The Distributive and Welfare Effects of Product and Labour Market Deregulation

G. Cardullo

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Institut de Recherches Économiques et Sociales
de l'Université catholique de Louvain

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Gabriele Cardullo†

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Abstract

This paper studies the effects of product and labour market deregulation on wage inequality and welfare. By constructing an analytically tractable model in which the level of product market competition and the wages are endogenously distributed, I show that even though deregulation in labour markets raises the aggregate level of employment and the average real wage, the welfare of trade unions may decrease in sectors with a low level of competition. Moreover, removing barriers to entry in the goods market has mixed effects on inequality: the wage variance and the Gini index are lower, but the ratio of the highest over the lowest wage paid in the economy increases. Finally, an interesting result of the model concerns the wage density function. By parameterizing the rates of firms creation and destruction on the basis of Belgian data, the resulting shape of the wage distribution exhibits an empirically accurate form, unimodal and positively skewed.

Keywords: product market competition; wage distribution; barriers to entry.
JEL codes: E24, J5, L16

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†DIEM, University of Genoa, Italy. E-mail : cardullo@economia.unige.it.

1 Introduction

Although most of the theoretical and empirical literature stresses the positive effects of product and labour market deregulation on employment and the real wage, the implementation of such policies faces in many cases and in many countries the vehement opposition of lobbies, interests groups, unions, and simple citizens. The aim of this paper is to offer an interpretation for these apparently conflicting facts.

The issue may be put in these terms. First, the call for less restrictions in product and labour markets has a sounding theoretical rationale. As Blanchard and Giavazzi (2003), Carlin and Soskice (2006), and Ebell and Haefke (2009) clearly spell out, removing barriers to operate in product markets entails more competition, a lower mark-up that can be chosen by each single producer, and larger output. This in turn raises labour demand. Real wages are higher both because of the increased demand and because fiercer competition also reduces the price of the consumption goods. A positive shift of labour demand may also spur from a less regulated labour market, if the employment protection schemes are so tight to dampen firms' incentive to hire workers.

Recent empirical studies confirm these theoretical predictions. For the OECD (2006), liberalization in goods market is one decisive factor that helps to explain why some countries (Ireland, Austria, Scandinavia, and the Netherlands) experience high employment rates even though their labour markets remain very regulated. Nicoletti and Scarpetta (2005), Griffith *et al.* (2007), and Fiori *et al.* (2008) also confirm the beneficial effects on employment and the real wage of product market deregulation in some OECD countries [1].

But then a natural question arises: if deregulation is such a panacea both for the unemployed (since it raises the employment) and for the employees (for the real wage increases), why do so many citizens look at these reforms with diffidence or hostility?

There is plenty of evidence that confirms such a negative mood. According to the Eurobarometer (2008) (a survey conducted in Europem on a regular basis), 43% of the European citizens dislike free trade originated by the globalization because it is considered a threat for their jobs and for the incumbent firms. Another significant

[1] Griffith *et al.* (2007) and Fiori *et al.* (2008) also show that the positive impact of a more deregulated product market on employment is greater the more regulated is the labour market or the stronger the workers' bargaining power.

example is the recent (late 2007) choice made by the European governments not to include in the Lisbon Treaty the text relating to free and undistorted competition present the Principles of of the European Community since the Treaty of Rome of 1956.[2]

Several arguments can be brought to explain this attitude. For instance, some of the benefits that deregulation may deliver are not immediate or difficult to perceive, at least *ex ante*[3]. Or it may simply be a natural tendency towards the maintenance of the *status quo*, as Boeri *et al.* (2001) document in an analysis of a survey about the Europeans' attitude with respect to their welfare state[4].

In this paper I offer an alternative view. Workers' (more precisely, trade unions') diffidence may stem from the fact that such policies may reduce their welfare even though they have a positive impact on the average real wage and the level of employment. Moreover, product market deregulation also has mixed effects on inequality.

These results are obtained by constructing an analytically tractable model that incorporates two distinctive features: (i) the level of competition is not the same in each sector of the economy and (ii) it varies according to an endogenous stochastic process[5]. The economy is composed by a large number of intermediate goods sectors, identical *ex ante* and in which firms compete *à la* Cournot; the final consumption good is produced in a competitive market. As respects to labour market, unions of firms and workers bargain over the wage at sectoral level. The creation and the destruction of firms in each intermediate market follow a continuous-time Markov Chain. A new firm enters the sector at a certain rate, determined by a zero-profit condition in entry behaviour. In addition, at a certain exogenous rate, an incumbent firm exits the sector. At the steady-state equilibrium, the level of competition (i.e. the number of

[2]Such a reference has been moved to Protocol 6 "On the Internal Market and Competition".

[3]For instance, some workers may focus only on nominal wage variations and neglect the positive changes in the real wage caused by some income effect that makes the consumption goods cheaper.

[4]One could also emphasize the active intervention of lobbies and interest groups that try to influence the public opinion by giving more weight to the risks of the reforms and less to the their opportunities.

[5]These are also the crucial differences with respect to the previous literature on the interactions between product and labour market (for instance, Blanchard and Giavazzi, 2003; or Ebell and Haefke, 2009) that modeled an economy with monopolistic competition in which, at the equilibrium, each sector is indistinguishable from the others and the degree of competition takes an endogenous deterministic value.

firms competing), the real wage, and the level of employment are not the same among the intermediate markets but follow an endogenous distribution.

Comparative static results confirm that a reduction in the cost of entry in the product market or a decrease in the bargaining power of workers' unions raises the aggregate level of employment and the real wage[6]. Yet, squeezing entry costs has ambiguous effects on inequality too. On the one hand, it allows the existence of sectors with stronger competition[7], in which the rents extracted by workers under the form of wages are low. These low-paid jobs widen the distance between the highest and the lowest wages in the economy. On the other hand, since competition gets tougher, more workers are employed in sectors with many firms, implying that more employees earn the same wage. As numerical simulations show, the wage variance and the Gini index decrease[8].

Numerical results also allow to assess the impact of these reforms on the welfare of the unions of workers. While product market reforms enhance the expected utility of all workers, the effects of labour market deregulation vary with the level of competition of the market. Unions operating in sectors with low competition are worse off, whereas those working in more competitive markets benefit from the labour market deregulation. The reason is that a reduction in the bargaining power of workers' unions raises employment and the real wage but also makes more likely the entry of new firms in the future. In low competitive sectors the capital loss the incumbents incur when a new firm enters the market is so high to outweigh the positive employment and real wage effects.

Finally, an interesting feature of the model concerns the wage distribution. Identical workers earn different salaries only because they operate in sectors with different degrees of product market competition. By parameterizing the rates of creation and

[6]More precisely, the effects of a lower bargaining power of workers' unions on the real wage are ambiguous at comparative statics level but result slightly positive in the numerical simulations.

[7]For the free entry condition, the expected costs of entry are equal to the expected profits. If costs are high, profits must be high as well, meaning that sectors cannot afford a too fierce degree of competition.

[8]The relationship between product market competition and wage inequality has been studied by Guadalupe (2007). The underlying theoretical mechanism is different however. More competition makes firms eager to hire high-skilled workers that are more capable to produce at lower costs. So the returns to skill increase, widening wage inequality.

destruction of firms on the basis of Belgian data, I get a wage distribution of empirically accurate shape: unimodal and positively skewed. Without the pretension to consider competition in goods market the main source of earnings inequality, I think however that this result is an useful piece of information that helps to understand the well-known characteristics of the wage density functions in most Western countries.

The structure of the paper is the following. Section 2 and 3 illustrate the model and the steady-state equilibrium. Section 4 presents the comparative static results. Section 5 shows the numerical simulations. Section 6 concludes.

2 The model

2.1 Preferences and technology

I consider an economy with one final consumption good and a large integer number I of intermediate goods. The final good market is perfectly competitive, whereas Cournot competition is assumed within each intermediate sector. The final good production function takes a CES form:

$$Y = \left[\sum_{i=1}^{I} Q_i^{\frac{s-1}{s}} \right]^{\frac{s}{s-1}} \tag{1}$$

in which Q_i is the amount of intermediate good i used by the production process of the final good and $s > 1$ to allow a situation in which some Q_i are equal to zero. Cost minimization in the final good sector leads to the inverse demand for each intermediate good i:

$$\begin{aligned} p(Q_i) &\equiv \frac{P_i}{P} = \left(\frac{Q_i}{Y} \right)^{-\frac{1}{s}} \\ \text{with } P &\equiv \left[\sum_{i=1}^{I} P_i^{1-s} \right]^{\frac{1}{1-s}} \end{aligned} \tag{2}$$

P is the price index. Parameter s is the elasticity of the demand for good i.

Time is continuous. In each intermediate sector there is a measure normalized to 1 of workers; they can be employed only in that industry, so there are I perfectly

segmented labour markets[9]. The number of firms competing in each intermediate sector follows a Markov chain that will be described in the next section. The intermediate firm production function is identical in each sector and is given by $l_{j,i}$, the labour input for firm j in sector i. The total amount of good i produced is therefore equal to the level of employment in that sector and is denoted by $Q_i = \sum_j l_{i,j}$.

In each intermediate sector unions of workers and firms bargain over the wage. Workers' unions enjoy an instantaneous utility equal to $w_i \cdot \sum_j l_{i,j} + z \cdot \left(1 - \sum_j l_{i,j}\right)$, with w_i and z respectively being the real wage paid in sector i and the value of home production. The instantaneous utility of the unions of firms is given by the sum of each firm's profits. The wage is bargained by the unions of firms and workers at a sectoral level. Conditional on the results of such a negotiation, any competitor decides the optimal level of the labour input to operate in the market.

A crucial assumption of the paper is that the the value of what is produced at home by the unemployed is a fraction of the total market output: $z = \alpha \cdot Y$, with $0 < \alpha < 1$. The motivation is twofold. First, as we will see later on, such an assumption allows to have in relatively simple way an income effect on workers' utility. Second, it is important for an accurate analysis of their welfare changes in response to some product and labour market policies. If the income of the unemployed was unaffected by Y, any intervention that raises the output of the final good and reduce its price would not have any impact on their instantaneous utility[10].

