period t multiplied by the survival rate

$$N_{+1}^{3+} = \pi(l^+)N^{2+} \text{ and } N_{+1}^{3-} = \pi(l^-)N^{2-}. \quad (56)$$

We can use this equation in period $t+1$ to get the following

$$N_{+1}^{101+} = nN^{2+} - nN_{+1}^{3+} \text{ and } N_{+1}^{101-} = nN^{2-} - nN_{+1}^{3-}. \quad (57)$$

The numbers of junior adults of the two human capital endowments in period $t+1$ are

$$N_{+1}^{2+} = \lambda(e_{+1}^+)(pN^{1+} + cqN^{101+}) + \lambda(e_{+1}^-)(pN^{1-} + cqN^{101-}), \quad (58)$$

$$N_{+1}^{2-} = \lambda(e_{+1}^+)(N^{1+} + cN^{101+}) + \lambda(e_{+1}^-)(N^{1-} + cN^{101-}) - N_{+1}^{2+}. \quad (59)$$

3.3 Dynamics and long run equilibrium

In this section we investigate the dynamics of equations (55)-(59). Later we develop the characterization of these demographic dynamics when the economic environment remains unchanged¹.

3.3.1 The dynamics of population

As mentioned previously, there are two kinds of junior adults, N^{2+} and N^{2-} , alive in period $t > 0$ with high and low human endowment respectively. Each of them has n children. These children will either become N_{+2}^{2+} junior adults with earnings equal to h at period $t + 2$ or they die at the period $t + 1$. Also, D_{+2} represents the additional number of junior adults who would exist if children had not died before reaching the age of junior adult. This means that λ would be equal to one. Our interest is focused on the dynamics of the model for $t \geq 2$. The states of the economy in periods 0 and 1 are considered as given. Consequently, the fundamental matrix is:

$$\begin{bmatrix} N_{+2}^{2+} \\ N_{+2}^{2-} \\ D_{+2} \end{bmatrix} = \begin{bmatrix} N^{2+} \\ N^{2-} \\ D \end{bmatrix} Mn = n \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 1 - a_{11} - a_{21} & 1 - a_{12} - a_{22} & 1 \end{bmatrix} \begin{bmatrix} N_{+2} \\ N_{+2} \\ D \end{bmatrix} \quad (60)$$

with

$$\begin{aligned} a_{11} &= \lambda(e^+) \{ \pi(l^+)p + [1 - \pi(l^+)]cq \} \\ a_{21} &= \lambda(e^+) \{ \pi(l^+)(1 - p) + [1 - \pi(l^+)]c(1 - q) \} \\ a_{12} &= \lambda(e^-) \{ \pi(l^-)p + [1 - \pi(l^-)]cq \} \\ a_{22} &= \lambda(e^-) \{ \pi(l^-)(1 - p) + [1 - \pi(l^-)]c(1 - q) \} \end{aligned}$$

¹We need to mention that we use similar notation with Boucekkine and Laffargue (2010).

and with $N^{2+}(0)$, $N^{2-}(0)$ and $D(0)$ given if t is even and $N^{2+}(1)$, $N^{2-}(1)$ and $D(1)$ given if t is odd. Lemma 1, 2 and 3 that are also valid in the extension model impose that the above parameters satisfy the following constraints

$$0 < a_{12} < a_{11} < 1,$$

$$0 < a_{22} < a_{21} < 1,$$

$$a_{12} + a_{22} < a_{11} + a_{21} < 1$$

and

$$a_{11}a_{22} - a_{12}a_{21} = c(p - q)\lambda(e^-)\lambda(e^+)[\pi(l^+) - \pi(l^-)] > 0.$$

All the elements of matrix M are positive and the sum of each column of this matrix is identical to 1. Consequently, they can be equivalent to conditional probabilities. Investigating the elements of this matrix, we notice that $a_{11}-a_{12}$ is the difference between the probabilities for a child to obtain a high level of human capital if his parents have high capital versus his parents have low human capital. Also, $a_{21}-a_{22}$ is the difference between the probabilities for a child to obtain a low level capital if his parents are highly endowed versus if his parents are poorly endowed. Furthermore, $(a_{12}-a_{11})+(a_{22}-a_{21})$ is the difference between the probabilities for a child to die if his parents have high human capital versus if his parents have low human capital. The fate of children is independent of the social position of their parents when $a_{12}-a_{11}=a_{22}-a_{21}=0$.

It is crucial to mention that the elements of matrix M depend on health spending decisions of junior adults, l^+ , l^- , e^+ , e^- for themselves and their children respectively. These spending functions are determined by a number of exogenous variables in period t : The productivity Ψ , the parameters that define the survival rates of children and young adults A , A' , B , B' , α and β , the earnings of the junior adults h^+ and h^- and n , the number of children.

Equation (60) shows the dynamics of the population of young junior adults N^{2+} , N^{2-} and of the dead, D for $t \geq 2$, when the values of these variables are given in periods 0 and 1. Equations $N^{3+} = \pi(l^+) N_{-1}^{2+}$, $N^{3-} = \pi(l^-) N_{-1}^{2-}$ define the dynamics for the number of senior adults for $t \geq 1$. Furthermore, the number of orphans in period $t \geq 1$ can be given by the equations $N_{+1}^{101+} = nN^{2+} - nN_{+1}^{3+}$ and $N_{+1}^{101-} = nN^{2-} - nN_{+1}^{3-}$ from well and low endowed parents respectively. Also, the dynamics for the number of children are defined by the equations $N^{1+} = nN^{3+}$ and $N^{1-} = nN^{3-}$ for $t \geq 1$.

The equation $P = N^{2+} + N^{2-} + D$ can be characterized as the potential population of young junior adults. More precisely, this equation gives us the potential number of junior adults if all children have reached the age of junior adults. Consequently, the number of dead junior adults is $D = P - N^{2+} - N^{2-}$. Thus, we need to focus on the dynamics of living junior adults which are given by the following equations :

$$\begin{bmatrix} N^{2+}(t+2) \\ N^{2-}(t+2) \end{bmatrix} = \begin{bmatrix} N^{2+} \\ N^{2-} \end{bmatrix} M n = n \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} N^{2+}(t) \\ N^{2-}(t) \end{bmatrix} \quad (61)$$

with $N^{2+}(0)$ and $N^{2-}(0)$ are given if t is even and $N^{2+}(1)$ and $N^{2-}(1)$ given if t is odd. In our simulations we consider that the epidemic shock in generation 3 occurs and we define this generation as period 0, so we assume that t is even.

3.4 Characteristics of demographic dynamics

We suppose that all the parameters and exogenous variables remain constant over time for $t \geq 0$. As mentioned previously, we suppose that t is even. Hence, the matrix M will remain

constant over time. We introduce a new variable that is the discriminant of matrix M , that we call

$$\Delta = (a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21}) = (a_{11} - a_{22})^2 + 4a_{12}a_{21} > 0. \quad (62)$$

We introduce a lemma similar to Boucekkine and Laffargue(2007)

Lemma 3.3 *a) The eigenvalues of matrix M , ρ_1 and ρ_2 are real eigenvalues and such that $0 < \rho_2 < \rho_1 < 1$. Their values are given by the following equations:*

$$\rho_1 = (a_{11} + a_{22} + \sqrt{\Delta})/2 \text{ and } \rho_2 = (a_{11} + a_{22} - \sqrt{\Delta})/2 \quad (63)$$

b) Let us denote by

$$V_1 = \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix}$$

and

$$V_2 = \begin{bmatrix} v_{12} \\ v_{22} \end{bmatrix}$$

the right- based eigenvectors of M and by $V = (V_1 \ V_2)$ the matrix of these eigenvectors. A determination of these eigenvectors is

$$V = \begin{bmatrix} 2a_{12} & -a_{12} \\ a_{22} - a_{11} + \sqrt{\Delta} & -a_{22} + a_{11} + \sqrt{\Delta} \end{bmatrix} \quad (64)$$

V_1 can be normed such that its components are positive and sum to 1. V_2 can be normed such that its first component is negative, its second component is positive and the sum of both components is equal to 1.

c) Let

$$W = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$$

be the inverse of V : $V W = 1$. Then we have

$$W = \frac{1}{4a_{12}\sqrt{\Delta}} \begin{bmatrix} -a_{22} + a_{11} + \sqrt{\Delta} & 2a_{12} \\ -a_{22} + a_{11} - \sqrt{\Delta} & 2a_{12} \end{bmatrix} \quad (65)$$

d) The elements of matrix W satisfy the constraints

$$w_{11} > w_{12} > 0 \text{ and } w_{21} < 0 < w_{22} \quad (66)$$

Proof See Boucekkine and Laffargue(2007).

