A Model of Ideological Transmission with Endogenous Paternalism

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Abstract

We develop a dynamic framework of ideological evolution in a two-trait population of individuals with perfect foresight. We model how children are educated to a specific ideological trait, liberal or traditional, which later in life will influence their level of economic activity and therefore the well-being of the family. Our aim is to study the dynamics of ideological traits when an exchange matching process takes place. We show that the ideological distance between groups, namely the *taste for similarity* within the family, determines the long-run distribution of traits as well as the intertemporal parental behaviour in the intergenerational transmission process. Compared to the existing research on cultural transmission, the singularity of our model appears through the situation in which parents' paternalism in children's education is a necessary but not a sufficient condition to guarantee diversity or the preservation of heterogeneity in the long-run distribution of traits. In particular, our model supports the possibility of a reversal in the parental evaluation of traits and allow us to understand the changes in parents' behaviour over time, showing why, in particular contexts, it has changed from ideologically protective to non-protective. When the opportunity cost of having children with the same ideology is too high, altruistic parents can behave in a non-paternalistic way. Assuming myopic agents does not change the qualitative results of the model; however, paternalism persists for longer than in perfect foresight case.

Key Words: Ideological transmission, Taste for similarity, Paternalism, Diversity.

JEL classification: D10, J19, Z13.

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1 Introduction

The achievement of social norms, cultural paradigms, religious beliefs and moral customs is crucial in the characterization of societies. Anthropologists, sociologists and economists have long been interested in explaining the success of certain groups and the disappearance of others during their intergenerational evolution[1]. The groups' affiliation to different ideological paradigms is another important element that governs individuals' behaviors, because it determines the intensity of the effort made to preserve values in social life.

The concept of ideology is controversial in social science and myriads of definitions have been given. In this study we consider ideology as *the process of implementation and preservation of beliefs, norms, and values in social life by different groups that characterize the individuals' behaviors*[2]. We are not interested in the process of formation or in the normative evaluation of different ideologies, but we concentrate on the way in which ideologies are transmitted through generations, and the inter-temporal evolution of families' socialization behaviors. These two factors govern the preservation or the disappearance of ideologies in societies. In our opinion, both the perception of the social relations with members of the same or different ideological group, and the behaviour of families in the parent-to-children transmission process, play an important role in explaining the evolution of traits.

History presents several examples of social groups that have remained attached to their own ideological traits, but also examples of other groups which have gradually accepted the principles behind different ideological schemes. The Jewish culture is a typical example of the striking persistence of the conflict between agents of the same cultural group who have adopted an orthodox or a moderate paradigm. The conflict in Waziristan[3], the electoral success of Hamas in the Gaza strip and the Basque and Catalan extremists' claims, give evidence of the existence of ideological conflicts inside homogeneous ethnic groups.

The purpose of this paper is to study the dynamics of beliefs in a model that combines ideological and socio-economic factors, showing that diversity (that is a heterogeneous distribution of traits) is not guaranteed, even though parents are biased towards their own ideological beliefs. More formally, we want to show that the formal assumption that parents are willing to have children with the same trait is a necessary but not a sufficient condition for diversity in the long-run. This, in our opinion, is very important because it allows

[1] From an anthropological and sociobiological point of view this topic has been studied in the seminal works of Cavalli-Sforza and Feldman (1981) and Boyd and Richerson (1985). In the field of economics, the process of intergenerational transmission has been motivated by the evidence of the persistence of cultural diversity reported by Borjas (1995). Moreover, it has been theoretically modeled by Bisin and Verdier (1998, 2001) with a powerful framework that has been applied in different contexts.

[2] This definition is close to the formulation established by Hall (1986) who defines ideology as *'the mental frameworks - the languages, the concepts, categories, imagery of thought, and the system of representation - which different classes and social groups deploy in order to make sense of, define, figure out and render intelligible the way society works'*.

[3] The Waziristan conflict is the war between Pakistan and the Waziri tribes, which began in 2004.

for a reversal of the parental assessment of ideological beliefs during the intergenerational socialization process. In other words, the framework developed in this paper implies that diversity must be driven by parental demand for ideological pluralism, but also by the fact that this demand for pluralism does not necessarily guarantee a heterogeneous distribution of traits in the long-run.

The basic idea behind our model is that ideology can be viewed as a reliable signal of the relative trustworthiness of the exchange partners, and it might be conducive to different social and economic behaviors in society. Given that the value of any social interaction is determined by the agents' calculation of the costs and rewards of that interaction, people choose the group that provides them the maximum number of valued rewards and the smallest costs, that is, the group who share common languages, beliefs, norms, values, concepts, categories, imagery of thought, and systems of representation. According to the *social exchange theory*[4], it can be argued that agents tend to assign higher value to the exchange when they perceive more *cohesiveness* and easier *interaction*, that is, when the exchange partner shares common values, ideological beliefs and social norms[5].

Starting from the main statements of this theory, we develop a dynamic framework of ideological evolution in a two-trait (traditional and liberal) population of individuals in which a random matching process takes place. More precisely, agents are randomly engaged in socio-economic activities modeled in the shape of a trust-matching model. Ideology is crucially important in our approach because it generates changes in the form of costs and benefits facing economic actors, thereby inducing changes in matching outcomes. To capture the idea that people tend to interact with members with the same ideological beliefs, we assume that the essential variable in the matching process is given by the value that agents assign to their random match in the society. A match between members of the same ideological group provides a lower cost or a higher productivity level than a match between agents with differing ideologies. Moreover, traditional agents give greater value to interactions with people who share their values and social norms because, by definition, they consider their own roots and ideological beliefs crucially important. We interpret this claim assuming both that the productivity parameters of a single match between traditional agents is larger that between liberals, and that only traditional agents face a cost when involved

[4] *Social exchange theory* is due to George H. Homans and differs from *economic exchange theory*; it views the exchange relationship between specific actors as actions contingent on rewarding reactions from others, (Blau, 1964). One of the main statement of this theory is that social behaviour is an exchange of goods, material goods, but also non-material ones, such as the symbols of approval or prestige, (Homans, 1958). Buunk and van Yperen (1991) observe that relation satisfaction also depends on the respective opinions of partners on the fairness of the relation exchange. Emerson (1976) shows that *social exchange theory* is essentially a market economic framework for approaching non-economic phenomena by suggesting that groups' pressure and members' conformity are to be regarded as the two sides of transactions involving the exchange of utility or reward.

[5] Cohesiveness is a value variable; it refers to the degree of reinforcement people find in the activities of the group. See Homans (1958)

in mixed matches.

The process of ideological transmission between generations is related to the literature on cultural transmission and to the theory of endogenous preferences formation. Following the basic setup developed by Bisin and Verdier (1998, 2001)[6], we model how children are taught to develop a specific ideological trait which later in life will influence adult behaviour in the transmission of traits and the well-being of the family. The socialization process occurs during childhood through the education channel. Family is the primary agent of ideological transmission, but school, peers and social institutions also play a fundamental role in moulding the ideological orientation of individuals[7]. We assume that all children are born without defined ideological beliefs and acquire preferences through imitation and observation of existing ideological models in their social environment. They are first exposed to their parents' ideological traits (vertical transmission). If they are not directly socialized, they adopt the ideology of other adults with whom they are randomly matched in the society (oblique transmission). Belonging to one ideology rather than another will determine the matching outcome and, therefore, the welfare of the family.

Parents are assumed to be altruistic and they care about their offspring's well-being, which is measured by the utility deriving from the matching process of the child. Moreover, agents have perfect foresight and rational expectations on the distribution of traits and, therefore, on children's matching outcomes. In order to capture the parents' desire for ideological homogeneity within the family, we assume that the children's well-being expected by parents is discounted by a parameter, namely *taste for similarity*, which can be interpreted as the degree of ideological intolerance of parents when children deviate from their parents' trait, as well as a measure of the relative distance between ideologies[8]. Whenever this parameter is positive valued, parents are biased towards their own ideological beliefs.

We will consider two possibilities of parents' behaviour in the education of children: either parental preferences are strongly ideology dependent, in the sense that parents make efforts to have children with the same ideological traits, implying *paternalistic* behaviour; or parental preferences are weakly ideology dependent, and parents behave in a *non-paternalistic* way, in the sense that they maximize the expected well-being of their children,

[6]The powerful framework developed by Bisin and Verdier (1998, 2001), has been applied to different contexts in the literature, from labor-market discrimination (Sáez Martí and Zenou, 2007), to religious intermarriage and the evolution of ethnic traits (Bisin and Verdier, 2000, and Bisin et al., 2004), corruption (Hauk and Sáez Martí, 2002), identity and integration problems (Bisin et al., 2008) and fertility transition (Baudin, 2009), to name just a few.

[7]For instance, several sociological studies suggest that ideological transmission within the family was one of the factors that kept Basque nationalism active during the Franco dictatorship as well as during the reinstatement of the republic (see Gatti et al., 2005). In our framework we will assume an inter-generational model of trait evolution. This approach considers the evolution of ideas that offspring learn during childhood from adults (Richerson and Boyd, 1978). This model contrasts with the intra-generational model that applies to ideas which can spread throughout a population within a single generation (Werren and Pulliman, 1981).