2.2 The Stochastic Environment

The creation and destruction of firms in each intermediate market i follows a continuous time Markov chain that takes values in the set $L = \{0, 1, 2, ...L\}$, L being the maximum number of firms that can compete in a sector. I assume that in small interval of time *dt at most* one firm can enter or leave a sector. So, if x_i is the number of firms active in

[9]The assumption of perfect segmentation in the labour markets is essentially done for tractability reasons. Still, one can think that the tasks and abilities required for a job are so different among intermediate sectors that workers cannot switch from one market to another.

[10]Due to lack of times series data for home production, it is difficult to have a clear view on its relationship with market output. Recent studies (Blankenau and Kose, 2006) do not find a clear correlation between home production and GDP while suggesting that the former may be more volatile than the latter. The assumption of α constant is done only for simplicity reasons.

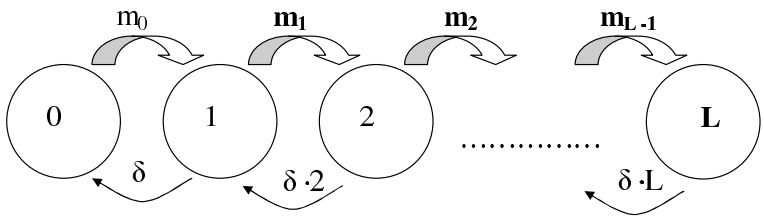

Figure 1: Creation and destruction of firms. In bold the endogenous variables.

sector i, there is a probability $m_{x_i}\,dt$ that a new firm enters and a probability $\delta \cdot x_i\,dt$ that one firm exits (see Figure 1)[11].

I impose $m_{x_i} = q \cdot V_{x_i}$, with V_{x_i} being the number of firms that want to enter in sector i when there are already x incumbents. At $x = L$, firms have no incentive to enter the market and $m_L = 0$. Both $V_{x_i}\,\forall x_i \in [1, 2, ...L]$ and L are endogenously determined by zero profit conditions in entry behaviour. As it will be more clear later on, for simplicity reasons I find convenient to impose m_0 as a constant.

Since intermediate sectors are identical *ex-ante* - they have the same level of labour force, the same preferences, and technology - the subscript i can be removed. Let $\pi_{x,t}$ be the probability that a time t there are x active firms in a generic intermediate market. Then:

$$\begin{aligned}
\pi_{x,\,t+dt} &= [\,1 - \delta \cdot x dt - m_x dt\,] \cdot \pi_{x,t} \;+\; m_{x-1} dt \cdot \pi_{x-1,\,t} \\
&\quad + \delta \cdot (x+1) dt \cdot \pi_{x+1,\,t} \qquad \forall x \in [1, 2, ..., L-1], \\
\pi_{L,\,t+dt} &= [\,1 - \delta \cdot L dt\,] \cdot \pi_{L,t} \;+\; m_{L-1} dt \cdot \pi_{L-1,\,t} \\
\pi_{0,\,t+dt} &= [\,1 - m_0 dt\,] \cdot \pi_{0,t} \;+\; \delta dt \cdot \pi_{1,\,t}.
\end{aligned}$$

The steady-state probability distribution is such that $\pi_{x,\,t+dt} = \pi_{x,t}$, $\forall t$. Expressing π_x in terms of π_{x-1} yields:

$$\pi_x = \frac{m_{x-1}}{\delta \cdot x} \cdot \pi_{x-1} \quad \text{with } x \in [\,1, 2, ., L\,], \tag{3}$$

[11] This Markov chain is called a birth-death process. See Taylor and Karlin (1998) for details.

Since $\pi_0 = 1 - \sum_{n=1}^{L} \pi_n$, one obtains:

$$\begin{aligned} \pi_x &= \frac{\left(\frac{1}{\delta}\right)^x \cdot \prod_{n=0}^{x-1} \frac{m_n}{n+1}}{1 + \sum_{j=1}^{L} \left(\frac{1}{\delta}\right)^j \cdot \prod_{n=0}^{j-1} \frac{m_n}{n+1}} \quad \text{with } x \in [1, 2, ., L] \\ \pi_0 &= \frac{1}{1 + \sum_{j=1}^{L} \left(\frac{1}{\delta}\right)^j \cdot \prod_{n=0}^{j-1} \frac{m_n}{n+1}} \end{aligned} \tag{4}$$

The probability π_x that in one intermediate sector there are x firms competing in the market depends on δ and the endogenous values L and $m_n = q \cdot V_n \quad \forall n \in [0, 1, 2, ... L-1]$.

With I is sufficiently large, I can apply the law of large numbers and define the aggregate level of employment

$$E = \sum_{x=0}^{L} l_x \cdot x \cdot \pi_x \cdot I. \tag{5}$$

2.3 Wage determination

In each intermediate sector, unions of firms and workers bargain over the wage. Such an assumption seems plausible for many countries in Continental Europe, where the sectoral level of negotiation often plays a major role[12].

Let r be the discount rate common to all agents. The expected discounted utility of the unions of workers is:

$$\begin{aligned} rU_W(x) &= w_x \cdot l_x \cdot x + \alpha \cdot Y \cdot (1 - l_x \cdot x) + m_x \left[U_W(x+1) - U_W(x)\right] \\ &+ \delta \cdot x \left[U_W(x-1) - U_W(x)\right], \end{aligned} \tag{6}$$

with $x \in [0, 1, ..., L]$. Operating as a union of workers in market with x firms is like holding an asset that pays you a dividend equal to the sum of the wages paid to the employees and the income earned by the unemployed. The underlying assumption is that the trade union behaves in a utilitarian way, caring about the sum of members' incomes. At certain rates, the level of competition may decrease or increase by one

[12] In Belgium, the country chosen as a target for the calibration of the model, the sectoral level is predominant, even if in recent years new agreements have taken place. See Brock and Dobbelaere (2006).

unit, modifying the asset value to $U_W(x-1)$ and $U_W(x+1)$. The expected discounted utility of a union of firms is:

$$\begin{aligned} rU_F(x) \;=\; & x\,[\,p(Q_x)\,l_x \;-\; w_x l_x\,] + m_x\,[U_F(x+1) - U_F(x)] \\ & + \delta \cdot x\,[U_F(x-1) - U_F(x)]\;, \end{aligned} \tag{7}$$

with $x \in [1, 2, ..L]$. Function $p(Q_x)$ is expressed in (2) and represents the price of the intermediate good when x firms are competing in the market. The instantaneous utility of the union of firms is is given by the sum of the revenues of each firm.

I consider an axiomatic Nash solution. The threat points for the unions of firms and workers are denoted respectively by $\bar{U}_F$ and $\bar{U}_V$:

$$r\bar{U}_F \;= 0. \tag{8}$$

$$r\bar{U}_W \;=\; \alpha \cdot Y. \tag{9}$$

If no agreement is concluded, the employees in that sector do not work and and earn the same fraction of the consumption good of the unemployed workers. The firm does not produce and does not pay any wage[13]. The real wage received by the employees in a sector with x active firms solve the problem:

$$\begin{aligned} & w_x \;=\; \text{argmax}\;\left[U_W(x) - \bar{U}_W\right]^{\beta}\left[U_F(x) - \bar{U}_F\right]^{1-\beta} \\ & \text{s.t.} \\ & U_W(x) \;>\; \bar{U}_W \\ & U_F(x) \;>\; \bar{U}_F \qquad \text{with}\, x \in [1, 2, ..L]. \end{aligned} \tag{10}$$

The constraints imposed in the maximization mean that both parties have always the possibility to abandon the negotiation if this choice makes them better off. I assume, as Rosen (1997) and Hall and Milgrom (2006) do, that such constraints are not binding: no player has an incentive to quit the negotiation and this holds for any value of x.

Computing the F.O.C. and using (6), (7),(8), and (9) yields :

$$w_x \;=\; \beta\, p(Q_x) \;+\; (1-\beta)\,\alpha \cdot Y \qquad \forall x \;\in [1, 2, ..., L]. \tag{11}$$

[13]Such threats points are similar to those introduced by Rosen (1997) and Hall and Milgrom (2008). The idea is that a disagreement in the negotiation between unions usually implies a delay in the production, strikes, not massive lay-offs or quits. Actually, in the paper of Hall and Milgrom, the delay in the production involves a flow cost for the firm. For simplicity, I impose it equal to zero.

This equation has a straightforward interpretation. The wage is a weighted average of the total revenues obtained in the intermediate sector ($p(Q_x)l_x$) and the opportunity cost of employment ($\alpha \cdot Y$). The weights are given by the bargaining power of workers and firms, β and $1-\beta$. If the union of workers has no bargaining power, each employee receives an instantaneous utility from being employed exactly equal to $\alpha \cdot Y$. On the other hand, when $\beta = 1$, all the profits earned in the market accrue to the employee. In this limit case, firms cannot recoup the entry costs and nobody is willing to enter a market.