Proposition 3.4 *Assume, to fix the ideas that $N^{2+}(0) + N^{2-}(0) = 1$. Then: The dynamic paths followed by the sizes of the cohorts of both kinds of junior adults are linear combinations of two geometric series with rates equal to the growth rate of potential population n times the eigenvalues of matrix M*

$$N^{2+}(t+2) = (\rho_1 n)^{t/2+1} v_{11} [w_{11} N^{2+}(0) + w_{12} N^{2-}(0)] + (\rho_2 n)^{t/2+1} v_{12} [w_{21} N^{2+}(0) + w_{22} N^{2-}(0)].$$

$$N^{2-}(t+2) = (\rho_1 n)^{t/2+1} v_{21} [w_{11} N^{2+}(0) + w_{12} N^{2-}(0)] + (\rho_2 n)^{t/2+1} v_{22} [w_{21} N^{2+}(0) + w_{22} N^{2-}(0)].$$

In the long run the populations of both kinds junior adults will grow at a rate equal to the growth rate of the potential population of junior adults times the largest eigenvalue of matrix M (which is smaller than one). The long run size of each group depends on the initial condition, $N_{2+}(0)$. However, the long run proposition of the two groups of junior adults are independent of the initial conditions, and are precisely proportional to the two components of the eigenvector associated to the largest eigenvalue of matrix M .

Proof See Boucekkine and Laffargue (2007).

This proposition shows that initial demographic shocks diminish after few periods.

Also, if the vector of the initial values of the population of the two kinds of junior adults is equal to the eigenvector of M matrix associated to the largest eigenvalue V_1 , the population of junior adults will follow the balanced growth path,

$$\begin{bmatrix} N^{2+}(t+2) \\ N^{2-}(t+2) \end{bmatrix} = (\rho_1 n)^{t/2+1} V_1. \quad (67)$$

According to the previous proposition, the steady state is asymptotically stable. This will concern our balanced growth path. The total domestic output in this model will be given by

$$Y(t) = N^{2+}(t)h^+ + N^{2-}(t)h^-. \quad (68)$$

Let us investigate more closely the equation 55. The relative variations in the populations of well endowed and poorly endowed junior adults in the period $t=2$ are given by the differentiation of Eq. 55

$$\frac{dN^{21}(2)}{N^{21}(2)} = \frac{v_{11}d\alpha_{11} + (1 - v_{11})d\alpha_{12}}{\rho_1 v_{11}}, \quad (69)$$

$$\frac{dN^{22}(2)}{N^{22}(2)} = \frac{v_{11}d\alpha_{21} + (1 - v_{11})d\alpha_{22}}{\rho_1 v_{11}}. \quad (70)$$

The relative changes in terms of the total young adults population and output will be

$$\frac{dN^{21}(2) + dN^{22}(2)}{N^{22}(2) + N^{21}(2)} = \frac{v_{11}(d\alpha_{11} + d\alpha_{21}) + (1 - v_{11})(d\alpha_{12} + d\alpha_{22})}{\rho_1}.$$

$$\frac{dY(2)}{Y(2)} - \frac{dN^{21}(2) + dN^{22}(2)}{N^{22}(2) + N^{21}(2)} = \frac{N^{21}(2)N^{22}(2)(h^+ - h^-)}{[N^{21}(2)h^+ + N^{22}(2)h^-][N^{21}(2) + N^{22}(2)]} \left[\frac{dN^{21}(2)}{N^{21}(2)} - \frac{dN^{22}(2)}{N^{22}(2)} \right].$$

The three different effects are present in the α_{11} , v_{11} , α_{12} and α_{22} . Therefore, it is impossible to conclude which effect dominates and simulations should be provided.

4 Quantitative analysis

We define the epidemic as a drop in survival probabilities of junior adults because of one period increase death rate of a given generation. We consider the shock on the parameter B' , inherent health characteristics.

HIV invades and ruins the immune system by damaging the CD4 lymphocytes¹. These cells are produced by the thymus and control the functions of the immune system. CD4 lymphocytes are also called helper lymphocytes because of the help that they provide to other types of lymphocytes. A normally immune system prevents the infections and the development of malignancies.

HIV infection results in a fall of in the number of CD4 lymphocytes with the consequence that the immune system cannot function normally. As a result, the high risk of infections and the number of cases with cancer increases(see Gebo et al(2004), Gebo et al (2006)).

This shock has two consequences. The first consequence is the reduction in productivity of junior adults and the second is that the mortality shock is diffused from them to their infants.

We assume that the epidemic hits people independently of their human capital and social background. Since, an epidemic like AIDS does not discriminate its victims. Our explanation about this abstraction emanates from the fact that HIV is a virus that uses material from our CD4 cells to make more copies of itself. This virus approaches one particular cell in our immune system called CD4 cell or T-cell. When it uses that cell's genetic material, it destroys the T-cell making and as a result the immune system is unable to protect us from other infections, thereby involving infected people more vulnerable to disease. When we are sick our productivity falls (see Glomm, Mendez and Corrigan (2005)).

Furthermore, UNAIDS (2009, November), 'AIDS epidemic update' refers that "In 2008, around 430,000 children under 15 became infected with HIV, mainly through mother-to-child transmission. About 90 percent of these MTCT infections occurred in Africa where AIDS is beginning to reverse decades of steady progress in child survival". So, an infected mother by a shock has high probability to infect her offspring either by breasting or during her pregnancy.

Also, we suppose that nothing can stop this epidemic even if young junior adults have the possibility to increase their health investment or the one of their children. As mentioned above, we consider a one generation epidemic. This idea is inspired by Boucekkine and Laffargue (2010) because it captures the main mechanism and it helps identifying the effect of the epidemic for a given age-profile. At the meantime, considering a shock parting more than one period leads to more complicated results.

Moreover, inspired by Boucekkine and Laffargue (2010) we assume that the total population of junior adults is equal to 1 (see table 1 and table 2 in appendix). Based on this intuition, we suppose that the total population of senior adults, children and orphans is normalized equal to 1 in the benchmark and extension model (see table 1). We used data for enrollment rates of population in education from the Barro and Lee data set. Then, we choose data sample from countries from sub-Saharan Africa, China and India. More precisely, we chose countries with the high ranking AIDS deaths from United Nations sources. In this list are countries that either do not exist in the above data set or have missing values that we replace them by other countries from sub-Saharan Africa which also have high rate of AIDS deaths.

¹Lymphocytes are a type of white blood cell that helps the body fight infection. There are two main types of lymphocytes; B-lymphocytes and T-lymphocytes. There are two kinds of T-cells: 1) CD4 Cells. These cells help the immune system to protect the body from infectious invaders such as viruses and bacteria. The second type is CD8 Cells. These cells destroy the infected cells and produce antiviral substances that fight off infectious organisms.

