# Income inequalities and innovation by incumbents

H. Latzer

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de l'Université catholique de Louvain

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# Income inequalities and innovation by incumbents *

Hélène Latzer†

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**Abstract**

Our paper presents a new rationale for innovation by incumbents. We show that the possibility to price-discriminate between consumers having different levels of wealth is a sufficient incentive for the industry leader to overcome the Arrow (1962) effect and keep investing in R&D, even in the absence of any incumbent advantage in the R&D field. We model an economy composed of two distinct groups of consumers, differing in their wealth endowment and subject to non-homothetic preferences, obtained through unit consumption of the quality good. We demonstrate that in such a framework, there exists a unique steady state equilibrium with positive innovation rates of both incumbents and challengers. Beyond its novelty, this result then also allows us to analyze the effect of the extent of income inequalities on both the challenger and incumbent innovation rates, and by extension on the economic growth rate. We demonstrate that a higher share of the population being poor is detrimental to the rate of economic growth, while a redistribution of wealth from rich to poor consumers increases the challenger innovation rate and has ambiguous effects on the incumbent's investment in R&D.

*Keywords:* Growth, Innovation, Income inequalities.

**JEL classification:** O3, O4, F4.

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†IRES, Université Catholique de Louvain. E-mail: helene.latzer@uclouvain.ac.be.

# 1 Introduction

In the first-generation quality-ladder models (Segerstrom et al., 1990; Grossman and Helpman, 1991b; Aghion and Howitt, 1992), quality leaders do not participate to the next innovation race. This absence of R&D investment by incumbents, known as the Arrow (1962) effect and justified by the fact that innovating quality leaders would only cannibalize their own business, is at odds with overwhelming evidence of leading firms still significantly investing in R&D.[1] Acknowledging the counterfactual nature of innovation races leaving out the current quality leaders, a growing literature has hence developed models of Schumpeterian growth featuring innovation by incumbents. Several mechanisms have been used to overcome the Arrow (1962) effect and explain a positive investment in R&D by quality leaders. Cozzi (2007) has shown that in the particular case of constant returns at the firm level, the incumbent is indifferent to its own R&D investment. Segerstrom and Zolnierek (1999) as well as Segerstrom (2007) introduce R&D cost advantages for the incumbents stemming from the expertise granted by quality leadership, while Etro (2004, 2008) and Aghion et al. (2001) feature leaders investing in R&D in order to escape competition pressure, whether it be in the innovative field or on the product market.[2]

This paper introduces a new rationale for investment in R&D by incumbents, based on another stylized fact usually overlooked in standard quality ladder models: all the quality leaders in high-tech sectors do not only invest positive and significant amounts in R&D, but also produce and sell more than one quality-differentiated version of their core products. Indeed, Intel currently sells three different families of microprocessors (Core, Pentium and Celeron), displaying different levels of speed and performance; Microsoft commercializes simultaneously Windows XP, Vista and 7; Nokia sells numerous quality-differentiated mobile phones, displaying significant variations in offered functionalities. This feature has so far been ignored by standard quality-ladder models, where homothetic preferences drive the result that only the version of the quality good displaying the highest price-adjusted quality is consumed at the equilibrium. However, the opportunity to offer different price-quality bundles in order to discriminate between consumers having different quality valuations represents a significant incentive for the leading-edge firms to invest in R&D in order to expand their product range. Hence, while so far the incentives for innovation by quality leaders have been argued to be mainly on the supply side (R&D cost advantages, Stackelberg leadership), this paper provides a *product-driven* incentive for investment in R&D by incumbents.

Our modeling framework builds on models of endogenous growth such as the ones of Li (2003), Zweimuller and Brunner (2005) or Zweimuller and Foellmi (2006), all allowing for more than one quality to be consumed at the equilibrium through differences in wealth endowment and non-homothetic preferences. By contrast, in the standard quality-ladder models (Grossman and Helpman, 1991a; Segerstrom et al., 1990), the quality good is divisible and the preferences of the consumers are homothetic, meaning that only the

[1] Segerstrom (2007) presents R&D expenditures of selected industry leaders for the year 2000, with a net sales to R&D investment ratio going as high as 11.5% for Intel or 16.4% for Microsoft.

[2] Etro (2004, 2008) models sequential patent races where the incumbents, acting as Stackelberg leaders, are given the opportunity to make a strategic precommitment to a given level of R&D investment: quality leaders have then an incentive to invest in order to escape the innovative pressure of outsiders. Product market competition drives the results of Aghion et al. (2001), where the perspective to lessen the competition pressure (and broaden the market share) provides the incentive for the incumbent to carry out positive R&D investments.

highest price-adjusted quality will be consumed at the equilibrium, even when differences in wealth endowments are allowed for: the poorest consumers will only consume a lower share of the top quality good. Our framework on the contrary allows for the level of a consumer's income to determine his willingness to pay for the highest quality,[3] this property being obtained through unit consumption of the quality good while the rest of the expenditures are spent on a standard (composite) good. The economy is composed of two distinct groups of consumers, differing in their wealth endowment. This feature, associated to non-homothetic preferences, yields different possible market structures for the quality good at the equilibrium, depending on the extent of inequalities in the distribution of wealth.

In models displaying this feature but without allowing for innovation by incumbents (Zweimuller and Brunner, 2005), the successful innovator was always a challenger before winning the R&D race, and has at its disposal a *unique* quality level: we then have either a monopoly with the quality leader capturing the whole market, or a duopoly with the producers of the first- and the second-best qualities sharing the market. The quality leader is the one that decides the market structure that is optimal for him and then sets the adequate price, depending on the wealth distribution within the economy. Hence, in those models, whether or not the second-best quality is being sold is once and for all deterministically determined by the extent of inequalities in the economy, and the successive quality jumps never change the market structure. In our model allowing for innovation by incumbent, a third case is possible: the last innovation race has been won by the incumbent, who has then successfully innovated *twice in a row*. The quality leader then has at its disposal two successive quality levels, and faces a monopoly price discrimination problem (Mussa and Rosen, 1978): he will then always optimally choose to offer two different price-quality bundles. Hence, in our model, the equilibrium market structure and the number of active producers *do* depend on the outcome of the successive innovation races, and can be considered as a stochastic process.[4] In such a framework, we then model R&D races with decreasing marginal productivity of R&D investment, and show that without any R&D cost advantages, the incumbent still has an incentive to invest in R&D, stemming from the increment between the profits realized when price discrimination is possible and the ones realized when selling the same quality to everybody.

In order to thoroughly analyze the steady state of the defined model, we then first restrict ourselves to the case of a **monopoly** market structure for the quality good in the case the innovation race has been won by a challenger.[5] We demonstrate the existence under certain parametric conditions of an interior solution for the steady-state equilibrium with *positive* R&D investment by the incumbent in the next innovative race. This result, beyond its novelty, enables us to then study the effects of income inequalities on innovations rates of both challengers and incumbents, as well as on the economy growth rate. We show that a growing part of the population being poor is always detrimental to both innovation rates, while redistribution of income from rich to poor people is benefic to the

[3] We choose to model different levels of preferences for the highest quality through differences in endowment rather than differences in quality valuation in order to be able to introduce the problematic of the effects of inequality.

[4] As it will be demonstrated, the presence of such a stochastic process does however not prevent us from proceeding to a steady state analysis, since we demonstrate it to be a Markov process with a stationary distribution.

[5] Indeed, our aim is to first clearly isolate the incentive for the monopolist to engage in the next R&D race stemming from the perspective of possible price discrimination. A wealth distribution resulting in a duopoly market structure in the case a challenger has won the R&D race adds a further incentive for the incumbent to invest in R&D, i.e. to escape competition along the lines of the Aghion et al. (2001) model.

innovation by challengers rate, and has ambiguous effects on the innovation by incumbent rate. We then finally define the steady state equilibrium for the cases where the distribution of wealth induces a duopoly market structure in the case the latest innovation race has been won by a challenger, and show that this case occurs rarely when allowing for innovation by incumbent.

As already mentioned, our paper is related to the literature studying the effects of inequalities in the product market on growth (Zweimuller, 2000; Li, 2003; Zweimuller and Brunner, 2005; Zweimuller and Foellmi, 2006). Those models use the feature of non-homothetic preferences and more than one quality being consumed to study the impact of income inequality on the equilibrium market structure (i.e. the number of price-competing producers of different qualities with positive market share) and by extension on the rate of economic growth.[6] However, our paper strongly differs from the latter by allowing for innovation by incumbent: in that case, beyond the determination of the market structure, consumption of more than one quality at the equilibrium also represents an opportunity for a monopolist to price-discriminate between consumers having different tastes for quality (in our framework, because of differences in levels of income). Our model hence also relates to the monopoly price discrimination literature (Salop and Stiglitz, 1977; Mussa and Rosen, 1978; Stokey, 1979). Finally, our paper is also obviously part of the growing body of literature concerning innovation by incumbents (Aghion et al., 2001; Cozzi, 2007; Segerstrom, 2007; Etro, 2004, 2008). It is however to the best of our knowledge the first one to model a product-driven, price-discrimination incentive for innovation by incumbents. Beyond its novelty, this approach also makes it possible to study the effects of income inequalities on the innovation by incumbents rate, and by extension on the global growth rate.

The rest of the paper is organized as follows. Sections 2 to 4 present our model, while section 5 studies the steady state equilibrium. Section 6 then studies the effects of the extent of inequalities on the innovation rates and economic growth rate. Section 7 finally studies the properties of the equilibrium in the case a duopoly occurs when the successful innovator is a challenger. Section 8 concludes.

## 2 Consumers

There is a fixed number $L$ of consumers that live infinitely and supply one unit of labor each period, paid at a constant wage $w$. While all consumers have the same wage income, they are assumed to differ with respect to asset ownership $\omega_i(t)$: along Zweimuller and Brunner (2005), we assume a two-class society with rich (R) and poor (P) consumers, being distinguished by their wealth (respectively $\omega_R$ and $\omega_P$).

The share of poor consumers within the population is denoted by $\beta$. The extent of inequalities within the economy is determined by this share, as well as by the repartition between rich and poor of the aggregate wealth $\Omega$. $d \in (0,1)$ is defined as the ratio of the value of assets owned by a poor consumer *relative* to the average per-capita wealth: $d = \frac{\omega_P}{\Omega/L}$. Given $d$, the wealth position of the rich can be computed as $d_R = \frac{1-\beta d}{1-\beta}$. We hence have $\omega_P = d\frac{\Omega}{L}$ and $\omega_R = \frac{1-\beta d}{1-\beta}\frac{\Omega}{L}$.

[6] Their analysis of the market structure at the equilibrium is then based on the static, partial equilibrium literature of price competition between quality-differentiated goods (Gabszewicz and Thisse, 1979; Shaked and Sutton, 1982).

Current income $y_i(t)$ of an individual belonging to the group $i$ ($i = P, R$) is then of the form:

$$y_i(t) = w + r\omega_i(t) \tag{1}$$

with $r$ being the interest rate.

Current income is then spent for the consumption of a single unit of a quality good with price $p_i(t)$ (depending on the quality $q_i(t)$ chosen by the consumer at time $t$), and of $c_i(t)$ units of a standardized good with price 1. Preferences are non-homothetic, with the instantaneous utility of a consumer of type $i$ being described by the following utility function:

$$u_i(t) = \ln c_i(t) + \ln q_i(t) = \ln(y_i(t) - p_i(t)) + \ln q_i(t) \tag{2}$$

As shown by Zweimuller and Brunner (2005), at time $\tau$ the intertemporal decision problem of the consumer is then of the form:

$$\max_{c_i(t), q_i(t)} \int_\tau^\infty (lnc_i(t) + lnq_i(t))e^{-\rho(t-\tau)} \quad s.t. \quad \omega_i(\tau) + \int_\tau^\infty we^{-r(t)(t-\tau)}dt \geq \int_\tau^\infty c_i(t)e^{-r(t)(t-\tau)}dt + \int_\tau^\infty p_i(t, q_i(t))e^{-r(t)(t-\tau)}dt$$

with $\rho$ being the rate of time preference. Given an expected time path for both the interest rate $r(t)$ and the relation between quality and price $p_i(t, q_i(t))$, it is then possible to determine the optimal time path of $c_i(t)$, the consumption of the standardized good, and of $q_i(t)$, the chosen quality of the unit consumption good.

