# Unemployment equilibrium and economic policy in mixed markets

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# Unemployment equilibrium and economic policy in mixed markets

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**Abstract**

This paper considers a simple static Cournot-Nash model of an exchange economy with two productive sectors at flexible prices and wages. The traders in the atomless sector are price-takers, while the atoms behave strategically. We focus on the consequences of strategic interactions on the market outcome. Firstly, strategic interactions create underemployment on the labor market. Secondly, when the number of atoms increases without limit, the underemployment equilibrium coincides with the competitive equilibrium. Thirdly, we compare the welfare reached by traders at both equilibria. Fourthly, we consider the implementation of a tax levied on strategic supplies. Finally, we compare the approach retained with the usual monopolistic competition framework.

## 1 Introduction

A vast literature has been devoted to un(der)employment equilibrium without rigidities, especially in a partial equilibrium framework (see for instance Cahuc and Zylberberg (2001))[1]. The motivations in this paper are twofold. Firstly, it aims at providing a conceptual framework into which the determination of market general equilibrium outcome is based on strategic interactions at flexible prices and wages. Secondly, it analyzes the working and the consequences on a perfectly competitive labor market of the strategic interactions taking place on the output markets. This paper therefore considers a simple static Cournot-Nash model of an exchange economy with productive sectors.

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[1]Without being exhaustive, one could make reference to search theory, matching approaches, wage bargaining theories...).

Two main approaches modeled strategic interactions in general equilibrium. First, the strategic market games were elaborated in order to circumvent the auctioneer (Dubey-Shubik (1978), Sahi-Yao (1989)). Second, the Cournot-Walras equilibrium (CWE) approach initially developed by Gabszewicz-Vial (1972), and pursued by Codognato-Gabszewicz (1991), (1993), Gabszewicz-Michel (1997) and by d'Aspremont et al. (1997) for pure exchange economies, was conceived to analyze the consequences of market power in general equilibrium. The CWE models make the equilibrium prices and the allocations the results of a market price mechanism in strategic multilateral exchange. As a consequence, the market demand which addresses to each producer is made endogenous. Consequently, we propose to investigate the question of underemployment equilibrium in strategic multilateral exchange *à la* Cournot-Walras.

Many models deal with un(der)employment in general equilibrium under imperfect competition without rigidities. Models of cooperation failures put forward un(der)employment equilibrium. Inefficiencies may be caused by local market power of firms and consumers, which stems from the fact that all goods (and all labors) are imperfect substitutes (Blanchard-Kiyotaki (1987), Layard et al. (1991)), or that deficiency of aggregate demand occurs (Hart (1982), d'Aspremont et al. (1989), (1990)). Monopolistic competition models do not provide microfoundations which explain why monopolistic agents could not interact strategically. In addition, there is no market price mechanism which determines equilibrium prices. In other (oligopolistic) models, each seller either objectively knows or must conjecture subjectively the demand which addresses to him (Bénassy (1991), Negishi (1961)). Otherwise, models of coordination failures feature indeterminacy (Heller (1986), Manning (1990), Roberts (1987)), so multiple equilibria complicate the study of the economic policy (Cooper (1999)).

In this paper, we consider a model in which the equilibrium prices are determined by a market mechanism and the demand functions are micro-founded. We extend the basic model of an exchange economy with a productive sector of Gabszewicz and Michel (1997). The economy includes two productive sectors with a competitive labor market[2]. In one sector (the atomless sector), all the agents are price takers, while the agents in the other sector (the atomic sector) behave strategically. We therefore refer to the concept of "mixed markets" rationalized by Shitovitz (1973) in a pure exchange economy. The following results are obtained. First, there is a CWE with underemployment at flexible prices and wages[3]. Second, when the number of atoms increases unboundedly, the underemployment CWE coincides with the full employment competitive equilibrium. Third, we compare the individual welfare reached at both equilibria. Fourth, we consider economic policy by introducing a tax levied on strategic supplies in order to reduce market distortions caused by strategic behaviors. In addition, we compute the Chamberlin-Walras equilibrium for the same basic economy. Thus,

[2] The production activities in time were considered in a bileteral oligopoly model in Cordella and Datta (2002), but with storage functions and without a labor market.

[3] The existence and the uniqueness analysis are beyond the scope of this paper, which aims at computing the market outcome. The existence of a general oligopoly equilibrium usually rises specific problems (Bonnisseau and Florig (2003), Gabszewicz (2002)).

the CWE is not Pareto dominated by the Chamberlin-Walras equilibrium. And, the tax policy has more impact on the market outcome at the CWE.

The paper is organized as follows. In section 2, we describe the basic economy. Section 3 is devoted to the CWE with underemployment. Section 4 considers a tax policy. Section 5 computes the Chamberlin-Walras equilibrium and compares it with the results previously obtained. In section 6, we conclude.

# 2 The economy

Consider an exchange economy with two productive sectors. The first sector (the atomless sector) includes two continua of agents represented by the intervals of mass 1 $T_i = [0,1]$, $i = 0,1$, with the Lebesgue measure $\mu$, where $T_0 = [0,1]$ is the set of negligible firms, and where $T_1 = [0,1]$ is the set of negligible consumers. These both sets include agents who behave competitively as price takers. The second sector (the atomic sector) embodies $n$ atoms $a_1, a_2..., a_n$, with typical element $\{a_j\}$, each of measure $\mu(\{a_j\}) = 1$, $j = 1,...,n$. Let us denote $T_2 = \{1,...,n\}$ the finite set of atoms (indifferently the large traders or the oligopolists) who behave strategically.

There are two produced consumption goods, and one nonproduced good, labor. Both consumption goods and labor are perfectly divisible. Let us denote $p_1$, $p_2$ and $w$ respectively the prices of good 1, of good 2, and the wage rate. We assume that good 1 is the *numéraire*, so $p_1 = 1$. As a consequence both relative prices shall be denoted $\frac{p_2}{p_1} = p$ and $\frac{w}{p_1} = w$. The preferences toward consumption goods are assumed to be represented by a Cobb-Douglas specification. We thus consider preferences that feature generalization in consumption activities[4].

The utility function of trader $t \in T_1$ defined as $U_t$: $T_1 \times \mathbb{R}^3_+ \to \mathbb{R}$, with $U(t,x,l) = U_t(x,l)$ is measurable. It is assumed to have the desired properties (continuity, monotonicity and strict concavity). It is also assumed to be additively separable in consumption demands $x(t)$ and leisure, and homogenous of degree 1 in consumption of both goods[5]. So we consider the following specification[6] for any $t \in T_1$:

$$U_t(x(t), l(t)) = \left[\frac{x_1(t)}{\alpha}\right]^{\alpha} \left[\frac{x_2(t)}{1-\alpha}\right]^{1-\alpha} - \frac{1}{1+\varepsilon}\left[l(t)\right]^{1+\varepsilon}, \ \alpha \in (0,1), \ \varepsilon > 0, \quad (1)$$

where $x_1(t)$, $x_2(t)$ and $l(t)$ denote respectively the demand of commodities 1 and 2 and the labor supply for $t \in T_1$. Additionally, $\alpha \in (0,1)$ is the constant elasticity of utility with respect to consumption, which also measures the strength of the demand linkage across both sectors. Additionally, $\varepsilon$ represents the Frisch elasticity of labor supply (constant marginal utility of wealth)[7].

[4] Some models assume agents do not consume the good they produce in order to circumvent "Ford effects" (Diamond (1982), Heller (1986), Roberts (1987) and Weitzman (1982)).

[5] The separation property simplifies welfare comparisons.

[6] We can notice that for $\alpha = 0$ the economy is autarkic; and for $\alpha = 1$ commodity 2 is a pure input and agents only consume good 1.

[7] Moreover, $\varepsilon - 1$ is the elasticity of marginal disutility with respect to work.

The utility function of any atom $\{a_j\}$, $j \in T_1$ defined as $U_{a_j} : \mathbb{R}^2_+ \to \mathbb{R}$ is assumed to have the desired properties (continuity, monotonicity and strict quasi-concavity). It is also continuous and homogenous of degree 1 in consumption of both goods. It may thus be written:

$$U_{a_j}(x) = \left(\frac{x_1(a_j)}{\alpha}\right)^{\alpha} \left(\frac{x_2(a_j)}{1-\alpha}\right)^{1-\alpha}, \ \alpha \in (0,1) \ \text{ for } j \in T_2, \tag{2}$$

where $x_1(a_j)$ and $x_2(a_j)$ represent the demand functions for goods 1 and 2 of atom $a_j$, $j \in T_2$.

Neither consumption good is initially possessed by any type of agents, whereas the consumers in the atomless sector are endowed with one unit of the nonproduced good, so the structure of the initial endowments is given by:

$$\begin{aligned} \omega(t) &= (0,0,0),\ t \in T_0, \\ \omega(t) &= (0,0,1),\ t \in T_1, \\ \omega(\{a_j\}) &= (0,0,0),\ j \in T_2. \end{aligned} \tag{3}$$

Hence all producers have zero vector of initial endowments, while consumers $t \in T_1$, are initially endowed with one unit of time to allocate between leisure and labor. In addition, they receive profits from firms $t \in T_0$.

