Is History a Blessing or a Curse ? International Borrowing without Commitment, Leapfrogging and Growth Reversals

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Is History a Blessing or a Curse? International Borrowing without Commitment, Leapfrogging and Growth Reversals *

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Abstract: We develop a simple open-economy AK model with collateral constraints that accounts for growth-reversal episodes, during which countries face abrupt changes in their growth rate that lead to either growth miracles or growth disasters. Absent commitment to investment by the borrowing country, imperfect contract enforcement leads to an informational lag such that the debt contracted upon today depends upon the past stock of capital. The no-commitment delay originates a *history* effect by which the richer a country has been in the past, the more it can borrow today. For (arbitrarily) small deviations from perfect contract enforcement, the history effect offsets the growth benefits from international borrowing and dampens growth, and it leads to leapfrogging in long-run levels. When large enough, the history effect originates growth reversals and we connect the latter to leapfrogging. Finally, we argue that the model accords with the reported evidence on growth disasters and growth accelerations. We also provide examples showing that leapfrogging and growth reversals may coexist, so that currently poor but fast-growing countries experiencing sharp growth reversals may end up, in the long-run, significantly richer than currently rich but declining countries.

Keywords: Growth Reversals, Leapfrogging, International Borrowing, Open Economies

Journal of Economic Literature Classification Numbers: F34, F43, O40,

1 Introduction

Since the 1950s, growth reversals have been experienced by most economies, especially, though not only, developing countries. During these ubiquitous episodes, countries have faced abrupt changes in their growth rate leading to either growth miracles - when the growth rate suddenly goes from below to above trend - or growth disasters (Hausmann, Pritchett and Rodrik [11], Jones and Olken [12], Cuberes and Jerzmanowski [7]). It is a striking observation that many of the countries which are susceptible to growth reversals are also heavily indebted. For instance, countries like Kenya, Trinidad and Tobago, Jordan or the Philippines have been through severe growth trend breaks over the last decades (Cuberes and Jerzmanowski [7]). At the same time, they have relied more and more heavily on international borrowing and hold net foreign asset positions that are negative and around half of their GDPs (Lane and Milesi-Ferreti [14]). Although this fact alone suggests that access to international financial markets might affect the occurrence of growth reversals, the literature has largely overlooked such a potential relationship.

In this paper, we show that international borrowing may trigger growth reversals. More precisely, we assume that due to imperfect enforcement of borrowing contracts, the richer a country has been in the past, the more it can borrow today, which originates a *history* effect. Such a dependence on history implies that the growth rate today depends on the entire growth path followed by the economy in the past, and not simply the initial capital stock as in a standard AK model. One key contribution of this paper is to show that this history effect offsets the growth benefits of international borrowing and dampens growth, and that it may lead to both leapfrogging and growth reversals.

Similar to the large literature showing that foreign borrowing might be detrimental to macroeconomic stability (see, among many others, Paasche [16], Aghion *et al.* [1], Mendoza [15], Devereux and Yetman [8]), we focus on capital equipment as pledgeable

collateral. However, our main departure from the existing body of research is that we relax the assumption of commitment to investment. We posit that the debtor country cannot commit to an investment strategy at the time the lender decides about the level of debt (that is expected to be repaid next period). Absent commitment, it is then natural to assume that in order to implement legal enforcement, the debt contract has to depend upon past values of the capital stock (e.g. the value reported in the debtor's balance sheets). In other words, imperfect contract enforcement implies an informational lag. We introduce such a lag by assuming that the debt contracted upon at date t depends upon the stock of capital at $t - \tau$.

Although highly stylized, our modeling captures in a simple way the broader view that in a world with potential debtor default and collateral constraints, the borrowing country's past growth performances and reputation matter for determining its ability to borrow. In fact, our formulation is reminiscent of a conjecture in Cohen and Sachs [6], who have pointed out in a discrete-time model that absent commitment, the lender might set a borrowing limit that depends upon last period's capital stock. However, we show in appendix A that the discrete-time version of our model with a one-period lag cannot explain growth reversals. In addition, longer lags lead to higher-dimensional, nonlinear difference equations that are not easily amenable to analysis, contrary to our continuous-time formulation which gives rise to a linear differential equation with lagged capital and its lagged rate of change (Neutral delay Differential Equation, or NDE for short).

We introduce international borrowing without commitment in a simple open-economy version of the AK setting with collateral constraints. With a constant savings rate, the dynamics of the economy follows a linear NDE with an exogenous delay.[1] First,

[1] As a robustness check, we show in Appendix E that the optimal control problem delivers the same mathematical structure, with dynamics still given by a linear NDE.

we show that there is a unique balanced-growth path (BGP for short), such that the positive growth rate declines with the no-commitment delay. Because how much the economy borrows today depends upon its lagged stock of collateral (capital), a history effect materializes and offsets the growth-enhacing effect of foreign debt, and ultimately it dampens growth. Essentially, a slow-growing country that has been rich in the past has better access to international debt markets than a fast-growing but still catching-up country. The history effect may significantly dissipate the growth benefits from foreign borrowing for small values of the no-commitment delay. This is because the level of the open-economy BGP growth rate converges exponentially fast to the autarkic one when the delay increases from zero.

Second, we prove that the BGP is stable and an important by-product of our stability analysis is that we characterize the long-run level of capital and output as well. We show that the long-run level depends upon the initial growth path, which is a striking departure from the standard AK model in which what determines the long-run level is the initial capital stock only. Moreover, we show that leapfrogging in long-run levels occurs for (arbitrarily) small delays, that is, for small deviations from perfect contract enforcement: the growth-enhancing effect then dominates the history effect, so that the fast-growing (but poor) country eventually leapfrogs the slow-growing (but rich) country.

We then move on to study short-run dynamics and provide a simple necessary and sufficient condition for growth reversals to occur, which turns out to be met if the delay is large enough and provided that the economy is not declining too fast initially. As both long-run effects and short-run dynamics depend on the initial growth path, we connect leapfrogging and growth reversals by showing that the absence of the former implies the latter: when the history effect is sufficiently large to rule out leapfrogging, it dominates the growth effect of foreign borrowing and leads to growth reversals.

Fourth, we study the quantitative implications of our analytical results. In our sug-

gestive numerical examples, financial openness with commitment increases the growth rate by 2 percentage points over and above the growth rate under autarky. However, these growth benefits dissipate quite fast as the no-commitment delay increases from zero. Then we show that the model's predictions regarding the changes in the growth rate at break dates accord with the evidence documented, e.g., by Cuberes and Jerzmanowski [7]. In particular, the model predicts growth disasters and miracles that are of the correct magnitude. Last but not least, we provide the first application (to our knowledge) of NDEs to economics, which allows us to uncover some powerful economic mechanisms that may cause both leapfrogging and growth reversals, such as the ones mentioned above.

The paper is organized as follows. Section 2 presents the credit-constained open-economy AK model and shows uniqueness and stability of the BGP with positive growth. Section 3 derives conditions for leapfrogging and growth reversals to occur and it also discusses quantitative implications. Finally, section 4 concludes and is followed by appendices containing proofs.

2 The Open AK Economy

The economy produces a tradeable good Y by using physical capital K, according to the following technology:

$$Y = AK, \tag{1}$$

where $A > 0$ is total factor productivity. Whereas output is tradeable, labor and capital are not.[2]

[2] Our results are virtually unchanged under capital mobility, which sets the net marginal product of capital to the world interest rate.