2.4 The Cournot game

Conditional on the wage equation (11), at each point in time, a firm decides the optimal level of labour input to play the Cournot game. The expected lifetime income for a firm producing in a market with $x-1$ competitors solves the following problem:

$$\begin{aligned} rJ_E(x) \;=\; & \max_{l_x} \; p(Q_x)\, l_x - w_x l_x \;+\; \delta(x-1)\,[J_E(x-1) - J_E(x)] \\ & \qquad + m_x\,[J_E(x+1) - J_E(x)] - \delta J_E(x) \\ \text{s.t. } \; w_x \;=\; & \beta\, p(Q_x) \;+\; (1-\beta)\,\alpha \cdot Y \end{aligned} \tag{12}$$

Operating in a such a sector pays you a dividend of $p(Q_x)\, l_x - w_x l_x$, the revenues net of the wage bill. At a rate $\delta \cdot (x-1)$, one of the competitors exits the market, implying a shift in the asset value from $J_E(x-1)$ to $J_E(x)$, while at a rate m_x, the number of firms active in the market increases by one unit and the new value function is $J_E(x+1)$. Finally, at a rate δ the firm itself exits the market, experiencing a capital loss equal to $-J_E(x)$.[14]

The F.O.C. of the problem is:

$$\begin{aligned} \alpha \cdot Y \quad &= \quad [\,p'(Q_x)\, l_x \;+\; p(Q_x)\,] \\ &= \quad p(Q_x)\left[1 \;-\; \frac{1}{x \cdot s}\right] \end{aligned} \tag{13}$$

The second line in (13) is obtained by using equation (2). Equation (13) is a standard solution of a $x-$players Cournot game. Each firm maximizes its surplus, given the

[14]This is tantamount to saying that the expected discounted utility of a firm that is outside the market is equal to zero.

optimal strategy of the other players. In equilibrium, the marginal revenue of a firm must be equal to the marginal utility of unemployment[15]. Notice also that from a single firm's viewpoint the total amount of the final good, Y, is given. This is due to the fact that, with I large enough, a single firm's decision has an impact only within each sector but does not affect the price index P and quantity Y.

Since all the firms in the same sector hire the same level of labour input, $Q_x = x \cdot l_x$. From (2) and (13), the equilibrium level of l_x is:

$$l_x = \frac{Y}{x} \cdot \left(\alpha \cdot Y \frac{xs}{xs - 1} \right)^{-s} \quad \forall x \in [1, 2, ..., L]. \tag{14}$$

By using equation (13), I can write the wage equation as a function of Y only:

$$w_x = \alpha \cdot Y \cdot \frac{\beta + x \cdot s - 1}{x \cdot s - 1} \quad \forall x \in [1, 2, ..., L]. \tag{15}$$

Finally, using (11) and (13), firms'revenues can be written as

$$p(Q_x) l_x - w_x l_x = (1 - \beta) \alpha \cdot Y \frac{l_x}{xs - 1} \quad \forall x \in [1, 2, ..., L]. \tag{16}$$

2.5 Zero-profit condition

Entering a market is a costly activity that requires time. Each firm deciding to compete in a sector with x incumbents must pay a flow cost $\frac{h}{x+1}$ until it enters the market[16]. The assumption of a cost decreasing in the level of competition seems plausible (it involves more effort to break a monopoly than entering a very competitive sector) and allows to have a finite number of potential entrants for each level of competition $x \in [0, 1, ..., L]$.[17] The expected utility of a firm willing to enter a market with x competitors is:

$$\begin{aligned} rJ_V(x) &= -\frac{h}{x+1} + q\left[J_E(x+1) - J_V(x)\right] \\ &+ \left[m_x - q\right]\left[J_V(x+1) - J_V(x)\right] + \delta \cdot x\left[J_V(x-1) - J_V(x)\right], \end{aligned} \tag{17}$$

[15] Differentiating the first line of equation (13) with respect to l_x, I obtain $[p''(Q_x)l_x + 2p'(Q_x)]$ that is negative if $\frac{1+s}{s} < 2x$. This is always true for any $x \geq 1$ and $s > 1$.

[16] This way of structuring firms' entry is the same used in an equilibrium matching framework *à la* Pissarides (2000) to model the posting of a job vacancy.

[17] If the expected cost of entering a market was the same for any x, firms would only try to produce in sectors with no competition, $x = 0$, because they ensure the highest expected profits.

with $x \in [0, 1, ...L-1]$.

To find V_x, the number of firms that want to enter a market with x competitors, I introduce a zero-profit condition. Firms enter one intermediate market as long as the expected return is nonnegative:

$$rJ_V(x) \;=\; 0 \qquad \forall\, x \in [0, 1, ..L-1] \tag{18}$$

From (12), the expected discounted value of a firm with x competitors must be equal to the expected cost of entry:

$$J_E(x+1) \;=\; \frac{h}{q \cdot (x+1)} \qquad \forall x \in [0, 1, 2, ..L-1] \tag{19}$$

Using (12), (16), (18), and (19) one gets:

$$\frac{m_x}{x\,+\,1} \;=\; \frac{q}{h}\,(1-\beta)\,\alpha \cdot Y\,\frac{l_x \cdot x}{xs\,-\,1} \;-\; r \qquad \forall\, x\;\in\,[\,1, 2, ..L\,]. \tag{20}$$

Conditional on Y, the rate m_x at which the number of active firms shifts from x to $x+1$ decreases with β and h, two parameters capturing respectively the intensity of regulation in the labour and product markets[18].

In Appendix A, I show that there exists a finite number of incumbents L such that no one wants to enter that sector anymore and $m_L = 0$.[19] Thus, evaluating both (14) and (20) at $x = L$ and rearranging, one gets:

$$Y^{2-s} \;=\; \frac{r \cdot h}{q}\,\frac{Ls}{1\,-\,\beta} \cdot \left[\alpha\,\frac{Ls}{Ls\,-\,1}\right]^{s-1} \tag{21}$$

[18] Differently from a standard search and matching framework, in this model there are no congestion effects, because each firm faces a constant entry rate, q, that is not affected by the number of potential entrants. In fact, a high value for $m_x = q \cdot V_x$ has a negative impact not on the entrants but on the incumbents, since it raises the probability that a new competitor will arrive; for instance, the higher m_1 is, the shorter will be the period the incumbent can enjoy its monopolist rent. If there are too many entrants at $x = 1$, the monopolist' expected revenues are not high enough to repay the cost of entry, and the expected discounted value $rJ_E(1)$ is negative. In turn, this implies that $rJ_E(2)$ is also negative, so no additional firm will enter a market with $x = 1$. This also explains why I need to impose m_0 exogenous: at $x = 0$ there are no incumbents, and the zero-profit condition does not apply.

[19] More correctly, L is an integer number such that the RHS of (20) is strictly positive when evaluated at $L-1$ and nonpositive at $x = L$. For simplicity reasons, I neglect this integer constraint. I take it into account only in the numerical analysis.

Substituting Y in equation (20) allows to make the rate m_x depending only on one endogenous variable, L:

$$\frac{m_x}{r \cdot (x\,+\,1)} \;=\; \frac{L \cdot s\,-\,1}{x \cdot s\,-\,1} \cdot \left[\frac{L}{x} \cdot \frac{x \cdot s\,-\,1}{L \cdot s\,-\,1}\right]^{s} \,-\,1, \tag{22}$$

with $x\,\in\,[\,1, 2, ..L\,]$.[20]

3 Equilibrium

At the equilibrium, the amount of the final good produced is not fixed. Applying the law of large numbers and using (14), Y can be written as:

$$\begin{aligned} Y \;&=\; \left[\sum_{i=1}^{I} Q_i^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}} \;=\; \left[I\pi_1 Q_1^{\frac{s-1}{s}} \,+\, ... + I\pi_x Q_x^{\frac{s-1}{s}} \,+ ... \,+ I\pi_L Q_L^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}} \;= \\ &=\; I^{\frac{s}{s-1}} \cdot \left[\sum_{x=1}^{L} Q_x^{\frac{s-1}{s}}\,\pi_x\right]^{\frac{s}{s-1}} \;=\; \left[I \cdot \alpha^{1-s} \cdot \sum_{x=1}^{L} \left(\frac{xs\,-\,1}{xs}\right)^{s-1} \pi_x\right]^{\frac{1}{s-1}}. \end{aligned} \tag{23}$$

Definition *A long-run general equilibrium is defined as a vector* $[\,l_x,\, w_x,\, V_x,\, P(Q_x)\,]\;\;\forall x\,\in\,[1, 2, ..., L]$, *a probability distribution* $[\pi_0, \pi_1, \pi_2, ..., \pi_L]$, *and a value* Y *of the final good satisfying:*

1. *The F.O.C.s (15) of the bargaining problem,* $\forall x\,\in\,[1, 2, ..., L]$.
2. *The Nash equilibrium (14) of the Cournot game.*
3. *The zero profit conditions (22),* $\forall x\,\in\,[1, 2, ..., L]$.
4. *The steady-state distribution (4).*
5. *The F.O.C. in the final good sector (2) and the equation for* Y *in (23).*

[20] The derivative of m_x with respect to L is

$$\frac{d\,m_x}{d\,L} \;=\; \frac{s}{x \cdot s\,-\,1}\left(\frac{x \cdot s\,-\,1}{x}\right)^{s}\left(\frac{L}{L \cdot s\,-\,1}\right)^{s} \cdot \left(1\,-\,\frac{1}{L}\right) \;>\; 0.$$

The F.O.C.s (15) and the solutions of the Cournot game (14) are expressed as a function of Y only. The zero profits conditions (22) determine the values of V_x as a function of L. Thus, the steady state distribution $[\pi_0, \pi_1, ..., \pi_L]$ also depends on L only. Therefore, the equilibrium of the model can be characterized by a system of two equations, (21) and (23), in (L, Y) space:

$$\begin{cases} Y^{2-s} - \frac{r \cdot h}{q} \frac{Ls}{1-\beta} \cdot \left[\alpha \frac{Ls}{Ls-1}\right]^{s-1} = 0 \\ Y - \left[I \cdot \alpha^{1-s} \cdot \sum_{x=1}^{L} \left(\frac{xs-1}{xs}\right)^{s-1} \pi_x \right]^{\frac{1}{s-1}} = 0 \end{cases} \tag{24}$$

If a solution to this system exists, then the equilibrium values of Y and L uniquely determine the value of the wage w_x, the level of employment $l_x \cdot x$, the number of potential entrants V_x, and the steady state distribution.

Proposition 1 *If $s \geq 2$, the system (24) has a unique positive solution in the space (L, Y). If $1 < s < 2$, the system admits at least two solution in the space (L, Y), one at the origin.*

Proof. See Appendix B.