In addition, each firm $t \in T_0$ and each atom $\{a_j\}$ have inherited a technology which specifies how to produce some amounts of only one good. This assumption features specialization in production. Let the production set of any agent $t \in T_0$ be $Y(t) = \left\{(y(t), n(t)) \in \mathbb{R}^2_+ \mid y(t) \leq \frac{1}{\beta}\left[n(t)\right]^{\beta}\right\}$, where $y_1(t)$ and $n(t)$ represent the production of good 1 and the demand of labor. The production set is assumed to be strictly convex, so $\beta \in (0,1)$, where $\beta$ measures the productivity of labor. Therefore, the production function of any agent $t \in T_0$, is defined by all vectors $(y(t), n(t)) \in \sup Y(t)$, and may be written:

$$y(t) = \frac{1}{\beta}\left[n(t)\right]^{\beta},\ \beta \in (0,1),\ t \in T_0. \tag{4}$$

Let $Y_{a_j} = \left\{(y(a_j), (z(a_j)) \in \mathbb{R}^2_+ \mid y(a_j) \leq \frac{1}{\gamma} z(a_j)\right\}$ be the production set of $a_j \in T_2$, where $y(a_j)$, $z(a_j)$ and $\gamma > 0$ represent respectively the amount of output 2, the demand of output 1 as an input and the inverse of the productivity[8]. Thus, the production function of firm $j$ is of the same type as in Gabszewicz and Michel (1997). It is defined by all vectors $((y(a_j), (z(a_j)) \in \sup Y_{a_j}$, and may consequently be written:

$$y(a_j) = \frac{1}{\gamma} z(a_j),\ j \in T_2. \tag{5}$$

[8] We consider a constant return to scale in order to simplify the computation of the general oligopoly equilibrium.

The atoms have two decisions to make: which quantities $y_j$ of good 2 to produce (which determines through (4) the amount $z_j$ of good 1 to be bought as an input), and which amounts $s_j$ of good 2 to supply in exchange for good 1 on the market. The only strategic decision concerns the quantity $s_j$ brought to the market. The production activities do not involve strategic interactions.

The convex strategy set of each type of traders may be written:

$$S_{a_j} = \{s(a_j) \in \mathbb{R}_+ \mid 0 \leq s(a_j) \leq y(a_j)\}, \, j \in T_2, \tag{6}$$

$$S_t = \{\varnothing\}, \, t \in T_0. \tag{7}$$

As a producer, a plan for any atom $\{a_j\}$ is a vector $(y(a_j), s(a_j))$, whose components are respectively an amount of production and a pure strategy, with $y(a_j) \in Y_{a_j}$ and $s(a_j) \in S_{a_j}$, $j \in T_2$. Hence, the profit of each oligopolist may be written $\Pi_{a_j}(s(a_j), y(a_j)) = p_2 s(a_j) - z(a_j)$, $\forall j \in T_2$. The equilibrium strategy and equilibrium production of any oligopolist $a_j$ will be denoted respectively $\tilde{s}(a_j)$ and $\tilde{y}(a_j)$, with $\tilde{s}(a_j) \in S_{a_j}$ and $\tilde{y}(a_j) \in Y_{a_j}$, $\forall j \in T_1$. As a producer, each agent $t \in T_0$ has two decisions to make: which amount $y_1(t)$ of good 1 to produce and which quantity of labor $n(t)$ to buy in order to produce. A plan for a non strategic agent $t \in T_0$ is a pair $(y(t), n(t))$, with $(y(t), n(t)) \in Y(t)$. The corresponding profit is denoted by $\Pi_t$, with $\Pi_t(y(t), n(t)) = y(t) - wn(t)$, $t \in T_0$. The equilibrium production and labor demand will be denoted respectively $\tilde{y}(t)$ and $\tilde{n}(t)$, with $(\tilde{y}(t), \tilde{n}(t)) \in Y(t)$, $t \in T_0$.

Some assumptions are made regarding the way markets perform the allocations. First, there is a complete set of markets. Second, all trades occur only at equilibrium. Third, trade is costless and without delay. The allocations resulting from the exchanges on markets follow. As a consumer, trader $\{a_j\}$ obtains in exchange of $s(a_j)$ a quantity $p_2 s(a_j)$ of good 1, and finally consumes an amount $x_2(a_j) = \Pi_{a_j}$ of this good, and a quantity $x_2(a_j) = y(a_j) - s(a_j)$ of good 2 (see thereafter). The equilibrium allocation of any atom will be denoted $(\tilde{x}_1(a_j), \tilde{x}_2(a_j))$, $\forall j \in T_2$. As consumers, the competitive agents have only two types of decisions to make: which quantities of the two goods $(x_1(t), x_2(t))$ they want to consume and which amount of labor they want to supply, taking the price system $(1, p_2, w)$ and the profits $\Pi_t$ they received from all agents $t \in T_0$ as given. The allocation $(x_1(t), x_2(t), l(t))$ of such traders satisfies $x_1(t) + p_2 x_2(t) \leq wl(t) + \int_0^1 \Theta(t)\Pi_t d\mu(t)$, $t \in T_0$, where $\Theta(t)$ represent the share of profits distributed in the atomless sector by any firm $t \in T_0$, among any negligible trader $t \in T_1$, with $\int_0^1 \Theta(t)\Pi_t d\mu(t) = 1$, $t \in T_1$.

**Definition 1** *An economy $\xi$ is a collection of agents (atoms and negligible traders), initial endowments, utility functions, production sets and strategy sets* $\xi = \left\{(\omega(t), S(t), Y(t))_{t \in T_0}, (U(t), \omega(t), \Theta(t))_{t \in T_1}, (\omega(\{a_j\}), U_{a_j}, Y_{a_j}, S_{a_j})_{j \in T_2}\right\}$.

# 3 Cournot-Walras unemployment equilibrium

The model features an equilibrium concept which is based on a market price mechanism which include all the traders and one games between the only atoms. The strategies of the atoms are their supplies and the payments are the utility levels they reach.

**Definition 2** *A CWE for $\xi$ is given by a vector of prices $\tilde{\mathbf{p}} = (1, \tilde{p}_2(\tilde{\mathbf{s}}), \tilde{w}(\tilde{\mathbf{s}}))$, a vector of strategies $\tilde{\mathbf{s}} = (\tilde{s}(a_1), ..., \tilde{s}(a_j), ..., \tilde{s}(a_n))$, with $\tilde{s}(a_j) \in S_{a_j}$, $\forall a_j \in T_2$, and allocations $(\tilde{y}(t), \tilde{n}(t)) \in IR^2_+$ for $t \in T_0$, $(\tilde{x}_1(t), \tilde{x}_2(t), l(t)) \in \mathbb{R}^3_+$ for $t \in T_1$ and $(\tilde{x}_1(a_j), \tilde{x}_2(a_j)) \in IR^2_+$ for $a_j \in T_2$ such that: (i) $\tilde{x}(a_j) = x(\tilde{s}(a_j), \tilde{\mathbf{s}}(a_{-j}), \tilde{\mathbf{p}})$, $\forall j \in T_2$, where $\tilde{\mathbf{s}}(a_{-j})$ is the vector of equilibrium strategies of all traders who are different from trader $j$, (ii) $(\tilde{y}(t), \tilde{n}(t))$ solves $Max$ $\Pi_t = y(t) - wn(t)$, $t \in T_0$ and $(\tilde{x}_1(t), \tilde{x}_2(t), l(t))$ solves $Max$ $U(x(t))$ s.t. $x_1(t) + p_2 x_2(t) \leq wl(t) + \int_0^1 \Theta(t)\Pi(t)d\mu(t)$, $t \in T_1$, (iii) $\int_{t\in T_1} x_1(t)d\mu(t) + \sum_{j\in T_2} x_1(a_j) = \int_{t\in T_0} y_1(t)d\mu(t)$, $\int_{t\in T_1} x_2(t)d\mu(t) = \sum_{j\in T_2} s(a_j)$, $\forall s(a_j) \in S_{a_j}$ and $\int_{t\in T_1} n(t)d\mu(t) = \int_{t\in T_0} l(t)d\mu(t)$, and (iv) $U_{a_j}(\tilde{x}(\tilde{s}(a_j), \tilde{\mathbf{s}}(a_{-j}), \tilde{\mathbf{p}})) \geq U_{a_j}(x(s(a_j), \tilde{\mathbf{s}}(a_{-j}), \mathbf{p}))$, $\forall s(a_j) \in S_{a_j}$.*

This equilibrium concept is based on a market-clearing price mechanism and a game under complete but imperfect information. Its computation implies a two-step procedure: the one is competitive and the other is stategic (Busetto et ali (2008)). Therefore, the CWE depends on competitive and strategic decisions. In a first step each trader determines his competitive plans for given strategies. Then the relative equilibrium prices, which clear all markets are thus determined. In a second step the atoms determine their equilibrium strategies[9].

Let us now determine the Cournot-Walras equilibrium. The program of any trader $t \in T_0$ may be written:

$$\underset{(y(t),n(t))}{Max} \ \Pi_t\left(y(t), n(t)\right) = y(t) - wn(t),\ t \in T_0 \tag{8}$$

$$\text{s.t. } y(t) = \frac{1}{\beta}\left[n(t)\right]^\beta$$

$$\text{with } n(t) \geq 0,\ y(t) \geq 0.$$

This leads to the demand of labor and the supply of output for good 1:

$$n(t) = \left(\frac{1}{w}\right)^{\frac{1}{1-\beta}},\ t \in T_0, \tag{9}$$

$$y(t) = \frac{1}{\beta}\left(\frac{1}{w}\right)^{\frac{\beta}{1-\beta}},\ t \in T_0. \tag{10}$$

[9] Thus, the equilibrium prices is determined for given strategies and the equilibrium allocation results from strategic interactions between reaction functions within quantity spaces.

These functions are continuous, homogeneous of degree zero with respect to the absolute prices and strictly decreasing with $w$.