[8]When a conflict between ideologies exists, it is more likely that parents with a high *taste for similarity* will try to avoid a deviation of children from their own ideological traits.

disregarding the similarity of their children. More precisely, we say that a society is paternalistic when parents of both groups actively promote their own ideological variant. Conversely, a society is non-paternalistic when parents of both groups agree on the trait that guarantees higher expected well-being, so that parents of one group do not promote their own trait, leaving the job of children's socialization to peers and oblique transmission. Parents always promoting their own ideological trait amounts to assume *imperfect* or *degenerate* altruism. For this reason, in our story, parents' paternalism is endogenously determined and depends on the trade-off between protection of the ideology and the objective well-being of the children.

The main literature on cultural transmission is based on two important assumptions: first, that the utility to a type-j parent of a type-z child is independent of the distribution of the ideological traits; second, that parents suffer of a particular form of myopia called *imperfect empathy.* These assumptions are crucial in the analysis and imply that parents always want to turn children into copies of themselves. These models are not able to explain why, in some situations, parents do not promote their own variant. In fact, '[...] although there are obvious example of culturally transmitted traits where parents do have an interest in promoting their own variant, e.g. language, religion, this interest is far less obvious when it comes to cultural traits and values associated with low status and poor market outcome' (Sáez Martí and Sjögren, 2008). The standard cultural transmission model predicts that when parental influence in their offspring's education satisfies imperfect empathy, we observe paternalism in children's transmission process and diversity in the long-run[9]. The innovation in our model appears through the situation in which, despite the fact that paternalistic behaviour is observed for several periods, the long-run dynamics can converge to *conformism*, i.e. a situation in which we observe an homogeneous stationary distribution of ideological traits. In particular, in our framework paternalism at time t is a necessary but not a sufficient condition to guarantee the preservation of ideological heterogeneity in the long-run.

This result relies on the fact that parents' paternalism might change over time when the taste for similarity is sufficiently low. More precisely, when the differences between groups' beliefs are weak, there exists a trade-off between protection of an ideological paradigm and exchange level in the matching process, so that a reversal of parental evaluation of the trait can be observed when the distribution of traits evolves towards the long-run equilibrium. This mechanism in the intertemporal socialization process allow us to explain why in some societies the ideological differences between groups have gradually disappeared over time, even when, for some periods, parents tried to protect their own trait by acting as pater-

[9] Bisin and Verdier (2001) show that as a consequence of imperfect empathy altruistic parents tend to prefer children with their own trait. This class of socialization mechanisms generates cultural substitutability and therefore the preservation of cultural heterogeneity.

nalistic agents. This outcome is due to the fact that when the opportunity cost of having children with the same beliefs is high, altruistic parents do not promote their own trait, but behave in a non-paternalistic way. A historical example of this reversal of parental evaluation related to language can be found in Ireland during the $XIXth$ century: many Irish parents discouraged their children from speaking their native tongue, and encouraged the use of English instead, because most economic opportunities at that time existed within the British Empire and the US[10]. On the other hand, this parent behaviour is not observable when the differences between ideologies are very strong and, therefore, the taste for similarity is sufficiently high. In this case, for instance in the Jewish culture or in the Basque country, altruistic parents always rationally promote their own ideological variant so that heterogeneity between ideological groups will persist over time.

This paper is organized as follows. First, we develop a dynamic model of ideological transmission in which a trust-matching process takes place. Second, we study the dynamics of parents' behaviour in children's education as an endogenous behaviour showing that paternalism is a necessary but not a sufficient condition to guarantee the preservation of ideological diversity in the long-run. Third, we compare the dynamics occurring under the assumption of perfect foresight with those of myopic foresight in order to give robustness to our results. Our conclusions are presented in the last section.

2 The Model

We consider an overlapping generations model (OLG) in which each individual lives for two periods. Total population is normalized to one and is composed of a continuum of agents with a specifical ideological trait. We model the optimal choice of parents' homogeneity effort in the ideological transmission process when families are composed of one parent and one child. In particular, we assume that there are two types of agents: traditional, T, and liberal, L. During the first period, as a child, the agent is educated in a specific ideological paradigm. In the second period, as an adult, he or she observes his or her type and randomly engages in socio-economic activities through a match with another individual. Belonging to one ideology rather than another will determine the matching outcomes and, therefore, the families' well-being. We will use the index $\{j, z\} \in \{T, L\}$ to indicate individual ideological orientations.

[10]See Buttimer (2004).

2.1 The Matching Framework

We model socio-economic activity as a trust-matching process in which the trade between individuals is facilitated when agents share the same ideological behaviour. In this formulation the level of well-being of families is determined by the match between adult agents belonging to different ideologies, assuming that each individual randomly encounters only one individual in each period. The essential variable in this matching process is given by the value that agents assign to their random match in the society. Ideological affiliation play a crucial role in this process because it can be viewed as a reliable signal of the relative trustworthiness of the exchange partner. For this reason, a common ideology implies an easier interaction between agents and provides a lower cost or a higher productivity level of the social exchange. According to social exchange theory, we assume that the expected value to one individual of meeting an individual with the same ideology is larger than the expected value of meeting an individual with a different ideological trait[11].

To formalize the idea that people adhering to a specific ideology prefer to encounter agents who share their values, as well as the fact that traditional agents perceive easier interaction or higher productivity in traditional relationships[12], we assume that the expected value for a traditional (or liberal) individual of encountering an individual of the same type is given by $\alpha+\beta$, (for liberal $\alpha+\gamma$), with $\beta > \gamma > 0$, whereas the expected value of meeting an individual with a different ideology is given by $\alpha > 0$ for liberal and $\alpha - \epsilon$ for traditional agents, with $\alpha > \epsilon > 0$. The parameter ϵ captures the degree of intolerance of traditional agents towards liberals and can be view as a psychological cost for traditional individuals in mixed matches. The larger the parameter ϵ, the more traditional agents are intolerant of a match with liberal agents. Moreover, we require this cost to be lower than the productivity gap between traditional and liberal agents in homogeneous matches, i.e. $\epsilon < \beta - \gamma$. The matrix in Table 1 sums up these assumptions and describes the outcomes of the matches between agents.

Table 1: The matching outcomes

Agents	Traditional	Liberal
Traditional	$\alpha+\beta$; $\alpha+\beta$	$\alpha-\epsilon$; α
Liberal	α; $\alpha-\epsilon$	$\alpha+\gamma$; $\alpha+\gamma$

Let $q_t \in [0,1]$ be the proportion of traditional individuals in the population at time t.

[11] As shown in Lazear (1999) this is also valid for other cultural elements that characterize social groups. A common language, for instance, facilitates trade between individuals, because the exchange without intermediaries produces a level of income higher than when individuals need a translator to negotiate.

[12] Traditional agents assign higher value to interactions with members of the same ideological variant because, by definition, they consider the preservation of their own cultural roots and ideological principles crucially important.

The expected gains for traditional and liberal agents will be given by:

$$y_t^T = (\alpha + \beta)q_t + (\alpha - \epsilon)(1 - q_t)$$

$$y_t^L = \alpha q_t + (\alpha + \gamma)(1 - q_t).$$

The well-being of families will depend on the productivity parameters and on the distribution of ideologies in the society. It is easy to show that when the proportion of individuals with different ideologies is the same, the richer group is determined by the values of the parameters. Whenever the proportion of traditional agents is larger than $\frac{\epsilon+\gamma}{\beta+\epsilon+\gamma}$ traditional agents are richer than liberals in welfare terms, and the reverse is true.

2.2 Ideological Transmission, Preferences Evolution and the Taste for Similarity

In this section we develop an economic model of ideology evolution in a two-trait population of individuals. It explains how children are educated to a specific trait which later in life will influence their adult behaviour and family's well-being. We will draw from the model of cultural transmission developed by Cavalli-Sforza and Feldman (1981), and formalized by Bisin and Verdier (1998, 2001). We assume that all children are born without defined ideology and acquire preferences through the imitation and observation of existing ideological paradigms in their social environment. They are first exposed to their parents' ideological trait (vertical transmission), which they adopt with probability τ_t^j with $j \in \{T, L\}$[13]. With probability $1 - \tau_t^j$ a child from a family with ideology j is not directly socialized and he or she adopts, via imitation and learning, the ideology of other adults he or she is randomly matched with (oblique transmission)[14]. The ideology transmission process takes place in childhood; however ideology is an important determinant of family well-being during adulthood when the matching process takes place.

Parents are assumed to be altruistic and they care about their offspring well-being, which is measured by the future expected utility perceived by parents at time t. We relax two important assumptions of the main literature on cultural transmission: first, that the utility to a type-j parent of a type-z child is independent of the distribution of the ideological traits;

[13]We observe that τ_t^j also represents the *ideological homogeneity effort* made by a parent with orientation j to socialize the child to the same ideological trait.