For any given time path of expenditures for the quality good $p_i(t, q_i(t))$ that does not exhaust life-time resources, the optimal path of consumption expenditures on the standardized good has to fulfill the standard first order condition of the maximization problem:

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho \tag{3}$$

The optimal time path of $q_i(t)$, on the other hand, cannot be characterized by a differential equation, since the quality choice is discrete. We notice however that the choice in $q_i(t)$ is made simultaneously along with the decision on $p_i(t)$ by profit-maximizing firms. We hence set aside the choice of quality on the part of consumers until having defined the market and price structure for the quality good.

# 3 Market structure and pricing

There is a linear technology for the production of the standardized good, with labor as the unit input. We use the price of this standardized good as the numeraire, and since the market is assumed to be competitive, unit labor input is $1/w$.

The market for the quality good is non-competitive. At any date $t$, we assume that a continuum of qualities $q_j(t)$, $j = 0, -1, -2, ...$ exist and can be produced, with $q_0(t)$ being the best quality, $q_{-1}(t)$ the second-best, etc. Labor is the only input, with constant unit labor requirement $a < 1$.[7] Two successive quality levels differ by a fixed factor $k > 1$:

[7] Given the fact we have unit consumption of the quality good, $a$ necessarily has to be inferior to 1.

$q_j(t) = k.q_{j-1}(t)$.

We will now define more precisely the structure of the quality good market. The quality good being characterized by unit consumption and fixed quality increments, firms use prices as strategic variables. We assume they know the shares of groups $P$ and $R$ in the population, the respective incomes $y_R$ and $y_P$ as well as the preferences of the consumers, but that they *cannot distinguish individuals by income*. In order to describe the strategic decisions operated by firms, we now define $p^T_{i,\{j,j-m\}}$, the "threshold" price for which a consumer belonging to group $i$ is indifferent between quality $j$ and quality $j-m$, i.e. $\ln(y_i - p^T_{i,\{j,j-m\}}) + \ln q_j = \ln(y_i - p_{j-m}) + \ln q_{j-m}$. Considering the fact that $q_j = k^{j-m} q_{j-m}$, solving for $p^T_{i,\{j,j-m\}}$ in the above equality yields:

$$p^T_{i,\{j,j-m\}} = y_i \left( \frac{k^m - 1}{k^m} \right) + \frac{p_{j-m}}{k^m} \tag{4}$$

The price $p^T_{i,\{j,j-m\}}$ is the maximum price that the firm selling the quality $j$ can charge to a consumer belonging to the group $i$ in order to have a positive market share, when facing the firm selling quality $j-m$. As one can see, this threshold price depends on the income $y_i$ of consumer $i$, as well as on the price charged by the competitor $p_{j-m}$. Having defined this threshold price, and along with Zweimuller and Brunner (2005), we can state:

**Lemma 1**: *If $p_j \geq wa$ holds for the price of some quality $q_j$, $j = -1, -2, ...$, then for the producer of the any higher quality $q_{j+m}$, $1 \leq m \leq -j$, there exists a price $p_{j+m} > wa$, such that any consumer prefers quality $q_{j+m}$ to $q_j$.*

*Proof:* For a given group of consumers $i$, $p^T_{i,\{j+m,j\}} = y_i \left(\frac{k^m-1}{k^m}\right) + \frac{p_j}{k^m}$ is a weighted average of $y_i$ and $p_j$. Given the fact that only prices being below their income are taken into account by consumers $i$, we have that $p_j < y_i$, and we can hence conclude that $p^T_{j+m,j} > p_j$. Hence, it is always possible for the producer of the quality $j+m$ to set a price $p_{j+m} > p_j \geq wa$ such that $p_{j+m} \leq p^T_{i,\{j+m,j\}}$, i.e. such that quality $q_{j+m}$ is preferred to quality $q_j$ by the consumers of group $i$. This ends the proof. □

Hence, if we take for granted that a producer never sells its quality at a price below the unit production cost $wa$, it is always possible for the producer of a higher quality to drive him out of the market, while still making strictly positive profits. Along this result, any firm entering the market with a new highest quality $q_0$ has to consider the following trade-off concerning the pricing of its product: setting the highest possible price for any given group of clientele, vs. lowering its price in order to capture a further group of consumers.

It is then possible to show that in an economy characterized by two distinct groups of consumers (R and P), the equilibrium has the following properties:

**Lemma 2**: *At equilibrium,*
*(1) The highest quality is produced,*
*(2) At most the two highest qualities $q_0$ and $q_{-1}$ are actually produced,*
*(3) The equilibrium price $p_{-1}$ fulfills $wa \leq p_{-1} \leq p^T_{P,\{-1,-2\}}$, with $p^T_{P,\{-1,-2\}}$ denoting the maximum price the producer of the $q_{-1}$ quality can set in order to deter the producer of the $q_{-2}$ quality from entry.*

The proof is made in Zweimuller and Brunner (2005). The intuition is that since there are only two distinct groups of consumers, at most two distinct qualities can be sold, and at least one is always consumed, since it is assumed every individual buys one unit of the quality good. By Lemma 1, higher qualities drive out lower ones, hence the two qualities being still possibly active are $q_0$ and $q_{-1}$. At equilibrium, no firm can make a loss, hence the price $p_j$ being charged for any quality $q_j$ active on the market is necessarily superior or equal to the production cost $wa$. Finally, $p_{-1} \leq p^T_{P,\{-1,-2\}}$ follows from the fact that otherwise the producer of quality $q_{-2}$ could enter the market.

As it can be seen from lemma 2, two different situations are possible for the equilibrium market structure and associated prices: either only the top quality good $q_0$ is sold to both groups of consumers (groups P and R), *or* the top quality good is sold only to the rich consumers (group R) while the second best quality good is sold to the poor consumers (group P). Lemma 1 shows that the decision regarding the market structure belongs to the producer having at its disposal the highest quality $q_0$, considering that he is always able to set a price that will drive its competitors out.

In Zweimuller and Brunner (2005), as well as other models studying the effects of inequality on growth through the product market (Zweimuller, 2000; Li, 2003; Zweimuller and Foellmi, 2006), when the second-best quality is sold on the market, it is produced by a producer *distinct* from the producer of the top quality. Indeed, in those models the incumbent does not engage in the next R&D race: when a new innovation occurs, the successful challenger becomes the quality leader, the previous quality leader becomes the producer of the second-best quality (whether he is still active or not depends on the pricing decision taken by the new quality leader), while the producer of the previous second-best quality is anyway driven out of the market. These papers hence bear a close relationship with the static models of price competition in oligopoly markets where consumers buy a single unit of a given quality good (Gabszewicz and Thisse, 1980; Shaked and Sutton, 1982, 1983). In this case, whether or not the second-best quality is being sold is solely determined by the distribution of income in the considered economy, since the latter determines the optimal pricing chosen by the quality leader. In other words, the equilibrium market structure in those models is set once and for all depending on the values assigned to the parameters $\beta$ and $d$, and the successive quality jumps do not alter it in any way.

In our model however, a further possibility has to be taken into account: it is possible that a *unique* producer does have at its disposal the *two* highest qualities, since we leave the opportunity for the incumbent to still engage in R&D races. In the case the incumbent does innovate, he then faces the monopoly pricing problem of a firm having at its disposal a spectrum of quality-differentiated goods (Mussa and Rosen, 1978).

Hence, in our model, the equilibrium market structure *does* depend on the outcome of the successive innovation races. In other words, the equilibrium market structure is a random process that we denote by $M(t)$. We define the state space of this stochastic process as $\{(SC), (SI)\}$, with the possible states (SC) and (SI) being characterized in the following way:[8]

- **"Successful Challenger" (SC) state**: a challenger is the winner of the last R&D race, i.e. the new quality leader is *different* from the former quality leader. The new

[8] As it will be further demonstrated, the stochastic process $M(t)$ is a continuous time Markov process for which it will be possible to determine a stationary distribution.

quality leader then only has the highest quality at its disposal. One or two qualities can then be sold on the market, depending on the pricing strategy chosen by the new quality leader (which will itself depend on the wealth distribution in the economy). The market structure can then either be a *monopoly* (only quality $q_0$ is sold), with the new quality leader charging a price that enables him to capture the whole market, or a *duopoly* (both qualities $q_0$ and $q_{-1}$ are sold), with the new quality leader charging a higher price and serving only the upper part of the market, leaving the lower part to the second-best quality producer.[9]

- **"Successful Incumbent" (SI) state**: the former quality leader, still carrying out R&D, is the winner of the last R&D race, and hence has two successive qualities at its disposal. According to lemma 2, the market structure is then necessarily a *monopoly*. However, as we will show, the quality leader will then offer two different quality/price bundles in order to *discriminate* between the groups P and R of the population (Mussa and Rosen, 1978), and hence both qualities $q_0$ and $q_{-1}$ are sold.

We will now describe in more details the possible market structures, prices and associated profits in the two existing states.

## 3.1 (SC) state

As already stated, the market structure in the (SC) state is deterministically either a monopoly or a duopoly, depending on the extent of inequalities in wealth distribution in the economy.

### 3.1.1 Monopoly case

It corresponds to the case where the wealth structure makes it optimal for the quality leader to set a price enabling him to sell the unique quality he has at its disposal to the entire market, driving the former quality leader out of the market.

$p^T_{i,\{0,-1\}}$ is the maximum price the producer of quality $q_0$ can set in order to capture the consumers of the group $i$ for a given price $p_{-1}$ of quality $q_{-1}$. Hence, optimal price chosen by a quality leader willing to capture the whole market is $p^T_{P,\{0,-1\}}$, given that the producer of quality $q_{-1}$ engages in marginal cost pricing (i.e. $q_{-1} = wa$). Indeed, setting a price that captures the consumers belonging to the group P automatically ensures that the rich consumers will consume the highest quality $q_0$ too.

We denote by $p_M$ the price being charged in the monopoly case of the "successful challenger" (SC) state. $p_M$ is thus of the form:

$$p_M = y^P_M \frac{k-1}{k} + \frac{wa}{k}$$

with the associated profits:

$$\pi_M = L(p_M - wa)$$

$y^P_M$ being the income of the poor consumers in the monopoly case of the (SC) state, of the form $y^P_M = w + rd\frac{\Omega_M}{L}$.

[9] It is important to notice that though either a monopoly or a duopoly, the market structure *inside* the (*SC*) case is *not* a stochastic process: for given values of the parameters $\beta$ and $d$, it is deterministically determined.

### 3.1.2 Duopoly case

It corresponds to the case where the wealth structure makes it optimal for the new quality leader to set a price capturing only the upper part of the market, abandoning the lower part to the producer of the second-best quality. The two highest qualities $q_0$ and $q_{-1}$ are then sold at the equilibrium, being produced by two different firms.

Zweimuller and Brunner (2005) have defined a possible equilibrium in that case, under the condition on the punishment strategies of the infinitely repeated pricing game that no firm is punished if it changes its price without affecting the other firm's profit (Proof: cf Zweimuller and Brunner (2005), p. 242). At this equilibrium, the new quality leader will charge the highest possible price enabling him to capture the group of rich consumers $p^T_{R,\{0,-1\}}$, given the expected strategy of the producer of the second-best quality. The former quality leader charges the highest possible price enabling him to capture the poor group of consumers $p^T_{P,\{-1,-2\}}$, given that the producer of quality $q_{-2}$ engages in marginal cost pricing. It is however important to notice that this strategy, chosen by the producer of quality $q_{-1}$, is only made possible because of the decision of the new quality leader to charge a higher price, capturing only the upper part of the market. Hence, as already previously stated, the quality leader's optimal decision, depending itself on the wealth distribution of the economy, always plays the decisive role in determining the market structure.