The plan of any trader $t \in T_1$:

$$\underset{(x_1(t),x_2(t),l(t))}{Max} \left[\frac{x_1(t)}{\alpha}\right]^{\alpha} \left[\frac{x_2(t)}{1-\alpha}\right]^{1-\alpha} - \frac{1}{1+\varepsilon}\left[l(t)\right]^{1+\varepsilon}, \; t \in T_1, \tag{11}$$

$$\text{s.t. } x_1(t) + p_2 x_2(t) \leq wl(t) + \int_{t \in T_0} \Theta(t)\Pi_t d\mu(t).$$

This leads to the following demand functions for goods and labor supply function (see Appendix 1):

$$(x_1(t), x_2(t)) = \left(\alpha\Omega(t), (1-\alpha)\frac{\Omega(t)}{p_2}\right), \tag{12}$$

$$l(t) = w^{\frac{\alpha}{\varepsilon}} \left(\frac{w}{p_2}\right)^{\frac{1-\alpha}{\varepsilon}}, \tag{13}$$

where $\Omega(t) = wl(t) + \int_{t \in T_0} \Theta(t)\Pi_t d\mu(t)$, $t \in T_1$. These functions have the desired properties (continuity, monotonicity and homogeneity of degree zero with respect to prices and wage).

Given a price system $(1, p_2)$ and given $(s(a_j), y(a_j))$ the profit of oligopolist $\{a_j\}$ is defined by $\Pi_{a_j}(s(a_j), y(a_j)) = p_2 s(a_j) - \gamma y(a_j)$. As a consumer, atom $\{a_j\}$ can buy any bundle $(x_1(a_j), x_2(a_j))$, the value of which does not exceed $\Pi_{a_j}(s(a_j), y(a_j))$. Thus, for a price system $(1, p_2)$ and given $(s(a_j), y(a_j))$, atom $\{a_j\}$ solves the problem:

$$\underset{(x_1(a_j),x_2(a_j))}{Max} \left(\frac{x_1(a_j)}{\alpha}\right)^{\alpha} \left(\frac{x_2(a_j) + y(a_j) - s(a_j)}{1-\alpha}\right)^{1-\alpha}, \; \forall j \in T_2 \tag{14}$$

$$\text{subject to } \; x_1(a_j) + p_2 x_2(a_j) \leq \Pi_{a_j}(s(a_j), y(a_j))$$

$$\text{and } x_1(a_j) \geq 0, \; x_2(a_j) \geq 0.$$

Following a similar procedure as in Gabszewicz and Michel (1997), it can be shown that the solution to (14) is given by (see Appendix 2):

$$(x_1(a_j), x_2(a_j)) = \left(\Pi_{a_j}, 0\right), \; \forall j \in T_2. \tag{15}$$

Consequently, the utility level reached by any oligopolist $\{a_j\}$ as a consumer can be written as the payoff function:

$$V_{a_j}(s(a_j), y(a_j)) = \left(\frac{\Pi_{a_j}}{\alpha}\right)^{\alpha} \left(\frac{y(a_j) - s(a_j)}{1-\alpha}\right)^{1-\alpha}, \; \forall j \in T_1. \tag{16}$$

The computation of the oligopoly equilibrium needs the equilibrium relative prices to be determined. Given an $n$-dimensional vector of strategies $(s(a_1), ..., s(a_j), ..., s(a_n))$, the market-clearing conditions may be written:

$$\int_{t\in T_1} x_1(t)d\mu(t) + \sum\nolimits_{j\in T_2} \Pi_{a_j} + \sum\nolimits_{j\in T_1} z(a_j) = \int_{t\in T_0} y_1(t)d\mu(t)$$

$$\int_{t\in T_1} x_2(t)d\mu(t) = \sum\nolimits_{j\in T_2} s(a_j) \tag{17}$$

$$\int_{t\in T_1} n(t)d\mu(t) = \int_{t\in T_0} l(t)d\mu(t)$$

The equilibrium relative prices follow (see Appendix 3):

$$p_2 = \left[\left(\frac{1-\alpha}{\beta}\right)\frac{1}{\sum_{j\in T_2} s(a_j)}\right]^{1-\theta}, \tag{18}$$

$$w = \left[\left(\frac{1-\alpha}{\beta}\right)\frac{1}{\sum_{j\in T_2} s(a_j)}\right]^{\left(\frac{1-\beta}{\beta}\right)\theta}, \tag{19}$$

where $\theta \equiv \frac{(1-\alpha)\beta}{1-\alpha\beta+\varepsilon}$, with $\theta < 1$.

From (16), the optimal choice $(\tilde{s}(a_j), \tilde{y}(a_j))$ for any atom $\{a_j\}$, $j \in T_2$ is the solution to the problem:

$$\underset{(\tilde{s}(a_j),\tilde{y}(a_j))\in\tilde{S}(a_j)\times Y(a_j)}{Arg\max} V_{a_j}(s(a_j), y(a_j)) = \left(\frac{p_2 s(a_j) - \gamma y(a_j)}{\alpha}\right)^{\alpha}\left(\frac{y(a_j) - s(a_j)}{1-\alpha}\right)^{1-\alpha}. \tag{20}$$

The first-order conditions are given by $\partial V_{a_j}/\partial s(a_j) = 0$ and $\partial V_{a_j}/\partial y(a_j) = 0$ and lead to:

$$\left(\frac{p_2}{p_1} + \frac{\partial p_2}{\partial s(a_j)} s(a_j)\right)\left(\frac{y(a_j) - s(a_j)}{1-\alpha}\right) - \frac{p_2 s(a_j) - \gamma y(a_j)}{\alpha} = 0, \tag{21}$$

$$-\frac{\gamma}{1-\alpha}\left(y(a_j) - s(a_j)\right) + \frac{p_2 s(a_j) - \gamma y(a_j)}{\alpha} = 0. \tag{22}$$

At the symmetric general equilibrium, one has $\tilde{s}(a_j) = \tilde{s}(a_{-j})$ for all $a_{-j} \neq a_j$, $\forall -j \neq j$, with $\sum_{j\in T_2} s(a_j) = s(a_j) + (n-1)s(a_{-j})$. From (19), one has $\frac{\partial p_2}{\partial s(a_j)} = (1-\theta)\left(\frac{1-\alpha}{\beta}\right)^{1-\theta}\left(\frac{1}{\sum_{j\in T_2} s(a_j)}\right)^{2(1-\theta)}$. Using (22)-(23) lead to the optimal strategy $\tilde{s}(a_j)$ and to the optimal production $\tilde{y}(a_j)$ of trader $\{a_j\}$, $\forall j \in T_2$:

$$\tilde{s}(a_j) = \left(\frac{1-\alpha}{\beta n}\right)\left[\frac{1}{\gamma}\frac{n-(1-\theta)}{n}\right]^{\frac{1}{1-\theta}}, \tag{23}$$

$$\tilde{y}(a_j) = \left(\frac{1-\alpha}{\beta n}\right)\left(\frac{1}{\gamma}\right)^{\frac{1}{1-\theta}}\left[\frac{n-\alpha(1-\theta)}{n}\right]\left[\frac{n-(1-\theta)}{n}\right]^{\frac{\theta}{1-\theta}}. \tag{24}$$

We deduce the equilibrium relative prices:

$$\tilde{p} = \frac{\gamma n}{n-(1-\theta)}, \tag{25}$$

$$\tilde{w} = \left[\frac{\gamma n}{n-(1-\theta)}\right]^{\left(\frac{1-\beta}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)}. \tag{26}$$

The level of production and the quantity of labor traded sector follow:

$$\tilde{n}(t) = \left\{\frac{1}{\gamma}\left[\frac{n-(1-\theta)}{n}\right]\right\}^{\frac{1}{\beta}\left(\frac{\theta}{1-\theta}\right)}, \; t \in T_0, \tag{27}$$

$$\tilde{y}(t) = \frac{1}{\beta}\left\{\frac{1}{\gamma}\left[\frac{n-(1-\theta)}{n}\right]\right\}^{\frac{(1-\alpha)\beta}{1-\beta+\varepsilon}}, \; t \in T_0. \tag{28}$$

The equilibrium allocations are given by:

$$(\tilde{x}_1(t), \tilde{x}_2(t)) = \left(\frac{\alpha}{\beta}\psi, \left(\frac{1-\alpha}{\beta}\right)\left[\frac{1}{\gamma}\frac{n-(1-\theta)}{n}\right]\psi\right), \; t \in T_1 \tag{29}$$

$$(\tilde{x}_1(a_j), \tilde{x}_2(a_j)) = \left(\frac{1-\alpha}{\beta n}\right)(1-\theta)\,\psi.\left(\frac{\alpha}{n}, \frac{1-\alpha}{\gamma n}\right), \; j \in T_2, \tag{30}$$

where $\psi \equiv \left[\frac{1}{\gamma}\frac{n-(1-\theta)}{n}\right]^{\frac{\theta}{1-\theta}}$.