The table assists us to construct the proportion of poor and well endowed population for our simulations. We assume that the poor people do not have school education. And we find that in average 73 percent of female population does not attend school in 1985. Moreover, 65 percent of the total population does not attend school the same period. We chose the year 1985 for the reason that AIDS appeared for the first time in April 1984. We impose the initial proportion 70 percent for the low human capital population and 30 percent for the well endowed human capital population. Our assumption is also supported by table 2 if we notice that the average human capital of people that they do not attend school at each country. The total average of human capital at no school education of all the countries is 67,5 percent.

The model is calibrated under the assumption that one period (or generation) lasts 25 years. Both models are simulated for 3 generations. We choose 3 generations for two important reasons. First, we investigate the short and medium terms of an epidemic in these exercises and second considering one shock, we capture the main consequences of this epidemic shock. We apply simulations on three different exercises. We will first simulate the predictions of the extension model under five different epidemic scenarios, assuming that every adult is endowed with the same human capital h . We will then run those simulations again, but allowing for two different levels of human capital, h^+ and h^- . Last, we will simulate the extension model when allowing differential fertility, i.e. allowing for the number of children of adults belonging to the two existing classes to differ from one another. We choose the parameters of the model such that the probabilities of survival of children and junior adults are between zero and one:

$$\lambda(e_{+1}) = \min \left\{ (Ae_{+1} + A')^{1-\alpha} / (1-\alpha), 1 \right\} \quad (71)$$

$$\pi(l) = \min \left\{ (Bl + B')^{1-\beta} / (1-\beta), 1 \right\}. \quad (72)$$

The table below summarizes all the values that we used to calibrate both of models.

Table 1
calibration

parameters	initial values
$B'=0.4$	$L^+=1$
$B=0.1$	$e^+=1$
$\alpha=0.1$	$L^-=0.5$
$\beta=0.4$	$e^-=1$
$c=0.999$	$N^{1+}=0.3$
$v=1$	$N^{1-}=0.7$
$p=0.5$	$N^{2+}=0.3$ and $N^{2-}=0.7$
$q=0.4$	$N^{3+}=0.3$ and $N^{3-}=0.7$
$h^+=1500$	$N^{101+}=0.3$ and $N^{101-}=0.7$
$h^-=500$	

The inherited health characteristics of junior adults, B' , is set to $B'=0.4$.² and B , which is the proportion of their health investment that has effect on their health is imposed equal to 0.1. The value is small for four reasons.

- We suppose that nothing can stop this epidemic even if young junior adults have the possibility to increase their health investment or of their children consequently the effect of B is supposedly small.
- According to the definition of probability, we need the survival probability of young junior adults to be lower than one.
- We manage to construct the probability of young junior adults as Corrigan, Gloom and Mendez (2005) who consider that a healthy person has survival rates around 96 percent and the people affected by an epidemic have the lowest survival rate is 80-87 percent in their simulations. Also, this explanation leads us to define the initial value of their health investment l equal to one and $\beta=0.4$.
- We needed the B' to cover the condition $0 < B' < (1 - \beta)^{1/(1-\beta)}$. Consequently, imposing $\beta=0.4$, it is necessary to choose B' less than 0.73.
These values concern the well endowed young adult. We suppose here that the values B and B' are the same for the poor endowed young adult. So, the difference in both classes concerning the survival probability is the different investment expenditures. More precisely, we know that a young adult poorly endowed with human capital has lower survival probability than the well endowed adult. The main cause in our simulations are the health expenditures. Consequently, we impose as initial value $l^-=0.5$ and their survival probability to be 0.82 .

Furthermore, as it is mentioned in previous paragraphs, we assume that the epidemic hits the inherited health characteristics. According to Gebo et al (2004) and Gebo et al (2006) the absolute CD4³ count and CD4 percentage⁴ can predict HIV progression. The same studies found that when the CD4 percentage is less than 15 percent, CD4 percentage should be considered along with the absolute CD4 count when determining illness risk and when to start HIV treatment. Consequently, we set the decrease in B' around 15 percent the period(or generation) that the epidemic takes place.

In all the simulation exercises, the productivity of all workers is assumed to be equal to 1. When the epidemic takes place the productivity of infected workers will be 0.5 (see Corrigan Gloom and Mendez (2005)). Also, this corresponds to AIDS related productivity losses mentioned in Guinness and Alban (2000), who find losses in agricultural output around 60 percent. It is worth to report that in the second model, we determine different productivity losses. The reason is that the job of low skilled low workers is based on their productivity and the epidemic affects it more than high skilled. As a result, we determine for the high skilled workers' productivity around 0.7.

²In the beginning of our calibration, we imposed many different values either on B' or B such that t the survival probability of young adult remained between 0.96 and 0.82 and our results are robust.

³The absolute CD4 count is a measure of how many functional CD4 T-cells are circulating in the blood. The lower the absolute CD4 count, the weaker the immune system. The absolute CD4 count is measured by a simple blood test and is reported as the number of CD4 cells per cubic millimeter of blood. HIV-negative people typically have absolute CD4 counts between 600 and 1200 CD4 cells per cubic millimeter. HIV-infected people have counts that are less than 500 CD4 cells per cubic millimeter.

⁴CD4 percentage represents the percentage of total lymphocytes that are CD4 cells. The CD4 percentage is measured using the same blood test as the absolute CD4 count.

Furthermore, we set that $A=0.004$ and the inherited characteristics of children are $A'=0.4$. The reasons are the following:

- We suppose here that the children's production of CD4 is the same as the young adults, regardless the fact that CD4 is expressed in higher levels and on more cells in young thymocyte populations than in old populations in the thymus (see Kitchen et al (1997). This assumption is based on the idea that the senior adults are the old population in our model. Also, our assumption is enhanced by medical literature (see Beisel (1996) and Hegde et al (1999)) that mention reduction of CD4 cells due to malnutrition. Usually, countries with high ranked AIDS deaths are faced with problems of malnutrition. Again these values are imposed in order to keep the survival rate of children below one and therefore less than their parents. The first is based on the definition of probability. The latter is based on the assumption that a child's survival probability is lower than the one from a young adult.
- We also satisfy the condition $0 < A' < (1 - \alpha)^{1/(1-\alpha)}$.

Finally, we choose the initial health expenditure of children to be $e=1$. This choice is based on the desire to keep their survival probability below 1. Therefore, we keep the same initial expenditure for children and young adults under our assumption that parents are wholly altruistic. Similar, we impose these parameters on the population endowed low human capital except the for expenditures on children's health that are lower $e^-=0.5$. Moreover, we impose arbitrary that $h^+=1500$ dollars and $h^-=500$ dollars. These values discriminate the wages of junior adults with high and low human capital. Also, they are imposed to satisfy the constraints of the model.

A woman with HIV can transmit the virus to her offspring either during pregnancy, labour or after childbirth via breastfeeding. The United Nations refers "Approximately one fourth to one third of children born to HIV-positive women are likely to acquire infection from their mothers. Paediatric HIV infection is expected to have a substantial impact on mortality during infancy and childhood, particularly among older children (above age one). Children who acquire the HIV virus from their mothers during childbirth or breast feeding usually do not survive long enough to enroll in school. Children die young from HIV owing to mother-to-child transmission and to the weakened ability of infected mothers to care for their infants and young children".