We call $p^R_D$ the price being charged to the rich consumers in the duopoly case of the (SC) state, while $p^P_D$ is the price charged to the poor consumers. They are of the following form:

$$p^P_D = y^P_D \frac{k-1}{k} + \frac{wa}{k}, \quad p^R_D = y^R_D \frac{k-1}{k} + y^P_D \frac{k-1}{k^2} + \frac{wa}{k^2}$$

with the associated profits $\pi^R_D$ for the quality leader and $\pi^P_D$ for the producer of the second-best quality:

$$\pi^R_D = (1-\beta)(p^R_D - wa), \quad \pi^P_D = \beta(p^P_D - wa)$$

and $y^P_D$ and $y^R_D$ being of the form:

$$y^P_D = w + rd\frac{\Omega_D}{L}, \quad y^R_D = w + r\frac{1-\beta d}{1-\beta}\frac{\Omega_D}{L}$$

Having defined $\pi_M$ (profits of the quality leader in the monopoly case of the (SC) state) and $\pi^R_D$ (profits of the quality leader in the duopoly case of the (SC) state), we can now clearly state the criteria governing the quality leader's decision concerning its pricing strategy in the (SC) state. Indeed, as already stated, the current quality leader plays the decisive role in the determination of the market structure, and takes his decision by comparing the generated profits in both cases, i.e. $\pi_M$ and $\pi^R_D$. If $\pi^R_D < \pi_M$, the leader will choose to be the only firm active on the market, and will charge the price $p_M$ that captures the two populations. Otherwise, the leader will leave the lower part of the market to the follower, charging the higher price $p^R_D$. Even if a fully analytical characterization of the parametric cases for which each situation occurs is not possible,[10] we can still state that the monopoly case occurs when the two groups are sufficiently similar to each other: $d$ sufficiently close to 1, and $\beta$ sufficiently large.

[10] Indeed, in order to determine the optimal price strategy, firms have to determine the endowments $\omega_R$ and $\omega_P$ in each regime, which in turn depend on the endogenous equilibrium value of overall wealth $\Omega_s$ in the two cases.

### 3.2 (SI) state

Two qualities are systematically sold in the (SI) state. Indeed, a leader having at its disposal two successive qualities and facing two groups of consumers having different levels of income will always find it optimal to offer two distinct price-quality bundles in order to maximize its profit (Mussa and Rosen, 1978). The market structure is then a **monopoly**. The price charged by the monopolist for its second-best quality will be the maximal price enabling him to capture the poor group of consumers $p^T_{P,\{-1,-2\}}$, given that the producer of quality $q_{-2}$ engages in marginal cost pricing. The price charged for the highest quality will then be the maximal price $p^T_{R,\{0,-1\}}$, given $p^T_{P,\{-1,-2\}}$.

We call $p^R_I$ the price being charged to the rich consumers in the (SI) state, while $p^P_I$ is the price charged to the poor consumers. They are of the following form:

$$p^P_I = y^P_I \frac{k-1}{k} + \frac{wa}{k}, \quad p^R_I = y^R_I \frac{k-1}{k} + y^P_I \frac{k-1}{k^2} + \frac{wa}{k^2}$$

with the associated profits for the discriminating monopolist:

$$\pi_I = \beta L(p^P_I - wa) + (1-\beta)L(p^R_I - wa)$$

and $y^P_I$ and $y^R_I$ being of the form:

$$y^P_I = w + rd\frac{\Omega_I}{L}, \quad y^R_I = w + r\frac{1-\beta d}{1-\beta}\frac{\Omega_I}{L}$$

We hence notice that the prices charged for the two qualities in the duopoly case of the (SC) state and in the (SI) state are strongly similar, even if the number of active firms are different. However, the overall wealth ($\Omega_s$, $s \in (D, I)$) is different depending on the state the economy finds itself in, hence making it necessary to clearly differentiate the prices charged in the two possible cases in which 2 qualities are sold.

Having defined the possible market structure, prices and profits in every possible state, we can now move to the description of the R&D process, which is the engine of growth in our model. It is however important to signal right away that for the sake of clarity, we will restrict ourselves to the case where the market structure in the $(SC)$ state is a monopoly in the following sections. We will study the duopoly case in the last section of the paper.

## 4 R&D sector

Firms carry out R&D in order to discover the next quality level. Two types of firms engage in R&D races: the current quality leader (incumbent), and followers (challengers). There is free entry for follower firms, and each one of them has access to the same R&D technology. Innovations are random, and occur for a given firm $i$ according to a Poisson process of hazard rate $\phi_i$. Labor is the only input, and we assume decreasing returns to R&D at the firm level, with the R&D cost function $\psi(\phi_i)$ being an increasing and convex function of R&D intensity $\phi_i$.

In order to have an immediate probability of innovating of $\phi_M$, the incumbent needs to hire $F\phi^\alpha_M$ labor units, with $\alpha > 1$. As for followers, we assume along with Segerstrom and Zolnierek (1999) that in order to have an immediate probability of innovating of $\phi_{Ci}$, a challenger needs to hire $F\phi^\alpha_{Ci}m^{\alpha-1}$ labor units, with $m$ being the number of firms entering the R&D race. We then have the global probability of a challenger to discover the next quality

being $\phi_C = m\phi_{Ci}$ and the amount of labor hired being $m(F\phi_{Ci}^{\alpha}m^{\alpha-1}) = F(m\phi_{Ci})^{\alpha} = F\phi_C^{\alpha}$.

Investment decisions in R&D are taken considering the associated costs and rewards in the case of a successful innovation. The reward accruing to a successful challenger in the $(SC)$ state depends on the optimal market structure chosen by the new leader, that itself depends on the distribution of wealth within the economy. As already signaled, in this first part of our paper, where we clearly want to isolate the incentive for incumbents to invest in R&D being linked to the possibility for them to discriminate between different groups of consumers, we will focus on the case where we have a **monopoly market structure** in the (SC) state. Indeed, in the case where we have a duopoly market structure, one could identify a further incentive for the incumbent to innovate along the line of the Aghion et al. (2001) model, i.e. to escape the competitive pressure applied by the producer of quality $q_{-1}$.

We define $v_M$ as the expected present value of a quality leader having innovated once ((SC) state), and $v_I$ as the expected present value of a quality leader having innovated twice, and hence having at its disposal two successive qualities ((SI) state). $v_{CM,i}$ and $v_{CI,i}$ are the corresponding expected present values of a challenger firm $i$, both in the (SC) and (SI) state. Free entry in the R&D races implies that we focus on the limit case where $m = \infty$, and that the individual contribution of any particular follower firm $i$ to the aggregate challenger innovation rate is negligible: hence, we have $v_{CM,i} = v_{CI,i} = 0$. The free entry condition in the R&D race further imposes the traditional equality constraint between expected profits and engaged costs for the challengers, both in states (SC) and (SI) of our economy:

$$\phi_{CM,i}v_M = wF\phi_{CM,i}^{\alpha}m^{\alpha-1} \tag{5}$$

$$\phi_{CI,i}v_M = wF\phi_{CI,i}^{\alpha}m^{\alpha-1} \tag{6}$$

And rearranging (5) and (6), we get the following equality:

$$v_M = wF\phi_{CM}^{\alpha-1} = wF\phi_{CI}^{\alpha-1} \tag{7}$$

Hence, considering equation (7) we get the result that $\phi_{CM}$ and $\phi_{CI}$ are equal. This stems from the fact that the expected reward $v_M$ for a challenger that innovates is the same, whether you are in state (SC) or in state (SI). Hence, from now on we will consider that $\phi_{CM} = \phi_{CI} = \phi_C$.

The incumbent on the other hand participates to the race with the advantage of having already innovated at least once, and hence being the current producer of the leading quality. It is hence not subject to the free entry constraint of equality between engaged costs and expected profits. In the (SC) state, he faces the following Hamilton-Jacobi-Bellman equation:

$$rv_M = \max_{\phi_M \geq 0}\{\pi_M - wF\phi_M^{\alpha} + \phi_M(v_I - v_M) + \phi_C(v_{CM,i} - v_M)\} \tag{8}$$

The incumbent in the (SC) state earns the monopoly profits $\pi_M$ (since we are restricting ourselves to the monopoly case), and incurs the R&D costs $wF\phi_M^{\alpha}$. With instantaneous probability $\phi_M$, the leader innovates once more, and its value jumps up to $v_I$. The economy then jumps to the state (SI), and two qualities start to be produced. However, with overall instantaneous probability $\phi_C$, some R&D challenger innovates, and the quality leader

becomes an R&D follower: its value falls to $v_{CM,i} = 0$. The economy then remains in the state (SC), and only one quality is produced.

In the (SI) state, the incumbent faces the following Hamilton-Jacobi-Bellman equation:

$$rv_I = \max_{\phi_I \geq 0} \{\pi_I - wF\phi_I^\alpha + \phi_I(v_I - v_I) + \phi_C(v_{CM,i} - v_I)\} \tag{9}$$

The incumbent in the (SI) state earns the profits $\pi_I$ of a monopolist being able to discriminate between rich and poor consumers by offering two distinct price/quantity bundles. He incurs the R&D costs $wF\phi_I^\alpha$. With instantaneous probability $\phi_I$, the incumbent innovates once more, in which case its value remains $v_I$, since we have established with Lemma 2 that at most two successive quantities are sold at equilibrium. Hence, the incumbent will still be the producer of the two qualities being sold, but he will drive himself out of the market for the former quality $q_{-1}$, that has become quality $q_{-2}$ with the latest quality jump. The economy then remains in state (SI). With instantaneous probability $\phi_C$, some R&D follower innovates, and the quality leader then falls back to being an R&D challenger: its value falls to $v_{CM,i} = 0$. The economy then jumps to the state (SC), and only the new highest quality is sold.

In both states, the incumbent firm chooses its R&D effort so as to maximize the right-hand side of its Bellman equation. Using the fact that $v_{CM,i} = v_{CI,i} = 0$, (8) and (9) then yield the following first order conditions:

$$v_I - v_M = \alpha wF\phi_M^{\alpha-1} \tag{10}$$

$$-\alpha wF\phi_I^{\alpha-1} = 0 \quad \Rightarrow \quad \phi_I = 0 \tag{11}$$

Hence, we obtain a relation between the R&D effort and the incremental value that would result from innovating in both states. Given that the incremental value of a further innovation for an incumbent in the (SI) state is null in our benchmark case of an economy with only two distinct population groups, we obtain that the optimal investment in R&D in that state is zero.[11]

Using the optimality constraints (10) and (11) in (8) and (9), we obtain the following expressions for the expected values $v_M$ and $v_I$:

$$v_M = \frac{\pi_M + (\alpha - 1)wF\phi_M^\alpha}{r + \phi_C} \tag{12}$$

$$v_I = \frac{\pi_I}{r + \phi_C} \tag{13}$$

The right-hand side of equation (12) deserves particular attention. Indeed, the expected value $v_M$ is not only composed of the actualized profits $\frac{\pi_M}{r+\phi_C}$, but also of a second term $\frac{(\alpha-1)wF\phi_M^\alpha}{r+\phi_C}$. This term represents a kind of second reward (beyond the realized profits) when becoming the quality leader: you gain access to the possibility of innovating for a

[11] It would of course be possible to generalize the result to more than two groups of population, and even a continuum of quality valuations as in Mussa and Rosen (1978). We believe the incumbent would then keep investing in R&D beyond the second innovation in a row.

second time *in a row*, and hence of becoming a discriminating monopolist. It represents the actualized possible gains of entering the "private" R&D race reserved to the quality leader (the entry cost being the requirement of having already innovated once).