The utility level reached by any type of traders may be written:

$$\tilde{U}_t = \left[\frac{1-\beta+\varepsilon}{\beta(1+\varepsilon)}\right]\left[\frac{1}{\gamma}\frac{n-(1-\theta)}{n}\right]^{\left(\frac{1+\varepsilon}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)}, \; t \in T_1, \tag{31}$$

$$\tilde{U}_{a_j} = \left(\frac{1-\alpha}{\beta n^2}\right)\left(\frac{1}{\gamma}\right)^{\left(\frac{1+\varepsilon}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)}(1-\theta)\left[\frac{n-(1-\theta)}{n}\right]^{\frac{\theta}{1-\theta}}, \; j \in T_2. \tag{32}$$

**Proposition 1** *When the number of atoms becomes arbitrarily large, the CWE with underemployment coincides with the competitive equilibrium.*

**Proof.** From (25)-(26) we deduce $\lim_{n\to\infty} \tilde{p}_2 \to \gamma$ and $\lim_{n\to\infty} \tilde{w} \to \gamma^{\left(\frac{1-\beta}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)}$. Additionally, one has $(\tilde{x}_1(a_j), \tilde{x}_2(a_j)) = (0,0)$ for $j \in T_2$ and $(\tilde{x}_1(t), \tilde{x}_2(t)) = \left(\frac{1}{\gamma}\right)^{\frac{1}{1-\theta}} \left(\frac{\alpha}{\beta}, \frac{1-\alpha}{\beta}\right)$ for $t \in T_1$ and $\tilde{s}(a_j) = \left(\frac{1-\alpha}{\beta n}\right)\left(\frac{1}{\gamma}\right)^{\frac{1}{1-\theta}}$, $\forall j \in T_1$, $(\tilde{y}(t), \tilde{n}(t)) = \left(\left(\frac{1}{\gamma}\right)^{\frac{1}{\beta}\left(\frac{\theta}{1-\theta}\right)}, \left(\frac{1}{\gamma}\right)^{\frac{\theta}{1-\theta}}\right)$, $t \in T_0$. We now compare those equilibrium relative prices and equilibrium allocations with those obtained at the competitive equilibrium. Each trader $\{a_j\}$, $j \in T_2$ now behaves as a price taker. Thus, atom $\{a_j\}$ solves $Max \left(\frac{x_1(a_j)}{\alpha}\right)^{\alpha} \left(\frac{x_2(a_j)}{1-\alpha}\right)^{1-\alpha}$ s.t. $x_1(a_j) + p_2 x_2(a_j) \leq \Pi_{a_j}$, $x_1(a_j) \geq 0$ and $x_2(a_j) \geq 0$, where $\Pi_{a_j} = y(a_j)\,(p_2 - \gamma)$, $\forall j \in T_2$. The supply correspondence is $y(a_j) = \{0$ for $p_2 < \gamma$, $y(a_j) \in [0, \bar{y}(a_j)]$ for $p_2 = \gamma$ and $\bar{y}(a_j)$ for $p_2 > \gamma\}$, with $\bar{y}(a_j) = \max y(a_j)$ for $y(a_j) \in [0, \bar{y}(a_j)]$, $\forall j \in T_2$. The market equilibrium price and the level of activity are determined by the aggregate demand function, where the supply of good 2 is perfectly elastic, i.e. for $\left(\frac{1}{p_2}\right)^{*} = \gamma$ and $s^*(a_j) = y^*(a_j) = \int_{t \in T_1} \left(\frac{1-\alpha}{\beta n}\right)\left(\frac{1}{\gamma}\right)^{\frac{1}{1-\theta}} d\mu(t)$, $\forall j \in T_2$. Moreover, one deduces the competitive firms allocations $(y^*(t), n^*(t)) = \left(\left(\frac{1}{\gamma}\right)^{\frac{1}{\beta}\left(\frac{\theta}{1-\theta}\right)}, \left(\frac{1}{\gamma}\right)^{\frac{\theta}{1-\theta}}\right)$, $t \in T_0$. The corresponding equilibrium allocations are given by $(x_1^*(a_j), x_2^*(a_j)) = (0,0)$, $\forall j \in T_2$, and $(x_1^*(t), x_2^*(t)) = \left(\frac{\alpha}{\beta}\left(\frac{1}{\gamma}\right)^{\frac{1}{1-\theta}}, \left(\frac{1-\alpha}{\beta}\right)\left(\frac{1}{\gamma}\right)^{\frac{1}{1-\theta}}\right)$, $t \in T_1$. *QED.* ■

When the number of strategic traders becomes arbitrarily large, the market power of any atom $\{a_j\}$, $j \in T_2$ vanishes. The $n$ atoms, each being of mass 1, have been splitted into an atomless continuum of traders, each of measure zero.

As a corollary, imperfectly competitive behaviors create market distortions in the allocation of resources on the labor market. The real wage at the CWE $\tilde{w}$ is higher than the competitive equilibrium wage $w^*$. Therefore, strategic interactions on the output markets create underemployment in the labor market since all mutually advantagenous trades are not fully exhausted[10]. This cooperation failure could disappear if the number of atoms became arbitrarily large or if the oligopolists would cooperate. Therefore, the cause of underemployment in the labor market relies on the non-cooperative behaviors sustained by strategic interactions.

**Proposition 2** *The CWE is not Pareto dominated by the competitive equilibrium.*

**Proof.** Consider the utility of any trader as a measure of individual welfare. We compare the levels of welfare reached by traders at both equilibria. Little algebra lead to $\tilde{U}_t - U_t^* = \left[\frac{1-\beta+\varepsilon}{\beta(1+\varepsilon)}\right]\left(\frac{\alpha}{\gamma}\right)^{\left(\frac{1+\varepsilon}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)} \left\{\left[\frac{n-(1-\theta)}{n}\right]^{\left(\frac{1+\varepsilon}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)} - 1\right\}$. As $\beta \in (0,1)$ and $\epsilon > 0$, we have $0 < 1 - \frac{1-\theta}{n} < 1$. Then $\tilde{U}_t < U_t^*$, $\forall t \in T_1$. And,

[10] Such unemployment might be conceived as underemployment of resources. Nevertheless, there is no Keynesian unemployment because the labor market does feature any rationing.

$\tilde{U}_{a_j} - U^*_{a_j} = \left(\frac{1-\alpha}{\beta n^2}\right)\left(\frac{1}{\gamma}\right)^{\left(\frac{1+\varepsilon}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)}(1-\theta)\left[\frac{n-(1-\theta)}{n}\right]^{\frac{\theta}{1-\theta}} - 0$, then $\tilde{U}_{a_j} > U^*_{a_j}$. Finally, one deduces $\left(U^*_t - \tilde{U}_t\right) - \left(U^*_{a_j} - \tilde{U}_{a_j}\right) > 0$ and $\left(\tilde{U}_t - U^*_t\right) - (\tilde{U}_{a_j} - U^*_{a_j}) < 0$. *QED.* ■

The utility of the oligopolists is higher at the CWE than at the competitive equilibrium, while the converse is true for the traders in the atomless sector. The atoms have market power and partially influence the relative equilibrium relative prices. When strategic interactions are replaced by anonymous interactions, all traders behave as price takers and the terms of trade become more favorable to traders in the atomless sector (the relative price decreases). In addition, the difference sign between the gains reached by both types of agents at the two equilibria is indeterminate, so both equilibria may not be Pareto-ranked.

# 4 Economic policy

In order to regulate market distortions caused by strategic behaviors, consider a taxation policy on strategic supplies as in Gabszewicz and Grazzini (1999). We assume that when exchange takes place a per unit tax $\tau$, with $\tau \in (0,1)$, is levied on the supplies of good 2. After exchanges have occured, the total product of the tax $\sum_{j\in T_2} \tau s(a_j)$ is transfered among all traders $t \in T_1$.

## 4.1 Tax and welfare

After little algebra, the computation of the CWE leads to (see Appendix 4):

$$\tilde{s}(a_j,\tau) = (1-\tau)^{\frac{\theta}{1-\theta}}\tilde{s}(a_j),\ j \in T_2, \tag{33}$$

$$\tilde{y}_j(a_j,\tau) = (1-\tau)^{\frac{\theta}{1-\theta}}\tilde{y}(a_j),\ j \in T_2. \tag{34}$$

The equilibrium relative prices follow:

$$\tilde{p}(\tau) = \left(\frac{\gamma}{1-\tau}\right)\left[\frac{n}{n-(1-\theta)}\right], \tag{35}$$

$$\tilde{w}(\tau) = \left\{\left(\frac{\gamma}{1-\tau}\right)\left[\frac{n}{n-(1-\theta)}\right]\right\}^{\left(\frac{1-\beta}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)}. \tag{36}$$

We deduce:

$$\tilde{n}(t,\tau) = \left\{\left(\frac{1-\tau}{\gamma}\right)\left[\frac{n-(1-\theta)}{n}\right]\right\}^{\frac{1}{\beta}\left(\frac{\theta}{1-\theta}\right)},\ t \in T_0, \tag{37}$$

$$\tilde{y}(t,\tau) = \frac{1}{\beta}\left\{\left(\frac{1-\tau}{\gamma}\right)\left[\frac{n-(1-\theta)}{n}\right]\right\}^{\frac{\theta}{1-\theta}},\ t \in T_0. \tag{38}$$

The corresponding allocations are respectively:

$$(\tilde{x}_1(t,\tau),\tilde{x}_2(t,\tau)) = \Psi(\tau).\left(\frac{\alpha}{\beta},\left(\frac{1-\alpha}{\beta}\right)\left[\frac{1}{\gamma}\frac{n-(1-\theta)}{n}\right]\right),\ t\in T_1, \tag{39}$$

$$(\tilde{x}_1(a_j,\tau),\tilde{x}_2(a_j,\tau)) = \left(\frac{1-\alpha}{\beta n}\right)(1-\theta)\,\Psi(\tau).\left(\frac{\alpha}{n},\frac{1-\alpha}{\gamma n}\right),\ j\in T_2, \tag{40}$$

where $\Psi(\tau)\equiv(1-\tau)^{\frac{\theta}{1-\theta}}\psi$.