Cohen (1998) writes on Southern Africa: "Infant mortality rates are already rising sharply in countries with mature epidemics. Children born to mothers who are HIV positive have a 30-60 percent chance of becoming positive themselves". WHO, UNAIDS and UNICEF, Towards Universal Access: Scaling up priority HIV/AIDS interventions in the health sector, Progress Report 2009 mentions that: "Each year, many children are newly infected with HIV, mainly through mother-to-child transmission. An overwhelming majority or more than 90 per cent of HIV infections in infants and children are passed on by mothers during pregnancy, labour, delivery or breastfeeding. Without any intervention, between 15 per cent and 45 per cent of infants born to mothers living with HIV will become infected (5 or 10 per cent during pregnancy, 10 or 20 per cent during labour and delivery and 5 or 20 per cent through breastfeeding). Approximately 50 per cent of infants infected with HIV from their mothers die before their second birthday." In our analysis, we assume that the chance of the transmitting HIV from a mother to children is around 45 percent for poorly endowed mothers and 25 percent for well endowed without treatment⁴.

⁴These numbers are inspired from Reports and WHO and HIV Handbooks and infant feeding technical consultation, Chronic HIV Care with ARV therapy and prevention. Integrated management of adolescent

At the end, the human capital is chosen in such way as to follow the assumption and restriction that e and l must be positive.

Concerning the fertility, we take the case of China. In China, the number of children n is selected equal to one accordingly to the one's child Policy. This law continues to exist through the period 2006-10 and we will be reconsidered after this period. We find that the number of children in China is $n=1.8$ from the United Nations Population department database. The reason of existence of this number is that the law of one child in China is voluntary. Government imposes heavy fines and punishments on couples which do not respect this law. Also, we need to remark that many parents with more than two children do not declare all their children. Moreover, in 17 provinces couples are permitted to have second child if the first is girl. In particular, in wealthy provinces such as Guangdong and Hainan rural families are allowed to have two children regardless of the sex of the first. Also, minority groups like Miao and Mongols are generally allowed to have three children if the first two are girls.⁵ In our simulations, we are based on the United Nations Population database.

Here we suppose the shock to be anticipated: it could be so in the case of a chronic disease (like malaria) or because the economy has been experiencing an epidemic hitting adults before $t = 0$, which is likely to be transmitted to children (like AIDS). Let us base the reasoning on the assumption that junior adults are struck by an epidemic at period $t=0$ which is the first generation in our quantitative analysis. We consider 5 scenarios:

- Scenario 1: No epidemic shock across generations.
- Scenario 2: Shock in B' .
- Scenario 3: Shock in B' affects the productivity of workers.
- Scenario 4: Shock in B' transmitted to children.
- Scenario 5: All the shocks together.

4.1 Benchmark Model

In this section, we examine the health investment decisions of young junior adults. Also, we focus on the structure of the population of young adults, senior adults, children and orphans in the defined scenarios. Their number in each scenario is always compared to the first scenario.

and adult illness; Integrated management of childhood illness. Interim guidelines for health workers at health centre of district hospital outpatient clinic. World Health Organisation and Handbook of HIV Medicine. Oxford University Press, Southern Africa. 5th Edition 2005. Chapter 34 (Infant Feeding), Chapter 41

⁵New York Times, Washington Post, Los Angeles Times, Times of London, National Geographic, Reuters Compton's Encyclopedia are the mainly sources for these acts

Table 3 Health investment for Junior adults (Benchmark model)			
Scenarios	<u>Generations</u>		
	1	2	3
	Expenditures		
Scenario 1	78.9710	78.9710	78.9710
Scenario 2	79.5603	78.9710	78.9710
Scenario 3	70.2279	78.9710	78.9710
Scenario 4	77.1079	78.9710	78.9710
Scenario 5	67.5792	78.9710	78.9710

Table 4 Health investment for children (Benchmark model)			
Scenarios	<u>Generations</u>		
	1	2	3
	Expenditures		
Scenario 1	511.6828	511.6828	511.6828
Scenario 2	511.3554	511.6828	511.6828
Scenario 3	238.7623	511.6828	511.6828
Scenario 4	512.7178	511.6828	511.6828
Scenario 5	240.2338	511.6828	511.6828

Table 3 shows that having a shock in B' leads to an increase in the investment of junior adult's own health expenditures (amount in dollars). This confirms our Lemma 2(b). Continuously, when this shock influences the productivity of workers, a junior adult takes the opposite decision which is the decrease of family's health investment. The intuition behind this is that less productive people receive lower wages and they invest less for their own and their children's health. This reduction is presented in scenario 3 in tables 3 and 4 the period of that epidemic takes place. A similar result can be noticed when there is a diffusion of the mortality shock from parents to children. More precisely, they decrease their investment but at the same time they increase the health expenditures of their children. In the last scenario, we incorporate the diffusion of the mortality shock from parents to children, the reduction in productivity of worker and the shock in inherited characteristics. Here, we can notice that the parents decrease their own and also they decrease their children's health investment. Consequently, we can conclude that the diffusion of disease from young adults to children and the reduction of productivity dominates the direct shock as it concerns their decisions for their own health expenditures (see scenario 5 in Table 3). On the other hand the reduction in productivity and the decrease of inherited characteristics, which reduce the survival rate of young adults, dominate the diffusion as a result the drop in health investment of their children (see table 4 scenario 5).

Finding 1. (a) A junior adult invests less in his health when the effect of an epidemic in his productivity is high. (b) A junior adult invests more in his health when the diffusion of epidemic to his children is low.

Table 5			
Health investment for Junior adults (Benchmark model)			
Scenarios	Generations		
	1	2	3
Productivity			
$\Psi=0.5$	70.2279	78.9710	78.9710
$\Psi=0.7$	74.0454	78.9710	78.9710
$\Psi=0.9$	77.7482	78.9710	78.9710
$\Psi=1$	79.5603	78.9710	78.9710

Table 6			
Health investment for Junior adults (Benchmark model)			
Scenarios	Generations		
	1	2	3
Diffusion			
$\varphi = 0$	79.5603	78.9710	78.9710
$\varphi=0.2$	77.9305	78.9710	78.9710
$\varphi=0.45$	77.1079	78.9710	78.9710
$\varphi=1$	74.6078	78.9710	78.9710

The previous tables illustrate the above finding. Table 5 displays the decrease of productivity because of the epidemic. We let the productivity of infected workers be in the interval 0.5 to 1. The $\Psi=1$ is the case where the productivity of young junior adults have not affected by the epidemic. The numbers are chosen as been explained previously in quantitative analysis section. Roughly speaking, an infected worker will spend less in his health in any case when his productivity is below one (see table 5). The abstraction behind this result is that obtaining smaller wage as his productivity falls he will spend less money to his own and his children's health, as we see later on. Furthermore, table 6 shows the percentage of transmission from the mother to her offspring starting with the case of no transmission to the case of total trasmission of the shock. Investigating the behavior of young adults in this case, we find that the young junior adults decrease their health investment as the transmission of shock increases (see table 6). This result coincides with our assumption that the parents are wholly altruistic.

Finding 2. (a) A junior adult invests more in his children's health when the effect of an epidemic on his productivity is small. (b) A junior adult invests more in his children's health when the diffusion of the shock of epidemic to his children is high.