The Ramsey households are defined by their utility:

$$\int_0^\infty e^{-\rho t}\frac{C(t)^{1-\theta}-1}{1-\theta}dt, \tag{2}$$

where $C > 0$ is consumption, $\theta \geq 0$, and $\rho \geq 0$ is the discount rate. The budget constraint is:

$$\dot{K}(t) - \dot{D}(t) = AK(t) - \delta K(t) - rD(t) - C(t), \tag{3}$$

where D is the amount of net foreign debt and the initial stocks $K(0) > 0$, $D(0)$ are given to the households.

We focus on collateral-constrained borrowing without commitment to investment and, following Cohen and Sachs [6], we posit that the creditor lends up to some fraction of the past value of collateral $\lambda K(t-\tau)$, for some exogenous (no-commitment) delay $\tau \geq 0$ and $\lambda > 0$.

Assumption 2.1 *Foreign borrowing is subject to a limit such that $D(t) = \lambda K(t-\tau)$, with $\lambda > 0$ and $\tau \geq 0$.*

Replacing D by its expression from Assumption 2.1, the budget constraint (3) can be written as:

$$\dot{K}(t) = \lambda\dot{K}(t-\tau) + (A-\delta)K(t) - r\lambda K(t-\tau) - C(t), \tag{4}$$

We explore the simplest case of constant savings rate and we show, in Appendix E, that the dynamics of the economy obtained by maximizing (2) subject to (4) has the same mathematical structure. Therefore, our stability analysis applies to the optimal control problem as well.

We suppose that consumption is a constant fraction of output, that is, $C = (1-s)Y$, where $1 > s > 0$ is the savings rate. This implies that the budget constraint (4) is a linear *Neutral delay* Differential Equation (NDE for short), as both K and $\dot{K}$ are delayed

(see Bellman and Cooke [4, chap. 6]):

$$\dot{K}(t) = \lambda \dot{K}(t-\tau) + \varepsilon K(t) - r\lambda K(t-\tau), \tag{5}$$

where $\varepsilon \equiv sA - \delta > 0$. Note that under autarky, the economy does not borrow - that is, $\lambda = 0$ - so that the corresponding growth rate is $g_a \equiv \varepsilon > 0$. The remaining part of this section is devoted to the analysis of the dynamics and asymptotic properties of equation (5) when $\lambda > 0$. This is the first application of NDEs to economics we know of, which is presumably explained by the fact that the mathematical literature on this topic is scant.

2.1 Balanced Growth Paths

A balanced growth path (BGP for short) is a trajectory $K(t) = e^{gt}$ that solves (5), that is, such that g is a solution to $g = \varepsilon + \lambda(g - r)e^{-g\tau}$. In the benchmark case of borrowing under commitment, $\tau = 0$ and solving the above equation gives the expression of the no-delay growth rate $g_0 \equiv (\varepsilon - r\lambda)/(1-\lambda)$. It is straightforward to prove the following.

Proposition 2.1 (No-Delay BGP)

Assume that $\varepsilon \equiv sA - \delta > r$ and $1 > \lambda$. It follows that for $\tau = 0$, there exists a no-delay BGP with $g_0 = (\varepsilon - r\lambda)/(1-\lambda) > g_a$ and such that $dg_0/d\lambda > 0$.

The assumption that $sA - \delta > r$ means that the economy is productive enough to afford foreign borrowing, given the level of the world interest rate, and it implies that foreign debt then fosters growth. This captures the growth-enhancing effect coming from access to international financial markets. In addition, λ is constrained to be smaller than one to reflect the fact that only a fraction of capital can be seized in case the debtor defaults, because contract enforcement is costly (see Djankov *et al.* [9] for some evidence).[3] Under

[3] Alternatively, our analysis goes through if $\lambda > 1$ and $\varepsilon < r\lambda$, with similar results.

commitment - that is, when $\tau = 0$ - the model boils down to the standard AK model and there is no transitional dynamics, as the economy jumps to the no-delay BGP at the initial date. Therefore, the growth rate equals g_0 forever and growth reversals cannot occur.[4]

Absent commitment, however, we assume that an information lag due to imperfect contract enforcement implies $\tau > 0$. We define the characteristic function as $Q(x) \equiv x - \varepsilon + \lambda(r - x)e^{-x\tau}$, whose roots give the BGP growth rates. Different from standard characteristic polynomials, Q is a transcendental function, hence it admits infinitely many roots in the complex plane. The following technical lemma shows existence of two real roots.

Lemma 2.1 (Characteristic Roots with Delay)

Under the assumptions of Proposition 2.1 and $\tau > 0$, there exist a unique characteristic real root $g_\tau > 0$ and a unique characteristic real root $h_\tau < 0$ that solve $g = \varepsilon + \lambda(g - r)e^{-g\tau}$.

Proof: see Appendix B.

Given that the two roots in Lemma 2.1 have opposite signs, we can establish existence and uniqueness of a BGP with positive growth.

Proposition 2.2 (Growth Rate with Delay)

Under the assumptions of Proposition 2.1, the NDE (5) governing the dynamics with delay $\tau > 0$ admits two BGPs associated with:

(i) a positive growth rate $g_\tau > 0$, such that $g_0 > g_\tau > g_a$, where $g_a \equiv \varepsilon$ is the growth rate

[4] Whether there is commitment or not, the model's BGP with positive growth is such that $g > r$. However, this property does not mean that dynamic inefficiency prevails, as technology is linear in the AK model.

under autarky and g_0 *is the no-delay growth rate, and* $\lim_{\tau\to 0} g_\tau = g_0$, $\lim_{\tau\to\infty} g_\tau = g_a$.

(ii) *a negative growth rate* $h_\tau < 0$ *such that* $\lim_{\tau\to 0} h_\tau = -\infty$, $\lim_{\tau\to\infty} h_\tau = 0$.

Although Proposition 2.2 establishes existence of a BGP with negative growth associated to h_τ, the next section shows that it is unstable. Therefore, we focus on the BGP associated to g_τ, whose comparative statics properties can be studied graphically. To do that, it is useful to rewrite the characteristic equation as $(g-\varepsilon)e^{g\tau} = \lambda(g-r)$ and to graph both sides of it.

In figure 1, the left-hand side of $(g-\varepsilon)e^{g\tau} = \lambda(g-r)$ shifts up (respectively down) with τ (respectively ε, that is, with s and A). It follows that $dg_\tau/d\tau < 0$. In addition, the right-hand side goes up with λ in figure 1, which implies that $dg_\tau/d\lambda > 0$. In summary, access to international borrowing fosters growth but the no-commitment delay is detrimental to growth.[5]

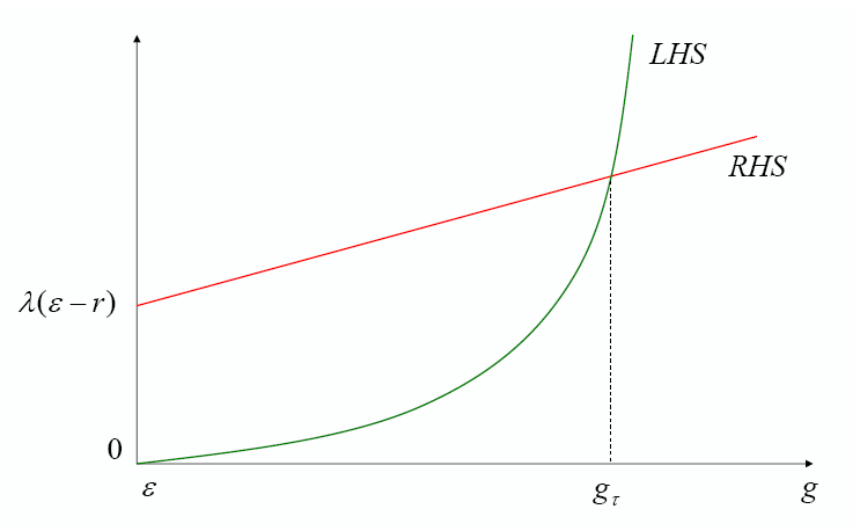

Figure 1: comparative statics of the positive growth rate g_τ

[5]It also follows from figure 1 that $dh_\tau/d\tau > 0 > dh_\tau/d\lambda$.