The utility levels reached may be written:

$$\tilde{U}_t(\tau) = \frac{1+\varepsilon-\beta(1-\tau)^{1-\alpha}}{(1-\beta+\varepsilon)\,(1-\tau)^{1-\alpha}}(1-\tau)^{\left(\frac{1+\varepsilon}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)}\tilde{U}_t,\ t\in T_1, \tag{41}$$

$$\tilde{U}_{a_j}(\tau) = (1-\tau)^{\frac{\theta}{1-\theta}}\tilde{U}_{a_j},\ j\in T_2. \tag{42}$$

**Proposition 3** *The level of welfare of any agent $t\in T_1$ is higher at the CWE with a tax than the welfare reached at the CWE without the tax for low values $\tau$, while the converse is true for any atom $\{a_j\}$, $j\in T_2$, $\forall\tau\in(0,1)$.*

**Proof.** First, from (41), one has to show that $\frac{1+\varepsilon-\beta(1-\tau)^{1-\alpha}}{(1-\beta+\varepsilon)(1-\tau)^{1-\alpha}}(1-\tau)^{\left(\frac{1+\varepsilon}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)} > 1$ for low values of $\tau$. Consider $\varphi(\tau)\equiv\frac{1+\varepsilon-\beta(1-\tau)^{1-\alpha}}{(1-\beta+\varepsilon)(1-\tau)^{1-\alpha}}(1-\tau)^{\left(\frac{1+\varepsilon}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)}-1$. One has $\lim_{\tau\to0^+}\varphi(\tau)>0$ and $\lim_{\tau\to1^-}\varphi(\tau)<0$. Moreover, one has $\frac{\partial\varphi(\tau)}{\partial\tau}<0$ as $\tau\in(0,1)$. Then, there exists $\bar{\tau}\in(0,1)$ such that $\varphi(\bar{\tau})=0$. Therefore $\hat{U}_t(\tau)>\hat{U}_t$, $t\in T_1$ for $\tau\in[0,\bar{\tau}]$. Second, from (42), one has $\tilde{U}_{a_j}(\tau)-\tilde{U}_{a_j}=\left[(1-\tau)^{\frac{(1-\alpha)\beta}{1-\beta+\varepsilon}}-1\right]\tilde{U}_{a_j}<0$, $j\in T_2$, $\forall\tau\in(0,1)$. *QED.* ■

A tax on strategic supplies enhances the relative prices as given by (36) and (37). This creates a decrease in labor demand and leads consequently to a decrease in the quantity of good 1 brought to the market. The same mechanism prevails for good 2. But, according to the redistribution of good 2 among the small consumers, the consumption of good 2 by negligible traders increases. This positive quantity effect overcomes the negative effect on prices, so the utility of agents in the atomless sector increases. The effect is reversed for the atoms.

## 4.2 Optimal tax

Let us now determine the equilibrium value of the per unit tax. Any Pareto optimal allocation, which would follow from commodity tax $\tau$ and provide any trader $t$, $t\in T_0^+$ with utility level $\bar{U}$, must solve the problem:

$$\underset{\{\tau\}}{Max}\sum\nolimits_{j\in T_2}U_j(\tau) \tag{43}$$

$$\text{s.t.}\ \int_{t\in T_1}U(t,\tau)d\mu(t)\leq\int_{t\in T_1}\bar{U}d\mu(t).$$

From (41)-(42), simple calculations lead to:

$$\frac{1}{1+\varepsilon}(1-\tau)^{\left(\frac{1+\varepsilon}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)}-\frac{1}{\beta}(1-\tau)^{\frac{\theta}{1-\theta}}+\left[\gamma\frac{n}{n-(1-\theta)}\right]^{\left(\frac{1+\varepsilon}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)}\bar{U}=0. \quad (44)$$

Consider the simple case for which[11]: $\alpha=\beta=\gamma=\frac{1}{2}$, $\varepsilon=0$, so $\theta=\frac{1}{3}$. Therefore, the optimal tax $\tilde{\tau}$ is the solution to $(1-\tau)-2(1-\tau)^{\frac{1}{2}}+\frac{1}{2}\left(\frac{n}{n-\frac{2}{3}}\right)\bar{U}=0$. Little algebra lead to:

$$\tilde{\tau}=1-\left[1-\sqrt{1-\frac{1}{2}\left(\frac{n}{n-\frac{2}{3}}\right)\bar{U}}\right]^2. \quad (45)$$

**Proposition 4** *Suppose the product of the tax is transferred among all traders $t\in T_1$. The post-tax allocation does not lead to an overall-Pareto optimal allocation.*

**Proof.** Consider the marginal rates of substitution of any trader $\{a_j\}$, $j\in T_2$ and of any trader $t\in T_1$. Substituting the value $\tilde{\tau}$ into (40) and (41) reveals that the marginal rate of substitution between good 1 and good 2 is equal to $\frac{1}{\gamma}$ for the atoms and to $\frac{1}{\gamma}\frac{n-(1-\theta)}{n}$ for the negligible traders. As these marginal rates vary across all traders, the resulting allocation is not Pareto optimal. *QED.* ■

The tax scheme is not sufficiently powerful to eliminate the distortions caused by the strategic behaviors between the atoms. The reason stands on the fact that the presence of the tax on equilibrium prices mitigates the consequences of strategic interactions, without neutralizing the market power of the atoms.

# 5 Comparison with the monopolistic competition framework

Consider the monopolistic competition model developed by Blanchard-Kiyotaki (1987), and based on Dixit–Stiglitz (1977). Therefore, each atom produces a good that is an imperfect substitutes to the others. There are three consequences for the economy we consider. There are now $n$ varieties for good 2. The price index is $p_2=\left(\frac{1}{n}\sum_{j\in T_1}p_2(a_j)^{1-\eta}\right)^{\frac{1}{1-\eta}}$, where $\eta>1$ is the constant elasticity of substitution between goods[12]. The price level $p_2$ is homogenous of degree 1 in $p(a_j)$, $j=1,...,n$. Good 2 is now defined as a consumption index $x_2(.)=n^{\frac{1}{1-\eta}}\left(\sum_{j\in T_1}(x_{j2}(.))^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$, where all goods enter the utility functions symmetrically. All traders have a demand for all the varieties, which reflects a preference for diversity. Third, the atoms do not interact strategically.

[11] When the parameter $\varepsilon$ is equal to $\frac{1}{2}$, the equation has multiple solutions.
[12] If $\eta$ is large, all varieties are close substitutes.

## 5.1 Equilibrium without tax

Atom $\{a_j\}$ determines the price for his variety $p_2(a_j)$, his demand for good 1 as an input $z(a_j)$ and its level of production $y(a_j)$ by taking the price level for all varieties $p_2$ and the price of the good 1 as given, under the constraint (5) and the objective demand which addresses to it (see Appendix 5), so its program (15) may now be written:

$$\underset{\{p_2(a_j),z(a_j),y(a_j)\}}{Max} \Pi_{a_j} = p_2(a_j)y(a_j) - z(a_j),\ \forall j \in T_2 \tag{46}$$

$$\text{s.t. } y(a_j) = \frac{1}{\gamma}z(a_j),\ \gamma > 0$$

$$\text{and } y(a_j) = (1-\alpha)\left\{\int_{t\in T_1}\frac{\Omega(t)}{np_2}d\mu(t) + \sum\nolimits_{j\in T_2}\frac{\Pi(a_j)}{np_2}\right\}\left(\frac{p_2(a_j)}{p_2}\right)^{-\eta}.$$

The equilibrium relative prices are derived from the preceding program and according to the equilibrium on the labor market (see Appendix 6):

$$\hat{p}_2 = \gamma\left(\frac{\eta}{\eta-1}\right), \tag{47}$$

$$\hat{w} = \left[\gamma\left(\frac{\eta}{\eta-1}\right)\right]^{\left(\frac{1-\beta}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)}. \tag{48}$$

We deduce:

$$\hat{n}(t) = \left\{\frac{1}{\gamma}\left[\frac{\eta-1}{\eta}\right]\right\}^{\frac{1}{\beta}\left(\frac{\theta}{1-\theta}\right)},\ t\in T_0, \tag{49}$$

$$\hat{y}(t) = \frac{1}{\beta}\left\{\frac{1}{\gamma}\left[\frac{\eta-1}{\eta}\right]\right\}^{\frac{\theta}{1-\theta}},\ t\in T_0. \tag{50}$$

$$\hat{y}(a_j) = \left(\frac{1-\alpha}{\beta}\right)\left[\frac{\eta}{\eta-(1-\alpha)}\right]\left[\frac{1}{\gamma}\left(\frac{\eta-1}{\eta}\right)\right]^{\frac{1}{1-\theta}},\ j\in T_2. \tag{51}$$

The equilibrium allocation are respectively:

$$(\hat{x}_1(t), \hat{x}_2(t)) = \left(\frac{\alpha}{\beta}\phi, \left(\frac{1-\alpha}{\beta}\right)\left(\frac{1}{\gamma}\frac{\eta-1}{\eta}\right)\phi\right), \tag{52}$$

$$(\hat{x}_1(a_j), \hat{x}_2(a_j)) = \left(\frac{1-\alpha}{\beta}\right)\left[\frac{\phi}{\eta-(1-\alpha)}\right]\cdot\left(\alpha, \frac{(1-\alpha)}{\gamma}\left(\frac{\eta-1}{\eta}\right)\right), \tag{53}$$

where $\phi \equiv \left[\frac{1}{\gamma}\left(\frac{\eta-1}{\eta}\right)\right]^{\frac{\theta}{1-\theta}}$.