Table 7
Health investment for children
(Benchmark model)

Scenarios	<u>Generations</u>		
	1	2	3
Productivity			
$\Psi=0.5$	238.7623	511.6828	511.6828
$\Psi=0.7$	347.7525	511.6828	511.6828
$\Psi=0.9$	456.8066	511.6828	511.6828
$\Psi=1$	511.3554	511.6828	511.6828

Table 8
Health investment for children
(Benchmark model)

Scenarios	<u>Generations</u>		
	1	2	3
Diffusion			
$\varphi=0$	511.3554	511.6828	511.6828
$\varphi=0.2$	512.2608	511.6828	511.6828
$\varphi=0.45$	512.7178	511.6828	511.6828
$\varphi=1$	514.1068	511.6828	511.6828

When the productivity of young junior adults is not affected by the epidemic, they decrease slightly the investment in their children. Starting from the assumption that a fall in their productivity (as resulting from the shock), we observe that they decrease their children's health expenditure more and when their productivity is almost half of what they had before the shock the expenditures for their children are almost half of what they would have spent without epidemic. As it is discussed before this finding is coming from the fact that less productive people would have a lower wage and would spend less for their children's health too. Also, in table 8 the parents will invest more in their children's health as the percentage of their health shock transmitted to their children is increased. We observe that when there is full diffusion they will invest more in the health of their children during the period of epidemic than they would had the epidemic not take place.

Table 9 Senior adults' Population (Benchmark Model)			
Scenarios	<u>Generations</u>		
	1	2	3
Scenario 1	100	100	100
Scenario 2	100	98	99
Scenario 3	93	84	94
Scenario 4	98	95	98
Scenario 5	91	79	92

Table 9 illustrates the structure of the population of senior adults in the different scenarios. Introducing an epidemic that affects the inherited characteristics of young junior adults, B' , reduces the number of senior adults in period 1 (see scenario 2, generation 2). Progressively, adding the two indirect effects, productivity and transmission of the shock from parents to their infants (scenarios 3 and 4 respectively), we observe that their number falls during the epidemic period and the period 2. The reduction of productivity of young junior adults decreases their number more than the two other effects mainly in odd periods. At the end, scenario 5 contains all the shocks and explains how the structure of their population is determined. It is notable that the highest drop in their population happens in period 1 (generation 2).

Table 10 Junior adults' Population (Benchmark Model)			
Scenarios	<u>Generations</u>		
	1	2	3
Scenario 1	100	100	100
Scenario 2	99	100	99
Scenario 3	84	94	84
Scenario 4	95	98	95
Scenario 5	79	92	79

As mentioned previously, the chosen period for the epidemic shock is the third generation. Scenario 2 shows that the number of junior adults slightly decreases at period 0 (epidemic takes places, in our table generation 1). And also there is a drop of their population in period 2 (generation 3). This result coincides with Boucekkine and Laffargue (2010). Introducing that the epidemic also affects their productivity (scenario 3) it is notable that all generations are affected after the shock. Decreasing productivity leads to the junior adults receive lower wages so they invest less not only for their health but also for their children's health expenditures than they spend if there is only the shock in B' or no shock (see scenario 1 and 2 in tables 3 and 4). Scenario 4 illustrates that the transmission of the disease has also

massively affected in the structure of their population. Their number is reduced less than in scenario 3 but their decrease is still high. This result can be explained by the fact that they decrease their health expenditures (scenario 4, generation 1 in table 4) and increase those of their children. Consequently their survival probability drops and their number is reduced. We notice that in the last three scenarios the effects affect more the even than odds periods after the epidemic introduction. At the end, scenario 5 shows the effect of all the effects in their number. The structure of their population is similar with the one in scenario 3.

Table 11
Orphans' Population
(Benchmark Model)

Scenarios	Generations		
	1	2	3
Scenario 1	100	100	100
Scenario 2	100	99	99.9
Scenario 3	100	84	94
Scenario 4	100	95	98
Scenario 5	100	79	92

The number of orphans decreases in generations 2 and 3 in all the scenarios that epidemic takes place except scenario 2. The transmission shock causes high drop on their number. Of course this table provides only what happens to their population with the different effects. The most important is to investigate the ratio orphans to children and then we will be able to understand the effects in terms of inequalities and on the evolution of domestic output across the generations.

Table 12
Children's Population
(Benchmark Model)

Scenarios	Generations		
	1	2	3
Scenario 1	100	100	100
Scenario 2	100	99	100
Scenario 3	93	84	94
Scenario 4	98	95	98
Scenario 5	91	79	92

Table 12 presents similar results with table 5. Again, we observe that the diffusion of the shock is mainly the effect that reduces the number of children. Also, it is worth to mention that their number highly drops in odd periods after the epidemic.

4.2 Extension Model

In this section, we apply the different scenarios to the extension model having two different human capitals, h^+ and h^- . Given that in terms of population structure the results are similar with those of the benchmark model, our analysis will focus on terms of inequality. In the beginning, we examine the structure of the population of senior adults and junior adults and later we mainly discuss the ratios of well-endowed versus poorly endowed junior adults and orphans versus children.

Table 13
Senior adults' population
(Extension Model)

Scenarios	Generations		
	1	2	3
High human capital			
Scenario 1	100	100	100
Scenario 2	100	99	100
Scenario 3	98	89	97
Scenario 4	98	94	98
Scenario 5	94	85	94

We consider again the generation 1 as the period 0 in which the epidemic takes place. Table 13 displays the senior adults with high human capital. An epidemic hitting junior adults does not affect the number of senior adults alive in period 0 and period 2 but their number will be lower as the result of epidemic in period 1 (see scenarios 1 and 2). Conversely, we notice that the number of senior adults changes not only during epidemics but also in the remaining periods (scenarios 3, 4 and 5). Also, scenarios 3 and 5 show the number of senior adults decreases more in odd periods than in even periods. The same results are also presented in table 14 for the population of senior adults with low human capital.

Table 14
Senior adults' Population
(Extension Model)

Scenarios	Generations		
	1	2	3
Low human capital			
Scenario 1	100	100	100
Scenario 2	100	99	100
Scenario 3	95	87	98
Scenario 4	98	94	98
Scenario 5	93	81	94

Table 15
Junior adults' Population
(Extension Model)

Scenarios	<u>Generations</u>		
	1	2	3
High human capital			
Scenario 1	100	100	100
Scenario 2	99.8	100	99.7
Scenario 3	89	98	88
Scenario 4	96	98	95
Scenario 5	85	96	83

In scenarios 2 and 4, an epidemic affecting junior adults has small effects on their number at period 0 (generation 1). On the other hand, scenarios 3 and 5 show that there is a high drop in the population of junior adults comparing to the other three scenarios. In scenarios 3 and 4, the reduction in productivity has mainly effects in period 0 and even periods . Table 16 shows similar results for the number of junior adults of low human capital for the first two scenarios. On the other hand, in scenarios 3 and 5, the reduction of productivity creates a different structure of their poorly endowed junior adults. More precisely, we notice that there is a huge drop in the number of poorly endowed adults in period 0 (see table 20 scenario 3 and 5, generation 1). We can conclude that obtaining lower wages than before, the poorly endowed will have lower income than without epidemic. Consequently, they will invest less for their health and this decreases their survival probability.

Table 16
Junior adults' Population
(Extension Model)

Scenarios	<u>Generations</u>		
	1	2	3
Low human capital			
Scenario 1	100	100	100
Scenario 2	99	100	99
Scenario 3	85	98	88
Scenario 4	95	98	95
Scenario 5	80	95	83

Table 17
Junior adults's Ratio of Human Capital
(Extension Model)

Scenarios	<u>Generations</u>		
	1	2	3
High human capital/Low human capital			
Scenario 1	0.6070	0.6268	0.6333
Scenario 2	0.6070	0.6268	0.6332
Scenario 3	0.6249	0.6262	0.6356
Scenario 4	0.6203	0.6278	0.6333
Scenario 5	0.6396	0.6274	0.6356

Table 17 displays the ratio of well and poorly endowed junior adults. During the epidemic period (generation 1) there is an increase of the ratio. Consequently, this means that the well educated people are more numerous than the less educated when the epidemic takes place. In period 2, the number of junior adults who were orphans will increase and the number of those who were brought up by their parents will decrease (tables 13, 14 and 18 respectively). In scenario 2, more young adults will get less educated two periods after the epidemic and output per worker goes down: the economy is clearly impoverished (with respect to the reference balanced growth path) at this time. Before starting to analyze the remaining scenarios, we need to mention that the proportion of young adults with low human capital changes under two opposite effects. First, the model generates a mechanism that leads to an increase in the proportion of young adults with high level of human capital. This mechanism is due to the fact that the proportion of children surviving falls by a lower percentage if the parents are wealthy than if they are poor. Also, children of wealthy families have a higher probability of reaching a high level of human capital than the children of poor families. Moreover, the model generates another mechanism opposite to the previous one. In particular, the number of junior adults who were orphans will increase. Orphans have a lower probability to be well-endowed. Thus, the proportion of young adults with a high human capital diminishes. After explaining these two mechanisms, we are able to present which of the above mechanism dominates the other in the remaining scenarios.