[14]For an exhaustive discussion on vertical and oblique transmission see Bisin and Verdier (1998, 2001)

second, that parents suffer of a particular form of myopia called *imperfect empathy*[15]. These assumptions are crucial in the analysis and entail parents always wanting to socialize their children to their own cultural traits.

In our model parents have perfect foresight, so that in the period t they have rational expectations about the distribution of ideologies in the next period, $t+1$. The assumption of perfect foresight about the path of ideological preferences, i.e. $q_{t+1}^{e,j} = q_{t+1}^{j}$, implies that parents know their children's future matching outcomes y_{t+1}^{j}[16]. In order to capture the role of ideological orientation, we assume that children's well-being expected by parents at time t is discounted by a parameter, $d \in [0,1]$, namely the taste for similarity within the family[17]. This parameter can be interpreted as the degree of ideological intolerance of parents when children deviate from their own trait, or as a measure of the relative distance between ideologies. When a conflict between ideologies exists, it seems reasonable to assume that parents are less willing for their children to deviate from their own ideological trait. Parents are always biased towards children's ideology, so that they evaluate the income the child will obtain taking into account the intensity of the taste for similarity within the family, that is, $\hat{y}_{t+1}^{e,j} = y_{t+1}^{j}(1-d)$ with $0 < d < 1$.

Let P_t^{jz} define the probability that the child of a parent with ideology j will adopt ideology z, with $\{j,z\} \in \{T, L\}$. The transition probabilities at time t are given by the following equations[18]:

$$P_t^{jj} = \tau_t^j + (1-\tau_t^j)q_t^j \tag{1}$$

$$P_t^{jz} = (1-\tau_t^j)(1-q_t^j). \tag{2}$$

The fraction of agents adhering to ideology j in period $t+1$, will be given by the following equation on differences that describes the dynamic of the distribution of traits within groups:

$$q_{t+1}^j = q_t^j + (1-q_t^j)q_t^j(\tau_t^j - \tau_t^z). \tag{3}$$

[15]In the literature, the hypothesis of imperfect empathy implies that parents are always paternalistic and willing to have children with their own ideology. The justification of imperfect empathy from an evolutionary perspective is provided in some empirical studies, as discussed by Bisin and Verdier (2001). On the other hand, for reasons discussed in the introduction, in our formulation paternalistic behaviour is not assumed as given but is endogenously determined.

[16]In Section 4 we prove that if parents have myopic foresight the main results do not change. Assuming parents with myopic foresight implies the same qualitative dynamic of ideological traits and the same long-run equilibrium, but a lower speed of convergence towards the stable equilibria compared to the model with perfect foresight. See Section 4 for more details.

[17]To simplify, we will assume that the taste for similarity is the same for parents displaying both ideological traits.

[18]By the law of large numbers P_i^{jz} also denotes the fraction of children with a type-j parent who adopt type-z ideology (see Bisin and Verdier, 1998, 2001).

Parents are altruistic and make efforts to maximize their children's expected well-being. Formally, j-type parents choose x_t^j and $\tau_t^j \in [0,1]$ that solves the following maximization problem:

$$\begin{cases} max_{x^j,\tau^j} \;\; u(x_t) + P_t^{jj}(\tau_t^j, q_t^j)V_t^{jj}(y_{t+1}^{e,j}) + P_t^{jz}(\tau_t^j, q_t^j)V_t^{jz}(\hat{y}_{t+1}^{e,z}) - C(\tau_t^j) \\ s.t. \;\; x_t^j = y_t^j \end{cases} \tag{4}$$

where x_t is consumption at time t and $C(\tau_t^j)$ denote the cost of the ideological homogeneity effort of a j-type parent. In order to simplify we consider a quadratic functional form[19], $C(\tau_t^j) = \frac{(\tau_t^j)^2}{2}$. V_t^{jj} and V_t^{jz} are the expected utilities a parent of type j attributes to the welfare of his or her child. By assumption, utility is linear and $\hat{y}_{t+1}^e = y_{t+1}^e(1-d)$ with $0 < d < 1$.

Let $j = T$ and $z = L$; as before q_t represents the proportion of traditional agents in the population. We define the utility relative gains that parents perceive from the ideological transmission process, taking into account their own ideological trait, with the following equations:

$$\Delta V_t^T \equiv V_t^{TT}(y_{t+1}^{e,T}) - V_t^{TL}(\hat{y}_{t+1}^{e,L}) = q_{t+1}[(\beta+\epsilon) + \gamma(1-d)] - (\epsilon+\gamma) + d(\alpha+\gamma) \tag{5}$$

$$\Delta V_t^L \equiv V_t^{LL}(y_{t+1}^{e,L}) - V_t^{LT}(\hat{y}_{t+1}^{e,T}) = q_{t+1}[(\beta+\epsilon)(d-1) - \gamma] + \epsilon(1-d) + (d\alpha+\gamma). \tag{6}$$

Depending on these utility gains and on the productivity parameters in the social exchange, we can have two kinds of parental behaviour in children's education: either parental preferences are 'strongly ideology dependent', in the sense that parents prefer to have children of the same ideological trait, or parental preferences are 'weakly ideology dependent', in the sense that parents are not willing to promote their specific trait. In particular, whenever both $\Delta V_t^T > 0$ and $\Delta V_t^L > 0$, all parents want to socialize their children to their own ideological trait and make positive effort in vertical transmission process. We define this behaviour as paternalism. Conversely, when one of the two utility gains is non-positive, some parents do not actively promote their own ideological variant inside the family. We refer to this behaviour as non-paternalism[20].

Maximizing (4) with respect to τ_t^T and τ_t^L we obtain the following first order conditions:

[19] It is possible to obtain similar results with any increasing and convex cost function.

[20] In the non-paternalistic scenario we consider that one of the two utility gains is zero. We exclude negative gains $\Delta V^{T,L} < 0$.

$$\frac{\partial P_t^{TT}(\tau_t^T, q_t)}{\partial \tau_t^T} V_t^{TT}(y_{t+1}^{e,T}) + \frac{\partial P_t^{TL}(\tau_t^T, q_t)}{\partial \tau_t^T} V_t^{TL}(\hat{y}_{t+1}^{e,L}) = \tau_t^T \tag{7}$$

$$\frac{\partial P_t^{LL}(\tau_t^L, q_t)}{\partial \tau_t^L} V_t^{LL}(y_{t+1}^{e,L}) + \frac{\partial P_t^{LT}(\tau_t^L, q_t)}{\partial \tau_t^L} V_t^{LT}(\hat{y}_{t+1}^{e,L}) = \tau_t^L. \tag{8}$$

Differentiating the transitional probabilities (1) and (2), we derive the optimal ideological homogeneity effort for T and L parents:

$$\tau_t^T = \begin{cases} \Delta V_t^T(1-q_t) & if \quad \Delta V_t^T > 0 \\ 0 & otherwise \end{cases} \tag{9}$$

$$\tau_t^L = \begin{cases} \Delta V_t^L q_t & if \quad \Delta V_t^L > 0 \\ 0 & otherwise \end{cases}. \tag{10}$$

The effect of the current distribution on parents' behaviour depends whether or not direct vertical socialization acts as a substitute or as a complement to oblique socialization. When vertical transmission acts as a cultural substitutes to oblique transmission, the effort of the parents is a strictly decreasing function of the size of their own ideological group. This means that the smaller group tends to socialize their children more intensely than the dominant group. When vertical and oblique transmission are complements, parents' effort is a strictly increasing function of the size. In this case the dominant group socialize their children more intensely[21]. The impact of the taste for similarity on the socialization effort is strictly positive for both types of agents, whereas the effect of ϵ is negative for traditional parents and ambiguous for liberal agents because it depends on the parameters and initial distribution of traits.

2.3 The Dynamics of Ideological Traits

The main literature on cultural transmission assumes that parents, while altruistic, are also paternalistic as a consequence of the imperfect empathy assumption. For this reason they prefer children with their own cultural traits and hence make positive efforts to socialize them into these traits[22].

[21]See Bisin and Verdier (2001) for an exhaustive discussion of substitutability and complementarity between the two transmission channels. They show that if vertical and oblique transmission are substitutes for both groups, the dynamics will converge to heterogeneous distribution of cultural traits.

[22]The fact that parents try to actively promote their own traits is well documented in the literature. The evidence of the persistence of ethnic, cultural and religious traits across generations motivates a large part of the literature on intergenerational transmission (see for instance Borjas, 1995; Fernandez and Fogli, 2009). In a recent study of fertility transition Baudin (2009), shows that agents who are more attached to their culture are less sensitive to asymmetric technological shocks and, therefore, make more efforts to help their

Bisin and Verdier (2008) state that 'given imperfect empathy on the parts of parents, $\Delta V^i > 0$. [...] It is straightforward to demonstrate that this class of socialization mechanisms generates cultural substitutability and therefore the preservation of long-run heterogeneity'[23]. In particular, when both parents are intolerant of their children's deviations from their own trait, i.e. $\Delta V_t^T > 0$ and $\Delta V_t^L > 0$, the long-run equilibrium will be characterized by diversity or ideological heterogeneity in the distribution of traits[24].