The corresponding payments follow:

$$\hat{U}_t = \left[\frac{1-\beta+\varepsilon}{\beta(1+\varepsilon)}\right]\left[\frac{1}{\gamma}\left(\frac{\eta-1}{\eta}\right)\right]^{\left(\frac{1+\varepsilon}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)}, \ t \in T_1, \tag{54}$$

$$\hat{U}_{a_j} = \left(\frac{1-\alpha}{\beta}\right)\left[\frac{1}{\eta-(1-\alpha)}\right]\left[\frac{1}{\gamma}\left(\frac{\eta-1}{\eta}\right)\right]^{\left(\frac{1+\varepsilon}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)}, \ j \in T_2. \tag{55}$$

**Proposition 5** *The Cournot-Walras equilibrium is not Pareto dominated by the Chamberlin-Walras equilibrium.*

**Proof.** We compare the utility levels reached by each trader at both equilibria. From (31) and (56): $\hat{U}_t - \tilde{U}_t = \Lambda\left[\left(\frac{\eta-1}{\eta}\right)^{\left(\frac{1+\varepsilon}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)} - \left[\frac{n-(1-\theta)}{n}\right]^{\left(\frac{1+\varepsilon}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)}\right]$, where $\Lambda \equiv \left[\frac{1-\beta+\varepsilon}{\beta(1+\varepsilon)}\right]\left(\frac{1}{\gamma}\right)^{\left(\frac{1+\varepsilon}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)}$. Moreover, $\frac{\eta-1}{\eta} < \frac{n-(1-\theta)}{n}$ since $\frac{1}{\eta} > \frac{1-\theta}{n}$ as $\theta < 1$. Then $\tilde{U}_t > \hat{U}_t$, $t \in T_1$. Additionally, one deduces from (32) and (57) $\tilde{U}_{a_j} - \hat{U}_{a_j} = \chi\left\{\frac{(1-\theta)}{n^2}\left[\frac{n-(1-\theta)}{n}\right]^{\frac{\theta}{1-\theta}} - \frac{1}{\eta-(1-\alpha)}\left(\frac{\eta-1}{\eta}\right)^{\left(\frac{1+\varepsilon}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)}\right\}$, where $\chi \equiv \left(\frac{1-\alpha}{\beta}\right)\left(\frac{1}{\gamma}\right)^{\left(\frac{1+\varepsilon}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)}$. Then $\hat{U}_{a_j} > \tilde{U}_{a_j}$, $\forall j \in T_2$. *QED.* ■

The monopolistic competition market structure makes the traders in the atomless sector achieve low utility level compared to the utility level they reach at the CWE. In the first case, the equilibrium real wage and the equilibrium underemployment are higher (we can verify that $\frac{\eta}{\eta-1} > \frac{n}{n-(1-\theta)}$ as $\theta < 1$). The degree of competition is lower in this case because each atom exerts some local monopoly power (when $\eta > 1$ but finite)[13]. The strategic interactions between the atoms is thus favorable to the small traders, and therefore lead to a better allocation for these remaining agents.

## 5.2 Equilibrium with tax

We assume that when exchange takes place a uniform tax $\tau$, with $\tau \in (0,1)$, is levied on the supplies of all varieties of good 2 sold by the monopolistic atoms. The total product of the tax $\sum_{j \in T_2} \tau s(a_j)$ is redistributed at equilibrium among all traders $t \in T_1$. The computation of the Chamberlin-Walras equilibrium with a tax leads to the following equilibrium prices:

$$\hat{p}_2(\tau) = \left(\frac{\gamma}{1-\tau}\right)\left(\frac{\eta}{\eta-1}\right), \tag{56}$$

$$\hat{w}(\tau) = \left[\left(\frac{\gamma}{1-\tau}\right)\left(\frac{\eta}{\eta-1}\right)\right]^{\left(\frac{1-\beta}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)}. \tag{57}$$

We deduce:

[13] When $\eta \to \infty$, the Chamberlin-Walras equilibrium coincides with the competitive equilibrium.

$$\tilde{n}(t,\tau) = \left\{ \left( \frac{1-\tau}{\gamma} \right) \left[ \frac{\eta - 1}{\eta} \right] \right\}^{\frac{1}{\beta}\left(\frac{\theta}{1-\theta}\right)}, \ t \in T_0, \tag{58}$$

$$\tilde{y}(t,\tau) = \frac{1}{\beta} \left\{ \left( \frac{1-\tau}{\gamma} \right) \left[ \frac{\eta - 1}{\eta} \right] \right\}^{\frac{\theta}{1-\theta}}, \ t \in T_0. \tag{59}$$

$$\hat{y}\left(a_j, \tau\right) = (1-\tau)^{\frac{\theta}{1-\theta}} \hat{y}\left(a_j\right), \ j \in T_2. \tag{60}$$

The equilibrium allocation are respectively:

$$(\hat{x}_1(t), \hat{x}_2(t)) = \Phi(\tau). \left( \frac{\alpha}{\beta}, \left( \frac{1-\alpha}{\beta} \right) \left( \frac{1-\tau}{\gamma} \right) \left( \frac{\eta-1}{\eta} \right) \left[ \frac{\eta - (1-\alpha)(1-\tau)}{(1-\tau)[\eta - (1-\alpha)]} \right] \right), \tag{61}$$

$$(\hat{x}_1(a_j), \hat{x}_2(a_j)) = \left( \frac{1-\alpha}{\beta} \right) \left[ \frac{\Phi(\tau)}{\eta - (1-\alpha)} \right]. \left( \alpha, \frac{(1-\alpha)}{\gamma} \left( \frac{\eta-1}{\eta} \right) \right), \tag{62}$$

where $\Phi(\tau) \equiv (1-\tau)^{\frac{\theta}{1-\theta}} \phi$. The payments follow:

$$\hat{U}_t(\tau) = \frac{\Delta(\tau)}{1-\beta+\varepsilon} (1-\tau)^{\left(\frac{1+\varepsilon}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)} \hat{U}_t, \ t \in T_1, \tag{63}$$

$$\hat{U}_{a_j}(\tau) = (1-\tau)^{\frac{\theta}{1-\theta}} \hat{U}_{a_j}, \ j \in T_2., \ t \in T_2, \tag{64}$$

where $\Delta \equiv \frac{(1+\varepsilon)[\eta-(1-\alpha)(1-\tau)]^{1-\alpha} - \beta\{[\eta-(1-\alpha)](1-\tau)\}^{1-\alpha}}{\{[\eta-(1-\alpha)](1-\tau)\}^{1-\alpha}}$.

**Proposition 6** *For low taxes, the level of welfare of any agent $t \in T_1$ is higher at the Chamberlin-Walras equilibrium with a tax than the welfare reached at the Chamberlin-Walras equilibrium without the tax, while the converse is true for any atom $\{a_j\}$, $j \in T_2$.*

**Proof.** First, using (63), one must verify that $\frac{\hat{U}_t(\tau)}{\hat{U}_t} > 1$, $t \in T_1$, i.e. $\frac{\Delta(\tau)}{1-\beta+\varepsilon}(1-\tau)^{\left(\frac{1+\varepsilon}{\beta}\right)\left(\frac{\theta}{1-\theta}\right)} > 1$. This leads to show $(1+\varepsilon)\left\{\left[1+\frac{(1-\alpha)\tau}{\eta-(1-\alpha)}\right]^{1-\alpha}(1-\tau)^{\frac{\theta}{1-\theta}} - 1\right\} + \beta[1-(1-\tau)^{\frac{\theta}{1-\theta}}] > 0$. Consider $\chi(\tau) \equiv (1+\varepsilon)\left\{\left[1+\frac{(1-\alpha)\tau}{\eta-(1-\alpha)}\right]^{1-\alpha}(1-\tau)^{\frac{\theta}{1-\theta}} - 1\right\} + \beta[1-(1-\tau)^{\frac{\theta}{1-\theta}}]$. We have $\lim_{\tau \to 0^+} \chi(\tau) > 0$ and $\lim_{\tau \to 1^-} \chi(\tau) < 0$. Moreover, $\frac{\partial \chi(\tau)}{\partial \tau} < 0$ as $1+\frac{(1-\alpha)\tau}{\eta-(1-\alpha)} > \beta^{\frac{1}{(1-\alpha)(1-\theta)}}$. Then, there exists $\check{\tau} \in (0,1)$ such that $\hat{U}_t(\check{\tau}) > \hat{U}_t$, $t \in T_1$. Second, $\hat{U}_{a_j}(\tau) - \hat{U}_{a_j} = \left[(1-\tau)^{\frac{\theta}{1-\theta}} - 1\right]\hat{U}_{a_j} < 0$, $j \in T_2$, $\forall \tau \in (0,1)$. *QED.* ■

The tax on strategic supplies creates a decrease in labor demand (and as a consequence in the quantity of good 1 brought to the market). The same happens for good 2. But a the same time, according to the redistribution of good 2 among the small consumers, the consumption of good 2 by negligible traders increases. This positive quantity effect overcomes the negative effect on prices, so the utility of agents in the atomless sector increases. This effect is reversed for the atoms.

Consider now the determination of the equilibrium tax at the Chamberlin-Walras equilibrium. Any Pareto optimal allocation, which would follow from commodity tax $\tau$ and provide any trader $t$, $t \in T_1$ with utility level $\bar{U}$, must solve the problem $\underset{\{\tau\}}{Max} \sum_{j\in T_2} U_j(\tau)$ s.t. $\int_{t\in T_1} U(t,\tau)d\mu(t) = \int_{t\in T_1} \bar{U} d\mu(t)$. Following the same procedure as in section 4, with $\alpha = \beta = \gamma = \frac{1}{2}$, $\varepsilon = 0$, one obtains the polynom $(1-\tau)^2 - \frac{4}{9}\left(\frac{4\eta}{\eta-1}\bar{U} - \eta\right)(1-\tau) + \left(\frac{2}{3}\frac{\eta}{\eta-1}\bar{U}\right)^2$. Little algebra lead to:

$$\hat{\tau} = 1 - \left\{ \frac{2}{9}\left(4\frac{\eta}{\eta-1} - \eta\right)\left[1 - \sqrt{1 + \frac{9\eta\bar{U}}{\left(\frac{4\eta}{\eta-1}\bar{U} - \eta\right)^2}}\right]\right\}. \tag{65}$$

As it stands for the Cournotian competition, the tax scheme is not sufficiently powerful to wipe out the distortions caused by strategic behaviors among the atoms: the marginal rate of substitution vary across traders since it equals $\frac{1}{\gamma}\frac{\eta-1}{\eta}$ for the atoms and $\frac{1}{\gamma}\frac{\eta-1}{\eta}\left[\frac{\eta-(1-\alpha)(1-\tau)}{\eta-(1-\alpha)}\right]$ for the negligible traders.