Table 18
Ratio Orphans/children
(Extension Model)

Scenarios	<u>Generations</u>		
	1	2	3
Ratio Orphans/children			
Scenario 1	0.0585	0.0614	0.0626
Scenario 2	0.0585	0.0615	0.0626
Scenario 3	0.0614	0.0625	0.0626
Scenario 4	0.0596	0.0622	0.0626
Scenario 5	0.0626	0.0633	0.0626

Henceforth, in scenarios 3, 4 and 5, the proportion of well educated people will be higher than that of less educated in the period $t=0$ (generation 1). We expected this kind of result because in scenario 3, the wage of low human capital drops more than that of high human capital. Consequently they have lower income and invest less in health expenditures.

Also, in scenario 4, the proportion of young junior adults of high human capital increases at period 0. This conclusion coincides with the results of feeding programs which show that women with low education, poor access to water and health services and strong cultural pressures to breast-feed are more likely to transmit the virus HIV to their children than well educated mothers (Coutsoudis et al (2001)). This point of view is also enhanced by reports of Unicef which mention that the education is the best defence against the disease. The more educated and skilled is the population, the more likely people are to protect themselves from infection; and those in school spend less time in risky situations. From the education the girls will be able to obtain critical analysis change their stereotypes and learn more about HIV/AIDS and its prevention. Also, the founder and President of One Bright World, Fay M. Vassilakis expresses the opinion that a good education will enhance a good prevention from diseases. More precisely she refers that : "One Bright World⁶ organization believes that Education is the key to a meaningful life as it unlocks the opportunities the world holds. Access to education is as fundamental to the human being as is health and nutrition. Education feeds the mind, the imagination and the human spirit. The children of the world deserve access to basic education without economic, gender or any other form of discrimination. Parents' educational needs are equally important and the programs of One Bright World include them in its focus upon basic education and health awareness. By providing parents and children with basic education and health awareness, we offer the language, math and life skills that will form the foundation of a more fruitful future for them and for generations to come".

Furthermore, our results assuming that the shock of epidemic is anticipated, are enhanced by empirical evidence (De Walque (2007), Stoneburner and Low-Beer (2004)) that points to the conclusion that less educated young adults will experience a disproportionately large number of new infections. For instance, De Walque (2007) analysing data from Uganda demonstrates the importance of schooling in an individual's response to a prevention campaign. He also demonstrates the evolving nature of the relationship between HIV and education. In 1990 he found no relationship between HIV prevalence and education. On the other hand, in

⁶<http://onebrightworld.org>

2000 having completed primary education was associated with a 5.1% reduction risk of HIV infection and secondary education was associated with an 8.8% reduction in the risk. Interestingly, the relationship between HIV and education was found for women and not men. These finding supports the thesis that more educated individuals are better able to mount a response to the HIV epidemic. This structure changes in the following periods. In scenario 4, the proportion of unskilled junior adults will also increase (compared to the balanced growth path) in period $t=1$. We conclude that the second mechanism, the increase of number of orphans (scenario 3, generation 3 table 18) dominates the first mechanism. Also, we can conclude that many of the survival orphans can be infected by HIV and die when they become junior adults. We know that a virus like HIV develops AIDS in ten years or fourteen if the patients follow therapies. The rate of progression can be much faster in those with weakened immunity from other causes like drugs. As consequence, the output per worker is lower that period (see table 19). We observe that the two effects are offset at period $t=2$. In table 18, the ratio orphans children increases in epidemic period and also in $t=1$ and in $t=2$ for the scenario 4. But we notice that the first mechanism dominates the second so the proportion of young junior adults of low human capital drops. As a result, we expect that the output per capita to increase but the total output to drop (table 19). In our last scenario where we incorporate all the effects, we notice the medium term distributional effects of epidemics. The first mechanism dominates all the periods, more young adults will get more education during these periods and the output per worker will increase regardless of the drop in total output.

Table 19
Income
(Extension Model)

Scenarios	<u>Generations</u>		
	1	2	3
Low human capital			
Scenario 1	100	100	100
Scenario 2	99	100	99
Scenario 3	88	96	88
Scenario 4	95	98	95
Scenario 5	83	98	83

We know that, in the long run, the shares of junior adults of well endowed and low endowed human capital, that is the income distribution will go back to their balanced growth values. Consequently, in contrast to other contributions in the AIDS-related literature (like Bell et al., (2003)), the model provides more corrective dynamics which will bring some key variables to the corresponding balanced growth values. But we cannot even conclude on the long run change in the total population of junior adults without more assumptions.

Table 20
Health investment for Junior adults
(Extension Model)

Scenarios	<u>Generations</u>		
	1	2	3
High human capital			
Scenario 1	103.2713	103.2713	103.2713
Scenario 2	103.8595	103.2713	103.2713
Scenario 3	94.7509	103.2713	103.2713
Scenario 4	102.0777	103.2713	103.2713
Scenario 5	92.8650	103.2713	103.2713

Table 21
Health investment for Junior adults
(Extension Model)

Scenarios	<u>Generations</u>		
	1	2	3
Low human capital			
Scenario 1	73.2148	73.2148	73.2148
Scenario 2	73.8019	73.2148	73.2148
Scenario 3	68.2819	73.2148	73.2148
Scenario 4	70.8500	73.2148	73.2148
Scenario 5	65.1991	73.2148	73.2148

Table 22
Health investment for children
(Extension Model)

Scenarios	<u>Generations</u>		
	1	2	3
	Low human capital		
Scenario 1	237.1029	237.1029	237.1029
Scenario 2	236.7767	237.1029	237.1029
Scenario 3	100.9545	237.1029	237.1029
Scenario 4	238.4167	237.1029	237.1029
Scenario 5	102.6672	237.1029	237.1029

Having a shock B' which is anticipated leads the parents to increase their investment (lemma 2.1) but we notice that in scenarios 3, 4 and 5 there is a decrease. The two effects affect their decision by decreasing their investment. Tables 20 and 21 show that the reducing productivity leads to parents to decrease more their health investment. On the other hand scenarios 3, 4 and 5 in both tables present that the diffusion of shock across generations affects more the investment of adults with high human capital. On the contrary, the reduction of productivity affects more the investment of poorly endowed capital young adults.

Table 23
Health investment for children
(Extension Model)

Scenarios	<u>Generations</u>		
	1	2	3
	High human capital		
Scenario 1	775.9604	775.9604	775.9604
Scenario 2	775.6336	775.9604	775.9604
Scenario 3	530.6939	775.9604	775.9604
Scenario 4	776.6235	775.9604	775.9604
Scenario 5	531.7417	775.9604	775.9604

Tables 22 and 23 illustrate three important results. First, an epidemic shock leads the parents to decrease their children's health expenditures in both classes (lemma1). Second, if the shock influences the productivity of the parents, they decide to decrease their children's health expenditures but having higher reduction for those of low human capital. Third, parents under the assumption that there are wholly altruistic decrease their investment and increase those of their children in both categories at the same proportion. Observing at scenario 5 we find that the diffusion shock leads to a small offset of the negative effect of productivity in the investment.