# 5 Steady state equilibrium

## 5.1 Labor market equilibrium

We have two possible equations describing the equilibrium on the labor market, whether we are in the (SC) or the (SI) state. The equilibrium on the labor market in the (SC) state is of the form:

$$L = F\phi_M^\alpha + F\phi_C^\alpha + aL + (L/w)(\beta(y_M^P - p_M) + (1-\beta)(y_M^R - p_M)) \tag{14}$$

with $\phi_M + \phi_C$ being the number of people hired in the R&D sector, $aL$ being the number of people hired for the production of the quality goods (we know there is unit consumption of each of them), and finally $L/w(\beta(y_M^P - p_M) + (1-\beta)(y_M^R - p_M))$ being the number of people devoted to the production of the standardized good.

The equilibrium on the labor market in the (SI) state is of the form:

$$L = F\phi_C^\alpha + aL + (L/w)(\beta(y_I^P - p_I^P) + (1-\beta)(y_I^R - p_I^R)) \tag{15}$$

with $\phi_C$ being the number of people hired in the R&D sector (the incumbent does not invest in R&D in the (SI) state), $aL$ being the number of people hired for the production of the quality goods, and finally $L/w(\beta(y_I^P - p_I^P) + (1-\beta)(y_I^R - p_I^R))$ being the number of people devoted to the production of the standardized good.

It will prove convenient to express (14) and (15) in terms of profit flows in both states, i.e. in terms of $\pi_M$ and $\pi_I$. Multiplying both sides by $w$ and replacing $y_M^P$, $y_M^R$, $y_I^P$ and $y_I^R$ by their values expressed in Section 3, (14) and (15) yield:

$$wL = wF\phi_C^\alpha + wF\phi_M^\alpha + waL + L(\beta(w + rd\frac{\Omega_M}{L} - p_M) + (1-\beta)(w + r\frac{1-\beta d}{1-\beta}\frac{\Omega_M}{L} - p_M))$$

$$wL = wF\phi_C^\alpha + waL + L(\beta(w + rd\frac{\Omega_I}{L} - p_I^P) + (1-\beta)(w + r\frac{1-\beta d}{1-\beta}\frac{\Omega_I}{L} - p_I^R))$$

Splitting $waL$ into $\beta waL + (1-\beta)waL$ and rearranging terms, we finally obtain the following two equations defining labor equilibrium in both states:

$$wF\phi_M^\alpha + wF\phi_C^\alpha = \pi_M - r\Omega_M \tag{16}$$

$$wF\phi_C^\alpha = \pi_I - r\Omega_I \tag{17}$$

## 5.2 Steady state analysis

In order to be able to proceed to a steady state analysis, we need to prove the existence (and, if possible, uniqueness) of a stationary distribution for the stochastic process $M(t)$, i.e. for the stochastic equilibrium market structure.

**Proposition 1**: *The market structure $M(t)$ is a Markov process with state space $\{(SC),(SI)\}$, transition rate matrix $Q = \begin{pmatrix} -\phi_M & \phi_M \\ \phi_C & -\phi_C \end{pmatrix}$, and transition probability matrix $P(\Delta t) = I + Q\Delta t$.*

*Proof:* The continuous stochastic process $M(t)$ satisfies the Markov property:

$$P(M(t+\Delta t) = k | M(t) = j, M(t_i) = x_i \forall i) = P(M(t+\Delta t) = k | M(t) = j)$$

with $k, j \in \{(SC),(SI)\}$, $t_0 < t_1 < ... < t_n < t$ and $x_0, ..., x_n \in \{(SC),(SI)\}$. Indeed, the current state of the market structure $M(t)$ contains all the information that is needed to characterize the future stochastic behavior of the process: at a given time $t$, we only need to know the realization of the random variable $M(t)$ to be able to compute the probabilities associated to the possible realizations of $M(t+\Delta t)$. We define as $q_{i,j}$ the probability per time unit that the system makes a transition from state $i$ to state $j$:

$$q_{i,j} = \lim_{\Delta t \to 0} \frac{P(M(t+\Delta t) = j | M(t) = i)}{\Delta t}$$

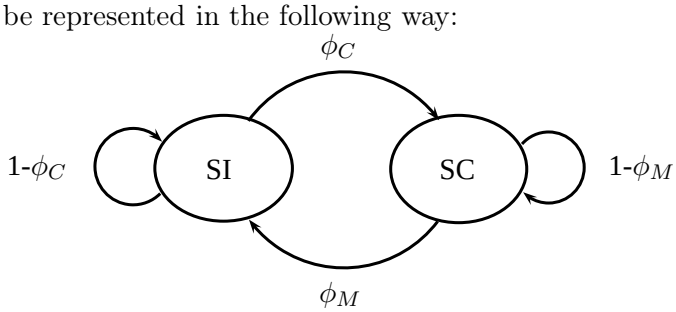
Considering the R&D races described in our model, we have $q_{SC,SI} = \phi_M$ and $q_{SI,SC} = \phi_C$. Indeed, $\phi_M$ corresponds to the immediate probability for the incumbent to innovate when in the (SC) state, while $\phi_C$ corresponds to the immediate aggregate probability for a challenger to innovate, whether it be in the (SC) or the (SI) state.

We define as $q_i$ the total transition rate out of state $i$, and $q_{i,i} = -q_i$.

The transition rate matrix $Q$ of such a Markov process is:

$$Q = \begin{pmatrix} q_{SC,SC} & q_{SC,SI} \\ q_{SI,SC} & q_{SI,SI} \end{pmatrix} = \begin{pmatrix} -\phi_M & \phi_M \\ \phi_C & -\phi_C \end{pmatrix}$$

and the transition probability matrix over time interval $\Delta t$ is $P(\Delta t) = I + Q\Delta t$. Finally, the embedded (discrete time) Markov chain of the continuous time Markov process $M(t)$ can be represented in the following way:




This ends the proof. □

Now that we have determined that $M(t)$ is a Markov process, we still need to prove that it admits a stationary distribution in order to be able to characterize a steady state for our economy.

**Proposition 2**: *The Markov process M(t), describing the market structure, admits a stationary distribution with stationary state probability vector $\pi = \begin{pmatrix} \frac{\phi_C}{\phi_C+\phi_M} & \frac{\phi_M}{\phi_C+\phi_M} \end{pmatrix}$.*

*Proof*: We define the state probability vector $\pi(t)$, being a function of time and evolving as follows: $\frac{d}{dt}\pi(t) = \pi(t) \cdot Q$. The stationary solution $\pi = \lim_{t\to\infty} \pi(t)$ is independant of

time, and thus satisfies $\pi \cdot Q = 0$. Being a probability distribution vector, it also satisfies $\pi \cdot e^T = 1$ with $e$ being a row vector with all elements equal to 1. Defining $E = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, we then have that $\pi \cdot E = e$.

Using all those results together, we have that $\pi(Q + E) = e$, and hence the stationary distribution is obtained by solving for $\pi = e \cdot (Q + E)^{-1}$, provided $Q + E$ is an invertible matrix. We have $Q + E = \begin{pmatrix} 1 - \phi_M & 1 + \phi_M \\ 1 + \phi_C & 1 - \phi_C \end{pmatrix}$. This matrix is indeed invertible, with $(Q + E)^{-1} = \frac{1}{-2\phi_C - 2\phi_M} \begin{pmatrix} 1 - \phi_C & -1 - \phi_M \\ -1 - \phi_C & 1 - \phi_M \end{pmatrix}$. Finally, we obtain $\pi = e \cdot (Q + E)^{-1} = \begin{pmatrix} \frac{\phi_C}{\phi_C + \phi_M} & \frac{\phi_M}{\phi_C + \phi_M} \end{pmatrix}$. This ends the proof. □

The steady state equilibrium is hence defined by the overall wealths in both states $\Omega_M$ and $\Omega_I$, as well as the stationary distribution of $M$, determined by the endogenous transition rates $\phi_M$ and $\phi_C$. As already stated, those transition rates are being determined by the R&D investment decisions of the incumbent (transition from the (SC) to the (SI) state) and the challengers (transition from the (SI) to the (SC) state), taken according to the rewards accruing to successful innovators, $\pi_M$ and $\pi_I$. Such a structure makes it impossible to solve separately for the two states (SC) and (SI): indeed, the reward of innovating a second time in a row (profits $\pi_I$) is taken into account by the incumbent when deciding its investment in R&D in the (SC) state, as shown by equation (10).

The equations defining the steady state hence are: the free-entry equality constraint between expected profits and incurred costs (7), equation (10) stating the relationship between marginal costs in R&D of the incumbent in the (SC) state and the incremental gain in its expected value, equations (12) and (13) expressing the expected value of an innovation in both states as a function of profits, and finally equations (16) and (17) describing equilibrium on the labor market in both states. Putting equations (7) and (12) as well as (10) and (13) together, we are left with the 4 following equations:

$$wF\phi_C^{\alpha-1} = \frac{\pi_M + (\alpha - 1)wF\phi_M^{\alpha}}{r + \phi_C} \tag{18}$$

$$wF\phi_C^{\alpha-1} + \alpha wF\phi_M^{\alpha-1} = \frac{\pi_I}{r + \phi_C} \tag{19}$$

$$wF\phi_M^{\alpha} + wF\phi_C^{\alpha} = \pi_M - r\Omega_M \tag{20}$$

$$wF\phi_C^{\alpha} = \pi_I - r\Omega_I \tag{21}$$

**Proposition 3**: *In the case the R&D cost function takes the quadratic form $\psi(\phi_i) = F\phi_i^2$ and for $(r, k)$ small enough, the system (18)-(21) has a unique solution in $(\phi_C, \phi_M, \Omega_M, \Omega_I)$, all strictly positive.*

*Proof:* cf Appendix A.

Some comments of Proposition 3 are in order. By introducing the possibility for the quality leader monopolist having innovated twice to discriminate among consumers having different levels of wealth, we have provided a rationale for the industry leader to overcome the Arrow (1962) effect and to keep investing in R&D. To the best of our knowledge,

this result is new in the innovation by incumbent literature. Indeed, the strictly positive innovation rate of the incumbent in the (SC) state, $\phi_M$, is here obtained with complete equal treatment in the R&D field between the industry leader and the challengers: we are not allowing for any R&D cost advantage of the incumbent over the followers (Segerstrom, 2007), nor for any sequentiality in the patent races (Etro, 2008).

Our result demonstrates that the incentive can be found on the demand side (quality good market structure), and not only on the supply side (R&D sector characteristics and R&D capabilities of challenger and incumbent firms). To that respect, our model can be compared to the one of Aghion et al. (2001), where the perspective to escape competitive pressure on the product market was driving the incumbent's investment in R&D. In their framework however, each industry is assumed to be duopolistic with respect to R&D as well as production, i.e. only the incumbent monopolist and an outsider participate to patent races. Our paper on the other hand allows for free entry in the R&D races, and sheds light on a totally different demand-sided motive for innovation by incumbent. Indeed, positive investment in R&D by the incumbent is not driven by the motive to escape competition, but solely stems from the positive increment between the two *monopolistic* profits $\pi_M$ and $\pi_I$, i.e. from the possibility to price discriminate.

Such a result, beyond its novelty, enables us furthermore to study the impact of income inequality on the innovation by incumbent rate, as well as on the overall economic growth rate. This is the aim of next section.

# 6 Growth rate and inequality

We now move to studying the impact of income inequality on innovation rates, whether it be of the challengers or the incumbent. Does a wider group of poor people (increasing $\beta$) or a more equal distribution of overall wealth between rich and poor (increasing $d$) increase or lower the innovation rates? We will hence be able to contribute to the existing literature analyzing the relationship between income inequality and growth, along the papers of Zweimuller (2000), Li (2003) or Zweimuller and Brunner (2005). As we will see, the picture obtained once allowing for innovation by incumbent is more complete, and sheds light on some new mechanisms. We first study the effects of inequality on the intensity of investment in R&D of both incumbents and challengers, before analyzing the effect of such shocks on the overall growth rate of the economy.

## 6.1 Wealth distribution and R&D intensity

We first start by studying the effect of a positive shock on $\beta$ on the different endogenous variables in our economy.