**Proposition 7** *The economic policy has more impact on market distorsions at the CWE than at the Chamberlin-Walras equilibrium.*

***Proof.*** *It suffices to compare the marginal rates of substitution at both equilibria. At the CWE, one has* $\tilde{\kappa} = \frac{n-(1-\theta)}{n}$, *while it is* $\hat{\kappa} = \frac{\eta-(1-\alpha)(1-\tau)}{\eta-(1-\alpha)}$ *at the Chamberlin-Walras equilibrium. Then* $\tilde{\kappa} < 1$ *and* $\hat{\kappa} > 1$, *so* $\hat{\kappa} > \tilde{\kappa}$. *QED.* ■

This result brings into light the gap between both rates is more significant in the case of the monopolistic competition, what reveals that the market distortions are more important. The main reason is that market distortions are more favorable to the atoms when they have local monopoly power and do not interact strategically with other competitors.

# 6 Conclusion

The previous model considered a mixed markets exchange economy with production which generates underemployment in the labor market. Inefficiencies on the competitive labor market are caused by market failures. Such failures stem from strategic interactions between many atoms. In addition, the tax policy is not sufficient to eliminate market imperfections caused by strategic interactions. Finally, inefficiencies are more significant under monopolistic competition: the market imperfections are more favorable to the atoms who do not interact strategically with other competitors.

The market structure associated with the CWE concept presents two advantages. First, the market demand addressed to the atoms is here made endogenous, which overcomes the lack of microfoundations, which is linked to the usual assumption of an exogenous information regarding the market demand function. Second, the model displays different kinds of heterogeneity and throws light on their consequences in terms of welfare. It integrates asymmetries across all markets, which cannot be captured in a partial equilibrium analysis.

# 7 Appendix

## 7.1 Appendix 1. Optimal plans of traders $t$, $t \in T_1$

Let the Lagrangian be $\pounds(x_1(t), x_2(t), l(t), \lambda) = \left[\frac{x_1(t)}{\alpha}\right]^{\alpha} \left[\frac{x_2(t)}{1-\alpha}\right]^{1-\alpha} - \frac{1}{1+\varepsilon}\left[l(t)\right]^{1+\varepsilon} + \lambda\{\Pi_t + wl(t) - [x_1(t) + p_2 x_2(t)]\}$, $\lambda > 0$. The first-order conditions may be written $\left[\frac{x_1(t)}{\alpha}\right]^{\alpha-1} \left[\frac{x_2(t)}{1-\alpha}\right]^{1-\alpha} = \lambda$, $\left[\frac{x_1(t)}{\alpha}\right]^{\alpha} \left[\frac{x_2(t)}{1-\alpha}\right]^{\alpha} = \lambda p_2$ and $\left[l(t)\right]^{\varepsilon} = \lambda w$. The first two conditions lead to $x_2(t) = \left(\frac{1-\alpha}{\alpha}\right)\frac{1}{p_2} x_1(t)$. The budget constraint gives the demand functions for both goods $x_1(t) = \alpha\left(\Pi_t + wl(t)\right)$ and $x_2(t) = (1-\alpha)\frac{\Pi_t + wl(t)}{p_2}$. Inserting these functions in the budget constraint leads to $\left(\frac{1}{p_2}\right)^{1-\alpha} = \lambda$. Therefore, from $\left[l(t)\right]^{\varepsilon} = \lambda w$, one deduces $l(t) = w^{\frac{\alpha}{\varepsilon}}\left(\frac{w}{p_2}\right)^{\frac{1-\alpha}{\varepsilon}}$.

## 7.2 Appendix 2. Restrictions on the strategy sets

Following a procedure similar to that in Gabszewicz and Michel (1997), it is possible to restrict the strategy set of any atom. The program of any atom $\{a_j\}$, $j \in T_2$, consists in solving $(*)$ $\underset{(x_1(a_j), x_2(a_j))}{Max} \left(\frac{x_1(a_j)}{\alpha}\right)^{\alpha} \left(\frac{x_2(a_j)+y(a_j)-s(a_j)}{1-\alpha}\right)^{1-\alpha}$ s.t. $x_1(a_j) + p_2 x_2(a_j) \leq \Pi_{a_j}(s(a_j), y(a_j))$, and $x_1(a_j) \geq 0$ and $x_2(a_j) \geq 0$, where $\Pi_{a_j}(s(a_j), y(a_j)) \equiv p_2 y(a_j) - \gamma(a_j)$. Firstly, the positivity constraints in profits imply that $\Pi_{a_j}(s(a_j), y(a_j)) \geq 0$, $\forall j \in T_2$, which leads to $s(a_j) \geq \gamma p_2 y(a_j) \equiv \underline{s}(a_j)$. Moreover, given $y(a_j)$, any utility level that can be reached choosing $s(a_j) \leq y(a_j)$ can also be reached by determining $s(a_j)$ in such a way that the quantity $y(a_j) - s(a_j)$ kept for later consumption is at least equal to the competitive demand of good 2, that is $(1-\alpha)\left(1 - \gamma\frac{1}{p_2}\right) y(a_j)$, in solving $(*)$. Accordingly, we consider only strategies $(s(a_j), y(a_j))$ satisfying the constraint $s(a_j) \leq \left[\alpha + (1-\alpha)\gamma\frac{1}{p_2}\right] y(a_j) \equiv \bar{s}(a_j)$, $\forall j \in T_2$. Consider then the strategy set $\bar{S}_{a_j} = \{s(a_j) \in \mathbb{R}_+ \mid \underline{s}(a_j) \leq s(a_j) \leq \bar{s}(a_j)\}$. If $s(a_j) \leq \left[\alpha + (1-\alpha)\gamma\frac{1}{p_2}\right] y(a_j)$, the solution to $(*)$ in $S_{a_j}$ coincides with the solution to $(*)$ in $\bar{S}_{a_j}$, and this latter solution is given by $(x_1(a_j), x_2(a_j)) = (\Pi_j, 0)$, $\forall j \in T_1$. If $s(a_j) \leq \left[\alpha + (1-\alpha)\gamma\frac{1}{p_2}\right] y(a_j)$, the solution to the problem $(*)$ in $S_{a_j}$ is given by $(x_1(a_j), x_2(a_j)) = \left(\alpha\left(p_2 - \gamma\right) y(a_j), s(a_j) - \left[\alpha + (1-\alpha)\gamma\frac{1}{p_2}\right] y(a_j)\right)$. Then

$U_{a_j}\left(x_1(a_j), x_2(a_j)+y(a_j)-s(a_j)\right)=\left[(p_2-\gamma)\right]^{\alpha}\left[\left(1-\gamma\frac{1}{p_2}\right)\right]^{1-\alpha} y(a_j)$, for $j\in T_2$. Now consider that if $s(a_j)>\left[\alpha+(1-\alpha)\gamma\frac{1}{p_2}\right]y(a_j)$, the strategy $s(a_j)$ is substituted by the strategy $s'(a_j)=\left[\alpha+(1-\alpha)\gamma\frac{1}{p_2}\right]y(a_j)$. Then $s'(a_j)\in\bar{S}_{a_j}$. Thus, the solution to $(*)$ in $\bar{S}_{a_j}$ is $(\alpha\,(p_2-\gamma)\,y(a_j),0)$ so $U_{a_j}(\alpha\,(p_2-\gamma)\,y(a_j), y(a_j)-s'(a_j))=\left[(p_2-\gamma)\,y(a_j)\right]^{\alpha}\left[\left(1-\gamma\frac{1}{p_2}\right)y(a_j)\right]^{1-\alpha}$, which corresponds to the utility level obtained at the optimum in $S_j$.

### 7.3 Appendix 3. Market-clearing conditions

Using (12), (13) and (15), the system given by (17) may be written, after post-multiplying equation (A2) by $p_2$, as:

$$\sum_{j=1}^{n} p_2 s_j+\alpha\int_1^2\left[\left(\frac{1-\beta}{\beta}\right)\left(\frac{1}{w}\right)^{\frac{\beta}{1-\beta}}+w^{\frac{\alpha+\varepsilon}{\varepsilon}}\left(\frac{w}{p_2}\right)^{\frac{1-\alpha}{\varepsilon}}\right]d\mu(t)=\frac{1}{\beta}\int_0^1\left(\frac{1}{w}\right)^{\frac{\beta}{1-\beta}}d\mu(t), \tag{A1}$$

$$(1-\alpha)\int_1^2\left[\left(\frac{1-\beta}{\beta}\right)\left(\frac{1}{w}\right)^{\frac{\beta}{1-\beta}}+p_2 w^{\frac{\alpha}{\varepsilon}}\left(\frac{w}{p_2}\right)^{\frac{1-\alpha+\varepsilon}{\varepsilon}}\right]d\mu(t)=p_2\sum_{j=1}^{n}s_j, \tag{A2}$$

$$\int_0^1\left(\frac{1}{w}\right)^{\frac{1}{1-\beta}}d\mu(t)=\int_1^2 w^{\frac{\alpha}{\varepsilon}}\left(\frac{w}{p_2}\right)^{\frac{1-\alpha}{\varepsilon}}d\mu(t). \tag{A3}$$

From the third market-clearing equation, one deduces $\frac{1}{w}=\left(\frac{w}{p_2}\right)^{\frac{(1-\alpha)(1-\beta)}{\alpha(1-\beta)+\varepsilon}}$. Substituting $\frac{1}{w}$ from $\left(\frac{w}{p_2}\right)^{\frac{(1-\alpha)(1-\beta)}{\alpha(1-\beta)+\varepsilon}}$ in (A1) and (A2) yields after rearrangement $\frac{w}{p_2}=p_2^{\frac{(1-\beta)\alpha+\epsilon}{1-\beta+\varepsilon}}$. Inserting these values of $\frac{1}{w}$ and $\frac{w}{p_2}$ in (A2) yields:

$$p_2=\left[\left(\frac{1-\alpha}{\beta}\right)\frac{1}{\sum_{j=1}^{n}s(a_j)}\right]^{\frac{(1-\alpha)\beta}{1-\beta+\varepsilon}}, \tag{A4}$$

$$w=\left[\left(\frac{1-\alpha}{\beta}\right)\frac{1}{\sum_{j=1}^{n}s(a_j)}\right]^{\frac{(1-\alpha)(1-\beta}{1-\alpha\beta+\varepsilon})}. \tag{A5}$$

Since $\theta\equiv\frac{(1-\alpha)\beta}{1-\alpha\beta+\varepsilon}$, one deduces (18) and (19).