4.3 Does differential fertility matters?

In this model, we considered the fertility exogenous. This is done in order to obtain an analytical representation of economic and demographic dynamics at any period. On theoretical ground a decline in life expectancy can increase or decrease the fertility depending on parental preferences and on the defined kind of mortality (see Hazan and Zoabi (2006)). Actually, a high child mortality may increase the willingness of obtain children (increase of insurance effect). On the contrary, a Barro-Becker model(1989) where mothers can freely supply labor would draw under a drop in labor supply, female participation in labor market will rise, leading to a decrease in fertility. There is a debate in empirical literature concerning the effect of epidemics on fertility. Young (2005) and Boucekkine, Desbordes and Latzer (2009) argue that the HIV is lowering the fertility in Sub-Saharan Africa. On the other hand, Kalemli- Ozkan (2006) using a context of the Solow model, finds a positive relationship between AIDS and total fertility rates which lowers per capita economic growth of current and future generations in Africa. In this paper, we adopt the viewpoint of Young (2005) and Boucekkine, Desbordes and Latzer (2009) that AIDS decreases the fertility in Sub-Saharan Africa. We analyse again our model but considering that the young adults well endowed have different number of children from the poorly endowed human capital. More precisely, we suppose that a junior adult living either having high human capital or low human capital in period 0 has at the end of this period, a number of children reduced by the amount $dn < 0$. Now, the junior adult's program is

$$Max_{l, e_{+1}} \lambda(e_{+1}) \{ \pi(l)r + 1 \} \quad (73)$$

$$\Psi h = ne_{+1} + l. \quad (74)$$

$$l, e_{+1} > 0. \quad (75)$$

Program has a unique solution defined by the two equations

$$\frac{B\Psi h + \frac{A'}{A}nB + B'}{Bl + B'} - \frac{1 - \alpha}{r(Bl + B')^{1-\beta}} = 1 + \frac{1 - \alpha}{1 - \beta} \quad (76)$$

$$\frac{\Psi h - l}{n} = e_{+1}. \quad (77)$$

Again, it is difficult to conclude the real effects of the shocks so we need to provide simulations. As it concerns the demographics, the number of junior adults in period 2 will change. It will tend to decrease because of the lower number of children born at the end of period 0, but we do not know what will happen in the decisions of parents for the investment of the health of each of their children. We tried to compute the total effect by differentiating the equation 55.

$$\begin{bmatrix} dN_{+2}^{2+} \\ dN_{+2}^{2-} \end{bmatrix} = [n \begin{bmatrix} d\alpha_{11} & d\alpha_{12} \\ d\alpha_{21} & d\alpha_{22} \end{bmatrix} + Mdn] \begin{bmatrix} N^{2+}(0) \\ N^{2-}(0) \end{bmatrix} \quad (78)$$

Again, we cannot infer anything for the structure of young junior adults without providing simulations. In our simulations, we use the same values as in previous exercises except for the number of children. Now, we consider the case of South Africa, a country that has large number of deaths because of AIDS. We impose different number of children in both classes and we assume as in scenario 5 that in period 1 junior adults decrease the number of infants. In scenario 6, we include all the scenarios together to find which shock affects the decisions of parents concerning the health investments and the demographics. We set $n=4.56$ the number of children of young adults of low human and $n=3.04$ for young adults

well endowed. Also, these numbers $n=3.1$ and $n=1.85$ are chosen for young adults of low and high human capital respectively. These numbers are taken from United Nations Department of Population data set. More precisely, we choose the fertility rate of 1980 when the disease appeared for the first time and the last numbers are predictions of fertility rates in 2020.

Table 24
Health investment for children
(Extension Model)

Scenarios	<u>Generations</u>		
	1	2	3
High human capital			
Scenario 1	451.9113	451.9113	451.9113
Scenario 2	451.7173	451.9113	451.9113
Scenario 3	306.3236	451.9113	451.9113
Scenario 4	452.5950	451.9113	451.9113
Scenario 5	451.7173	754.4578	451.9113
Scenario 6	307.2375	754.4578	451.9113

As it can be observed, the first four scenarios show similar results with those of extension model. In scenario 5, young adults decrease their health expenditures and increase those of their children in period 1 (see generation 2 in tables 23, 24, 25, 26 and 27). It is noticed that the child mortality

Table 25
Health investment for children
(Extension Model)

Scenarios	<u>Generations</u>		
	1	2	3
Low human capital			
Scenario 1	82.4387	82.4387	82.4387
Scenario 2	82.3093	82.4387	82.4387
Scenario 3	28.3807	82.4387	82.4387
Scenario 4	83.5393	82.4387	82.4387
Scenario 5	82.3093	129.3598	82.4387
Scenario 6	29.6347	129.3598	82.4387

and the decision of parents to decrease the number of their infants increase the insurance effect. In particular, the willingness of parents leads to an increase of the health expenditures of their offsprings.

Table 26
Health investment for Junior adults
(Extension Model)

Scenarios	<u>Generations</u>		
	1	2	3
Low human capital			
Scenario 1	124.0795	124.0795	124.0795
Scenario 2	124.6697	124.0795	124.0795
Scenario 3	120.5840	124.0795	124.0795
Scenario 4	119.0609	124.0795	124.0795
Scenario 5	124.6697	98.9847	124.0795
Scenario 6	114.8656	98.9847	124.0795

In table 28, we present the effects of the different effects cause by the epidemic in total output. For the first four scenarios, we find that we have similar results as our simulations in the previous sections. More precisely, we argue that in even periods the drop in total output is high because of epidemic in scenario 2, 3 and 4. Also, in scenario 5, where we include the decision of parents to decrease the number of their offspring because an epidemic like AIDS affects their inherited characteristics, it has massively results on generations 4 and 5. We notice two opposite results.

Table 27
Health investment for Junior adults
(Extension Model)

Scenarios	<u>Generations</u>		
	1	2	3
High human capital			
Scenario 1	126.1897	126.1897	126.1897
Scenario 2	126.7793	126.1897	126.1897
Scenario 3	118.7763	126.1897	126.1897
Scenario 4	124.1113	126.1897	126.1897
Scenario 5	126.7793	104.2531	126.1897
Scenario 6	115.9981	104.2531	126.1897

In generation 2, there is an increase of total income but a massively drop of it follows in generation 3. Trying to explain these two results, we find that by increasing the health expenditures of their children (tables 25 and 26 scenario 5 generation 3), the junior adults increase the survival probability of their children. Moreover, we note that the effects of epidemic in odd periods are small for the population of junior adults so their mortality is

low in this period. These two outcomes lead to a rise of the total output of economy in period 1. On the other hand, there is a huge drop of it in generation 3. This outcome is explained by three reasons.

Table 28
Income
(Extension Model)

Scenarios	Generations		
	1	2	3
Scenario 1	100	100	100
Scenario 2	99	100	99
Scenario 3	93	97	91
Scenario 4	93	97	92
Scenario 5	99	106	58
Scenario 6	86	99	51

The first reason is the effect of the epidemic. As we noticed usually an epidemic has a great impact of the population of junior adults. Also, this effect becomes even higher when junior adults decrease their expenditures for health and increase those of their infants. The second reason is the decision of the parents to have less children decrease the number of future junior adults as a consequence of the decrease in total output of the economy, as it would be without epidemic. The third reason is that looking in tables 29 and 30, we can argue that the proportion of junior adults of low human capital increase and as result the country has less educated people and the output per worker goes down the economy is clearly impoverished (with respect to balanced growth path) at this time (see generation 5 scenarios 5 and 6 on both tables).