**Proposition 4**: *In the case the R&D cost function takes the quadratic form form $\psi(\phi_i) = F\phi_i^2$ and under the parametric conditions allowing for a positive steady state equilibrium to exist, both the incumbent and the challenger R&D intensity decrease along $\beta$: $\frac{\partial \phi_M}{\partial \beta} < 0$ and $\frac{\partial \phi_C}{\partial \beta} < 0$. The overall wealth $\Omega_M$ in the (SC) state increases along $\beta$, while overall weatlh $\Omega_I$ in the (SI) state decreases along $\beta$: $\frac{\partial \Omega_M}{\partial \beta} > 0$ and $\frac{\partial \Omega_I}{\partial \beta} < 0$.*

*Proof:* cf Appendix B

We hence have that for a given fraction $d$ of average wealth per capita being attributed to poor consumers, a higher fraction $\beta$ of the population being poor is detrimental to the

rate of innovation of both the incumbent ($\phi_M$) and the challengers ($\phi_C$).

The analysis of these results revolves around the effect of an increasing $\beta$ on the innovation rewards $v_M$ and $v_I$. We first notice that for given $\Omega_M$ and $\Omega_I$, the profits $\pi_M$ are not impacted by an increase in $\beta$, while profits $\pi_I$ are decreasing.[12] Such a result will obviously impact $\phi_M$, since the increment between profits in the (SC) state and the (SI) state has decreased: the incumbent will hence have a smaller incentive to invest in R&D, and for given $\Omega_M$ and $\Omega_I$ we have that $\phi_M$ drops. Concerning $\phi_C$, we simply need to remember that $v_M$, the reward for innovating once, also includes a part linked to the benefits associated to entering the "private" R&D race only accessible to the incumbent: the term $(\alpha - 1)wF\phi_M^2$ in equation (12). This part of the reward associated to challenger innovation drops along $\pi_I$: the incremental gain linked to charging a higher price to the rich part of the population decreases when the share of those rich people in total population is diminishing. Hence, $\phi_C$ drops as well for an increasing $\beta$.

Concerning the impact of a rising $\beta$ on $\Omega_M$, we consider equation (20), defining the equilibrium constraint on the labor market in the (SC) state. In the case of an increase in $\beta$, less labor is dedicated to the R&D sector, while the amount being dedicated to the production of the quality good is fixed at $aL$ (we indeed impose unit consumption for the quality good). It hence means that the extra labor has been reassigned to the production of the standard good, implying that more of this good is now consumed. Given that the consumption of the standard good is independent from $\beta$ in the (SC) state for a given $\Omega_M$ and increasing in $\Omega_M$, we hence necessarily have that the overall wealth $\Omega_M$ has increased. We hence also have that $\pi_M$ increases, since the latter is not otherwise influenced by an increase in $\beta$. Concerning $\Omega_I$, the intuition is less straightforward. Considering equation (21) defining the labor equilibrium constraint in the (SI) state, we have that for a given $\Omega_I$, the consumption of the standard good increases along $\beta$ (since the profits $\pi_I$ associated to the sales of the quality good drop for a given level of overall wealth). Given the fact that $\phi_C$ drops for a rising $\beta$, labor is simply being reallocated from the R&D sector to the production of the standard good. The direction of variation of $\Omega_I$ is hence not obvious: proposition 4 states that $\Omega_I$ decreases along $\beta$. Hence, the incremental gain $v_I - v_M$ decreases both because of the direct effect of $\beta$ on $\pi_I$ and the generated effect on $\Omega_I$. The extent of this drop counteracts the increase in $\pi_M$, that would have gone in the direction of an increasing $\phi_C$. The overall effect on $\phi_C$ of an increase in $\beta$ remains negative.

Hence, in our model, a shift in the population shares impacts the innovation intensity, and hence the growth rate, even when the equilibrium market structure is a monopoly in the (SC) state. These results can be compared to the ones obtained in the more general literature studying the link between innovation and growth on one hand, and to the ones displayed in Zweimuller and Brunner (2005) on the other hand. In the standard quality-ladder models (Segerstrom et al., 1990; Aghion and Howitt, 1992), the extent of inequalities existing in the distribution of a given overall wealth does not impact the R&D rates, since the presence of homothetic preferences ensures that poorer people will only consume a smaller quantity of the top quality good.[13] In Zweimuller and Brunner (2005), when in the case of a monopoly in the (SC) state, the size of the population share $\beta$ being poor does not influence the innovation rate. Indeed, R&D is only carried out by challengers,

[12] Indeed, for a given $\Omega_M$ we have $\frac{\partial \pi_M}{\partial \beta} = 0$: in the (SC) state the whole population is being charged the same price, and hence the shares of poor and rich people do not matter for $\pi_M$. For a given $\Omega_I$ on the other hand, we have $\frac{\partial \pi_I}{\partial \beta} = -\frac{(k-1)(rd\Omega_I + (1-a)Lw)}{k^2} < 0$.

[13] What influences the equilibrium growth rate however in those models is the size of population $L$, which is a well-identified phenomenon known as the scale effect property. This is however not the effect we have put forward in our analysis.

whose only reward for innovating is the profits $\pi_M$, which are not influenced by $\beta$ for a given level of wealth. In our framework however, challengers also take into account in their R&D investment decision a second gain linked to becoming quality leader, which is the possibility to enter the incumbent private R&D race. If the gains linked to this possibility decrease, which is the case along an increasing $\beta$, we hence have that investment in R&D by both challengers and incumbent is negatively influenced.

**Proposition 5**: *In the case the R&D cost function takes the quadratic form $\psi(\phi_i) = F\phi_i^2$ and under the parametric conditions allowing for a positive steady state equilibrium to exist, the challenger R&D intensity increases along d: $\frac{\partial \phi_C}{\partial d} > 0$. The directions of variation of the incumbent R&D intensity $\phi_M$ as well as the overall wealths $\Omega_M$ and $\Omega_I$ in both possible states are ambiguous.*

*Proof:* cf Appendix B

For given $\Omega_M$ and $\Omega_I$, we have that both $\pi_M$ and $\pi_I$ increase in the case of an increase in $d$,[14] which in turn unambiguously increases the reward for innovating once $v_M$. Indeed, as already pointed out concerning our analysis of the effects of a rising $\beta$, $v_M$ takes into account *both* the profits accruing from being a simple monopolist in the (SC) state and the ones stemming from price discriminating in the (SI) state. Hence, considering equation (18), we get that investment in R&D by the challenger $\phi_C$ increases following this increase in the expected reward. Alternatively, one could have considered the labor market equilibrium constraint (21): we have that since for a given $\Omega_I$ profits $\pi_I$ have increased, less labor is being used to produce the standard good, and the freed workers are reallocated to the R&D sector, thus increasing $\phi_C$.

Concerning $\phi_M$, Proposition 5 states that the direction of its variation is ambiguous. This should not come as a surprise, considering that increasing $\pi_M$ and $\pi_I$ do not determine the sign of the variation of the increment *between* $\pi_M$ and $\pi_I$. Indeed, for given $\Omega_I$ and $\Omega_M$ we have that $\frac{\partial(\pi_I - \pi_M)}{\partial d} = \frac{(k-1)r(\Omega_I(1-\beta)-k\Omega_M)}{k^2}$, whose sign is ambiguous. We however notice that the increment has higher chances to increase for small values of $\beta$ and $k$. As far as $\Omega_M$ and $\Omega_I$ are concerned, their variation is ambiguous as well, as it can be seen from equations (20) and (21). Indeed, considering equation (21) and since the direct effect of $d$ on $\pi_I$ for a given $\Omega_I$ is positive, $\phi_C$ can still increase along $d$ even if $\Omega_I$ decreases. The extent of this decrease cannot however be strong enough to overcome the initial increase in $\pi_I$, otherwise $\phi_C$ would decrease, which is not possible along Proposition 5. Concerning $\Omega_M$, the ambiguous direction of variation of $\phi_M$ makes the analysis even more difficult.

Hence, we have that the effect of a redistribution of wealth from the rich to the poor consumers is always beneficial to the innovation rate $\phi_C$, while it has ambiguous effects on the incumbent innovation rate $\phi_M$. As a whole, we can conclude that diminishing income inequalities are beneficial for the overall economic growth rate, that we now define.

## 6.2 Utility and economic growth rate

Our model displays neither capital accumulation nor productivity improvements. However, consumers become better off due to the successive quality improvements of the quality consumption good.

[14] For given $\Omega_M$ and $\Omega_I$, we indeed have $\frac{\partial \pi_M}{\partial d} = \frac{(k-1)r\Omega_M}{k} > 0$ and $\frac{\partial \pi_I}{\partial d} = \frac{(k-1)(1-\beta)r\Omega_I}{k^2} > 0$.

**Proposition 6**: *The steady state utility growth rate of our economy is* $\gamma = (\ln k)\phi_C(1 + \frac{\phi_M}{\phi_M+\phi_C})$.

*Proof*: Considering equation (2) and the fact that in a given state, the consumption of the standard good c remains constant when at the steady state equilibrium, we have $\gamma_i = \frac{\dot{u}_i(t)}{u_i(t)} = \frac{\dot{q}_i(t)}{q_i(t)}$. In state (SC), we hence have that $\gamma_R = \frac{\dot{q}_R(t)}{q_R(t)} = (\ln k)(\phi_C + \phi_M)$ and $\gamma_P = \frac{\dot{q}_P(t)}{q_P(t)} = (\ln k)(\phi_C)$. Indeed, if the next innovation race is won by a challenger, the latter will sell to both population groups the *unique* quality he has at its disposal, having a quality increment $k$ with respect to the previous quality being consumed by both groups. However, if the next innovation race is won by the incumbent, the latter will sell the highest quality he has at its disposal to the rich consumers, whose utility will indeed increase. He will however keep selling the second-best quality to the poor consumers, whose utility will hence not increase following this quality jump. In state (SI), only challengers carry out R&D, and in the case they win the next innovation race, they will again sell the highest quality to the two groups of consumers. Hence we have that $\gamma_R = (\ln k)\phi_C$, while $\gamma_P = 2(\ln k)\phi_C$. Indeed, the poor consumers were consuming quality $q_{-1}$ before the quality jump. Hence, considering the stationary probability distribution of the market structure, we have that the average utility growth rate of rich consumers is $\gamma_R = (\ln k)((\phi_C + \phi_M)(\frac{\phi_C}{\phi_C+\phi_M}) + \phi_C(\frac{\phi_M}{\phi_C+\phi_M})) = (\ln k)\phi_C(1 + \frac{\phi_M}{\phi_M+\phi_C})$, while the average utility growth rate of poor consumers is $\gamma_P = (\ln k)\phi_C(\frac{\phi_C}{\phi_C+\phi_M}) + 2(\ln k)\phi_C(\frac{\phi_M}{\phi_C+\phi_M}) = (\ln k)\phi_C(1 + \frac{\phi_M}{\phi_M+\phi_C})$. This ends the proof. □

This result can be commented in the light of the literature analyzing price discrimination by a monopolist having at its disposal a product range including different quality levels. The seminal paper in this literature is Mussa and Rosen (1978), in which it is demonstrated that serving costumers who place smaller valuations on quality creates negative externalities for the monopolist, that limit the possibilities for capturing costumer surplus from those who have a higher valuation for quality. In their set-up, the monopolist does have access to a whole product line: he then internalizes the negative externalities by inducing less enthusiastic consumers to buy lower quality items charged at a lower price, opening the possibility of charging higher prices to more adamant buyers of high quality units.

In our model however, the monopolist only has access to as many qualities as R&D races he has won. The negative externalities stemming from having to serve two distinct groups of consumers having different quality valuation is then internalized by expanding the line of product towards *higher* qualities (and not lower qualities as in Mussa and Rosen (1978)), i.e. through R&D investment. Indeed, innovating once more and having at its disposal two successive qualities enables the monopolist to discriminate between rich consumers and poor consumers by offering two different price-quality bundles. Proposition 6 shows that such a mechanism is in favor of a higher global innovation rate, since innovation by incumbent adds itself to the traditional innovation by challengers, which is beneficial for the economy as a whole.