Proposition 2.3 (Comparative Statics of Positive Growth Rate with Delay)

Under the assumptions of Proposition 2.1, one has $dg_\tau/d\lambda > 0 > dg_\tau/d\tau$. That is, access to foreign borrowing fosters growth whereas the no-commitment delay dampens growth.

The impact of λ and τ on the BGP growth rate conforms with intuition. Given τ, a higher λ implies higher growth because it relaxes the borrowing constraint. This is the growth enhanging-effect of foreign borrowing. Moreover, in a growing economy where the stock of capital is increasing over time, the higher the delay τ in observing K, given λ, the lower the stock of collateral, hence the tighter the borrowing contraint and the lower the growth rate. This history effect is detrimental to growth and may possibly undo the growth benefits of financial integration. In fact, the BGP growth rate converges exponentially fast to the autarkic one - g_a - when τ increases from zero, as shown in figure 2. Therefore, small informational lags due to imperfect contract enforcement may undo the growth benefits of having access to foreign borrowing. The next section will provide numerical examples showing how fast the growth benefits of openness dissipate under no commitment.

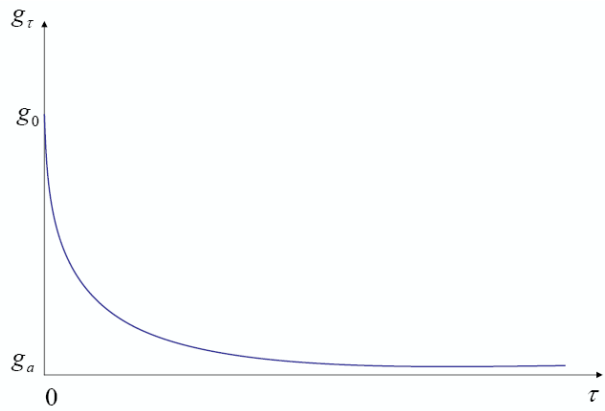

Figure 2: the positive growth rate g_τ as a function of the no-commitment delay τ

2.2 Stability of BGP

We define $x(t) = e^{-gt}K(t)$ as detrended capital stock and we perform this change of variable in (5). When either $g = g_\tau > 0$ or $g = h_\tau < 0$, this change yields the following detrended NDE:

$$\dot{x}(t) = \lambda e^{-g\tau}\dot{x}(t-\tau) + (g-\varepsilon)\{x(t-\tau) - x(t)\} \tag{6}$$

where $g_\tau - \varepsilon > 0 > h_\tau$ under the assumptions of Proposition 2.1. The following property is a key step in the process of studying the stability of the BGP with positive growth. It can be readily shown that the roots of the characteristic equation associated to the detrended equation (6) are obtained from those of the original NDE (5) by applying a translation of $-g$. This comes from the linearity of the NDE. In view of Lemma 2.1, it follows that both 0 and $h_\tau - g_\tau < 0$ are roots of the characteristic function corresponding to equation (6). We now make use of the null root to establish the stability of the positive BGP. In the proof, we extensively use results from Kordonis *et al.* [13] that apply to more general NDEs.

Proposition 2.4 (Stability of the Positive BGP)

Under the assumptions of Proposition 2.1, the following holds for all $\tau > 0$:

(i) the BGP of (5) associated to $g_\tau > 0$ is asymptotically stable,

(ii) the BGP of (5) associated to $h_\tau < 0$ is unstable.

Proof: see Appendix C.

Of some interest is the fact that Proposition 2.4 ensuring stability of the positive BGP also delivers an expression for the long-run level of K, that we now use. In contrast, most of the literature on economic applications of (non-neutral) delayed differential equation (e.g. Boucekkine *et al.* [5]) do not easily characterize the asymptotic level.

Corollary 2.1 (Long-Run Level)

Under the assumptions of Proposition 2.1, suppose that $\phi(t)$ is the initial function of the detrended NDE (6), that is, $x(t) = \phi(t)$ for $t \in [-\tau, 0]$. Then the BGP associated to $g_\tau > 0$ is such that $\lim_{t\to\infty} K(t) = \psi e^{g_\tau t}$, where:

$$\psi = \frac{\phi(0) - \lambda e^{-g_\tau \tau}\phi(-\tau) + (g_\tau - \varepsilon)\int_{-\tau}^{0}\phi(s)ds}{1 - \lambda e^{-g_\tau \tau} + \tau(g_\tau - \varepsilon)}. \tag{7}$$

Proof: Follows from the proof of Proposition 2.4. □

Notice that in contrast to the standard AK model, the long-run level ψ does not only depend on the initial condition $\phi(0)$ but also on the whole path for $t \in [-\tau, 0]$. This essentially means that two countries with the same $\phi(0)$ but different histories (in the sense that their integral terms in (7) differ) end up with distinct long-run levels. This feature and the dependence of ψ on λ and τ are studied in the next section.

3 Leapfrogging and Growth Reversals

3.1 Long-Run Level: Leapfrogging

As the expression of ψ in (7) is not easily amenable to analysis, we study the comparative statics through a simple example. An important feature of the long-run level is that it depends on the initial growth path. If $x(t) = \phi$, where $\phi > 0$ is a given constant, for $t \in [-\tau, 0]$, then $\psi = \phi$. In other words, if the economy starts right on the BGP, then such a path solves (6) and therefore it will stay there forever. Suppose instead that $x(t) = e^{\mu t}$ for $t \in [-\tau, 0]$ for some $\mu \neq 0$. Then the economy grows exponentially for $t \in [-\tau, 0]$ at some given rate μ, possibly negative if we want to account for slow-growing countries,

that is, countries which experience growth rates below trend. Then the expression in (7) becomes:

$$\psi = \frac{1 - \lambda e^{-\tau(g_\tau + \mu)} + (g_\tau - \varepsilon)(1 - e^{-\tau\mu})/\mu}{1 - \lambda e^{-g_\tau \tau} + \tau(g_\tau - \varepsilon)}. \tag{8}$$

We are interested in how the initial growth rate μ affects the long-run level of capital and output through ψ. Our main result is that leapfrogging occurs when the delay τ is small enough. For simplicity, we restrict the analysis to values of the initial growth rate that are small enough to make a second-order approximation accurate. This is acceptable in view of the fact that μ measures the growth rate so that its values are bound to be smaller than 10% in absolute value according to historical evidence.