Table 29
Junior adults's Ratio of Human Capital
(Extension Model)

Scenarios	Generations		
	1	2	3
Scenario 1	0.5788	0.5998	0.6001
Scenario 2	0.5788	0.5998	0.6000
Scenario 3	0.5803	0.5985	0.6003
Scenario 4	0.5877	0.6045	0.6013
Scenario 5	0.5788	0.6000	0.5938
Scenario 6	0.5890	0.6042	0.5951

Table 29 displays the structure of junior adults in the different scenarios. More precisely, in the first scenarios show the same results like the case of China. It means that AIDS as epidemic does not discriminate across the countries and it has similar effects on human beings. Our discussion focus on scenarios 5 and 6. We observe that the proportion of young junior adults of high human capital increases in generation 2 and it decreases in generation 3.

Table 30
Ratio Orphans Children
(Extension Model)

Scenarios	<u>Generations</u>		
	1	2	3
Scenario 1	0.0360	0.0360	0.0370
Scenario 2	0.0360	0.0361	0.0370
Scenario 3	0.0369	0.0361	0.0370
Scenario 4	0.0368	0.0365	0.0371
Scenario 5	0.0360	0.0377	0.0372
Scenario 6	0.0378	0.0382	0.0372

This outcome is due to two main reasons. In generation 2, the survival probability of children of high human capital is higher than the survival probability of children of low human capital. In contrast to this, in generation 5, the proportion of junior of poorly endowed increases. The proportion of orphans increases and also, the decision of less children in both classes dominates the first mechanism.

Finding 3. If the probability for an orphan to reach a high level of human capital, q , is low enough, and a child who has living parents and who stays alive has a low probability p of obtaining low human capital we have the following results

- The population of junior adults holding a high level of human capital decreases, and the population of junior adults with a low level of human capital increases. Thus, the proportion of junior adults with a low endowment of human capital in the total population increases. Consequently, domestic output per worker decreases for the next generations after the epidemic when we include the three effects.
- The proportion of junior adults with a low endowment of human capital in the total population increases. Consequently, domestic output per worker decreases for the next two generations after the epidemic when we include the three effects and changing the number of the children.

To prove this finding, we apply the case of South Africa. We present three scenarios. Scenario 1 presents the ratio of junior adults without the epidemic. Scenario 2 illustrates the proportion of junior adults affected by the three different effects. Scenario 3 displays the proportion of junior adults affected by the three different effects and the decision of them to decrease the number of their infants in period 1 after the epidemic. We keep all the values as we had imposed earlier and we change p and q . We suppose that $p=0.2$ and $q=0.1$.

Table 31
Ratio Wealthy vs Poor Human capital
(Extension Model)

Scenarios	<u>Generations</u>		
	1	2	3
Scenario 1	0.2298	0.2148	0.2175
Scenario 2	0.2339	0.2146	0.2175
Scenario 3	0.2339	0.2147	0.2164

When an epidemic takes place, well-endowed junior adults will spend more on the health of their children. In period 2(generation 3), the number of junior adults who were orphans will increase and the number of those who were brought up by their parents will decrease. If the probability for an orphan to reach a high level of human capital, q , is low enough, the number of junior adults with a high level of human capital, alive in period 2, will become lower. This finding is a crucial example of the medium term distributional effects of young junior adult epidemics. We observe that the number of orphans dominates the child mortality in both generations. Also, we argue that indirect effect of the reduction in productivity dominates the indirect effect of transmission for the period 1(generation 2) and also decisions of young adults concerning the number of their children have massive effects not only in their future demographic structure but also in the economy as a whole. More young adults will get less educated two periods after the epidemic and output per worker decreases: the economy is clearly impoverished (with respect to the reference balanced growth path) at this time horizon.

5 Main Results

In this subsection, we summarize the main results of our paper. In particular, we had three main findings. The first two findings concern the decisions of parents for their own and their children's health expenditures. The first finding mentions that a junior adult invests less in his health when the effect of epidemic in his productivity is high and he invests more as the diffusion of epidemic to his children is low (see table 5 and table 6). The intuition behind these results is that obtaining smaller wage as his productivity falls he will spend less money to his health and also since agents are wholly altruistic, they decrease their health investment as the transmission of shock to children increases.

The second finding refers that a junior adult invests more in his children's health when the effect of an epidemic in his productivity is small and invests more in his children health as the diffusion of the shock of epidemic to his children is high (see table 7 and table 8).

The third finding concerns the effects of an epidemic in the total economy and population in terms of inequality. It mentions that if the probability for an orphan to reach a high level of human capital is low enough, and a child who has living parents and who stays alive has a probability of obtaining low human capital is low then the following results exist:

- The population of junior adults holding a high level of human capital decreases, and the population of junior adults with a low level of human capital increases. Thus, the proportion of junior adults with a low endowment of human capital in the total population increases. Consequently, domestic output per worker decreases for the next generations after the epidemic when we include the three effects.

- The proportion of junior adults with a low endowment of human capital in the total population increases. Consequently, domestic output per worker decreases for the next two generations after the epidemic when we include the three effects and the decrease in the number of children.

6 Conclusions

In this paper, we presented a analytical dynamic theory of income distribution under AIDS. We develop a framework under three different effects of an AIDS epidemic. In particular, an epidemic hits the inherited characteristics of young adults. In the meantime, this effect implies two indirect effects, reduction in productivity of young adults and transmission of disease from them to their offsprings. We examine the economic and demographic effects as a result of these effects. Within this framework, we have several properties. First, transitory epidemiological shocks have permanent shocks on the size of population and total output. Nevertheless, income distribution is not altered in the long run. Second, we show that this distribution is changed in the short and medium run due to the identified mechanisms. More precisely, the survival probability of children with less educated people is too low as well as the ability of orphans to access high levels of education. We notice that in the short run, the proportion of young adults with high human capital increases and as a result the output per worker increases. When, we investigate the decision of parents to decrease their fertility due to epidemic then there is poverty in the medium run. As further research, we propose to introduce uncertainty in the epidemic shock and investigating another epidemic in this case. Also, an edogenization of fertility by applying some adjustments to this model could lead to interesting results in terms of inequalities.

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7 Appendix

Table 2

Countries with AIDS	Female (1985) No schooling	Total population (1985)No schooling	Population (1960-2000)No schooling	Ranked AIDS deaths
Benin	91,3	85,4	83,3	42
Botswana	46,2	50,0	54,9	2
Burundi	85,5	75,4	74,5	18
Cameroon	68,8	58,0	62,5	16
Central Africa	68,8	78,8	81,7	9
Congo	72,2	58,8	58,8	23
Gambia	94,1	90,0	85,8	52
Kenya	68,6	55,6	59,2	17
Liberia	87,8	76,7	78,9	19
Malawi	71,5	55,0	59,1	8
Mali	95,2	90,8	92,3	43
Mozambique	91,0	78,2	78,3	10
Nigeria	96,5	90,6	91	21
Rwanda	81,3	71,9	72,2	22
South Africa	26,5	24,4	32,5	5
Sudan	88,8	76,7	89,9	41
Swaziland	38,0	42,0	50,2	1
Togo	84,3	71,9	75,5	28
Uganda	76,2	63,1	61,3	29
Zambia	75,4	60,0	75,4 8	7
Zimbaboue	37,5	31,0	33,3 5	4
China	54,7	40,0	36,85	168
India	85,2	66,9	65,3	167
Total average	73,3	64,8	67,5	

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