However, the inequalities that are at the root of such a higher incentive to innovate still make the benefits of the innovation process unevenly spread following certain quality jumps. Indeed, in the case the incumbent wins the next innovation race, the poor consumers will not enjoy the new top quality: hence, in our model as in the model of Mussa and Rosen (1978), more than the optimal number of qualities for consumers survive, but only when in the (SI) state. The average steady state growth rate is then the same for the rich

and poor consumers' utility, but only because when switching back to the (SC) state, the increment in quality for the poor consumers is twice as big as the one for the rich consumers.

We have hence demonstrated the existence of a steady state equilibrium with positive investment of the incumbent in R&D races, and analyzed the various effects of income inequality on the growth characteristics of our model. We have however so far restricted ourselves to the case where the market structure in the (SC) state is a monopoly, arguing that this case would necessarily occur for $\beta$ and $d$ sufficiently close to 1. We will now complete the picture and define as well the steady state equilibrium in the case where the market structure in the (SC) case turns out to be a duopoly, in order to check that we indeed have cases where we have $\pi_M > \pi_D^R$.

# 7 Duopoly in the (SC) state

We now define the steady state equilibrium equations in the case we have a **duopoly** market structure in the (SC) state.

*R&D sector.* The costs of conducting R&D remain the same. However, as we have already stated, investment decisions in R&D are taken not only considering the associated costs, but also the rewards in the case of a successful innovation. Since those ones are different if the market structure in the (SC) case is a duopoly, we need to redefine them. We now define as $v_D$ the expected present value of the quality leader in the (SC) state, and $v_I$ as the expected present value of a quality leader having innovated twice in the (SI) state. $v_{CD,i}$ and $v_{CI,i}$ are the associated expected present values of a challenger firm $i$. Again, free entry in the R&D races implies that we have $v_{CD,i} = v_{CI,i} = 0$. Equality between expected profits and engaged costs for the challengers yields the two following equalities, similar to the ones of the monopoly in the (SC) state case:

$$\phi_{CD,i} v_D = wF\phi_{CD,i}^{\alpha} m^{\alpha-1} \tag{22}$$

$$\phi_{CI,i} v_D = wF\phi_{CI,i}^{\alpha} m^{\alpha-1} \tag{23}$$

And rearranging (22) and (23), we get the equality:

$$v_D = wF\phi_{CD}^{\alpha-1} = wF\phi_{CI}^{\alpha-1} \tag{24}$$

Hence, considering equation (24) we get again the result that the innovation rate by challenger is the same in states (SC) and (SI). From now on we will refer to this common rate as $\phi_{CD}$. The two Hamilton-Jacobi-Bellman equations in respectively the (SC) and (SI) state are then of the following form for the incumbent:

$$rv_D = \max_{\phi_D \geq 0} \{\pi_D - wF\phi_D^{\alpha} + \phi_D(v_I - v_D) + \phi_{CD}(v_F - v_D)\} \tag{25}$$

$$rv_I = \max_{\phi_I \geq 0} \{\pi_I - wF\phi_I^{\alpha} + \phi_I(v_I - v_I) + \phi_{CD}(v_F - v_I)\} \tag{26}$$

Indeed, the incumbent in the (SC) state now earns the profits $\pi_D^R$, capturing only the upper part of the quality good market, and incurs the R&D costs $wF\phi_D^{\alpha}$. The incumbent in the (SI) state still earns the profits $\pi_I$ of a monopolist being able to discriminate between rich and poor consumers by offering two distinct price/quantity bundles. A new value $v_F$ is introduced: it corresponds to the expected present value of the second-best quality

seller, having a positive market share in the duopoly case of the (SC) state. This expected present value is equal to the profits made by the second-best quality seller, corrected by the immediate probability to be driven out of the market by either a successful incumbent or a successful challenger. We hence have $v_F = \frac{\pi_D^P}{r+\phi_{CD}+\phi_D}$.

In both states, the incumbent firm chooses its R&D effort so as to maximize the right-hand side of its Bellman equation. Using the fact that $v_{CD,i} = v_{CI,i} = 0$, equations (25) and (26) then yield the following first order conditions:

$$v_I - v_D = \alpha w F \phi_D^{\alpha-1} \tag{27}$$

$$-\alpha w F \phi_I^{\alpha-1} = 0 \quad \Rightarrow \quad \phi_I = 0 \tag{28}$$

Using the optimality constraints (27) and (28) in (25) and (26), we obtain the following expressions for the expected values $v_D$ and $v_I$:

$$v_D = \frac{\pi_D^R + (\alpha-1)wF\phi_D^\alpha + \phi_{CD}v_F}{r+\phi_{CD}} \tag{29}$$

$$v_I = \frac{\pi_I + \phi_{CD}v_F}{r+\phi_{CD}} \tag{30}$$

Hence, compared to the case where the market structure in the (SC) state is a monopoly, we here have a further term on the RHS of the equalities (corresponding to the expected profits): the perspective of still having a positive market share as the second-best quality producer. As pointed out by Zweimuller and Brunner (2005), the effect on the R&D investment is not obvious.

*Labor market.* The two equations describing the equilibrium on the labor market in respectively the (SC) and the (SI) state are now of the form:

$$L = F\phi_D^\alpha + F\phi_{CD}^\alpha + aL + (L/w)(\beta(y_D^P - p_D^P) + (1-\beta)(y_D^R - p_D^R)) \tag{31}$$

$$L = F\phi_{CD}^\alpha + aL + (L/w)(\beta(y_I^P - p_I^P) + (1-\beta)(y_I^R - p_I^R)) \tag{32}$$

Again, it proves convenient to express (29) and (30) in terms of profit flows in both states, i.e. in terms of $\pi_D^P$, $\pi_D^R$ and $\pi_I$. Multiplying both sides by $w$, replacing $y_D^P$, $y_D^R$, $y_I^P$ and $y_I^R$ by their values expressed in Section 3, splitting $waL$ into $\beta waL + (1-\beta)waL$ and finally rearranging terms, (31) and (32) yield:

$$wF\phi_D^\alpha + wF\phi_{CD}^\alpha = \pi_D^R + \pi_D^P - r\Omega_D \tag{33}$$

$$wF\phi_{CD}^\alpha = \pi_I - r\Omega_I \tag{34}$$

*Stochastic long-run equilibrium.* Propositions 1 and 2 can be reformulated the following way in the duopoly case:

**Proposition 7**: *In the case the equilibrium market structure is a duopoly in the (SI) state, the market structure $M(t)$ is a Markov process with state space $\{(SC),(SI)\}$, transition rate matrix $Q_D = \begin{pmatrix} -\phi_D & \phi_D \\ \phi_{CD} & -\phi_{CD} \end{pmatrix}$, and transition probability matrix $P_D(\Delta t) =$*

*$I + Q_D\Delta t$. It admits a stationary distribution with stationary state probability vector* $\pi_D = \begin{pmatrix} \frac{\phi_{CD}}{\phi_{CD}+\phi_D} & \frac{\phi_D}{\phi_{CD}+\phi_D} \end{pmatrix}$

The 4 endogenous variables describing the steady state equilibrium are now the innovation rates $\phi_D$ and $\phi_{CD}$, as well as the overall wealth $\Omega_D$ in the (SC) state and $\Omega_I$ in the (SI) state. The equations defining the steady state equilibrium are now the following ones:

$$wF\phi_{CD}^{\alpha-1} = \frac{\pi_D^R + (\alpha-1)wF\phi_D^\alpha + \phi_{CD}\frac{\pi_D^P}{r+\phi_D+\phi_{CD}}}{r+\phi_{CD}} \tag{35}$$

$$wF\phi_{CD}^{\alpha-1} + \alpha wF\phi_D^{\alpha-1} = \frac{\pi_I + \phi_{CD}\frac{\pi_D^P}{r+\phi_D+\phi_{CD}}}{r+\phi_{CD}} \tag{36}$$

$$wF\phi_D^\alpha + wF\phi_{CD}^\alpha = \pi_D^P + \pi_D^R - r\Omega_D \tag{37}$$

$$wF\phi_{CD}^\alpha = \pi_I - r\Omega_I \tag{38}$$

Those expressions unfortunately make any analytical result concerning the existence and uniqueness of a steady state equilibrium with $(\phi_{CD}, \phi_D, \Omega_D, \Omega_I)$ all strictly positive hard to define. Indeed, even with quadratic R&D costs of the form $\psi(\phi_i) = F\phi_i^2$, the degrees of the polynomials that need to be solved have increased because of the presence of the term $\frac{\pi_D^P}{r+\phi_D+\phi_{CD}}$, expressing the fact that an incumbent that has lost its quality leadership gets anyway to survive during one more innovative race, selling quality $q_{-1}$ to the group of poor consumers.

Hence, providing results concerning the existence of a steady state equilibrium and the various effect of income inequalities on growth could only be done through simulations in the case the equilibrium market structure is a duopoly in the (SC) state, and without any rigorous analytical result. Since the aim of our paper was to shed light on a new mechanism possibly driving a positive investment in R&D by the incumbent, which has analytically been done in the dominant case of a monopoly market structure in the (SC) state, we hence stop our analysis here.

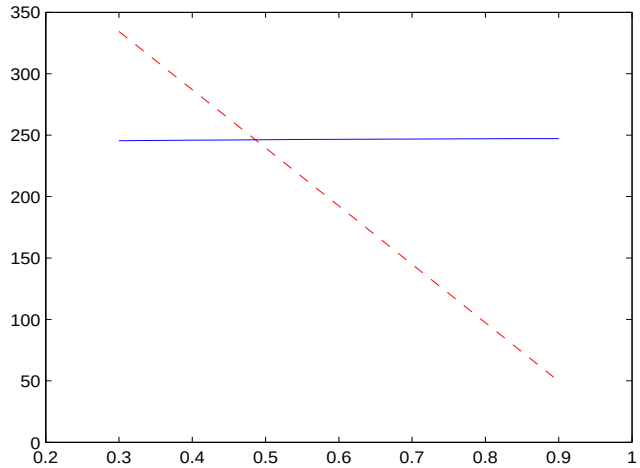
We however need to check that the duopoly case is not the dominant one, i.e. that there are indeed parametric cases for which we have $\pi_M > \pi_D^R$. We hence proceed to simulations in the case of R&D quadratic costs. As already stated, the decisive parameters that will determine the equilibrium market structure in the (SC) state are $d$ and $\beta$, defining the extent of inequalities in the wealth distribution. We hence focus on the effects of different values taken by those parameters on $\pi_M - \pi_D^R$, and check whether this difference is positive for some parametric cases.

We hence proceed to simulations for $\beta$ varying from $0,3$ to 1 and $d$ varying from $0,1$ to 1. We also proceed to a sensitivity analysis along different values of $F$, $k$, $a$ and $r$. Our numerical findings are the following:

*Numerical finding 1*: under a wide array of parametric cases, it is indeed possible to obtain a positive equilibrium for *both* the monopoly and duopoly cases.

*Numerical finding 2*: for almost every value of $\beta$ and $d$ for which a positive steady state equilibrium indeed exists, we have $\pi_M > \pi_D^R$, making Propositions 4 to 6 valid for a wide array of parametric cases.

Figure 1: Profits $\pi_M$ and $\pi_D^R$ in the (SC) state for $\beta$ varying from $0,3$ to $0,9$




Dashed line: profits $\pi_D^R$, Full line: profits $\pi_M$. Values for the other parameters: $r = 0,1$, $L = 200$, $a = 0,1$, $F = 50$, $d = 0,5$, $w = 10$, $k = 1,1$

Indeed, we find that if the duopoly case occurs, it is never for higher values of $\beta$ than $0,4$. For a given $\beta$, varying values of $d$ only matter in the sign of $\pi_M - \pi_D^R$ for very high values of $a$ ($a > 0,9$). Otherwise, for any wealth repartition between poor and rich consumers, it is the share of each population group that will determine the dominant case in the (SC) state. Hence, as soon as the share of poor people in the overall population is equal or superior to 1/2 and under the parametric conditions for which a positive steady state equilibrium exists, the equilibrium market structure in the (SC) state is a monopoly.