■

3.1 Properties of the Distribution

Proposition 2 *If $m_0 > \delta$, the steady-state distribution $[\pi_0, \pi_1, \pi_2, ..., \pi_L]$ is humped-shaped.*

Proof.
From equation (3) we know that $\pi_{x+1} > \pi_x \leftrightarrow m_x/(x+1) > \delta$. So imposing $m_0 > \delta$ implies that $\pi_1 > \pi_0$. From the zero profit condition (20), one also gets that $\frac{\Delta\,[m_x/(x+1)]}{\Delta\,x} < 0$.[21] So $m_x/(x+1)$ reaches the maximum at $x = 1$ and then it monotonically decreases until it is equal to zero at $x = L$.

[21] Ignoring for a moment the integer problem, one gets

$$\frac{d\,[m_x/(x+1)]}{d\,x} = r \cdot (L \cdot s - 1) \cdot \frac{L}{x} \cdot \frac{s}{x \cdot s - 1} \cdot \left(\frac{L}{x} \cdot \frac{x \cdot s - 1}{L \cdot s - 1}\right)^{s-1} \cdot \left(\frac{1}{x} - 1\right) < 0.$$

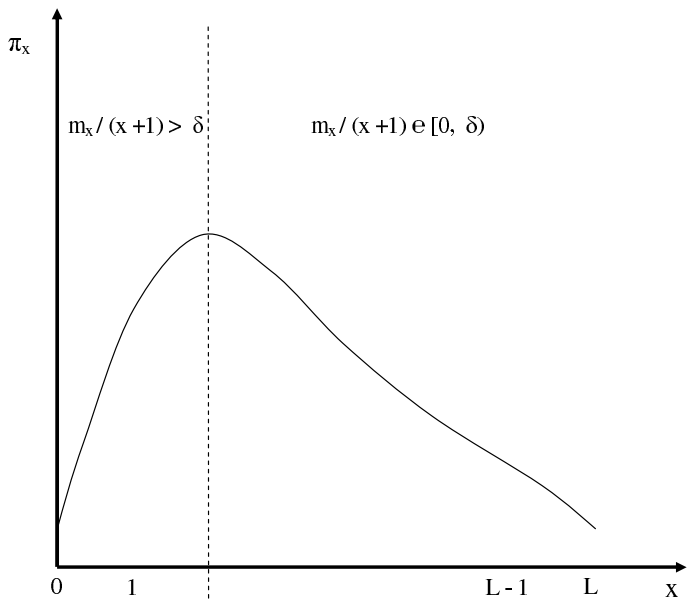

Figure 2: The endogenous distribution has an humped-shaped form.

Since $\pi_1 > \pi_0$, two cases are possible. If $m_x/(x+1) < \delta \ \forall x \in [1, 2, ..., L-1]$, then the maximum point of the distribution $[\pi_0, \pi_1, \pi_2, ..., \pi_L]$ is at $x = 1$ and then it monotonically decreases.
Otherwise, the distribution is increasing as long as $m_x/(x+1) > \delta$ and decreases for $m_x/(x+1) \in [0, \delta)$. Figure 2 shows it. In both cases, it is hump-shaped.

■

4 Deregulation in Products and Labour Markets

In this section I summarize the effects of product and labour market deregulation policies on employment and the real wage. I consider the parameter h, the flow cost of entry, as a proxy for the extent of the rigidity in the product market, and the bargaining power of the unions of workers β as an indicator of the level of regulation present in the labour market. Formally, I totally differentiate the system (24) and apply the implicit function theorem. Computations are presented in Appendix C.

4.1 Reducing the entry cost

Employment effects

Lowering the flow cost of entry h makes more firms eager to enter the market for each level of competition x. This in turn increases the rates m_x at which the level of competition increases in each sector. The maximum number of firms L that may compete in each market also goes up, because a lower h decreases the expected cost of entry. From equation (23), a higher L raises the amount of the final good Y produced in the economy: fiercer competition in the intermediate sectors augments the supply of these goods to the final representative firm.

Two countervailing effects intervene on the aggregate level of employment, E. On the one hand, a higher Y has a negative effect on the sectoral level of employment $l_x \cdot x$ (see equation 14). This stems from the assumption that unemployed workers receive a fraction α of the final good Y. A larger amount of the final good Y that accrues to the unemployed workers raises the opportunity cost of employment and make them more demanding in the wage negotiation. Higher wages discourage firms from hiring more workers. Such negative income effect tends to reduce the aggregate level of employment as h goes down.

On the other hand, in the economy there are more sectors with stronger competition. Employment in each intermediate sector increases with competition (notice from equation 14 that $l_x \cdot x$ is increasing in x), so this distribution effect tends to raise E. In the Appendix C, I show that a reduction in entry costs raises the aggregate level of employment: the distribution effect is stronger than the negative income effect.

Effect on the real wage

As shown in equation (15), the real wage is affected by a change in the entry cost only via Y. A reduction in h raises the real wage for any level of x because it increases the amount of the final good going to the employees. However, this does not imply that the *average* real wage, $\bar{w_x} = \sum_{x=1}^{L} w_x \, x \, l_x \, \pi_x \, I$, necessarily increases. The reason is that there is also a distribution effect going in the opposite direction. A reduction in entry costs augments the likelihood of being employed in more competitive sectors, where workers are paid less. It is impossible to say at the analytical level which of the two effect predominates.

4.2 Reducing unions' bargaining power

Employment effects

A decrease in the bargaining power of the unions of workers boosts the entry of new firms in the product market, because potential employers know they will get a higher fraction of the rents. Since expected profits are higher for any level of competition x, L also increases. The distribution $[\pi_0, \pi_1, ..., \pi_L]$ shifts to the right: it is more likely to be employed in competitive sectors.

As in the case of product market deregulation, two forces affect the aggregate level of employment. A lower β, by raising the amount of consumption good produced Y, increases workers' opportunity cost of employment. This in turn reduces the level of employment $l_x \cdot x$ for any given level of competition. But the change in the distribution outweighs this negative effect and the aggregate level of employment E goes up.

Effect on the real wage

A change in the unions' bargaining power has a twofold effect on the real wage. On the one hand, lowering the share of the rents that go to the employees exerts a downward pressure on the wage. On the other hand, a reduction in β raises the total amount of the final good produced in the economy, and the wage tends to appreciate in real terms. The net effect cannot be ascertained at the analytical level.

4.3 Distributive Effects

What are the effects of product and labour market deregulation on the probability distribution? A first assessment may be carried out by examining the ratio between the highest and the lowest wage paid in the economy. From equation (15), it is equal to:

$$\frac{w_1}{w_L} = \frac{s-1+\beta}{s-1} \cdot \frac{L \cdot s - 1}{\beta + L \cdot s - 1}$$

This expression is increasing in L. The higher is the maximum number of firms that can compete in a market, the wider the dispersion measured by this indicator. Since a decrease in the entry costs h raises L, product market deregulation has the effect of enlarging the gap between the highest paid workers and the lowest paid ones. When entry costs are high, only sectors with a low level competition can survive because

Variables	Interpretation	Source
s	measure of competition between sectors	u about 8%
δ	net destruction rate of a business	INS (2000); (2001); (2002).
N	labour force per sector	imposing full employment at $x = L + 1$.
β	trade unions' bargaining power	imposing $w_L/\alpha Y \cong 0.98$.
r	discount rate	5% on annual basis.
h/q	expected cost of entry	zero profit equation (21)
α	fraction of Y going to the unemployed	lowest wage close to minimum wage.

Table 1. Values of the variables for calibration.

firms need huge profits to repay them. So, deregulation in product markets allow the existence of markets with fiercer competition, small profits and low-paid jobs.

On the contrary, the consequences of a weakening of trade unions' power cannot be ascertained at the analytical level. A decrease in β has a twofold effect on the ratio w_1/w_L. The direct one - obtained by differentiating the ratio with L fixed - is negative, because a reduction in unions' bargaining power has a stronger negative impact on the wage earned by an employee in a monopoly compared to that earned by a worker in a sector with L firms. So a lower β reduces the wage dispersion. However, a reduction in β also enhances L, that in turn raises w_1/w_L. The complexity of the computations do not permit to verify which effect outnumbers the other.

The ratio w_1/w_L is only one possible measure of wage dispersion. For a fuller account of the distributive effects of product and labour market deregulation, it is necessary to evaluate the effects of a change in β and h on the whole distribution $[\pi_0, \pi_1, ..., \pi_L]$. Unfortunately the computations are too cumbersome to be discerned at the analytical level, so most of the results of the paper are obtained via numerical simulations and will be presented in the next section.

5 Quantitative Results

5.1 Calibration

I take the month as unit of time. Data refer to the period 2002-2003 in Belgium. Table 1 summarizes my calibration procedure. The discount rate is fixed at 0.004 (5% on an annual basis). The maximum number of firms in each sector, L, is set equal to 15. It may seem a small number. Recall however that firms compete both within each sector - *à la* Cournot - and among sectors, the intensity of this competition depending on the value of the elasticity s. Imposing $L = 15$ simply means that in the economy there are at most 15 firms that produce exactly the same type of good, competing with others companies that sell broadly similar items.

The firm's destruction rate δ is inferred by looking at the number of Belgian firms that lose each year their VAT code number [22]. The Belgian Institute of Statistics provide these data for the years 2000-2001-2002 (INS, 2000; 2001; 2002). The monthly destruction rate is about 0.004 in all the three years[23]. The exogenous rate of entry m_0 is normalized to 1.

In the model, the labour force in each sector is exogenous and normalized to 1. In the calibration, I impose that it is equal to the level of employment if $L+1$ firms were producing in the sector; this implies that even when the level of competition is highest (at $x = L$), there is still some frictional level of unemployment. This assumption allows to have the unemployment rate as a function of s, δ, m_0, L only.[24] Since I have

[22] I rule out all the firms that declared no employees.

[23] The same source also provides the firms' creation rate for the same years. It is about 0.0035, suggesting that the stock of active firms was fairly stable in that period.