## 7.4 Appendix 4. Market-clearing condition with a tax

The market clearing conditions now write:

$$\sum_{j\in T_2} p_2(1-\tau)s(a_j) + \int_{t\in T_1} x_1(t)d\mu(t) = \int_{t\in T_0} y(t)d\mu(t) \tag{A6}$$

$$\int_{t\in T_1} x_2(t)d\mu(t) = (1-\tau)\sum\nolimits_{j\in T_2} s(a_j) \tag{A6}$$

$$\int_{t\in T_1} n(t)d\mu(t) = \int_{t\in T_0} l(t)d\mu(t) \tag{A7}$$

Following the same procedure as in the Appendix 3 gives:

$$p_2(\tau) = \left[\left(\frac{1-\alpha}{\beta}\right)\left(\frac{1}{1-\tau}\right)\frac{1}{\sum_{a_j\in T_2} s(a_j)}\right]^{1-\theta} \tag{A9}$$

$$w(\tau) = \left[\left(\frac{1-\alpha}{\beta}\right)\left(\frac{1}{1-\tau}\right)\frac{1}{\sum_{a_j\in T_2} s(a_j)}\right]^{\left(\frac{1-\beta}{\beta}\right)\theta} \tag{A10}$$

The atom $\{a_j\}$, $j\in T_2$ now solves:

$$\underset{(\tilde{s}(a_j,\tau),\tilde{y}(a_j,\tau))}{Arg\max} \left(\frac{p_2(1-\tau)s(a_j)-\gamma y(a_j)}{\alpha}\right)^{\alpha}\left(\frac{y(a_j)-s(a_j)}{1-\alpha}\right)^{1-\alpha}. \tag{A11}$$

By following the same procedure as that developed in the section 3 yields (33) to (42).

## 7.5 Appendix 5. The market demand function addressed to atom $\{a_j\}$

There are two types of agents who formulate a demand for the good produced by any atom $\{a_j\}$, $j\in T_2$: the first emanates from the traders in the atomless sector, and the second comes from all the atoms. In each case, the procedure to derive the market demand function which addresses to the atom $\{a_j\}$, $j\in T_2$, relies on a two-step budgeting program. First, any trader $t\in T_1$ solves $\underset{(x_1(t),x_2(t),l(t))}{Max}\left[\frac{x_1(t)}{\alpha}\right]^{\alpha}\left[\frac{x_2(t)}{1-\alpha}\right]^{1-\alpha} - \frac{1}{1+\varepsilon}\left[l(t)\right]^{1+\varepsilon}$ subject to $x_1(t)+p_2x_2(t)\leq wl(t)+\int_0^1\Theta(t)\Pi_t d\mu(t)$, with $p_2 = \left(\frac{1}{n}\sum_{j\in T_1} p_2(a_j)^{1-\eta}\right)^{\frac{1}{1-\eta}}$. This leads to $(x_1(t),x_2(t)) = \left(\alpha\Omega(t),(1-\alpha)\frac{\Omega(t)}{p_2}\right)$ and $l(t) = w^{\frac{\alpha}{\varepsilon}}\left(\frac{w}{p_2}\right)^{\frac{1-\alpha}{\varepsilon}}$. At the same time, the atom $\{a_j\}$, $j\in T_2$ solves $\underset{(x_1(a_j),x_2(a_j))}{Max}\left(\frac{x_1(a_j)}{\alpha}\right)^{\alpha}\left(\frac{x_2(a_j)}{1-\alpha}\right)^{1-\alpha}$ subject to $x_1(a_j)+p_2x_2(a_j)\leq \Pi_{a_j}\left(s(a_j),y(a_j)\right)$ and $x_1(a_j)\geq 0$, $x_2(a_j)\geq 0$, where

$x_2(a_j) = n^{\frac{1}{1-\eta}} \left( \sum_{j \in T_1} (x_{j2}(a_j))^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$, $j \in T_2$. This leads to $(x_1(a_j), x_2(a_j)) = \left( \alpha \Pi_{a_j}, (1-\alpha) \frac{\Pi_{a_j}}{p_2} \right)$, $j \in T_2$. Second, the traders determine the demand for variety $j$. Thus, any trader $t \in T_1$ solves $\underset{\{x_{j2}(t)\}}{Min} \sum_{j \in T_2} p_2(a_j) x_{j2}(t)$ subject to $x_2(t) \leq n^{\frac{1}{1-\eta}} \left( \sum_{j \in T_2} (x_{j2}(t))^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$. At the optimum, one has $\frac{[x_{j2}(t)]^{-\frac{1}{\eta}}}{[x_{k2}(t)]^{-\frac{1}{\eta}}} = \frac{p_2(a_j)}{p_2(a_k)}$, $\forall j, k$. This yields $\frac{[x_{j2}(t)]^{-\eta}}{[x_{k2}(t)]^{-\eta}} = \left( \frac{p_2(a_j)}{p_2(a_k)} \right)^{-\eta}$, $\forall j, k$. Inserting the latter expression in $x_2(t) = n^{\frac{1}{1-\eta}} \left( \sum_{j \in T_2} (x_{j2}(t))^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$ and using the fact that $np_2^{1-\eta} = \sum_{j \in T_2} p_2(a_j)^{1-\eta}$, we find $x_{k2}(t) = \frac{1}{n} \left( \frac{p_2(a_j)}{p_2} \right)^{-\eta} x_2(t)$, $t \in T_1$. Moreover, since $x_2(t) = (1-\alpha) \frac{\Omega(t)}{p_2}$, one finally has $x_{k2}(t) = (1-\alpha) \frac{\Omega(t)}{np_2} \left( \frac{p_2(a_j)}{p_2} \right)^{-\eta}$, $t \in T_1$. Finally, the atom $\{a_j\}$, $j \in T_2$ solves $\underset{\{x_{j2}(a_j)\}}{Min} \sum_{j \in T_2} p_2(a_j) x_{j2}(a_j)$ subject to $x_2(a_j) \leq n^{\frac{1}{1-\eta}} \left( \sum_{j \in T_2} (x_{j2}(a_j))^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$ (in order to simplify, it is assumed here that any atom $\{a_j\}$ consumes an amount of the good it produces). The same procedure as that developed before for $t \in T_1$ yields $x_{k2}(a_j) = (1-\alpha) \frac{\Pi(a_j)}{np_2} \left( \frac{p_2(a_j)}{p_2} \right)^{-\eta}$, $j \in T_2$.

## 7.6 Appendix 6. Monopolistic competition prices

Let $y(a_j) = \frac{y}{n} \left( \frac{p_2(a_j)}{p_2} \right)^{-\eta}$, where $y = \frac{(1-\alpha)\left[ \int_{t \in T_1} \Omega(t) d\mu(t) + \sum_{j \in T_2} \Pi_{a_j} \right]}{p_2}$. The utlity function of any atom $\{a_j\}$ becomes $U_{a_j} \left( \alpha \Pi_{a_j}, (1-\alpha) \frac{\Pi_{a_j}}{p_2} \right) = (\frac{1}{p_2})^{1-\alpha} \Pi_{a_j}$, so $\frac{U_{a_j}}{p_2} = \left( \frac{1}{p_2} \right)^{2-\alpha} \frac{\Pi_{a_j}}{p_2}$, $\forall j \in T_2$. The program of $\{a_j\}$ may therefore be written $\underset{\{p_2(a_j), n(a_j), y(a_j)\}}{Max} \frac{\Pi_{a_j}}{p_2} = (1-\alpha) \frac{y}{n} \left( \frac{p_2(a_j)}{p_2} \right)^{1-\eta} - \gamma (1-\alpha) \frac{y}{n} \frac{1}{p_2} \left( \frac{p_2(a_j)}{p_2} \right)^{-\eta}$, $\forall j \in T_2$. The first-order condition $\frac{\partial \left( \frac{\Pi_{a_j}}{p_2} \right)}{\partial \left( \frac{p_2(a_j)}{p_2} \right)} = 0$ leads to the pricing rule $\left( \frac{p_2(a_j)}{p_2} \right) = \left( \frac{\eta-1}{\eta} \right) \gamma \left( \frac{1}{p_2} \right)$, where $\left( \frac{1}{\eta} \right)$ is the markup over constant marginal cost. At the symmetric general equilibirium, one has $p_2(a_j) = p_2$, $\forall j \in T_2$, so $\frac{p_2(a_j)}{p_2} = 1$. Therefore $\hat{p}_2 = \gamma \left( \frac{\eta-1}{\eta} \right)$. From the market clearing condition on the labor market, one deduces $\hat{w} = \left[ \gamma \left( \frac{\eta}{\eta-1} \right) \right]^{\frac{(1-\alpha)(1-\beta)}{1-\beta+\varepsilon}}$.

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