# 8 Conclusion

Our paper has presented a new, demand-driven rationale for innovation by incumbents. By allowing, through the feature of non-homothetic preferences, for the possibility of price discrimination of consumers having different levels of wealth, we have provided a sufficient incentive for the industry leader to overcome the Arrow (1962) effect and keep investing in R&D. The strictly positive innovation rate of the incumbent is here obtained without any advantage of the incumbent in the R&D field (supply side), but through income inequalities and the generated different quality valuation of poor and rich consumers (demand side). Beyond its novelty, this framework has also allowed us to analyze the effect of the extent of income inequalities on both the challenger and incumbent innovation rates, and hence on the growth rate of our economy. We have demonstrated that a higher share of the population being poor is detrimental to the rate of economic growth. A redistribution of wealth from rich to poor consumers is on the other hand beneficial for the challenger innovation rate, while it has ambiguous effects on the incumbent innovation rate.

A possible objection to our results would be that although positive, the incumbent R&D rate we obtain is in most simulated cases significantly lower than the challenger R&D rate. Such results are at odds with the ones of Etro (2008), that has found that patentholders carry out a major bulk of the overall R&D investment. However, our aim in this paper was to *isolate* a possible demand-driven incentive for incumbents to invest in

R&D, which is not incompatible at all with other mechanisms put forward in papers such as the ones of Segerstrom (2007), Etro (2008) or Aghion et al. (2001). Indeed, the fact that incumbents behave as Stackelberg leaders or have an advantage in the R&D process seems highly probable to us.

Some lines of further work can be quickly sketched. An obvious extension to our model would be to treat the more general case of more than two types of consumers, in order for the incumbent to keep investing in R&D after the second successful race.[15] A model such as ours can also be extended to a two-country framework, in order to contribute to the developing literature studying the role of multi-product firms in international trade (Brambilla, 2009; Fajgelbaum et al., 2009): an increasing $\beta$ could then be perceived as a globalization shock, and one could study its effects on the contribution of quality leaders to the economic growth rate. Finally, our modeling approach can also prove itself a useful tool in the durable good literature: it can be considered as a way of endogenizing the R&D decision of the seller of a durable good (Fischman and Rob, 2000; Nahm, 2004).

[15]Indeed, as already pointed out, the null investment in R&D by the incumbent in the (SI) state solely stems from the fact that we have only two distinct groups of consumers: once having offered two distinct price-quality bundles, the incumbent does not have any incentive to keep carrying out R&D.

# 9 Appendix A

In order to ease up notations in the rest of the proof, we define the following terms:

$\pi_M = A + B\Omega_M$ with $A = \left(\frac{k-1}{k}\right)(1-a)wL$ and $B = \left(\frac{k-1}{k}\right)dr$.

$\pi_I = C + D\Omega_I$ with $C = \left(\frac{k-1}{k^2}\right)(k+1-\beta)(1-a)wL$ and $D = \left(\frac{k-1}{k^2}\right)r(k+(1-\beta)d)$.

Considering the stationarized steady state Markovian equilibrium defined by equations (18) to (21), it is possible to express $\Omega_M$ and $\Omega_I$ as functions of $\phi_I$ and $\phi_M$ using equations (20) and (21). We hence obtain:

$$\Omega_M = \frac{A - wF(\phi_M^2 + \phi_C^2)}{r - B}$$

$$\Omega_I = \frac{C - wF\phi_C^2}{r - D}$$

Substituting back in equations (18) and (19), the equilibrium is now defined by the two following equations:

$$wF\phi_C(r+\phi_C) = A + B\left(\frac{A - wF(\phi_M^2+\phi_C^2)}{r-B}\right) + wF\phi_M^2 \tag{39}$$

$$(wF\phi_C + 2wF\phi_M) = C + D\left(\frac{C - wF\phi_C^2}{r-D}\right) \tag{40}$$

Rearranging the terms, we define two equations $G(\phi_M, \phi_C) = 0$ and $H(\phi_M, \phi_C) = 0$ defining the equilibrium of our model:

$$G(\phi_M,\phi_C) = -\frac{Ar}{r-B} + Frw\phi_C + \left(\frac{Frw}{r-B}\right)\phi_C^2 + \left(\frac{F(2B+r)w}{B-r}\right)\phi_M^2 = 0$$

$$H(\phi_M,\phi_C) = -\frac{Cr}{r-D} + Frw\phi_C + \left(\frac{Frw}{r-D}\right)\phi_C^2 + 2Fw\phi_C\phi_M + 2Fwr\phi_M = 0$$

Noticing that $r - B = r(1 - \left(\frac{k-1}{k}\right)d) > 0$ and $r - D = r(1 - \left(\frac{k-1}{k^2}\right)(k + (1-\beta)d)) > 0$ for $d < 1$, we have the following inequalities holding for $\phi_M > 0$ and $\phi_C > 0$:

$$\frac{\partial G}{\partial \phi_C} = Frw + \frac{2Frw}{r-B}\phi_C \quad > 0$$

$$\frac{\partial G}{\partial \phi_M} = -2\frac{F(2B+r)w}{r-B}\phi_M \quad < 0$$

$$\frac{\partial H}{\partial \phi_C} = Frw + 2\frac{Frw}{r-D} + 2wF\phi_M \quad > 0$$

$$\frac{\partial H}{\partial \phi_M} = 2wF\phi_C + 2wFr > 0$$

Using the implicit function theorem, we have the following expression and sign for $\frac{\partial \phi_M}{\partial \phi_C}$ in G:

$$\frac{\partial \phi_M}{\partial \phi_C} = -\frac{\partial G/\partial \phi_M}{\partial G/\partial \phi_C} = -\frac{2\frac{F(2B+r)w}{B-r}\phi_M}{Frw + \frac{2Frw}{r-B}\phi_C} > 0 \quad \text{for} \quad \phi_M, \phi_C > 0$$

and the following expression and sign for $\frac{\partial \phi_M}{\partial \phi_C}$ in H:

$$\frac{\partial \phi_M}{\partial \phi_C} = -\frac{\partial H/\partial \phi_M}{\partial H/\partial \phi_C} = -\frac{2wF\phi_C + 2wFr}{Frw + 2\frac{Frw}{r-B} + 2wF\phi_M} < 0 \quad \text{for} \quad \phi_M, \phi_C > 0$$

Hence, *provided* those two curves intersect in the plane $(\phi_M, \phi_C)$ for $\phi_M > 0$ and $\phi_C > 0$, it will be a unique intersection. *We have established that if a positive equilibrium exists, it is unique.*

We now are left to prove that those two curves indeed intersect for $\phi_M, \phi_C > 0$, and that for those values taken by $\phi_C$ and $\phi_M$ we have positive corresponding values for $\Omega_M$ and $\Omega_I$. We hence need to establish that:

1. Under certain parametric conditions, the $\phi_M$ for which $G(\phi_M, 0) = 0$ is inferior to the solution to $H(\phi_M, 0) = 0$,

2. Under certain compatible parametric conditions, the $\phi_C$ for which $G(0, \phi_C) = 0$ is inferior to the solution to $H(0, \phi_C) = 0$.

1. We start by solving for $G(0, \phi_C) = 0$ and $H(0, \phi_C) = 0$:

$$G(0,\phi_C) = -\frac{Ar}{r-B} + Frw\phi_C + \frac{Frw}{r-B}\phi_C^2 \Longleftrightarrow \phi_C = \frac{(r-B)(-Frw \pm \sqrt{(Frw)^2 + 4\left(\frac{FAr^2w}{(r-B)^2}\right)})}{2Frw}$$

$$H(0,\phi_C) = -\frac{Cr}{r-D} + Frw\phi_C + \frac{Frw}{r-D}\phi_C^2 \Longleftrightarrow \phi_C = \frac{(r-D)(-Frw \pm \sqrt{(Frw)^2 + 4\left(\frac{FCr^2w}{(r-D)^2}\right)})}{2Frw}$$

And we are left to compare the two solutions that are possibly positive:

$$\phi_C \mid_{G(0,\phi_C)=0} = \frac{(r-B)(\sqrt{(Frw)^2 + 4\left(\frac{FAr^2w}{(r-B)^2}\right)} - Frw)}{2Frw}$$

$$\phi_C \mid_{H(0,\phi_C)=0} = \frac{(r-D)(\sqrt{(Frw)^2 + 4\left(\frac{FCr^2w}{(r-D)^2}\right)} - Frw)}{2Frw}$$

Rearranging the terms, we have:

$$\phi_C \mid_{H(0,\phi_C)=0} - \phi_C \mid_{G(0,\phi_C)=0} =$$

$$\underbrace{(D-B)Frw}_{>0} + \sqrt{(Frw(r-D))^2 + 4FwCr^2} - \sqrt{(Frw(r-B))^2 + 4FwAr^2}$$

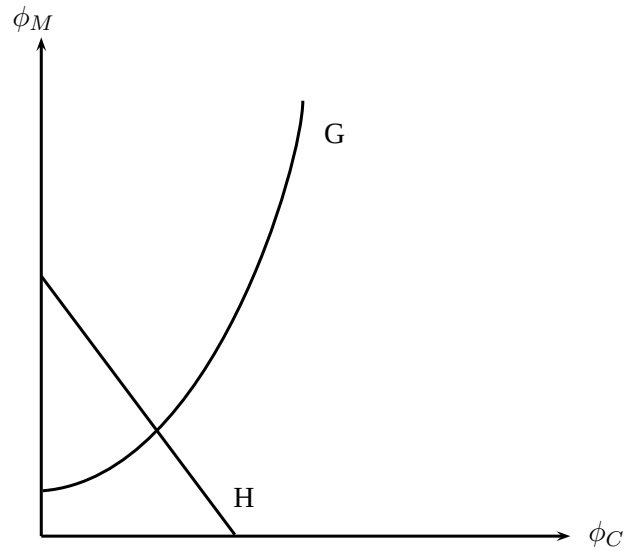
We have $r < 1$, and for $r$ small enough the terms $4FwCr^2$ and $4FwAr^2$ dominate the terms $(Frw(r-D))^2$ and $(Frw(r-B))^2$. Since $C > A$, we hence have that $\phi_C \mid_{H(0,\phi_C)=0} - \phi_C \mid_{G(0,\phi_C)=0} > 0$.

2. We then move on by solving for $G(\phi_M, 0) = 0$ and $H(\phi_M, 0) = 0$:

$$H(\phi_M, 0) = -\frac{Cr}{r-D} + 2wFr\phi_M = 0 \Longleftrightarrow \phi_M = \left(\frac{Cr}{r-D}\right)\left(\frac{1}{2wFr}\right)$$

$$G(\phi_M, 0) = -\frac{Ar}{r-B} - \frac{F(2B+r)w}{r-B}\phi_M^2 = 0 \Longleftrightarrow \phi_M^2 = -\frac{Ar}{F(2B+r)w}, \quad \text{impossible}$$

Hence, for $r$ small enough, G and H are of the following form in the plane $(\phi_M, \phi_C)$:




We are now left to determine the sign of $\Omega_M$ and $\Omega_I$ under those parametric conditions. The sign of those two depends on the sign of the expressions $A - wF\phi_C^2 - wF\phi_M^2$ and $C - wF\phi_C^2$. Given the fact we are not able to analytically solve for the values of $\phi_C$ and $\phi_M$, the sign of those expressions is a priori ambiguous. Those expressions will however be positive if $\phi_C$ and $\phi_M$, which are the immediate probabilities of innovating of the challengers and the incumbent, are small enough. We know that investment in R&D decreases along $k$. Indeed, $k$ denotes the quality increment when successfully innovating: a smaller $k$ decreases the incentives to invest in R&D, which in turn decreases the immediate probability to successfully innovate. As $k$ becomes closer to 1, $\phi_C$ and $\phi_M$ will decrease: $wF\phi_C^2$ and $wF\phi_M^2$ hence become negligible compared to $(\frac{k-1}{k})(1-a)wL$, since we assumed that $L > F$.