In this section we are interested in showing under what conditions our model is conducive to diversity in the long-run[25]. To this end we study the dynamics of ideological traits within groups, assuming that the parents are altruistic and have perfect foresight, plus a positive taste for similarity in the ideological transmission process. As before, we define q_t as the proportion of traditional agents. Substituting (5) and (6) into (9) and (10), using (3), after some algebraical manipulation, we determine the dynamics of the distribution of traditional traits in the population as:

$$q_{t+1} = \begin{cases} \frac{q_t\{1+\gamma[d(1-q_t)-1](1-q_t)+\alpha d(1-q_t)(1-2q_t)-\epsilon(1-q_t)(1-dq_t)\}}{1+(1-q_t)q_t[d(\beta+\epsilon-\gamma)q_t-\gamma(1-d)-\beta-\epsilon]} & if\ \Delta V_t^T > 0, \Delta V_t^L > 0 \\ \frac{\{(q_t-1)^2[\alpha d-\epsilon-(1-d)\gamma]+1\}q_t}{1-[\beta+\epsilon+\gamma(1-d)]q_t(q_t-1)^2} & if \Delta V_t^T > 0, \Delta V_t^L = 0 \\ \frac{q_t\{(1-q_t)[\alpha d+\epsilon(1-d)+\gamma]q_t-1\}}{(1-q_t)[(1-d)(\beta+\epsilon)+\gamma]q_t^2-1} & if \Delta V_t^T = 0, \Delta V_t^L > 0 \end{cases} \tag{11}$$

The asymptotic behaviour of our dynamic system depends upon the number of steady states that our dynamic equation possess which are admissible in the domain . Eliminating the temporal index and solving for the proportion of traditional parents, we observe that the dynamics always present at least two steady states admissible in the domain, $\bar{q} = 0$ and $\bar{q} = 1$, which are stable for $d < \frac{\gamma+\epsilon}{\gamma+\alpha}$ and $d \leq \frac{\beta}{\alpha+\beta}$ respectively.

Proposition 1. *There* $\exists$ *two thresholds* $d_1 = \frac{\gamma+\epsilon}{\alpha+\gamma}$, $d_2 = \frac{\beta}{\alpha+\beta}$ *and a critical value* $e^* = \frac{\alpha(\beta-\gamma)}{\alpha+\beta}$. *Diversity is observed in the long-run:*
(i) when $\epsilon < \epsilon^*$, $\forall\ 0 < q_0 < 1$ *iff* $d > d_2$.
(ii) when $\epsilon > \epsilon^*$, $\forall\ \bar{q}_{I^-} < q_0 < 1$ *iff* $d > d_2$.
(iii) when $\epsilon = \epsilon^*$, $\forall\ 0 < q_0 < 1$ *iff* $d > d^* = d_1 = d_2$.

culture survive.

[23] Bisin and Verdier (2001) show other micro-founded specifications in which cultural complementarity and tendency of cultural homogenization over time is observed.

[24] Conversely, cultural transmission mechanisms with perfect empathy imply dynamics of the distribution of traits which converge to degenerate distributions or *conformism*, that is, cultural homogenization in the long-run.

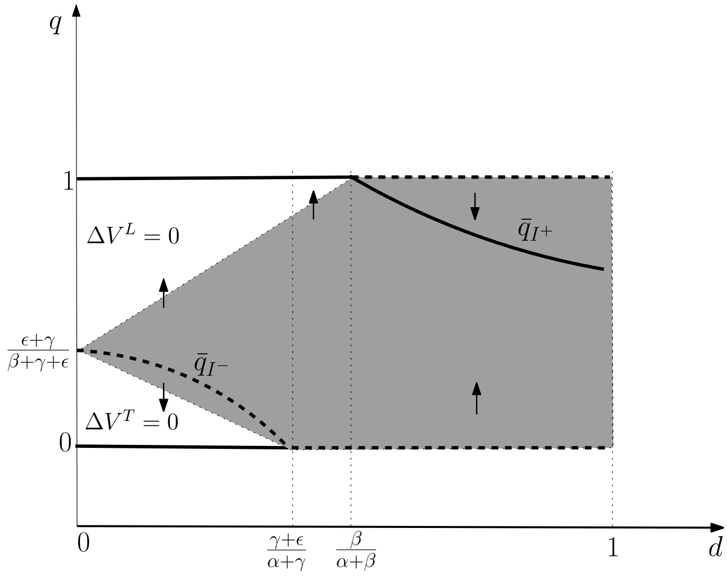
[25] In our framework it is important to understand that the ideological trait adopted by the child is expected to affect the child's welfare welfare in adult life. Parents have perfect foresight and care about their children's welfare, but they are also biased about the ideological orientation the children adopt because $d > 0$.

Proof. *See Appendix.*

■

In general, we claim that in our model diversity is ensured in the long-run when the taste for similarity (d) is sufficiently high. When this is not the case the dynamics might present multiple equlibria, so that the long-run distribution will depend on the initial distribution and on the value of the taste for similarity and the parameters. Interestingly enough, imperfect empathy is observed when the taste for similarity is sufficiently high. This means that at every period the type of empathy is endogenously determined.

Figure 1: Transcritical bifurcation when $\epsilon < \epsilon^*$



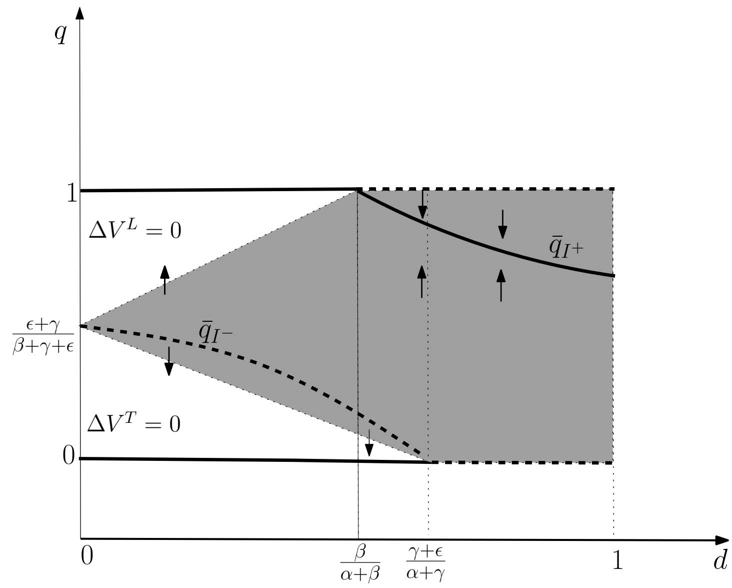
In order to state the proposition 1, let us define the r.h.s. of our dynamic equation (11) as $f(q_t)$. Firstly, assume that $\Delta V_t^i > 0 \quad \forall i = \{T, L\}$ and that $\epsilon < \epsilon^*$ such that $d_1 = \frac{\gamma+\epsilon}{\alpha+\gamma} < \frac{\beta}{\alpha+\beta} = d_2$ (see Figure 1)[26]. The limit of $\frac{f(q_t)}{q_t}$ when $q \to 0$ is given by $1 + \alpha d - \gamma(1-d)$. When $d > d_1$ this limit is grater than 1. This means that the function $f(q_t)$ passes above the 45° line as q tends towards zero. Solving (11) at the equilibrium we always find three steady states admissible in the domain except when $d \in [d_1, d_2]$. If $d > d_2$, the solution $\bar{q}_{I^+} = \frac{\beta-2\alpha d+\epsilon(1+d)+\gamma(1-2d)+\sqrt{\Lambda}}{2d(\beta-\gamma)}$ belongs to the domain and is always stable and smaller than 1[27]. In this case, for any initial $0 < q_0 < 1$ the trajectory converges

[26]The shaded area represents the situation in which both parents actively promote their ideological traits, $\Delta V_t^i > 0 \ \ \forall i \in \{T, L\}$, that is they are paternalistic in their children trait transmission. We will discuss in Section 3 the inter-temporal behaviour of parents in their children's education.

[27]With $\Lambda = \{\beta+\epsilon+d[\epsilon-2(\alpha+\gamma)]+\gamma\}^2+4d(\beta+\epsilon-\gamma)[\alpha d-\epsilon+\gamma(d-1)]$. Since $\epsilon < \beta-\gamma$ by assumption,

to the stable interior solution. When $d < d_1$ the solution admissible in the domain is given by $\bar{q}_{I^-} = \frac{\beta - 2\alpha d + \epsilon(1+d) + \gamma(1-2d) - \sqrt{\Lambda}}{2d(\beta-\gamma)}$. Since the limit of $\frac{f(q_t)}{q_t}$ when $q \to 0$ is smaller than 1, the trajectory converges to the origin, passing below the 45° line. This implies that the interior solution $\bar{q}_{I^-}$ is unstable. For any initial $0 < q_0 < 1$ we have multiple equilibria and the trajectory converges to the origin whenever $q_0 < \bar{q}_{I^-}$, and to 1 whenever $q_0 > \bar{q}_{I^-}$. Note that in this case $\bar{q}_{I^-}$ is always smaller than $\frac{\gamma+\epsilon}{\beta+\gamma+\epsilon}$. When $d \in [d_1; d_2]$ the dynamic only allows the two trivial steady states admissible in the domain and the system always converges to the stationary steady state $\bar{q} = 1$. When $\Delta V_t^T = 0$, the long-run dynamics do not change, since the non-trivial solution admissible in the domain $\bar{q}_{I^T} = \frac{\alpha d + \epsilon(1-d)+\gamma}{(1-d)(\beta+\epsilon)+\gamma}$ is always unstable. Similarly, when $\Delta V_t^L = 0$, then $\bar{q}_{I^L} = \frac{\epsilon+\gamma-d(\alpha+\gamma)}{\beta+\epsilon+\gamma(1-d)}$ is always unstable.