[24] From equation (14), the level of employment is

$$E = \sum_{x=1}^{L} x l_x \pi_x I \;=\; Y^{1-s}\alpha^{-s} \sum_{x=1}^{L} \left(\frac{xs}{xs-1}\right)^{-s} \pi_x \cdot I.$$

The total labour force in the economy is

$$N \cdot I \;=\; (L+1)\cdot l_{L+1} \cdot I \;=\; Y^{1-s}\alpha^{-s}\left(\frac{(L+1)s}{(L+1)s-1}\right)^{-s} \cdot I.$$

Thus, the unemployment ratio $u \equiv \frac{N \cdot I - E}{N \cdot I}$ depends only on L, s, m_0, and δ, the last two parameters appearing in the probability distribution π_x.

already attributed a value to the last three variables, I can find s by imposing that the unemployment rate should be close to 7/8%, the percentage that Belgium experienced in the last five years. The corresponding value of s is 2.1.

In Belgium, the gross monthly minimum wage is $1387,49$ euros; for a single employee that works full-time the net salary is about 1100 euros. The unemployment benefits are generous: a single beneficiary may receive a subsidy up to 1144 euros per month. So, to calibrate the bargaining power of workers'unions I impose that the ratio between the lowest wage in the economy and the income of unemployed workers - that, by using equation (15), is $(L \cdot s - 1)/(L \cdot s - 1 + \beta)$ - is close to 1. With such a ratio equal to 0.98, β is 0.62. The number of sectors in the economy is fixed to 2350. The parameter α is obtained by imposing that $w_L \cong 1100$ euros; the resulting value is 0.01.

Finally, the expected cost of entry h/q is inferred by imposing the zero profit condition (21). The resulting value is 143.79, a very tiny amount. This is because the construction of the model is such that the expected profits of a firm are small compared to the wages.

5.1.1 Wage Distribution

An interesting result that stems from the calibration concerns the wage distribution. By pinning down the variables δ, m_0, s, and L only on the basis of employment data, the resulting wage density function $w_x \rightarrow l_x \cdot x \cdot \pi_x \ \forall x$ is unimodal and positively skewed, just as the earning distributions observed in the data of most countries (see Figure 3).

Why does the wage density function have such a shape? The unimodal property depends on the structure of the probability distribution $[\pi_0, \pi_1, ..., \pi_L]$, proved in Proposition 2. The positive skewness property stems from the convexity of the wage function (15) with respect to x.[25] The wage loss caused by an increase in competition is larger the more oligopolistic is the sector. While workers employed in sectors with $L-1$, $L-2$, $L-3$ competitors earn broadly the same salary, the difference between the wage of the employee of a monopolist and that of duopolist is much wider. So, most of the earnings are concentrated on the left tail of the distribution.

[25] If we ignore the integer problem, this can be easily seen by computing the second derivative of w_x with respect to x and noting that is positive.

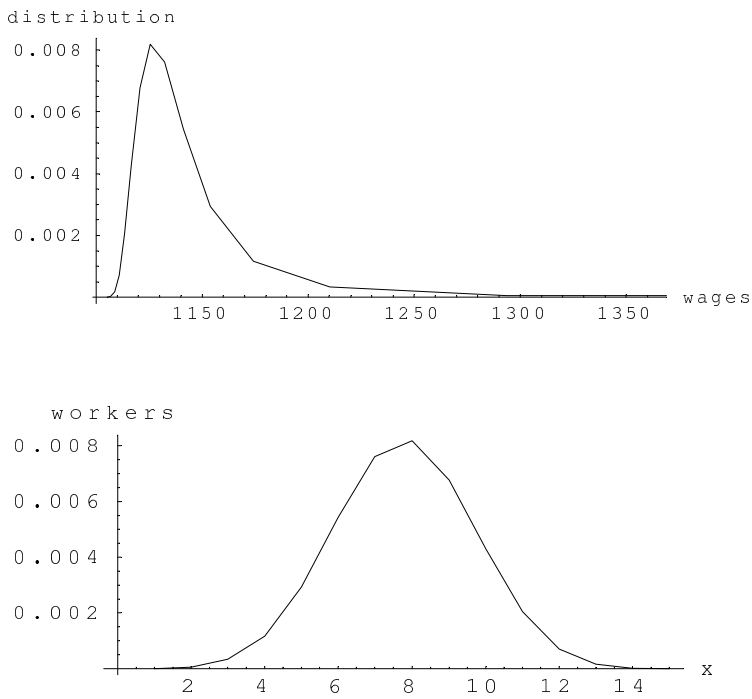

Figure 3: Above: Wage distribution $w_x \rightarrow l_x \cdot x \cdot \pi_x$ for $x \in [1, 2, ..., L]$. Below: distribution of workers per level of competition $x \rightarrow l_x \cdot x \cdot \pi_x$ for $x \in [1, 2, ..., L]$.

What the model is not able to capture is the large differences in absolute value between the richest and the poorest. The Gini index resulting from the calibration is very low, about 0.009. The obvious explanation for that is the lack in the model of most of the features that explain most of the income inequality in the industrialized countries, such as rents and revenues coming from the overall wealth of the individuals. An additional reason stems from the wage equation, that is not convex enough in x to display a large difference between the highest wages and the lowest ones.

5.2 Simulation Results

5.2.1 Lowering entry costs

I consider a reduction up to 25% in the expected costs of entry in the labour market, h/q. Figure 4 summarizes the main results. The maximum number of firms that may

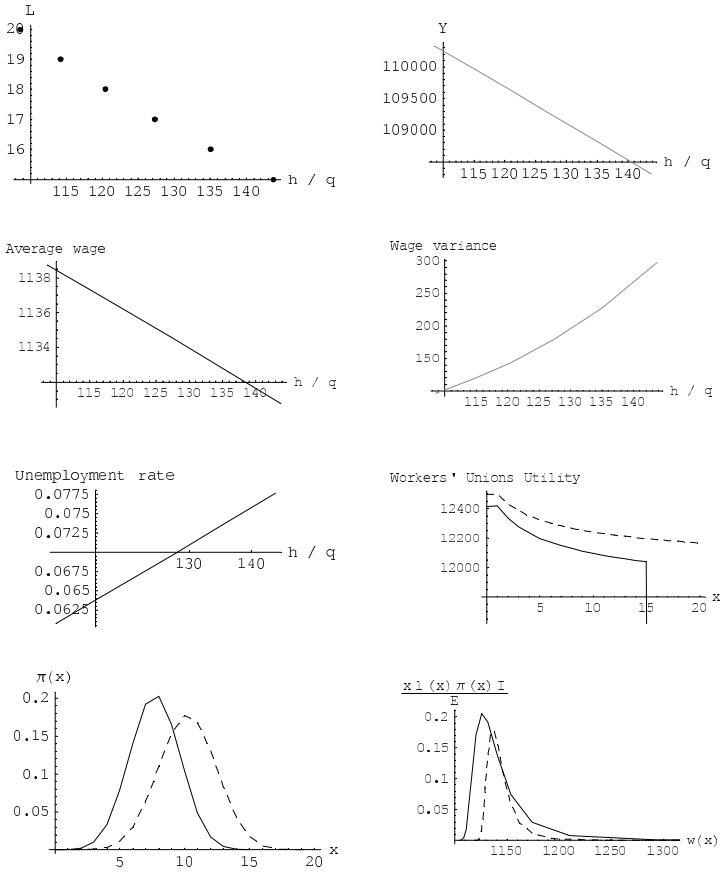

Figure 4: Numerical results: a reduction in the expected entry costs, h/q. Dotted lines: simulation with h/q 25% lower.

compete shifts from 15 to 20, and the output Y is raised by 1.8%. The unemployment rate decreases up to 1.5 percentage points. The average real wage goes up, but very slightly: the salary gain is about 8 euros, that corresponds to a 0.7% increase. Such a tiny improvement depends on the two potentially offsetting effects mentioned in section 4.1, the positive income effect and the negative distribution effect. The small increase in $\bar{w}_x$ means that the former slightly outnumbers the latter.

Inequality

Lower entry costs also affect the steady state distributions. The consequences in terms

of wage inequality are mixed. On the one hand, we know from section 4.3 that the ratio between the highest and the lowest wage, w_1/w_L, increases. However, the numerical results show that diminishing h/q lowers the wage variance and the Gini index[26]. The decline in the variance is considerable: once h/q is reduced by 25%, it plummets from 297.3 to 96.3 euros, about 68% lower. The Gini index also shrinks by about 39%.

How can such different results be explained? Tougher competition in the product market lowers the variance and the Gini index because there are now more workers employed in sectors with many firms, that implies more employees earning the same wage. So, the fact that the distance between the highest and the lowest wage paid in the economy widens does not entail an overall increase in wage inequality.

Welfare of trade unions

Decreasing the entry costs also has non trivial effects on the expected utility of a worker's union, $rU_W(x)$. Again, the presence of a negative distribution effect may offset the wage and employment gains obtained by a more competitive product market. This can be easily grasped by inspecting equation (6). A decrease in h/q raises the instantaneous utility of the union, equal to $w_x\, x\, l_x + (1 - x\, l_x)\, \alpha\, Y$, because w_x and Y go up. However, the rate m_x at which a new firm enters the market increases as well, making the union more likely to operate in a more competitive market, with lower rents to be shared.

As far as the change in h/q is concerned, the benefits caused by the increase in unions' instantaneous utility prevail. A decrease in entry costs make workers' unions better off for any level of competition. Even unions in which all the members are are unemployed (operating in sectors with $x = 0$) benefit from the reduction in h/q, because the income of the unemployed, αY, is higher.