We have hence demonstrated that for $(r, k)$ small enough, there exists a unique, positive equilibrium to the system (18)-(21) in $(\phi_C, \phi_M, \Omega_M, \Omega_I)$, all strictly positive.

# 10 Appendix B

## 10.1 Comparative statics with respect to $\beta$

*Direction of variation of $\phi_M$ and $\phi_C$.* Using the implicit function theorem, we have:

$$\begin{pmatrix} \frac{\partial \phi_C}{\partial \beta} \\ \\ \frac{\partial \phi_M}{\partial \beta} \end{pmatrix} = -\begin{pmatrix} \frac{\partial G}{\partial \phi_C} & \frac{\partial G}{\partial \phi_M} \\ \\ \frac{\partial H}{\partial \phi_C} & \frac{\partial H}{\partial \phi_M} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial G}{\partial \beta} \\ \\ \frac{\partial H}{\partial \beta} \end{pmatrix}$$

which yields:

$$\begin{pmatrix} \frac{\partial \phi_C}{\partial \beta} \\ \\ \frac{\partial \phi_M}{\partial \beta} \end{pmatrix} = -\frac{1}{\frac{\partial G}{\partial \phi_C}\frac{\partial H}{\partial \phi_M} - \frac{\partial G}{\partial \phi_M}\frac{\partial H}{\partial \phi_C}} \begin{pmatrix} \frac{\partial G}{\partial \beta}\frac{\partial H}{\partial \phi_M} - \frac{\partial G}{\partial \phi_M}\frac{\partial H}{\partial \beta} \\ \\ -\frac{\partial H}{\partial \phi_C}\frac{\partial G}{\partial \beta} + \frac{\partial G}{\partial \phi_C}\frac{\partial H}{\partial \beta} \end{pmatrix}$$

and we have:

$$\frac{\partial G}{\partial \beta} = 0$$

$$\frac{\partial H}{\partial \beta} = \frac{wk(k-1)((1-a)(1+d(k-1))L - dFk\phi_C^2)}{(k-d(k-1)(1-\beta))^2}$$

According to the signs defined in Appendix B, we know that $-\frac{1}{\frac{\partial G}{\partial \phi_C}\frac{\partial H}{\partial \phi_M} - \frac{\partial G}{\partial \phi_M}\frac{\partial H}{\partial \phi_C}} < 0$. We are left to study the sign of $\frac{\partial H}{\partial \beta}$, which itself depends on the sign of $(1-a)(1-d+dk)L - dFk\phi_C^2$. According to the labor market equilibrium condition, we know that we necessarily have $(1-a)L > F\phi_C^2$ (i.e. the available labor force once having taken out the $aL$ fraction devoted to the production of the quality good is strictly superior to the amount of labor hired by the challengers to carry out R&D if we have a strictly positive investment in R&D by the incumbent). It hence follows that we have $(1-a)(1-d+dk)L - dkF\phi_C^2 > dk(1-a)L - dkF\phi_C^2 > 0$. We hence have that $\frac{\partial H}{\partial \beta} > 0$.

Considering the signs defined in Appendix A, we can then conclude that $\frac{\partial \phi_M}{\partial \beta} < 0$ and $\frac{\partial \phi_C}{\partial \beta} < 0$.

*Direction of variation of* $\Omega_M$. Considering equation (20), i.e. the labor market equilibrium constraint in the (SC) state, we see that the left-hand side decreases following a positive shock on $\beta$. $\beta$ does not appear on the right-hand side of this equation, which is decreasing along $\Omega_M$. Hence, in order to re-establish the equality, $\Omega_M$ needs to increase: $\frac{\partial \Omega_M}{\partial \beta} > 0$.

*Direction of variation of* $\Omega_I$. Considering equation (21), we have that $\frac{\partial \Omega_I}{\partial \beta} = (-\frac{k-1}{k^2}(1-a)wL - 2wF\phi_C\frac{\partial \phi_C}{\partial \beta})(r-D) - dr\left(\frac{k-1}{k^2}\right)(C - wF\phi_C^2)$. The second term is negative, while the sign of the first term depends on the sign of $-\frac{k-1}{k^2}(1-a)wL - 2wF\phi_C \underbrace{\frac{\partial \phi_C}{\partial \beta}}_{<0}$. Considering the detailed expression obtained for $\frac{\partial \phi_C}{\partial \beta}$, we have that the second term of this difference is negligible, which means that the whole expression is negative. We hence obtain that $\frac{\partial \Omega_I}{\partial \beta} < 0$.

## 10.2 Comparative statics with respect to $d$

Using the implicit function theorem, we have:

$$\begin{pmatrix} \frac{\partial \phi_C}{\partial d} \\ \\ \frac{\partial \phi_M}{\partial d} \end{pmatrix} = -\begin{pmatrix} \frac{\partial G}{\partial \phi_C} & \frac{\partial G}{\partial \phi_M} \\ \\ \frac{\partial H}{\partial \phi_C} & \frac{\partial H}{\partial \phi_M} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial G}{\partial d} \\ \\ \frac{\partial H}{\partial d} \end{pmatrix}$$

which yields:

$$\begin{pmatrix} \frac{\partial \phi_C}{\partial \beta} \\ \\ \frac{\partial \phi_M}{\partial \beta} \end{pmatrix} = -\frac{1}{\frac{\partial G}{\partial \phi_C}\frac{\partial H}{\partial \phi_M} - \frac{\partial G}{\partial \phi_M}\frac{\partial H}{\partial \phi_C}} \begin{pmatrix} \frac{\partial G}{\partial d}\frac{\partial H}{\partial \phi_M} - \frac{\partial G}{\partial \phi_M}\frac{\partial H}{\partial d} \\ \\ -\frac{\partial H}{\partial \phi_C}\frac{\partial G}{\partial d} + \frac{\partial G}{\partial \phi_C}\frac{\partial H}{\partial d} \end{pmatrix}$$

and we have:

$$\frac{\partial G}{\partial d} = \frac{(k-1)w(Fk(\phi_C^2 + \phi_M^2) - (1-a)(k-1)L)}{(d+k(1-d))^2}$$

$$\frac{\partial H}{\partial d} = \frac{(k-1)w(1-\beta)(Fk^2\phi_C^2 - (1-a)(k-1)L(k+1-\beta))}{(k-d(k-1)(1-\beta))^2}$$

The sign of those two derivatives is necessarily negative when in the parameter conditions allowing for a strictly positive solution to the system (18)-(21). Indeed, $\Omega_M > 0$ implies that $(k-1)(1-a)wL - wF(\phi_M^2 + \phi_C^2) > 0$ and $(k-1)(k+1-\beta)(1-a)wL - wk^2F\phi_C^2 > 0$.

We then unambiguously have $\frac{\partial \phi_C}{\partial d} > 0$. The sign of $\frac{\partial \phi_M}{\partial d}$ is however ambiguous, and depends on the relative size of $-\frac{\partial H}{\partial \phi_C}\frac{\partial G}{\partial d} > 0$ and $\frac{\partial G}{\partial \phi_C}\frac{\partial H}{\partial d} < 0$.

The sign of the variations of $\Omega_I$ and $\Omega_M$ is ambiguous as well. For given $\phi_M$ and $\phi_C$, we have that both $\Omega_M$ and $\Omega_I$ increase along $d$, since the numerator of their expression obtained in Appendix A decreases. However, the fact that $\frac{\partial \phi_C}{\partial d} > 0$ counteracts this direct increase. The overall variation of $\Omega_I$ depends on the relative strength of those two effects. We indeed have that $\frac{\partial \Omega_I}{\partial \beta} = \underbrace{(-2wF\phi_C\frac{\partial \phi_C}{\partial d})(r-D)}_{<0} + \underbrace{\left(\frac{k-1}{k^2}\right)r(1-\beta)(C - wF\phi_C^2)}_{>0}$. The sign of the overall variation of $\Omega_M$ is even more ambiguous, given the fact that the sign of both $\frac{\partial \phi_C}{\partial d}$ and $\frac{\partial \phi_M}{\partial d}$ matters.

## References


**Aghion, Philippe and Peter Howitt**, "A Model of Growth through Creative Destruction," *Econometrica*, 1992, *50*, 323–351.

**_ , Christopher Harris, Peter Howitt, and John Vickers**, "Competition, Imitation and Growth with Step-by-step Innovation," *Review of Economic Studies*, 2001, *68*, 467–492.

**Arrow, Kenneth**, *Economic Welfare and the Allocation of Resources for Invention*, in Nelson, 1962.

**Brambilla, Irene**, "Multinationals, Technology, and the Introduction of Varieties of Goods," *Journal of International Economics*, 2009, *79*, 89–101.

**Cozzi, Guido**, "The Arrow Effect under Competitive R&D," *The B.E. Journal of Macroeconomics*, 2007, *7* (1).

**Etro, Federico**, "Innovation by Leaders," *The Economic Journal*, 2004, *114* (495), 281–310.

_ , "Growth Leaders," *Journal of Macroeconomics*, 2008, *30* (3), 1148–72.

**Fajgelbaum, Paolo, Gene M. Grossman, and Elhanan Helpman**, "Income Distribution, Product Quality and International Trade," *NBER Working Paper Series*, 2009, *15329.*

**Fischman, Arthur and Rafael Rob**, "Product Innovation by a Durable-Good Monopoly," *RAND Journal of Economics*, 2000, *31* (2), 237–252.

**Gabszewicz, Jaskold J. and J.-F. Thisse**, "Price Competition, Quality and Income Disparities," *Journal of Economic Theory*, 1979, *22*, 327–338.

**_ and _** , "Entry (and Exit) in a Differentiated Industry," *Journal of Economic Theory*, 1980, *22*, 327–338.

**Grossman, Gene M. and Elahan Helpman**, "Endogenous Product Cycles," *The Economic Journal*, 1991, *101*, 1214–1229.

**_ and _** , "Quality Ladders and Product Cycles," *Quarterly Journal of Economics*, 1991, *106*, 557–586.

**Li, Chol-Won**, "Income Inequality, Product Market and Schumpeterian Growth," 2003.

**Mussa, Michael and Sherwin Rosen**, "Monopoly and Product Quality," *Journal of Economic Theory*, 1978, *18*, 301–317.

**Nahm, Jae**, "Durable Goods Monopoly with Endogenous Innovation," *Journal of Economics and Management Strategy*, 2004, *13* (2), 303–319.

**Salop, Steven and Joseph Stiglitz**, "Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion," *Review of Economic Studies*, 1977, *44* (138), 465–492.

**Segerstrom, Paul S.**, "Intel Economics," *International Economic Review*, 2007, *48* (1), 247–280.

— **and James M. Zolnierek**, "The R&D Incentives of Industry Leaders," *International Economic Review*, 1999, *40* (3), 745–766.

**Segerstrom, Paul, T. Anant, and Elias Dinopoulos**, "A Schumpeterian Model of the Product Life Cycle," *American Economic Review*, 1990, *80*, 1077–1091.

**Shaked, Avner and John Sutton**, "Relaxing Price Competition through Product Differentiation," *Review of Economic Studies*, 1982, *49*, 3–13.

— **and** — , "Natural Oligopolies," *Econometrica*, 1983, *51* (5), 1469–1483.

**Stokey, Nancy L.**, "Intertemporal Price Discrimination," *Quarterly Journal of Economics*, 1979, *93* (3), 355–371.

**Zweimuller, Josef**, "Schumpeterian Entrepreneurs Meet Engel's Law: the Impact of Inequality on Innovation-Driven Growth," *Journal of Economic Growth*, 2000, *5* (2), 185–206.

— **and Johan Brunner**, "Innovation and Growth with Rich and Poor Consumers," *Metroeconomica*, 2005, *56* (2), 233–262.

— **and Reto Foellmi**, "Income Distribution and Demand-Induced Innovations," *Review of Economic Studies*, 2006, *73*, 941–960.

Institut de Recherches Économiques et Sociales
Université catholique de Louvain

Place Montesquieu, 3
1348 Louvain-la-Neuve, Belgique

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