Direct inspection of (8) shows that the numerator depends on μ whereas the (positive) denominator does not. It is straightforward to show that ψ is an increasing function of μ at $\mu = 0$ if and only if the delay is small enough, which proves the following result.[6]

Proposition 3.1 (Leapfrogging)

Under the assumptions of Proposition 2.1, suppose that the initial function of the detrended NDE (6) is $x(t) = e^{\mu t}$ for $t \in [-\tau, 0]$ and some μ real. Then if the initial growth rate μ is close to zero, there is leapfrogging if and only if $\tau < 2/(g_\tau - r) \equiv \tau_{lf}$.

Note that leapfrogging happens when the delay is arbitrarily close to zero. This can be further illustrated through an example, using figure 3. Suppose that two economies are similar except for their initial growth rates $\mu_l > \mu_s$. This means that both countries have the same BGP and differ only by their initial growth path for $t \in [-\tau, 0]$. In other words, country with μ_l has been initially poorer than country μ_s, though both end up with the same initial condition at $t = 0$. Leapfrogging can be explained as the outcome of

[6] More precisely, the derivative of the numerator of ψ in (8) can be approximated by $\tau\{1/(g_\tau - r) - \tau/2\}$ when $\mu \approx 0$, using that $e^{\tau\mu} \approx 1 + \tau\mu + (\tau\mu)^2/2$.

two conflicting effects, the growth-enhancing effect and the history effect that dampens growth. The growth effect is favorable to leapfrogging because country μ_l gets to $\ln x(0)$ at $t = 0$ at a faster growth rate, hence should leapfrog country μ_s for $t > 0$. In contrast, the history effect implies that country μ_l had less capital at $t = -\tau$ - that is, less collateral - hence faces at $t = 0$ a tighter borrowing constraint than country μ_s, which goes against leapfrogging. If the delay is small, then the latter effect is dominated by the former effect and there is leapfrogging, that is, $\psi_{\mu_l} > \psi_{\mu_s}$. As a consequence, country μ_l follows for $t > 0$ a path that will converge to ψ_{μ_l} while the path of country μ_s converges to the lower level ψ_{μ_s}. Eventually, therefore, country μ_l will lead.

This sharply contrasts with what happens under commitment: with $\tau = 0$, we are back to the standard AK model without delay in which both economies are identical and evolve on the same BGP with growth rate g_0. Although leapfrogging does not occur with $\tau = 0$, our result above shows that it does as long as $\tau > 0$, however arbitrarily small. In view of Proposition 3.1, leapfrogging also arises for intermediate values of τ, as τ_{lf} is expected to be large for reasonable values of $g_\tau - r$.

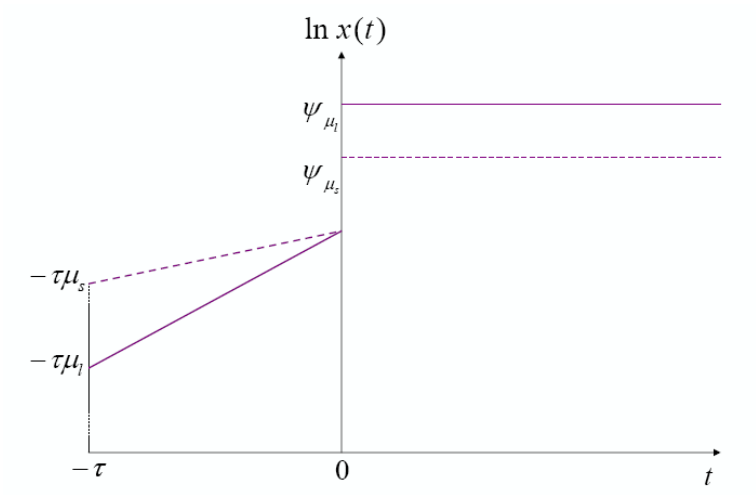

Figure 3: leapfrogging

3.2 Non-Monotonic Convergence to BGP: Growth Reversals

We now study the short-run dynamics by implementing the method of steps. Recall first that if $x(t) = \phi$ for $t \in [-\tau, 0]$, then such a path solves (6) so that there is no transitional dynamics. Therefore, we focus again, as in subsection 3.1, on exponential growth in the initial time interval and we provide a necessary and sufficient condition for growth reversals to occur. As in Cuberes and Jerzmanowski [7], we define a growth reversal as a situation such that the growth rate goes through the BGP growth rate g_τ at a break date, either from below (growth miracles) or from above (growth disasters). More precisely, a growth reversal occurs when the detrended growth rate right before $t = 0$ and the growth rate right after $t = 0$ have opposite signs.

Proposition 3.2 (Non-monotonic Convergence to Positive BGP)

Under the assumptions of Proposition 2.1, suppose that the initial function of the detrended NDE (6) is $x(t) = e^{\mu t}$ for $t \in [-\tau, 0]$ and some μ real. It follows that $\mu \dot{x}(0) < 0$, hence convergence to the positive BGP is non-monotonic, if and only if:

$$g_\tau > r + \frac{\mu}{e^{\mu\tau} - 1}, \tag{9}$$

The above condition is violated if $\tau = 0$, and it is met if $\tau = \infty$ provided that $\mu > r - g_\tau$. In addition, if $\mu \approx 0$, then condition (9) writes as:

$$\tau > 1/(g_\tau - r) \equiv \tau_{gr}. \tag{10}$$

Proof: See Appendix D.

The above condition characterizes growth reversals that occur at $t + \tau$. On the other hand, violating condition (9) does not preclude growth reversals that unfold at $t + n\tau$, $n > 1$. It is straightforward to extend Proposition 3.2 so as to derive conditions under

which this happens. For space consideration, however, we do not develop our analysis along these lines and simply conjecture that growth reversals at $t + n\tau$ are expected for smaller values of the no-commitment delay τ.

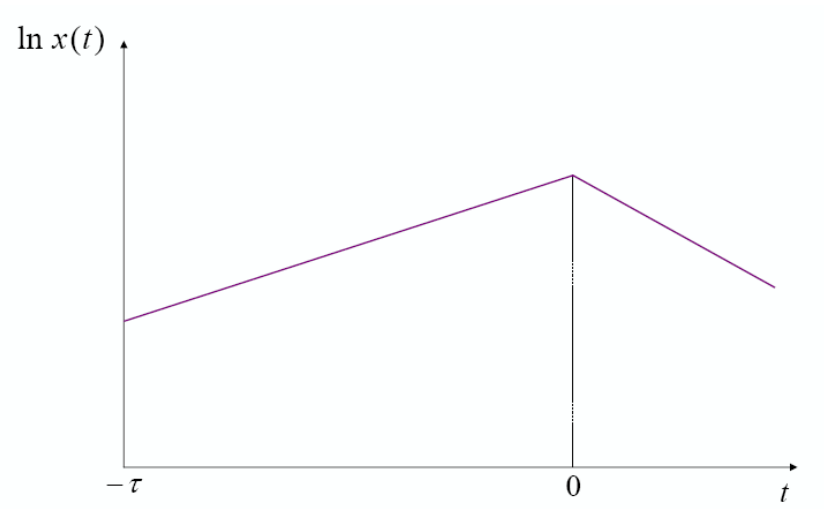

Figure 4: growth disaster

Figure 4 pictures a growth disaster that occurs when $\mu > 0$, under condition (9) in Proposition 3.2. At $t = 0$, there is a sudden fall in the growth rate, which goes from above to below trend. Similarly, a growth miracle occurs under condition (9) when $\mu < 0$. It is interesting to note that growth reversals at break dates are associated with endogenous sudden stops of capital inflows that are not, in our model, caused by shocks (as, e.g., Mendoza in [15]).