5.2.2 Lowering workers' unions bargaining power

I consider a reduction up to 20% in β. As far as the unemployment and the total output are concerned, squeezing β has the same effects of a reduction in h/q. Figure 5 shows the evolution of Y and u as β decreases. Even the orders of magnitude appear broadly similar: a reduction up to 20% of unions' bargaining power shrinks unemployment

[26] The Gini index is computed by taking the income of unemployed into account

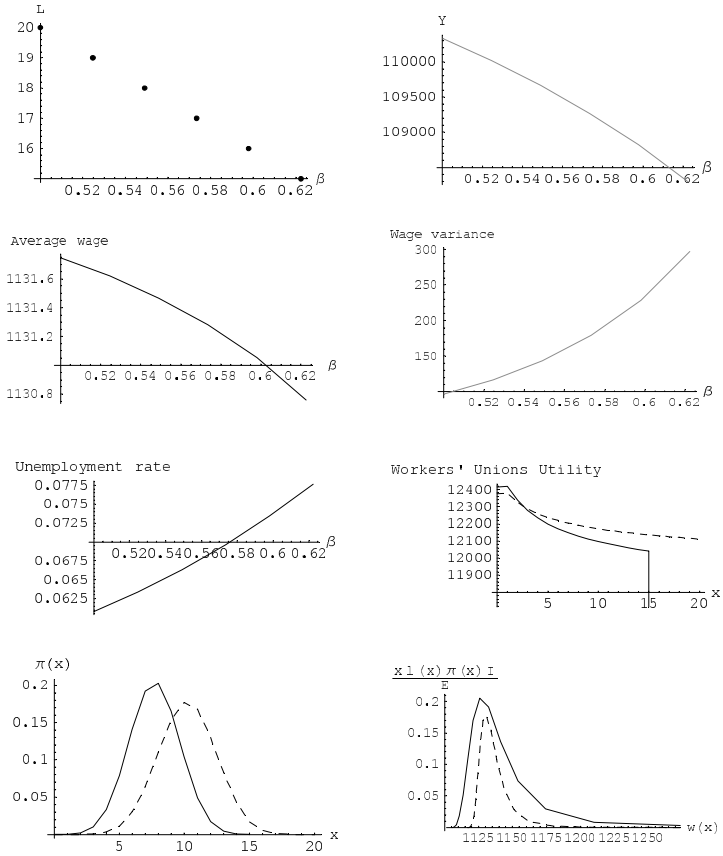

Figure 5: Numerical results: a reduction in the bargaining power of workers' unions, β. Dotted lines: simulation with β 20% lower.

as much as a decrease by 25% in entry costs. The effects on the average real wage are almost negligible: diminishing β by 20% raises $\bar{w}$ by 1 euro, that corresponds to a 0.08% increase. This is due to the fact that, besides the positive income and the negative distribution effect present in the case of lower entry costs, a change in β also affects the wage equation; from (15), a lower β decreases the w_x for any value of x. The two negative effects basically offset the income effect caused by a lower β.

Inequality

A lower unions' bargaining power squeezes the wage variance and the Gini index. A 20% reduction of β translates into a decrease in the wage variance of about 68%; the

size of the decline is identical to that caused by the 25% decrease in entry cost. The Gini index is also lowered by 50%. The explanations for such a sharp decline are the same expressed for the case of product marker deregulation. A lower workers' unions bargaining power triggers competition that in turn entails a larger share of workers having the same salary.

Welfare of trade unions

The effects of a reduction in β on the welfare of workers' unions depend on the level of competition in the intermediate sector. The sixth graphic in Figure 5 illustrates that; trade unions operating in a monopoly and in duopoly are worse off after the reduction in β, whereas those belonging to sectors with more than one competitor and the unemployed are better off. Consider equation (6). As in the case of lower entry costs, two countervailing forces affect unions' expected utility. On the one hand, a lower β raises the instantaneous utility of the union, $w_x\, x\, l_x + (1 - x\, l_x)\, \alpha\, Y$, because both Y and w_x are higher. On the other hand, the capital loss due to the increase in competition augments because of the higher probability a new firm will enter the market. Because firms' profits are decreasing and convex with respect to x, this second effect is stronger in sectors with low competition. Labour market deregulation worsens the welfare of trade unions in poorly competitive industries, even though both the aggregate real wage and the level of employment go up.

6 Conclusion

In Europe, reforms aimed to improve the functioning of the product and labour markets are at the centre of the political agenda. This paper does not argue that such changes must not be undertaken; rather, that the overall consequences for the economy are more mixed than an analysis uniquely focused on the employment and wage gains suggests.

Easing the cost for a firms to enter the market reduces the wage variance and the Gini index but enlarges the distance between the lowest and the highest pay in the economy. Labour market deregulation worsens the welfare of workers employed in markets with low competition, while it raises the utility of all the others; the theoretical prediction is that these reforms are harder to implement than those concerning the

functioning of the product market. This would leave room for some political economy reflections that I neglected.

Moreover, the conclusions on wage inequality and the welfare of trade unions have not been empirically tested. This can be left for future research.

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Appendix A: Existence of L

By definition, L is such that $m_L = 0$. Three conditions are sufficient for the existence of L. First, the term in the LHS of (20) is decreasing in x. Second, that it tends to a negative value as $x \to +\infty$. Third, that it is positive at $x = 1$. If these conditions are fulfilled, there exists only one $x = L$ such that $m_L = 0$ (and the LHS of (20) is equal to zero).

Ignoring for simplicity the integer problem, the derivative of the first term in the LHS of (20) with respect to x is:

$$\frac{q}{h}(1-\beta)\,\alpha \cdot Y \left[\frac{d\,l_x}{d\,x} \cdot \frac{x}{xs\,-\,1} \,-\, \frac{l_x}{(xs\,-\,1)^2}\right],$$

that is negative if $d\,l_x/d\,x$ is negative. This is the case in a Cournot model in which the amount of the good produced by each player decreases with the number of competitors. This can also be checked by differentiating (14).

Furthermore, $\lim_{x\to+\infty} \frac{q}{h}(1-\beta)\,\alpha \cdot Y \frac{l_x \cdot x}{xs-1} - r = -r$ because in case of perfect competition (i.e. $x \to +\infty$), both Y and $Q_x = l_x \cdot x$ take positive finite values.

Finally, evaluating (12) at $x = 1$ and knowing that $J_E(x) = \frac{h}{q\cdot(x+1)}$, one obtains:

$$r\frac{h}{q} \;=\; p(Q_1)\,l_1 - w_1 l_1 \,-\, \delta\frac{h}{q} \,+\, m_1\frac{h}{q}\left[\frac{1}{2} \,-\, 1\right].$$

Using the equation of firms' revenues 16 and rearranging, one gets:

$$\frac{q}{h}(1-\beta)\,\alpha \cdot Y\,\frac{l_1}{s\,-\,1} \,-\, r \;=\; \frac{m_1}{2} \,+\, \delta$$

The RHS is positive and this proves the third condition.

Appendix B: Existence of the Equilibrium

Before formally proving Proposition 1, I introduce the following Lemma:

Lemma 1 *The sum $S \equiv \sum_{x=0}^{L} g(x) \cdot \pi_x \cdot I$ is increasing (decreasing) in m_x for any function $g(.)$ increasing (decreasing) in x.*

It easy to check that $\frac{d\pi_x}{dm_n} > 0$ if $n \in [0, 1, 2, ..., x-1]$, and $\frac{d\pi_x}{dm_n} < 0$ if $n \in [x, x+1, ..., L-1]$. Hence:

$$\frac{d\,S}{d\,m_x} = \left[\sum_{n=0}^{x} g(n) \cdot \frac{d\pi_n}{dm_x} + \sum_{n=x+1}^{L} g(n) \cdot \frac{d\pi_n}{dm_x} \right] \cdot I.$$

The first term at the RHS is negative, the second one is positive. To check the sign of this derivative, notice that $\sum_{n=0}^{L} \pi_n = 1$. Then:

$$\begin{aligned} -\sum_{n=0}^{x} \frac{d\pi_n}{dm_x} &= \sum_{n=x+1}^{L} \frac{d\pi_n}{dm_x} \quad \Longleftrightarrow \\ -\sum_{n=0}^{x} \frac{d\pi_n}{dm_x} \cdot x &= x \cdot \sum_{n=x+1}^{L} \frac{d\pi_n}{dm_x} \quad \Longleftrightarrow \\ -\sum_{n=0}^{x} \frac{d\pi_n}{dm_x} \cdot g(n) &< \sum_{n=x+1}^{L} \frac{d\pi_n}{dm_x} \cdot g(n) \end{aligned}$$

The last inequality is verified if $g(x)$ is an increasing function.

■

To prove Proposition 1, I distinguish three case.

CASE 1: $s > 2$.

Consider the system (24). I write the first equation as an implicit relationship in (L, Y) space:

$$\mathbb{G}_1(L, Y) \equiv Y^{2-s} - \frac{r \cdot h}{q} \frac{Ls}{1-\beta} \cdot \left[\alpha \frac{Ls}{Ls-1} \right]^{s-1} = 0 \tag{25}$$

Applying the implicit function theorem, I get:

$$\frac{d\,Y}{d\,L} = \frac{s}{2-s} \cdot \left(\frac{h}{q} \cdot \frac{r \cdot \alpha^{s-1}}{1-\beta} \right)^{\frac{1}{2-s}} \cdot s^{\frac{s}{2-s}} \cdot (L-1) \cdot L^{\frac{2(s-1)}{2-s}} \cdot (Ls-1)^{\frac{1}{s-2}} < 0. \tag{26}$$

The implicit function $\mathbb{G}_1(L, Y) = 0$ is decreasing in the (L, Y) space. I denote $Y = g_1(L)$ the explicit function of $\mathbb{G}_1(L, Y) = 0$. Then, $\lim_{L \to 0} g_1(L) \to +\infty$ and $\lim_{L \to +\infty} g_1(L) = 0$.