Figure 2: Transcritical bifurcation when $\epsilon > \epsilon^*$



Assume now that $\epsilon > \epsilon^*$ so that $d_1 = \frac{\gamma+\epsilon}{\alpha+\gamma} > \frac{\beta}{\alpha+\beta} = d_2$ (see Figure 2). As before, when the taste for similarity is sufficiently high, $d \geq d_1$, and for any initial $0 < q_0 < 1$, the trajectory will converge to the stable interior solution in which diversity of ideological traits is observed. When $d \in]d_2; d_1[$ a multiplicity of equilibria appear and the long-run solution will depend on the initial distribution of traits. In particular if $q_0 > \bar{q}_{I^-}$ then diversity will be observed; however if $q_0 < \bar{q}_{I^-}$ the dynamics will converge to a homogeneous distribution of ideological traits. When $d \leq d_2$, for any initial $0 < q_0 < 1$ we have multiple equilibria and the trajectory converges to the origin whenever $q_0 < \bar{q}_{I^-}$, and to 1 whenever $q_0 > \bar{q}_{I^-}$.

The case in which the cost for traditional agents in mixed matches equals its critical level $\epsilon = \epsilon^*$ and, consequently $d_1 = d_2 = d^*$, is characterized by only two possible situations.

$\Lambda > 0$. See the Proof 1 for more details.

Whenever the taste for similarity is sufficiently high, $d > d^*$, the system converges to the stable internal solution. If this is not the case, the dynamic depends on the initial distribution of the ideological traits, and a multiplicity of equilibria appear.

From Figures 1 and 2 we can observe that our model presents two bifurcation points, in which two fixed points change their stability proprieties from stable to unstable and viceversa. This kind of dynamics, namely transcritical bifurcation, happens when four conditions are satisfied. A precise definition of *transcritical bifurcation* is useful.

Definition (Transcritical Bifurcation): *Let $\hat{d}$ be a value of the parameter d, and $\bar{q}$ be a non-hyperbolic steady state of the dynamic $q_{t+1} = f(q_t, d)$. If the following conditions are satisfied:*

- *(i)* $f(\bar{q}, \hat{d}) = \bar{q}$;
- *(ii)* $\frac{\partial f}{\partial d}(\bar{q}, \hat{d}) = 0$
- *(iii)* $\frac{\partial^2 f}{\partial q \partial d}(\bar{q}, \hat{d}) \neq 0$
- *(iv)* $\frac{\partial^2 f}{\partial q^2}(\bar{q}, \hat{d}) \neq 0$

then the dynamic system $q_{t+1} = f(q_t, d)$ undergoes a transcritical bifurcation where the equilibria pass through each other.

Proposition 2. *For parameters $d = \{d_1; d_2\}$ the steady state $\bar{q}$ is a non-hyperbolic steady state. Our dynamic system undergoes a transcritical bifurcation and presents two bifurcation points in which one equilibrium collides with another and the two equilibria exchange their stability proprieties, but continue to exist both before and after the bifurcation.*

Proof. *See Appendix.*

■

A bifurcation occurs at $d = d_1$ where the steady state $\bar{q} = 0$ changes its stability proprieties. In particular, the trivial steady state $\bar{q} = 0$ changes its proprieties from stable, for $d < d_1$, to unstable, for $d \geq d_1$. A bifurcation also occurs at $d = d_2$. At this point the trivial steady state $\bar{q} = 1$ changes its proprieties from stable, for $d \leq d_2$, to unstable, for $d > d_2$[28].

[28]The non-trivial steady states admissible in the domain, $\bar{q}_{I-}$ and $\bar{q}_{I+}$, change their stability proprieties at these values of d.

3 When Paternalism is not conducive to Heterogeneous Distribution of Traits

The assumption of imperfect empathy is commonly interpreted as a form of myopic or paternalistic altruism. As a consequence parents, while altruistic, want to have children with the same ideological trait. They evaluate the future welfare of their children only through the filter of their own preferences, that is, they behave in a paternalistic way. For this reason they always prefer children with their own ideological traits. In our framework parents' paternalism is not taken as given but is endogenously determined and depends on the trade-off between the protection of ideological traits and the exchange level in the matching process. More precisely, our model is consistent with both the evidence that families try to have children of the same type, and the fact that parents care about the possible consequences for their children's well-being. The combination of the intensity of the taste for similarity and matching outcomes will determine the opportunity cost of trait preservation and, thereby, the parents' behaviour in the ideological transmission process.

One of the reasons behind our formulation is, as we know, that parents try to actively promote their own traits. But the fact that parents in some particular situations, for instance in disadvantaged environments, continue to promote their own ideological traits at the expense of a higher well-being is not convincing and appears counterintuitive, in particular when parents seem not care too much about their children's ideological values. In our opinion, situations in which altruistic parents do not evaluate the consequences of their actions on their children's well-being are more likely when the desire for ideological homogenization within the family plays a crucial role in the transmission process, that is, when the taste for similarity is sufficiently high. Parents always promoting their own traits amounts to assuming *imperfect or degenerate* altruism.

Compared to the existing literature, our model yields two new results. First, parents' paternalism at time t is a necessary but not a sufficient condition to ensure diversity in the long-run. Second, parents' behaviour in the preference transmission process can change over time when the dynamics converge to the long-run equilibrium. Our approach could explain why some groups adhering to a specific ideological paradigm have changed their approach to children's education during their intergenerational evolution. Our model is able to reproduce historical events in which a reversal of parents' evaluation of a trait has been observed. An example of this reversal of behaviour in children's education is the decline of the Irish language in the $XIXth$ century: during the Great Famine many Irish parents discouraged their children from speaking Irish, and encouraged the use of English instead[29].

[29] See Buttimer (2004).

The reason behind this behaviour can be found in the economic opportunities related to the use of English, seen as the only way to a better life[30].

The intuition related to this outcome is that, when the distance between ideological paradigms is not too great, there is a trade-off between the taste for similarity and children's expected well-being, which may induce parents not to promote their variant even though their are biased towards their own ideological beliefs[31].

Proposition 3. *Paternalism at time t is a necessary but not a sufficient condition to guarantee diversity in the long-run.*
(i) Assume $\epsilon \leq \epsilon^*$: if $d \leq d_2$ (conformism) and q_0 is such that $\Delta V_0^i > 0$ with $i = \{T, L\}$, then paternalism disappears over time; if $d > d_2$ (diversity) then paternalism is observed at every t $\forall$ $0 < q_0 < 1$.
(ii) Assume $\epsilon > \epsilon^*$: if $d \leq d_2$ (conformism) and q_0 is such that $\Delta V_0^i > 0$ with $i = \{T, L\}$, then paternalism disappears over time; if $d \in]d_2, d_1]$ paternalism disappears only if $q_0 < \bar{q}_{I^-}$ (conformism); if $q_0 > \bar{q}_{I^-}$ (diversity) paternalism is observed at every t; if $d > d_1$ (diversity) then paternalism is observed at every t $\forall$ $0 < q_0 < 1$.

Proof. *See Appendix.*
■

Our results are a direct consequence of the assumptions that the utility to a type-j altruistic parent of a type-z child depends upon the distribution of ideological traits, and that parents' behaviour in children's socialization is driven by what we have called taste for similarity. For this reason, the contribution of this paper contrasts with the standard result in the literature in which it is shown that paternalism is a necessary and sufficient condition to ensure diversity in the long-run, but also with the result in which it is shown that paternalism in neither a necessary, nor a sufficient condition for diversity[32].

When there are differences between ideologies parameter d should be positively valued in our model and, in particular, the larger the differences between ideological beliefs, the higher the taste for similarity. However, given q_t, when d is large enough to lead to a paternalistic society, i.e. d is such that both $\Delta V_t^T > 0$ and $\Delta V_t^L > 0$, but not sufficiently large to lead to diversity in the long-run, then paternalistic behaviour at time t, does not guarantee

[30] Think for instance of the importance of the English language as the language of survival in the migrant's choices from Ireland to the British Empire or US.

[31] For instance, during the $XIXth$ century, Irish political leaders, such as Daniel O'Connell were critical of the Irish language, seing it as "backward", with English the language of the future.