Although short-run and long-run effects are different aspects, both depend on initial conditions and it is perhaps illuminating to relate them. This is now our goal, when μ is close to zero for simplicity. From Propositions 3.1 and 3.2, it follows that $\tau_{lf} = 2\tau_{gr} > \tau_{gr}$ when $\mu \approx 0$. This means that the absence of leapfrogging for μ close to zero implies growth reversals. Intuitively, the delay is so large that the history effect now dominates

the growth effect, as illustrated in figure 5. Suppose that we compare two countries with $\mu_l > 0 > \mu_s$. The country that is initially poor is catching up with the rich but declining country. The fact that there is no leapfrogging means that the capital and output levels of country μ_l will stay below that of country μ_s for t large. Figure 5 illustrates this result: no leapfrogging implies growth reversals because there must be a sharp break in the rate of change at $t = 0$ that reverts the direction of growth. The history effect now dominates the growth effect, so that country μ_s will enjoy at $t = 0$ the benefits of having had a lot of collateral at $t = -\tau$ and will start growing again above trend for $t > 0$. In contrast, country μ_l will now be punished for having had too low a stock of capital in the past. In that case, the history effect dampens growth and it both prevents leapfrogging and entails growth reversals.

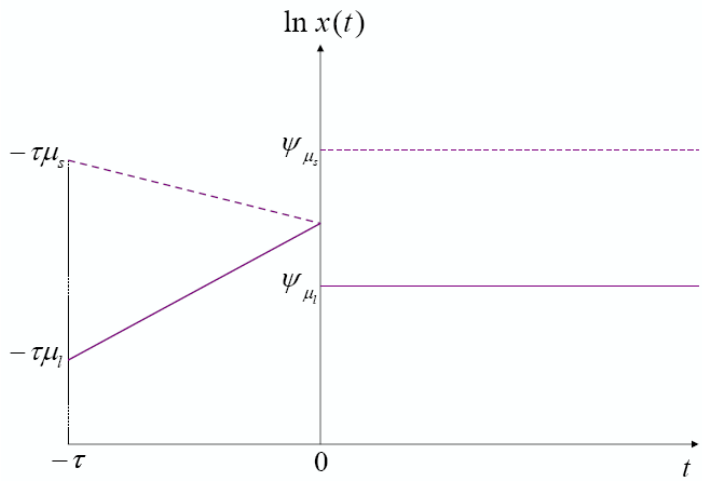

Figure 5: no leapfrogging implies growth reversals

The intermediate case such that leapfrogging and growth reversals coexist (when $\tau_{lf} > \tau > \tau_{gr}$) is omitted for brevity. A numerical example is given in the next section.

3.3 Quantitative Implications

The aim of this section is to show that the model's predictions regarding growth breaks and growth reversals accord with the empirical evidence. In addition, the analysis enables us to connect such episodes to long-run effects, as both features depend on initial conditions.

First, we ask whether or not the model is able to replicate the observed growth changes. Figure 6 below pictures the distribution of growth changes at break dates computed by Cuberes and Jerzmanowski [7, p. 1275] from the Penn World Table for the period 1950-2000. In particular, the mean change in trend growth is about -0.007 (that is, -0.7 percentage points) and the distribution is slightly right-skewed.

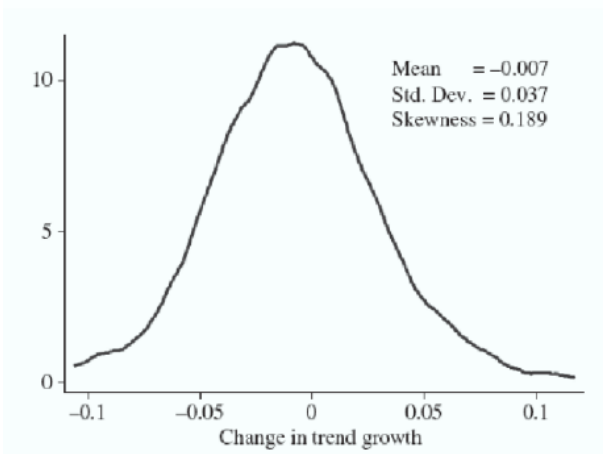

Figure 6: density of growth-rate changes at break dates, from Penn World Table, 1950-2000. Source: Cuberes and Jerzmanowski [7, fig. 2]

Going back to the model, suppose that the autarkic growth rate $g_a \equiv \varepsilon = 0.03$ and that the world interest rate is $r = 0.01$. Then if $\lambda = 0.5$ (within the range of estimates from Djankov *et al.* [9] for developing countries), the no-delay growth rate is $g_0 = 0.05$. With commitment, therefore, foreign borrowing would increase the BGP growth rate from 3% under autarky to 5% under financial openness. The no-commitment delay,

however, reduces the growth benefits from international borrowing and table 1 below shows how quickly g_τ converges to g_a when τ goes up.

Effect of delay τ on:	$\tau = 0$	$\tau = 0.1$	$\tau = 1$	$\tau = 10$	$\tau = 100$
growth rate with delay $g_\tau =$	5%	4.98%	4.8%	4%	3.05%
dissipation of growth benefits $(g_0 - g_\tau)/(g_0 - g_a) =$	0%	1%	10%	50%	98%

Table 1: Effect of delay on growth rate level and benefits

For example, the fifth column of table 1 shows that the growth benefits from openness dissipate by half for as low a value of the delay as $\tau = 10$. The history effect of borrowing without commitment hampers growth by a significant amount for small delays.

We now show that the growth rate changes at break dates predicted by the model accord with the evidence. Using the proof Proposition 3.2 (see Appendix D), the change in the growth rate at $t = 0$ is $\dot{x}(0) - \mu$, with $\dot{x}(0) = (g_\tau - \varepsilon)\{[1 + \mu/(g_\tau - r)]e^{-\tau\mu} - 1\}$. It follows that for τ close to zero, one has $\dot{x}(0) \approx \mu(g_0 - \varepsilon)/(g_0 - r)$. Given our chosen parameter values, $\dot{x}(0) - \mu \approx -\mu/2$. Essentially, this means that for small delays, the initial growth rate is divided by 2 at $t = 0$, which indicates abrupt growth breaks. The history effect hampers growth by a significant margin even for small deviations from perfect contract enforcement. It follows that $\mu = 0.014$ delivers a growth change at the break date $t = 0$ equal to $\dot{x}(0) - \mu \approx -0.007$. That is, the model replicates the mean change at break dates from Cuberes and Jerzmanowski [7, fig. 2], reported in figure 6, for small delays. See, for example, the third column of table 2 when $\tau = 0.1$.

Effect of initial growth rate μ on:	$\mu = -0.05$	$\mu = 0.014$	$\mu = 0.05$
growth rate changes $\dot{x}(0) - \mu =$	0.025	-0.007	-0.025

Table 2: Growth changes at break date, for $\tau = 0.1$

Cuberes and Jerzmanowski [7, p. 1275] also observe that the density of growth rate changes is slightly right-skewed, indicating that large negative changes are more common and can be as large as -0.025 for 10% of the sample (see figure 6 and also Cuberes and Jerzmanowski [7, fig. 8] for non-democracies). The last column of table 2 shows that such values occur also in the model when μ is larger. Essentially, the faster the economy was growing (over and above trend growth) prior to the break date, the larger the history effect and the more abrupt the growth rate change. In addition, the second column of Table 2 shows that small delays account for positive changes in the growth rate as well. For instance, the model predicts a value of 0.025 for the growth rate change at break date, which occurs at frequency around 7% according to figure 6.