I also write the second equation as an implicit relationship in (L, Y) space:

$$\mathbb{G}_2(L, Y) \equiv Y - \left[I \cdot \alpha^{1-s} \cdot \sum_{x=1}^{L} \left(\frac{xs-1}{xs} \right)^{s-1} \pi_x \right]^{\frac{1}{s-1}} = 0 \tag{27}$$

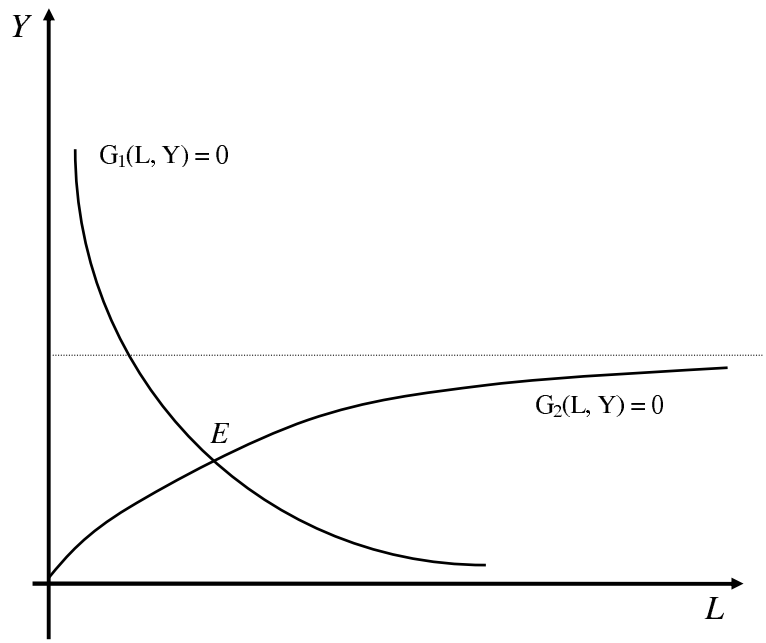

Figure 6: Equilibrium when $s > 2$.

Applying the implicit function theorem, I get:

$$\frac{d\,Y}{d\,L} = \frac{Y^{2-s}}{s-1} \cdot I \cdot \alpha^{1-s} \cdot \sum_{x=1}^{L} \left(\frac{x \cdot s - 1}{x \cdot s}\right)^{s-1} \frac{d\,\pi_x}{d\,L}. \tag{28}$$

Notice that $\frac{d\,\pi_x}{d\,L} = \sum_{n=1}^{L} \frac{\partial\,\pi_x}{\partial\,m_n} \cdot \frac{\partial\,m_n}{\partial\,L}$. The derivative $d\,m_n/d\,L$ is positive (see footnote 20). Therefore, the derivative in (28) is positive if the marginal increase in m_n on $\sum_{x=1}^{L} \left(\frac{x \cdot s-1}{x \cdot s}\right)^{s-1} \pi_x$ is positive. For Lemma 1, this is the case because the function $g(x) = \left(\frac{x \cdot s-1}{x \cdot s}\right)^{s-1}$ is increasing in x.
The implicit function $\mathbb{G}_2(L, Y) = 0$ is increasing in the (L, Y) space.
I denote $Y = g_2(L)$ the explicit function of $\mathbb{G}_2(L, Y) = 0$. Then, $\lim_{L \to 0} g_1(L) = 0$ and $\lim_{L \to +\infty} g_1(L) = k$, a positive and finite number, because $\lim_{L \to +\infty} \frac{Ls-1}{Ls} = 1$.
Figure 6 illustrates the equilibrium.

CASE 2: $s = 2$.

If $s = 2$, Y disappears from $\mathbb{G}_1 = 0$, that uniquely defines the value for L. $\mathbb{G}_1(L) = 0$ is a vertical line in the (L, Y) space. Figure 7 illustrates the equilibrium.

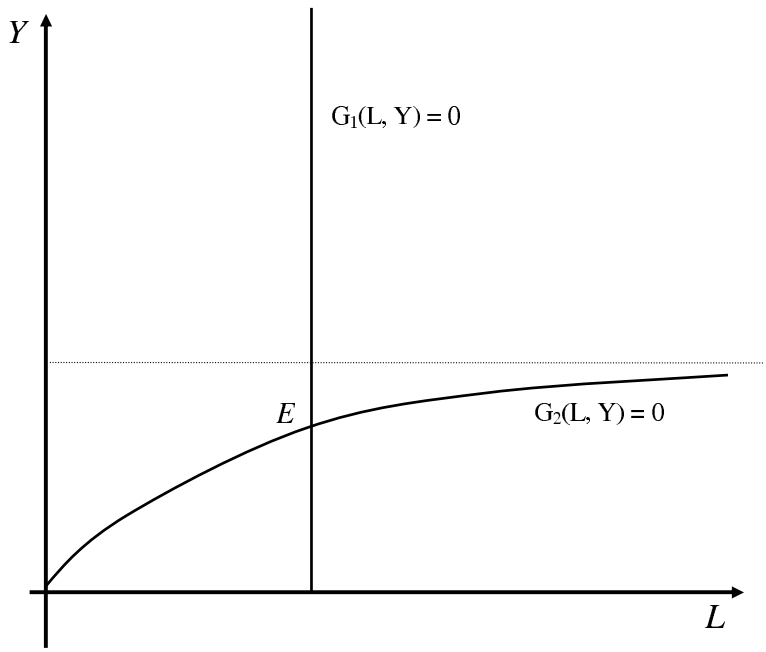

Figure 7: Equilibrium when $s = 2$.

CASE 3: $1 < s < 2$.

In this case, the derivative (26) is positive and $\mathbb{G}_1(L, Y) = 0$ is increasing in the (L, Y) space. Moreover, $\lim_{L\to 0} g_1(L) = 0$ and $\lim_{L\to +\infty} g_1(L) \to +\infty$.
Since $\mathbb{G}_2(L, Y) = 0$ is also verified at the origin, the point $(0, 0)$ is one equilibrium solution of the system.
Moreover, at $L \to +\infty$, $g_1(L) > g_2(L) = k$. So, if I show that in the interval $L \in [0, 1]$ $g_1(L) < g_2(L)$, the system admits at least another equilibrium, in which the function $g_1(L)$ intersects the function $g_2(L)$ from below. See Figure 8.
To prove this last point, I evaluates the derivatives in (26) and (28) at $L = 0$ and $L = 1$. It is easy to see that $g_1'(0) = g_2'(0) = 0$ and $g_1'(1) = 0$. On the contrary:

$$g_2'(1) = \frac{(g_2(1))^{2-s}}{s-1} \cdot I \cdot \alpha^{1-s} \cdot \left(\frac{s-1}{s}\right)^{s-1} \frac{d\,\pi_1}{d\,L} > 0.$$

So, in the interval $L \in [0, 1]$, the function $g_1(L)$ coincides with the horizontal axis and for $L \in (1, +\infty)$ becomes upward sloping. On the other hand, $g_2(L)$ is increasing for $L \in (0, +\infty)$. The function $g_1(L)$ intersects $g_2(L)$ in the positive hortant at least once.

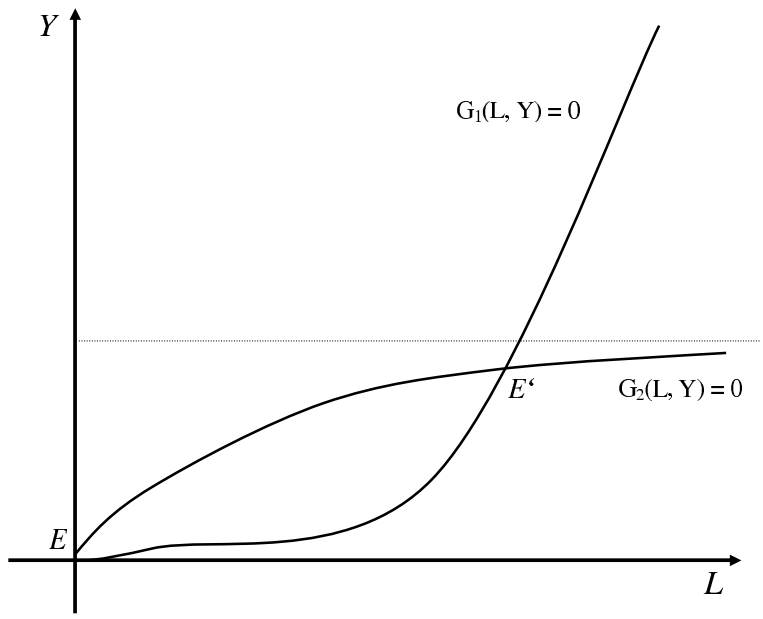

Figure 8: Equilibria when $1 < s < 2$.

Appendix C: Comparative Statics

CASE 1: $s > 2$.

Effects on L and Y

I consider the system composed by equations (25) and (27) and summarize the derivatives computed in Appendix B:

$$\begin{aligned}
\frac{\partial \mathbb{G}_1}{\partial L} &= -\frac{q}{h}(1-\beta)\alpha^{1-s} \cdot Y^{2-s}\frac{(Ls-1)^{s-1}}{(ls)^s} \cdot \frac{s(L-1)}{L(Ls-1)} < 0, \\
\frac{\partial \mathbb{G}_1}{\partial Y} &= \frac{q}{h}(1-\beta)\alpha^{1-s}(2-s) \cdot Y^{1-s}\frac{(Ls-1)^{s-1}}{(Ls)^s} < 0, \\
\frac{\partial \mathbb{G}_2}{\partial L} &= -I \cdot \alpha^{1-s} \sum_{x=1}^{L} \left(\frac{x \cdot s - 1}{x \cdot s}\right)^{s-1} \frac{\partial \pi_x}{\partial L} < 0, \\
\frac{\partial \mathbb{G}_2}{\partial Y} &= (s-1) \cdot Y^{s-2} > 0.
\end{aligned} \tag{29}$$

Moreover, it is easy to check that $\partial \mathbb{G}_1/\partial h < 0$, $\partial \mathbb{G}_2/\partial h = 0$, whereas $\partial \mathbb{G}_1/\partial \beta < 0$, and $\partial \mathbb{G}_2/\partial \beta = 0$.