[32] Sáez Martí and Sjögren (2008) observe that under *demand for pluralism*, i.e. $\Delta V^i > 0$, the standard result in the literature (namely that if parents promote their own trait and oblique transmission is linear, diversity is guaranteed) is confirmed. Assuming that oblique transmission is biased or frequency dependents implies that paternalism in neither a necessary, nor a sufficient condition for diversity.

heterogeneity, because the opportunity cost of preserving the group's ideology increases with the size of the other group and therefore with the outcome of the matching process. In other words when the taste for similarity is sufficiently low, there exists a trade-off between the protection of traits and future children's well-being.

Assume for simplicity $\epsilon = \epsilon^*$. Conflict between ideologies implies a high taste for similarity and, thereby, strong protection of trait in the ideological transmission process, so that we can assume $d > d^*$, with $d^* = d_1 = d_2$. In this specific case, parents are paternalistic at every $t \geq 0$ and both groups strive to protect their own variant, so that diversity is ensured in the long-run distribution. Only in this particular situation we observe imperfect empathy at every period of the intergenerational evolution of the families. For instance, in Jewish cultures traditional and liberal agents have never assimilated but continue to coexist. Traditional parents attribute great value to children's ideology and make strong efforts to promote their own trait. Liberal parents also prefer to have children of the same type, because they consider the traditional group too conservative with respect to their ideological paradigm. Given the fact that both types of parents exhibit a high taste for similarity, it is not surprising that the model predicts the persistence of ideological heterogeneity in the long-run.

On the other hand, when differences between ideologies are not important, parents are more interested in their children's market outcomes than the promotion of their own ideological variant. This is the case in which parents have a low taste for similarity in the model, i.e. $d < d^*$. The long-run equilibrium will depend on the initial distribution of ideologies and will converge to the equilibrium characterized by conformism or a homogeneous stationary distribution of traits. This parental behaviour in the transmission of traits may also explain why in some societies ideological differences between groups have disappeared over time, even though in some periods parents wanted to protect their own traits by acting as paternalistic agents. Think for instance of the history of the main Protestant countries of northern Europe. Parental paternalism towards different beliefs was strong for several centuries but more recently has become less protective and has gradually disappeared. The increasing size of the Protestant group as well as the influence of Protestant institutions has led to the acceptance of many basic Protestant principles, such as the attitude towards thrift and economic status. More precisely, the trade-off between protection of ideological traits and future children's well-being has discouraged parents from promoting their own ideological beliefs.

In order to give a numerical example we set the following values for parameters and initial condition on the distribution of the ideological traits: $\alpha = 0.7$, $\beta = 0.8$, $\gamma = 0.5$, $q_0 = 0.3$, and $\epsilon = \epsilon^* = 0.14$, so that $d^* = 0.533$. As shown in Table 2, when the level

Table 2: Paternalism versus diversity

	$d = 0.3$			$d = 0.8$		
time	ΔV^T	ΔV^L	q	ΔV^T	ΔV^L	q
0	0.08	0.48	0.30	0.68	0.85	0.30
1	0.05	0.51	0.28	0.73	0.82	0.35
2	0.03	0.53	0.26	0.76	0.79	0.39
3	0	0.56	0.24	0.80	0.77	0.43
4	0	0.59	0.21	0.82	0.75	0.46
5	0	0.63	0.18	0.84	0.74	0.48
6	0	0.66	0.15	0.86	0.73	0.50
∞	0	0.81	0	0.91	0.70	0.56

of the taste for similarity is below the threshold that guarantees diversity in the long-run, paternalistic behaviour, i.e. $\Delta V^i > 0$ with $i = \{T, L\}$ can evolve over time as the ideological trait changes. Consider for instance, $d = 0.3$: if we look at the trends in intergenerational behaviour as the dynamics converge to the stable steady state, we see that after 3 periods of time, traditional parents will behave in a non-paternalistic way and not promote their own trait to their children. Since $\tau_t^T = \Delta V_t^T (1 - q_t)$, when traditional parents are not paternalistic $\Delta V^T = 0$, they will not make any effort to have children of the same type. This outcome is a consequence of the fact that the opportunity cost of having children with the same ideology modifies parents' behaviour in the intergenerational transmission process. In particular parents give more weight to their children's well-being than to promoting their own ideological variant, that is, they behave in a non paternalistic way. In this example the traditional trait will disappear in the long-run since the size of this group will decrease monotonically as will the probability of socialization to this trait via oblique transmission. On the other hand, this behaviour is not observable when the differences between ideologies are very strong, as for instance in Catalonia or the Basque Country, and therefore the taste for similarity is sufficiently high. In this case we set $d = 0.8$, so that altruistic parents always promote their own ideological variant, imperfect empathy is observed in every period, and the long-run dynamics converge to the stationary solution in which diversity appears, as suggested in the main literature on cultural transmission.

4 Robustness of the Model: Parents with Myopic Foresight

In the section we relax the assumption of rational expectations in order to give robustness to the model presented above. Our aim is to compare the dynamics occurring under the assumption of perfect foresight with those of myopic foresight[33]. When parents have myopic foresight they cannot formulate rational expectations about the future distribution of traits in the population, so they evaluate their children's well-being using their own payoff matrix and the current distribution, that is, $y_{t+1}^{e,j} = y_t^j$, with $j = \{T, L\}$[34].

Let us consider the dynamics of the proportion of traditional agents, $j = T$. Substituting the myopic foresight condition $y_{t+1}^{e,j} = y_t^j$ into the utility gains (5) and (6) and into the parents' optimal efforts (9) and (10), from equation (3) we derive the dynamics of the distribution of traditional trait in the population:

$$q_{t+1} = q_t + (1-q_t)q_t\{(\alpha+\gamma)d - (\epsilon+\gamma) + [\beta+\alpha+d(\epsilon-2\alpha-2\gamma)+\gamma]q_t - d(\beta+\epsilon-\gamma)q_t^2\}. \quad (12)$$

Eliminating the temporal index and solving for the proportion of traditional agents in the population we find the same long-run equilibria as in the model with perfect foresight.

Proposition 4. *Assume myopic foresight expectations on ideologies' distribution in the population, that is, $q_{t+1}^{e,j} = q_t^j$. The dynamic under myopic foresight exhibits a lower speed of convergence towards the long-run equilibrium but the same qualitative structure of the dynamic under perfect foresight, in the sense that the steady states are the same and the first-order stability conditions are equivalent.*

Proof. *See Appendix.*

■

Although the two models have the same qualitative dynamics, one important difference is that the dynamic under perfect foresight has a higher speed of convergence towards the stable steady states. This means that, under rational expectations, the long-run equilibrium is reached more quickly, because parents can perfectly anticipate the children's future well being and choose optimally their effort in the ideological transmission process. The intuition behind this result is that in the myopic foresight case parents do not predict that

[33] Michel and de la Croix (2000) compare myopic and perfect foresight dynamics in a standard OLG model, showing that when both dynamics are monotonic, the steady states are the same.

[34] As before, the expected children's income should be discounted by the level of the taste for similarity within the family, since parents' utility depends on children's ideological orientation.

the distribution of trait will be different tomorrow.

To give a numerical example, let us consider an economy with the following productivity parameters in the matching process: $\alpha = 0.7, \beta = 0.8, \gamma = 0.5$ and assume that the initial proportion of traditional agents is 30% of total population. Given these parameter values we observe $\epsilon^* = 0.14$. Assuming $\epsilon = \epsilon^*$ we have $d^* = 0.533$. From the simulation presented in Table 3, we observe as in perfect foresight case, that the dynamic quickly converges towards its stationary steady state[35].

Table 3: Convergence of q_t

	Perfect foresight		Myopic foresight	
$\frac{taste}{time}$	$d < d^*$	$d > d^*$	$d < d^*$	$d > d^*$
0	0.400	0.400	0.400	0.400
1	0.381	0.445	0.387	0.436
2	0.353	0.476	0.371	0.464
3	0.315	0.497	0.350	0.484
4	0.264	0.511	0.323	0.499
5	0.202	0.520	0.289	0.510
6	0.137	0.526	0.248	0.517
7	0.081	0.530	0.201	0.523
∞	0	0.537	0	0.537

Under the myopic foresight assumption, Propositions 1, 2 and 3 are still valid and their analytical tractability is easier than in the case with perfect foresight. With myopic foresight, when a reversal of parents' educational strategy is observed, paternalism persists for longer than in the perfect foresight case. This result is a consequence of the optimal effort chosen by parents when transmitting vertically their own ideological beliefs. In particular, the protection of the trait is greater (lower) in the perfect foresight case as the dynamic tends towards the long-run equilibrium in which the distribution of traits is characterized by an increase (decrease) in the group's size. As in the perfect foresight case, paternalism a time t is not a sufficient condition to guarantee diversity in the long-run[36].

[35] As before we assume $d = 0.3$ and $d = 0.8$.

[36] At the same time it is straightforward to demonstrate that non-paternalism is a sufficient but not a necessary condition to ensure conformism in the long-run.