Turning now to growth reversals (see figure 4), we learn from Proposition 3.2 that it takes larger τ's to explain them. For example, condition (9) is met for any positive μ if and only if $\tau \geq 45$. Table 3 reports the growth changes associated with growth disasters, that is, when the sign of the growth rate turns negative at break date, for $\tau = 45$.[7]

Effect of initial growth rate μ on:	$\mu = 0.014$	$\mu = 0.05$
growth rate changes $x'(0) - \mu =$	-0.014	-0.052

Table 3: Growth disasters at break date, for $\tau = 45$

[7]For $\tau = 45$, the growth benefits from openness dissipate by a factor of about 87%.

Note that the growth rate changes at break date in table 3, when there are growth reversals, still fall within the range depicted in figure 6. That is, the model predicts growth disasters that fall within the ballpark. However, to account for growth miracles - such that the growth rate goes above trend from below - is more demanding. This is because condition (9) requires even larger τ's when $\mu < 0$. For example, when $\tau = 65$ and $\mu = -0.01$, the change in growth rate $x'(0) - \mu \approx 0.01$. Therefore, with delays that are large enough, growth reversals implying growth acceleration may be explained as well.

In addition to explaining growth reversals, we can use condition (8) to measure the magnitude of leapfrogging effects. Interestingly enough, growth reversal when $\tau = 65$, in our last example, does not prevent leapfrogging because $\tau_{lf} \approx 93 > \tau = 65 > \tau_{gr} \approx 45$ so that for such a value of the delay, poor countries growing fast may leapfrog rich but declining countries. For example, set $\tau = 65$ and suppose we compare two countries that are identical except for their growth rate μ for $t \in [-\tau, 0]$. Country $\mathcal{L}$ has $\mu_l = g_\tau$, that is, it has grown twice faster than the long-run growth rate in the past. In contrast, country $\mathcal{S}$ is richer but has not grown in the past, that is, has $\mu_s = -g_\tau$ so that its capital, output and consumption have been stagnating. Using (8), we compute that country $\mathcal{L}$ will end up with a long-run level of capital, output and consumption that is 30% higher than that of country $\mathcal{S}$. In other words, the fast-growing country will leapfrog the declining country by a significant amount. The price to pay, however, for country $\mathcal{L}$ is to go through (large) growth reversals that country $\mathcal{S}$ does not experience.

4 Conclusion

This paper has proposed a simple open-economy AK model of growth reversals due to collateral constraints under no commitment to investment. Although the model is

highly stylized, it explains leapfrogging and growth reversals. Both features occur as outcomes of two antagonistic forces unleashed by foreign borrowing without commitment to investment. Strikingly enough, the history effect hampers growth by a significant margin for small deviations from perfect contract enforcement. We believe that the key feature of the model - the no-commitment delay - may well capture important aspects of how actual international credit markets are being imperfect. More precisely, it embodies the idea that a slow-growing country that has been rich in the past has better access to international debt markets than a fast-growing but still catching-up country. This idea itself materializes in the actual working of debt markets because the former country would have better historical record of repayment rates and better reputation than the latter. One key result of this paper is to show that small delays matter, as they may at the same time contribute to undo the growth benefits from financial openness, favor leapgrogging and lead to growth breaks.

A Discrete-Time Model With One-Period Lag

The purpose of this section is to study the discrete-time version of the model with a one-period lag. The main result is that the lag creates a poverty trap (negative growth) BGP but generates monotonic convergence to the positive BGP.

The discrete-time analog of budget constraint (3) is:

$$K_{t+1} - D_{t+1} = (sA + 1 - \delta)K_t - (1 + r)D_t. \tag{11}$$

The benchmark case with commitment, that is, no delay occurs when $D_{t+1} = \lambda K_{t+1}$. Then the growth rate is $g_0 = (sA - \delta - r\lambda)/(1 - \lambda)$, as in the continuous-time model (see Proposition 2.1). Under the assumptions of Proposition 2.1, the comparative statics are identical and there is no transitional dynamics. In particular, one has $g_0 > g_a$ (that is, openness is good for growth).

Next, we follow Cohen and Sachs [6] and introduces a one-period lag: $D_{t+1} = \lambda K_t$. Then (11)

becomes:

$$K_{t+1} = (sA + 1 - \delta + \lambda)K_t - \lambda(1 + r)K_{t-1}. \tag{12}$$

Defining $g_{t+1} \equiv K_{t+1}/K_t - 1$, (12) can be written as

$$g_{t+1} = \gamma - \frac{\phi}{1 + g_t}, \tag{13}$$

where $\gamma \equiv sA - \delta + \lambda > 0$ and $\phi \equiv \lambda(1 + r) > 0$. It is straightforward to show that (13) admits two stationary solutions $g > 0$ and $0 > h > -1$. As in section 2, under the assumptions of Proposition 2.1, one has $dg/d\lambda > 0$: opening to international financial markets promotes growth. In addition, $g < g_0$. Here again, problems of contractual enforcement leading to an informational lag dampen growth. Figure 7 shows that there is monotonic convergence towards the positive BGP g provided that the initial growth rate is larger than the negative BGP rate h. Therefore, in contrast to the continuous-time version with delay studied in section 2, the discrete-time version with a one-period lag cannot explain growth reversals. This suggests that if non-monotonic convergence to the BGP is to be explained within the discrete-time model, it requires a longer lag. However, larger delays lead to non-linear, higher dimensional difference equations for which explicit solutions and conditions for global stability are very demanding. Essentially, increasing the lag by one period means that the non-linear dynamics goes up by one dimension. For instance, the model with a two-period lag, that is, $D_{t+1} = \lambda K_{t-1}$, leads to a 2-dimensional, nonlinear dynamical system for which analytical results hardly go beyond local stability properties.

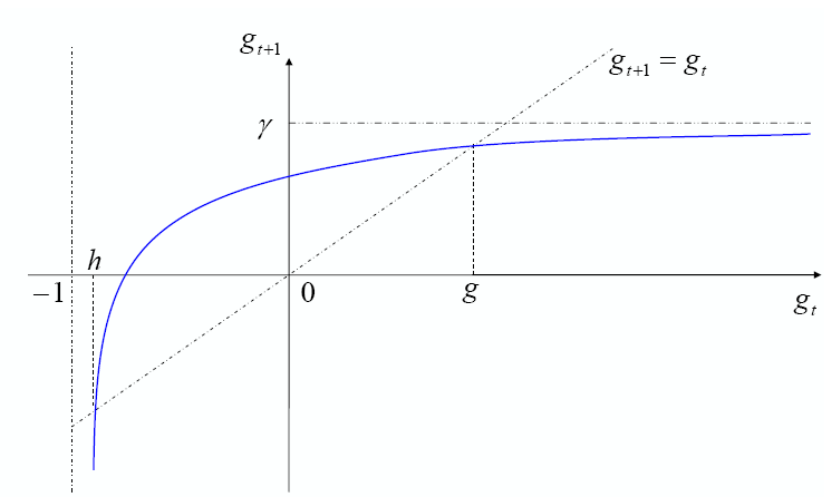

Figure 7: monotonic convergence in the discrete-time model with a one-period lag

B Proof of Lemma 2.1

The existence of a real root $g_\tau > 0$ of $Q(x) \equiv x - \varepsilon + \lambda(r-x)e^{-x\tau}$ follows from the intermediate value theorem, as $Q(x)$ is a continuous function with $Q(0) = r\lambda - \varepsilon < 0$ and $\lim_{x\to\infty} Q(x) = \infty$. To ensure uniqueness, we now show that $Q'(g_\tau) > 0$ for any such $g_\tau > 0$, which is equivalent to:

$$1 - \lambda e^{-g_\tau \tau} + \tau(g_\tau - r)e^{-g_\tau \tau} > 0$$

and is satisfied, as $1 > \lambda > \lambda e^{-g_\tau \tau}$, if $g_\tau > r$ which is equivalent to $g_\tau > \varepsilon$ in view of $g = \varepsilon + \lambda(g-r)e^{-g\tau}$. It is not difficult to show, using $g = \varepsilon + \lambda(g-r)e^{-g\tau}$ again, that the latter inequality writes $\varepsilon > r$ which is one of the assumptions of Proposition 2.1.

To show that there exists a unique real $h_\tau < 0$ solving $g = \varepsilon + \lambda(g-r)e^{-g\tau}$, we use again the intermediate value theorem and the fact that $Q(0) = r\lambda - \varepsilon < 0$ and $\lim_{x\to-\infty} Q(x) = \infty$. Then the condition that $Q'(h_\tau) < 0$ for any such $h_\tau < 0$ is equivalent to:

$$1 - \lambda e^{-g_\tau \tau} + \tau(h_\tau - r)e^{-h_\tau \tau} < 0$$

which is met, as $h_\tau - r < 0$ and $1 < \lambda e^{-h_\tau \tau}$, or using that $\lambda e^{-h_\tau \tau} = (h_\tau - \varepsilon)/(h_\tau - r)$, $\varepsilon > r$ which is met under the assumptions of Proposition 2.1. □

C Proof of Proposition 2.4

Our proof of (i) is based on corollaries 1 and 3 in Kordonis *et al.* [13]. The zero real root of (6) satisfies the condition that $a\tau + b < 1$, that is, $\tau(g_\tau - \varepsilon) + \lambda e^{-g_\tau \tau} < 1$ or equivalently $\tau e^{g_\tau \tau}(g_\tau - \varepsilon) + \lambda < e^{g_\tau \tau}$. To show this, recall that $e^x = \sum_{n=0}^{\infty} x^n/n!$ so that $\tau e^{g_\tau \tau}(g_\tau - \varepsilon) + \lambda < e^{g_\tau \tau}$ if and only if, using $Q(g_\tau) = 0$, $\tau\lambda(g_\tau - r) + \lambda < \sum_{n=0}^{\infty} (g_\tau \tau)^n/n!$. The latter inequality is then $-r\lambda\tau < (1 + g_\tau\tau)(1-\lambda) + \sum_{n=2}^{\infty} (g_\tau\tau)^n/n!$, which is met because $1 > \lambda$ under the assumptions of Proposition 2.1. Therefore, 0 satisfies condition (Q) in corollary 1 of Kordonis *et al.* [13, p. 461] and, as result, $\lim_{t\to\infty} x(\phi; t) = \psi$, for some constant ψ, where $\phi(t)$, for $t \in [-\tau, 0]$, is the initial function. Therefore, $\lim_{t\to\infty} K(t) = \psi e^{g_\tau t}$. From corollary 3 in Kordonis *et al.* [13, p. 463], it follows that the trivial solution (6) is uniformly stable, and, from corollary 1 of Kordonis *et al.* [13, p. 461] that if $\phi(t)$ is the initial function of

the detrended NDE (6), that is, $x(t) = \phi(t)$ for $t \in [-\tau, 0]$ then $\lim_{t\to\infty} x(\phi; t) = \psi$ where $\psi = \{\phi(0) - \lambda e^{-g_\tau \tau}\phi(-\tau) + (g_\tau - \varepsilon)\int_{-\tau}^{0}\phi(s)ds\}/\{1 - \lambda e^{-g_\tau \tau} + \tau(g_\tau - \varepsilon)\}$. It follows that $\lim_{t\to\infty} K(t) = \psi e^{g_\tau t}$, that is, the BGP associated to $g_\tau > 0$ is asymptotically stable.

To prove (ii), we cannot use corollary 3 in Kordonis *et al.* [13] because $h_\tau - g_\tau < 0$ does not satisfy their property $P(\lambda_0)$. Therefore, we rely on corollary 2.2 of Freedman and Kuang [10, p. 190]: because our parameter $b \equiv \lambda e^{-h_\tau \tau} > 1$, we are in the case such that the trivial solution of (6) and the BGP associated with h_τ are unstable. □

D Proof of Proposition 3.2

Rewrite (6) as:

$$\dot{x}(t) = -ax(t) + f(t) \tag{14}$$

and suppose that the initial function is $x(t) = e^{\mu t}$, so that $\dot{x}(t) = \mu e^{\mu t}$, for $t \in [-\tau, 0]$ and some real μ. Then from (6), $f(t) = ae^{\mu(t-\tau)} + \mu b e^{\mu(t-\tau)} = Ke^{\mu t}$, where $K \equiv (a + b\mu)e^{-\mu\tau}$. A solution to the ODE (14) is then of the form $x(t) = K_1 e^{-at} + K_2 e^{\mu t}$, for some K_1, K_2 to be determined. But the particular solution $K_2 e^{\mu t}$ solves (14) if and only if $\mu K_2 e^{\mu t} = -aK_2 e^{\mu t} + Ke^{\mu t}$ which gives $K_2 = K/(\mu + a)$. Moreover, from $x_0^-(0) = 1$ and $x_0^+(0) = K_1 + K_2$, one gets that $K_1 = 1 - K/(\mu + a)$. Therefore, one has that the general solution is such that $\dot{x}(t) = -a[1 - K/(\mu + a)]e^{-at} + \mu K e^{\mu t}/(\mu + a)$ so that $\dot{x}(0) = K - a$. It follows that if $\mu > 0$, then $\dot{x}(0) < 0$ if and only if $K < a$, which can be written as $g_\tau > r + \mu/(e^{\mu\tau} - 1)$. Similarly, if $\mu < 0$, one has that $\dot{x}(0) > 0$ under the same condition. Direct inspection of equation (9) shows that it is violated when $\tau = 0$, as $g_0 < \infty$, whereas it is met when when $\tau = \infty$, as $g_\infty = \varepsilon > r$ when $\mu > 0$ under the assumptions of Proposition 2.1 and provided that $\mu > r - g_\tau$ when $\mu < 0$. Finally, using that $e^{\tau\mu} \approx 1 + \tau\mu + (\tau\mu)^2/2$ when $\mu \approx 0$, condition (9) writes as $\tau > 1/(g_\tau - r)$. □

E Optimal Growth Model

Consider the maximization of the intertemporal welfare function $\int_0^\infty e^{-\rho t}\frac{C(t)^{1-\theta}-1}{1-\theta}\,dt$ under the state equation:

$$\dot{K}(t) = \lambda\dot{K}(t-\tau) + (A-\delta)K(t) - r\lambda K(t-\tau) - C(t), \tag{15}$$

where ρ and θ are the usual time preference and risk aversion positive parameters. Given an initial profile for capital, $K_0(t)$, on $[-\tau\ ,0]$, where $K_0(t)$ is piecewise differentiable, a trajectory $(C(t), K(t))$, $t \geq 0$, is optimal if it checks (15) with $C(t)$ positive and piecewise continuous, $K(t)$ positive and piecewise differentiable, if the integral objective function is convergent, and if the value of the latter is greater than or equal to its value along any other admissible trajectory.