Applying the implicit function theorem:

$$\frac{dL}{dh} = -\frac{det\begin{bmatrix} \partial\mathbb{G}_1/\partial h & \partial\mathbb{G}_1/\partial Y \\ \partial\mathbb{G}_2/\partial h & \partial\mathbb{G}_2/\partial Y \end{bmatrix}}{det\begin{bmatrix} \partial\mathbb{G}_1/\partial L & \partial\mathbb{G}_1/\partial Y \\ \partial\mathbb{G}_2/\partial L & \partial\mathbb{G}_2/\partial Y \end{bmatrix}} = -\frac{\frac{\partial\mathbb{G}_1}{\partial h}\cdot\frac{\partial\mathbb{G}_2}{\partial Y}}{\frac{\partial\mathbb{G}_1}{\partial L}\cdot\frac{\partial\mathbb{G}_2}{\partial Y}-\frac{\partial\mathbb{G}_1}{\partial Y}\cdot\frac{\partial\mathbb{G}_2}{\partial L}} < 0,$$

$$\frac{dY}{dh} = -\frac{det\begin{bmatrix} \partial\mathbb{G}_1/\partial L & \partial\mathbb{G}_1/\partial h \\ \partial\mathbb{G}_2/\partial L & \partial\mathbb{G}_2/\partial h \end{bmatrix}}{det\begin{bmatrix} \partial\mathbb{G}_1/\partial L & \partial\mathbb{G}_1/\partial Y \\ \partial\mathbb{G}_2/\partial L & \partial\mathbb{G}_2/\partial Y \end{bmatrix}} = \frac{\frac{\partial\mathbb{G}_1}{\partial h}\cdot\frac{\partial\mathbb{G}_2}{\partial L}}{\frac{\partial\mathbb{G}_1}{\partial L}\cdot\frac{\partial\mathbb{G}_2}{\partial Y}-\frac{\partial\mathbb{G}_1}{\partial Y}\cdot\frac{\partial\mathbb{G}_2}{\partial L}} < 0.$$

Similarly, comparative statics on β leads to:

$$\frac{dL}{d\beta} = -\frac{det\begin{bmatrix} \partial\mathbb{G}_1/\partial \beta & \partial\mathbb{G}_1/\partial Y \\ \partial\mathbb{G}_2/\partial \beta & \partial\mathbb{G}_2/\partial Y \end{bmatrix}}{det\begin{bmatrix} \partial\mathbb{G}_1/\partial L & \partial\mathbb{G}_1/\partial Y \\ \partial\mathbb{G}_2/\partial L & \partial\mathbb{G}_2/\partial Y \end{bmatrix}} = -\frac{\frac{\partial\mathbb{G}_1}{\partial \beta}\cdot\frac{\partial\mathbb{G}_2}{\partial Y}}{\frac{\partial\mathbb{G}_1}{\partial L}\cdot\frac{\partial\mathbb{G}_2}{\partial Y}-\frac{\partial\mathbb{G}_1}{\partial Y}\cdot\frac{\partial\mathbb{G}_2}{\partial L}} < 0.$$

$$\frac{dY}{d\beta} = -\frac{det\begin{bmatrix} \partial\mathbb{G}_1/\partial L & \partial\mathbb{G}_1/\partial \beta \\ \partial\mathbb{G}_2/\partial L & \partial\mathbb{G}_2/\partial \beta \end{bmatrix}}{det\begin{bmatrix} \partial\mathbb{G}_1/\partial L & \partial\mathbb{G}_1/\partial Y \\ \partial\mathbb{G}_2/\partial L & \partial\mathbb{G}_2/\partial Y \end{bmatrix}} = \frac{\frac{\partial\mathbb{G}_1}{\partial \beta}\cdot\frac{\partial\mathbb{G}_2}{\partial L}}{\frac{\partial\mathbb{G}_1}{\partial L}\cdot\frac{\partial\mathbb{G}_2}{\partial Y}-\frac{\partial\mathbb{G}_1}{\partial Y}\cdot\frac{\partial\mathbb{G}_2}{\partial L}} < 0.$$

Effects on E

I now evaluate the effects on the aggregate level of employment of a change in h or β.

Notice that E can be written as:

$$\begin{aligned} E &= \sum_{x=0}^{L} x \cdot l_x \cdot \pi_x \cdot I \\ &= Y^{1-s} \cdot I \cdot \alpha^{-s} \cdot \sum_{x=1}^{L} \left(\frac{xs-1}{xs}\right)^s \pi_x \quad \text{(by using equation 14)} \\ &= \alpha^{-1} \cdot \frac{\sum_{x=1}^{L} \left(\frac{xs-1}{xs}\right)^s \pi_x}{\sum_{x=1}^{L} \left(\frac{xs-1}{xs}\right)^{s-1} \pi_x} \qquad \text{(by using equation 23).} \end{aligned} \tag{30}$$

The level of employment is a function of the endogenous variable L. Since L decreases with h or β, it remains to show the effect of L on E.
To simplify the notations, I denote $f_x \equiv \frac{xs-1}{xs}$, $a_x \equiv \pi_x \cdot \left(\frac{xs-1}{xs}\right)^{s-1}$, and $a'_x \equiv \frac{\partial a_x}{\partial L}$. Then:

$$\frac{d\,E}{d\,L} = \frac{(f_1 \cdot a'_1 + ... + f_L \cdot a'_L) \cdot (a_1 + ... + a_L) - (a'_1 + ... + a'_L) \cdot (f_1 \cdot a_1 + ... + f_L \cdot a_L)}{(a_1 \ + \ a_2 + ... + a_L)^2}$$

At the numerator, the terms $f_x \cdot a'_x \cdot a_x$ cancel out. The numerator can be re-expressed in the following way:

$$\sum_{x=1}^{L} \sum_{k=1}^{L-x} (a_x \cdot a'_{x+k} - a'_x \cdot a_{x+k}) \cdot (f_{x+k} - f_x) \tag{31}$$

The term $f_{x+k} - f_x$ is positive $\forall x \in [1, ..., L]$ and $\forall k \in [1, ..., L-x]$ because f_x is increasing in x.
Consider now the term $a_x \cdot a'_{x+k} - a'_x \cdot a_{x+k} =$

$$\pi_x \left(\frac{xs-1}{xs}\right)^{s-1} \cdot \pi'_{x+k} \left(\frac{(x+k)s-1}{(x+k)s}\right)^{s-1} - \pi'_x \left(\frac{xs-1}{xs}\right)^{s-1} \cdot \pi_{x+k} \left(\frac{(x+k)s-1}{(x+k)s}\right)^{s-1},$$

with $\pi'_x \equiv \frac{d\pi_x}{dL}$. From equation (3), $\pi_{x+k} = g(m_{x+k-1}, m_{x+k-2}, ..., m_{x+1}) \cdot \pi_x$ with $g(.)$ being an increasing function of $m_{x+k-1}, m_{x+k-2}, ..., m_{x+1}$.[27] So:

$$\begin{aligned} \pi_x \cdot \pi'_{x+k} \ - \ \pi'_x \cdot \pi_{x+k} \ &= \ \pi_x \cdot [\, g'(.) \cdot \pi_x + g(.) \cdot \pi'_x \,] \ - \ \pi'_x \cdot g(.) \cdot \pi_x, \\ &= \pi_x^2 \cdot g'(.) \ > \ 0 \end{aligned}$$

[27] For instance, $\pi_2 = \frac{m_1}{2\delta} \cdot \pi_1$ or $\pi_4 = \frac{m_3}{4\delta} \cdot \frac{m_2}{3\delta} \cdot \pi_1$.

since $g'(.) \equiv \sum_{n=x+1}^{x+k-1} \frac{\partial g(.)}{\partial m_n} \cdot \frac{\partial m_n}{\partial L}$ is positive.
Then, the numerator (31) is positive and $d\,E/d\,L$ is positive. An increase in h or β reduces the aggregate level of employment via a decrease in L.

Effects on the real wage
Differentiating the wage equation (15) I get:

$$\frac{d\,w_x}{d\,h} = \alpha \cdot \frac{x \cdot s - 1 + \beta}{x \cdot s - 1} \cdot \frac{d\,Y}{d\,h} < 0$$
$$\frac{d\,w_x}{d\,\beta} = \alpha \cdot \frac{x \cdot s - 1 + \beta}{x \cdot s - 1} \cdot \frac{d\,Y}{d\,\beta} + \alpha \cdot \frac{Y}{x \cdot s - 1}.$$

Since $dY/d\beta < 0$, the effect of β on w_x cannot be ascertained.

CASE 2: $s = 2$.

The only difference with respect to the case $s > 2$ is that $\partial\,\mathbb{G}_1/\partial\,Y = 0$. It easy to verify that the sign of the derivatives $\frac{d\,L}{d\,h}$, $\frac{d\,L}{d\,\beta}$, $\frac{d\,Y}{d\,h}$, and $\frac{d\,Y}{d\,\beta}$ is the same as in the case $s > 2$. The effect of L on E does not change too.

CASE 3: $1 < s < 2$.

When $s > 2$, $\partial\,\mathbb{G}_1/\partial\,Y > 0$. Recall that the denominator of the derivatives $\frac{d\,L}{d\,h}$, $\frac{d\,L}{d\,\beta}$, $\frac{d\,Y}{d\,h}$, and $\frac{d\,Y}{d\,\beta}$ is:

$$\frac{\partial \mathbb{G}_1}{\partial L} \cdot \frac{\partial \mathbb{G}_2}{\partial Y} - \frac{\partial \mathbb{G}_1}{\partial Y} \cdot \frac{\partial \mathbb{G}_2}{\partial L}. \tag{32}$$

Two scenarios are possible. In the equilibrium points in which $g_1(L)$ intersects $g_2(L)$ from below (like the positive one in Figure 8), $g_1'(L) > g_2'(L)$. From the formula of the implicit function theorem, this is equivalent to say that the denominator (32) is negative. Then, the effects of β and h on L, Y, the real wage, and the level of employment have the same sign as in $s \geq 2$.
In Appendix B I proved that at least one equilibrium of this kind exists. But I cannot rule out the existence of equilibria in which in which $g_1(L)$ intersects $g_2(L)$ from above, $g_1'(L) < g_2'(L)$, and the denominator (32) is positive. It is easy to see that under this scenario an increase in β and h *raises* L and Y and have a *positive* impact on the real wage and the level of employment.

Institut de Recherches Économiques et Sociales
Université catholique de Louvain

Place Montesquieu, 3
1348 Louvain-la-Neuve, Belgique

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