5 Conclusions

In this paper we have analyzed how affiliation to ideological paradigms interacts with agents' inter-generational behaviour in the process of children's traits transmission and well-being production. We considered a model of trait transmission assuming altruistic agents with perfect foresight who care about both their offspring's welfare, and having children with the same ideological orientation as themselves. In our framework we supposed the existence of an exchange social market in which a random matching process takes place, modeled in the shape of mutual trust. Trait distribution and ideological affiliation to a traditional or a liberal paradigm determines the level of matching outcomes and therefore the family's well-being. We also assumed that parents are biased about children's traits according to their taste for similarity. This defines the intensity of their efforts to preserve the ideological orientation within the family. This parameter can alternatively be interpreted as the distance or the degree of conflict between ideologies.

The result of the main literature on cultural transmission for which imperfect empathy generates cultural substitutability and therefore the preservation of long-run heterogeneity, appears only if the taste for similarity is sufficiently high. When this is not the case, our model supports the possibility of a reversal in the parental evaluation of traits; in particular parents in disadvantaged environments would not actively promote their own trait if it was conducive to a poor future for their children.

Our framework describes the parents' paternalistic behaviour as an endogenous behaviour, that is driven by the trade-off between the preservation of the ideological trait and the social exchange level in the matching process. The fact that paternalism in not exogenously determined allow us to understand the changes in parents' behaviour over time, showing why, in particular contexts, it has changed from ideologically protective to non-protective. When the opportunity cost of having children with the same ideology is too high, altruistic parents can behave in a non-paternalistic way. Contrary to the standard model, our theory can explain why some ideological traits become extinct, even though parents was willing to actively promote them for several periods of time.

Finally we tested the robustness of the model assuming myopic foresight. We obtained the same qualitative results as with the benchmark model even though the speed of convergence towards the long-run equilibrium is faster in the perfect foresight case. This is due to the fact that myopic parents do not realize that the distribution of traits may be different tomorrow. Moreover, paternalism persists for longer than in perfect foresight case.

A natural extension is to assume a heterogeneous distribution of the taste for similarity between groups and explore the effect on the dynamics of traits and on the dynamics of paternalism in the socialization process.

Appendix

Proof of Proposition 1

Let $q_{t+1} = f(q_t, d)$ be function (11) defined on the interval $J \in [0,1]$ of $\Re$. In order to prove Proposition 1 we have to study the stability of the steady states of this dynamic function in three different cases. First we observe that when a steady state is hyperbolic, we can study its stability on the basis of its first derivative. Let $\bar{q}$ be a steady state $\in J$. We say that $\bar{q}$ is hyperbolic if $|f^{'}(\bar{q})| \neq 1$; when $|f^{'}(\bar{q})| < 1$ then $\bar{q}$ is locally stable; when $|f^{'}(\bar{q})| > 1$ then $\bar{q}$ is unstable.

Assume first that $\Delta V_t^i > 0 \;\; \forall \;\; i \in \{T, L\}$. Eliminating the temporal index in the dynamic equation and solving for the proportion of traditional families, we find four steady states, $\bar{q} = \{0, 1, \bar{q}_{I^-}, \bar{q}_{I^+}\}$ that can be admissible in the domain depending on the parameters, the initial distribution of traits and the taste for similarity.

We define $d_1 = \frac{\gamma+\epsilon}{\alpha+\gamma}$ and $d_2 = \frac{\beta}{\alpha+\beta}$. Assume $\bar{q} = 0$. When $|f^{'}(0)| = 1 + \alpha d - \epsilon - (1-d)\gamma$, it is straightforward to prove that the trivial steady state $\bar{q} = 0$ is hyperbolic for $d \neq d_1$. Furthermore $\bar{q} = 0$ is stable (i.e. $|f^{'}(0)| < 1$) if $d < d_1$, and unstable (i.e. $|f^{'}(0)| > 1$) if $d > d_1$. Assume now $\bar{q} = 1$. In this case non-hyperbolicity arises if and only if $d = d_2$, because $|f^{'}(1)| = 1 - \beta(1-d) + \alpha d$. It is easy to show that $\bar{q} = 1$ is stable for $d < d_2$ and unstable for $d > d_2$.

When $d < d_1$ the solution $\bar{q}_{I^-}$ is always admissible in the domain $q \in [0,1]$. Nonetheless, this steady state is always unstable because $|f^{'}(\bar{q}_{I^-})| > 1 \; \forall \, d < d_1$. Conversely, the other non-trivial solution, $\bar{q}_{I^+}$, is admissible in the domain, and is stable, if and only if $d > d_2$. This steady state is always stable since $|f^{'}(\bar{q}_{I^+})| < 1 \;\; \forall \;\; d > d_2$.

If we assume $\Delta V_t^T = 0$ (resp. $\Delta V_t^L = 0$), then the dynamic presents only one non trivial solution, $\bar{q}_{I^T} = \frac{\alpha d + \epsilon(1-d) + \gamma}{(1-d)(\beta+\epsilon)+\gamma}$ (resp. $\bar{q}_{I^L} \frac{\epsilon+\gamma-d(\alpha+\gamma)}{\beta+\epsilon+\gamma(1-d)}$) that is admissible in the domain if and only if $d < d_1$ (resp. $d < d_2$). This non trivial solution is always unstable because $|f^{'}(\bar{q}_{I^T})| > 1$ (resp. $|f^{'}(\bar{q}_{I^L})| > 1$). Moreover we can observe that $\bar{q}_{I^L} < \bar{q}_{I^-} \;\; \forall \;\; d \neq \{0; d_1\}$. When $\Delta V_t^T = 0$ (resp $\Delta V_t^L = 0$), the dynamics will converge to the steady state $\bar{q} = 0$ (resp. $\bar{q} = 1$) that is always stable since $|f^{'}(0)| < 1$ (resp $|f^{'}(1)| < 1$).

Given the parameters' domain three scenarios are possible depending on the value of ϵ. (i) When $\epsilon < \epsilon^*$ it follows that $d_1 < d_2$. In this specific case if $d \in [d_1; d_2]$ only the two trivial solutions are admissible in the domain $q \in [0,1]$ and the long-run dynamic always converges to the trivial steady state $\bar{q} = 1$. If $d > d_2$ then $q_t \rightarrow \bar{q}_{I^+}$. If $d < d_1$ we have multiplicity of equilibria and the long-run distribution depends on the initial trait distribution q_0. (ii) When $\epsilon > \epsilon^*$ we observe $d_1 > d_2$. If $d \in]d_2; d_1[$ the two non-trivial solutions are both admissible in the domain. In particular $\bar{q}_{I^-}$ is unstable (since $d < d_1$) and $\bar{q}_{I^+}$ is stable (since

$d > d_2$). The long-run dynamic will depend on the initial conditions of the distribution of traits in the population. (iii) When $\epsilon = \epsilon^*$, then $d_1 = d_2 = d^*$; $\forall\ d \neq \{d^*\}$ at least one of the two non-trivial solutions is admissible in the domain. In particular $\bar{q}_{I^-}$ is admissible if $d < d^*$ and it is always unstable $\forall\ q_t \in [0, 1]$. On the other hand, $\bar{q}_{I^+}$ is admissible but always stable if $d > d^*$ $\forall\ q_t \in [0, 1]$.

Proof of Proposition 2

Assume first that $\Delta V_t^i > 0\ \ \forall\ \ i \in \{T, L\}$. Let us the r.h.s. of equation (11) as $f(q_t, d)$. As already shown in Proof 1, we find two non-hyperbolic steady states, $\bar{q} = 0$ and $\bar{q} = 1$, that arise for $d = d_1 = \frac{\gamma+\epsilon}{\alpha+\gamma}$ and $d = d_2 = \frac{\beta}{\alpha+\beta}$ respectively. In order to prove that our dynamical system undergoes a transcritical bifurcation, we have to show that the four conditions for this type of bifurcation are satisfied at these equilibrium points.

Consider the non-hyperbolic steady state $\bar{q} = 0$ with $d = d_1$. Substituting these values into the equation $f(q_t, d)$, conditions (i) and (ii) follow immediately since $f(0, d_1) = 0$ and $\frac{\partial f}{\partial d}(0, d_1) = 0$. Conditions (iii) and (iv) are also satisfied because $\frac{\partial^2 f}{\partial q \partial d}(0, d_1) = \alpha + \gamma$ and $\frac{\partial^2 f}{\partial q^2}(0, d_1) = 2\beta - \frac{2(\alpha-\epsilon+\gamma)(\epsilon+\gamma)}{\alpha+\gamma}$, which is always positive given the assumption on parameters.

Consider now the other non-hyperbolic steady state $\bar{q} = 1$ with $d = d_2$. As before all the four conditions are satisfied, since (i) $f(1, d_2) = 1$, (ii) $\frac{\partial f}{\partial d}(0, d_2) = 0$, (iii) $\frac{\partial^2 f}{\partial q \partial d}(1, d_2) = \alpha + \beta$ and (iv) $\frac{\partial^2 f}{\partial q^2}(1, d_2) = 2(\beta - \gamma) - \frac{2\alpha\epsilon}{\alpha+\beta}$, which is always positive given the assumption on parameters. The same results arise when $\Delta V_t^T > 0$ and $\Delta V_t^L = 0$ or when $\Delta V_t^T = 0$ and $\Delta V_t^L > 0$.