Proposition E.1 *Let $\phi > A-\delta$ be the unique positive solution of the x-equation:*

$$1 - \frac{A-\delta}{x} - \lambda e^{-x\tau} = 0.$$

ϕ is a decreasing function of the delay τ: it tends to $A-\delta$ when τ tends to ∞, and it tends to $\frac{A-\delta}{1-\lambda}$ when τ tends to zero. The optimal control problem has a solution if $\phi(1-\theta) < \rho$.

Proof: Define by $L^1\left(e^{-\psi t}\right)$ the set of functions such that $\int_0^\infty |f|e^{-\psi t}\,dt < \infty$, and consider the topology $\sigma(L^1, L^\infty)$. The trickiest part of the proof is to identify a $\psi > 0$ such that all the variables lie in balls of $L^1\left(e^{-\psi t}\right)$. The hemi-continuity of the operator defining the objective function of the problem, that is $V(c) = \int_0^\infty e^{-\rho t}\frac{C(t)^{1-\theta}-1}{1-\theta}\,dt$, in the topology $\sigma(L^1, L^\infty)$, will complete the argument. More details can be found in Askenazy and Le Van [3] or Boucekkine et al. [5]. Here we concentrate on the identification of ψ. Using the law of motion of human capital and given that $-r\lambda < 0$, one starts with the inequality:

$$\lambda\dot{K}(t-\tau) + (A-\delta)K(t) \geq \dot{K}(t).$$

If $K(t)$ grows at rate x, the inequality implies that:

$$\lambda e^{-x\tau} \geq 1 - \frac{A-\delta}{x}.$$

It is readily shown that for x to check the latter inequality, x cannot exceed ϕ given by the Proposition. ϕ is therefore the maximal growth rate allowed by the accumulation technology of

the model. It is easy to prove that this maximal growth rate degenerates to $\frac{A-\delta}{1-\lambda}$ when the delay vanishes. As the delay increases, the maximal growth rate allowed shrinks, reaching the limit $A-\delta$ when the delay goes to infinity. It is then enough to notice that ϕ is also the maximal growth rate for Y and for C given the technological and resource constraints of the problem. In such a case, if one assumes $\psi = \rho - \phi(1-\theta) > 0$, all the variables are indeed in balls of $L^1\left(e^{-\psi t}\right)$. □

The computation of necessary (and sufficient) optimality conditions are adapted from Boucekkine et al. (2005) who handle the optimal control of a functional differential equations of the delayed type. The extension to the neutral case can be readily done. One gets the following first-order conditions.

Proposition E.2 *If $(C(t), K(t))$, $t \geq 0$, is an interior optimal solution, then there exists a piecewise differentiable function $q(t)$ such that for all $t \geq 0$:*

$$q(t) = e^{-\rho t} C(t)^{-\theta}, \tag{16}$$

$$(A-\delta)q(t) + \dot{q}(t) - r\lambda q(t+\tau) - \lambda \dot{q}(t+\tau) = 0 \tag{17}$$

Now we show that the stability properties of the induced dynamic system can be trivially adapted from section 2. First, one has to make the following observation.

Proposition E.3 *A balanced growth path for the system (16)-(17) is determined by $g_c = -\frac{g_q+\rho}{\theta}$, where g_x is the growth rate of x along the BGP. It follows that $g_q < 0$ is the opposite of the growth rate(s) encountered in Proposition 2.2 with $s = 1$.*

The proof is trivial. The relationship $g_c = -\frac{g_q+\rho}{\theta}$ immediately comes from (16). The second, and more important, property directly derives from looking balanced growth solutions to the adjoint equation (17). One finds:

$$g_q = \lambda g_q \, e^{\tau g_q} + r\lambda \, e^{\tau g_q} - (A-\delta),$$

where $A-\delta$ coincides with ε when $s = 1$. One obtains exactly the same characterization of BGPs in Proposition 2.4 (when $s = 1$), with $g = -g_q$. It is then possible to state the following stability result, directly adapted from Proposition 2.4.

Proposition E.4 *Under the assumptions of Proposition 2.4 (with $s = 1$), the BGP of (16)-(17) associated to $g_q < 0$ is asymptotically stable. The one associated with $g_q > 0$ is unstable.*

References

[1] Aghion, P., Bacchetta, P., Banerjee, A. (2004). Financial development and the instability of open economies. *Journal of Monetary Economics* 51: 1077-1106.

[2] Aguiar, M., Gopinath, G. (2007). Emerging market business cycles: the cycle is the trend. *Journal of Political Economy* 115: 69-102.

[3] Askenazy, P., Le Van, C. (1999). A model of optimal growth strategy. *Journal of Economic Theory* 85: 24-51.

[4] Bellman R., Cooke, K. (1963). Differential-difference equations. Academic Press: New-York.

[5] Boucekkine, R., Licandro, O., Puch, L., de Rio, F. (2005). Vintage capital and the dynamics of the AK model. *Journal of Economic Theory* 120: 39-72.

[6] Cohen, D., Sachs, J. (1986). Growth and external debt under risk of debt repudiation. *European Economic Review* 30: 526-560.

[7] Cuberes, D., Jerzmanowski, M. (2009). Democracy, diversification and growth reversals. *Economic Journal* 119: 1270-1302.

[8] Devereux, M., Yetman, J. (2010), Leverage constraints and the international transmission of shocks. *Journal of Money Credit and Banking* 42: 71-105.

[9] Djankov, S., Hart, O., McLiesh, C., Shleifer, A. (2008). Debt enforcement around the world. *Journal of Political Economy* 116: 1105-1149.

[10] Freedman, H.I., Kuang, Y. (1991). Stability switches in linear scalar neutral delay equations. *Funkcialaj Ekvaciоj* 34: 187-209.

[11] Hausmann, R., Pritchett, L., Rodrik, D. (2005). Growth accelerations. *Journal of Economic Growth* 10: 303-329.

[12] Jones, B., Olken B. (2008). The anatomy of start-stop growth. *Review of Economics and Statistics* 90: 582-587.

[13] Kordonis, I.-G.E., Niyianni, N.T., Philos, C.G. (1998). On the behavior of the solutions of scalar first order linear autonomous neutral delay differential equations. *Archiv der Mathematik* 71: 454-464.

[14] Lane, P., Milesi-Ferreti, G. (2007). The external wealth of nations Mark II: revised and extended estimates of foreign assets and liabilities, 1970-2004. *Journal of International Economics* 73: 223-50.

[15] Mendoza E. (2010). Sudden stops, financial crises and leverage. Forthcoming *American Economic Review.*

[16] Paasche, B. (2001). Credit constraints and international financial crises. *Journal of Monetary Economics* 48: 623-50.

[17] Rancière, R., Tornell, A., Westermann, F. (2008). Systemic crises and growth. *Quarterly Journal of Economics* 123: 359-406.

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