Since all the conditions are satisfied and $\bar{q} = \{0; 1\}$ yields a non-hyperbolic steady state for $d = \{d_1; d_2\}$, we conclude that our dynamical system undergoes a transcritical bifurcation.

Proof of Proposition 3

In order to prove Proposition 3, we start by showing the conditions under which we have paternalism, that is both ΔV_0^T and ΔV_0^L are positive at $t = 0$. $\Delta V_0^T > 0$ if and only if q_0 is such that $\frac{\beta q_0+\gamma(1-q_0)[d+\beta(d-2)dq_0^2-1]+\alpha d[1+(d-2)(\beta+\gamma+\epsilon)(1-q_0)q_0^2]-\epsilon(1-q_0)}{1+(1-(q_0))q_0[d(\beta-\gamma+\epsilon)q_0-(\beta+\gamma+\epsilon)+d\gamma]} > 0$. The dominator is always positive; the numerator is a parabola which is positive if and only if parameter d and q_0 are such that $1 > d > \hat{d}^T(q_0) \equiv \frac{1}{2[\beta\gamma+\alpha(\beta+\gamma+\epsilon)]q_0^2(q_0-1)}(\delta^T - \sqrt{\rho^T})$ with $\delta^T = 2\alpha q_0^2(\beta+\gamma+\epsilon)(q_0-1)+\alpha+\gamma(1-q_0)(2\beta q_0-1)$ and $\rho^T = 4[\beta\gamma+\alpha(\beta+\gamma+\epsilon)](q_0-1)q_0^2[(\epsilon+\gamma)(q_0-

$1)+\beta q_0]+\{\gamma(q_0-1)(2\beta q_0-1)+\alpha[1-2\gamma q_0^2+(2\beta+2\epsilon)(q_0-1)q_0^2+2\gamma q_0^3]\}^2$. $\Delta V_0^L > 0$ if and only if q_0 is such that $\frac{(1-d)\epsilon(1-q_0)+\gamma(q_0-1)[\beta(d-2)dq_0(q_0-1)-1]+\alpha d[1+(d-2)(\beta+\gamma+\epsilon)(1-q_0)^2 q_0]}{1+(1-(q_0))q_0[d(\beta-\gamma+\epsilon)q_0-(\beta+\gamma+\epsilon)+d\gamma]} > 0$. The dominator is always positive; the numerator is once again a parabola which is positive if and only if parameter d and the q_0 are such that $1 > d > \hat{d}^L(q_0) \equiv \frac{1}{2[\beta\gamma+\alpha(\beta+\gamma+\epsilon)]q_0(q_0-1)^2}(\delta^L - \sqrt{\rho^L})$ with $\delta^L = \epsilon(1-q_0) + \beta[2\gamma(q_0-1)^2 - 1]q_0 + \alpha[2(\beta+\epsilon+\gamma)(q_0-1)^2 q_0 - 1]$ and $\rho^L = 4[\beta\gamma + \alpha(\beta+\gamma+\epsilon)](q_0-1)^2 q_0[(\beta+\epsilon+\gamma)q_0 - (\epsilon+\gamma)] + \{\epsilon(1-q_0) + \beta[2\gamma(q_0 - 1)^2 - 1]q_0 + \alpha[2(\beta+\gamma+\epsilon)(q_0-1)^2 - 1]\}^2$. Given that $0 < d < 1$ by assumption, we have paternalism iff $1 > d > \hat{d}^J(q_0) > 0$ with $J = \{T, L\}$. $\hat{d}^J(q_0) > 0$ is the threshold for paternalism. Moreover, whenever $q_0 > \frac{\gamma+\epsilon}{\gamma+\beta+\epsilon}$, traditional parents are always paternalistic but liberal parents are paternalistic iff $1 > d > \hat{d}^L(q_0) > 0$; when $q_0 < \frac{\gamma+\epsilon}{\gamma+\beta+\epsilon}$, liberal parents are always paternalistic but traditional parents are paternalistic iff $1 > d > \hat{d}^T(q_0) > 0$.
Assume that $\epsilon < \epsilon^*$, so that $d_1 < d_2$. From Proposition 1 we know that when $d > d_2$ then $q_\infty \to \bar{q}_{I^+}$ and diversity is ensured in the long-run; when $d \in [d_1; d_2]$ then $q_\infty \to 1$; when $d < d_1$ then $q_\infty \to \{0, 1\}$ respectively for $q_0 < \bar{q}_{I^-}$ and $q_0 > \bar{q}_{I^-}$. The following different scenarios are possible, depending on the level of the taste for similarity:
(i) Let $d < d_1$ and $q_0 < \bar{q}_{I^-}$, with q_0 such that both ΔV_0^T and ΔV_0^L are positive and paternalism is observed at $t = 0$. Knowing from Proposition 1 that the dynamics will converge to $\bar{q} = 0$ in the long-run, the limit of ΔV_t^L when $t \to \infty$ is always positive, but the limit of ΔV_t^T when $t \to \infty$ will be positive if and only if $d > d_1$, which is excluded by assumption. This implies that traditional parents will be non-paternalistic in the long-run. Given monotonicity, there $\exists\, t < N$ for which traditional parents aim to protect their ideological trait in their children. Whenever $t \geq N$ (with N threshold in time) traditional parents no longer promote their own trait; they behave in a non paternalistic way. (Assuming $q_0 > \bar{q}_{I^-}$ and q_0 such that ΔV_0^T and ΔV_0^L are both positive, implies in the long-run the presence of non-paternalistic liberal parents but paternalistic traditional parents). This implies that paternalism at $0 \leq t < N$ is not a sufficient condition to have diversity in the long-run.
(ii) Assume now $d \in [d_1; d_2]$ and q_0 such that both ΔV_0^T and ΔV_0^L are positive and paternalism is ensured at $t = 0$. The long-run dynamics will converge to $\bar{q} = 1$. The limit of ΔV_t^T at $t = \infty$ will be positive, but the limit of ΔV_t^L at $t = \infty$ will be positive if and only if $d > d_2$ which is excluded by assumption. In this case liberal parents will be non paternalistic in the long-run even if they are paternalistic at $t = 0$.
(iii) Assume $d > d_2$. Given the parameters' domain we know that $d_2 > \hat{d}^J(q_0)$ so that $d > \hat{d}^J(q_0)$ with $j = \{T; L\}$ $\forall\, q_0 \in [0, 1]$. This implies that both ΔV_t^T and ΔV_t^L are positive in every period of time. As we know from Proposition 1, the long-run dynamics will converge to the internal solution $\bar{q}_{I^+}$ in which diversity is observed. The limit of ΔV_t^T and ΔV_t^L when

$t \to \infty$ will be positive given the assumptions on the parameters. This means that when the taste for similarity is sufficiently high parents will be paternalistic in every period. Since paternalism is monotonic in q, we conclude that paternalism at $0 \leq t < N$ is a necessary condition to have diversity in the long-run.
The same proof can be applied to the other scenarios: $\epsilon > \epsilon^*$ and $\epsilon = \epsilon^*$.

Proof of Proposition 4

First of all we have to observe that both the dynamics under perfect foresight (11) and under myopic foresight (12) are monotonic, because both functions are continuous and non-decreasing. Solving equation (12) at the steady state we find the same equilibria as with the model with perfect foresight. Taking the limit of the dynamic equation (12) when $q \to 0$, we observe that if $d < \frac{\gamma+\epsilon}{\alpha+\gamma}$ the dynamic converges to zero passing below the 45 line. Conversely, when $d > \frac{\gamma+\epsilon}{\alpha+\gamma}$ the dynamic converges to zero passing above the 45 line. Proceeding as in Proof 1 it is straightforward to show that the dynamics under myopic foresight presents the same qualitative structure as the dynamics under perfect foresight, in the sense that the steady states are the same and the first-order stability conditions are equivalent.
We want to prove that the dynamics under perfect foresight (PF) exhibit a higher speed of convergence than those under myopic foresight (MF). To this end we have to show that the ratio between the speed of convergence under PF and MF is greater than one, that is, $\hat{\sigma} = \frac{\sigma^{PF}}{\sigma^{MF}} > 1$. Let us define the speed of convergence as:

$$\sigma^i = \frac{q^i_{t+1} - q^i_t}{\bar{q} - q^i_t} \quad with \ i = \{PF, MF\}$$

Substituting the steady state values and the dynamics (11) and (12) (respectively for $i = \{PF, MF\}$) into the equation above, then, $\forall \ q_t \in [0, 1]$, we obtain $\hat{\sigma} = \{1 - (q_t - 1)q_t[d(\gamma - 1) - (\epsilon + \beta) + d(\beta + \epsilon - \gamma)q_t]\}^{-1}$. Given the parameters' domain it is straightforward to demonstrate that $\hat{\sigma} = \frac{\sigma^{PF}}{\sigma^{MF}} > 1 \ \forall \ q_{t \geq 0}$.

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