The Manufacturers' Choice of Brand Policy under Successive Duopoly

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Abstract

We propose a non-cooperative game in order to emphasize the strategic rationale in shaping the distribution systems. Compared with the received literature, we let manufacturers select which retailer(s) will market their respective brand. This, together with retailers possibly being multi-product dealers, enlarges the set of distribution systems. Whether manufacturers employ two retailers rather than one reflects the tradeoff between two conflicting effects, there is an output increase but more competition is established. High levels of product differentiation and not too large brand asymmetry are enough to incentive manufacturers introduce intra-brand competition. However, the well-known exclusive dealing system shows up for little product differentiation and low brand asymmetry.

It is worth emphasizing that, if any type of exclusivity relationship ever occurs, it is the equilibrium outcome of a non-cooperative game in which neither manufacturers nor retailers may impose any vertical clauses.

Keywords: Brand Policy, Distribution Systems, Intra-brand Competition.

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1 Introduction

This paper investigates what are the equilibrium distribution systems in a successive duopoly when manufacturers choose the number of retailers they wish to employ, and both retailers can be multiproduct sellers. In this framework the organization of distribution systems by producers of differentiated asymmetric brands is concerned not only with the terms of payment but also with how many retailers to contract with. These two strategic decisions have important implications for market outcomes and consequently for the existence and the intensity of both inter and intra-brand competition.

It is observed that some brands are exclusively sold at a particular store while other brands can be found in several stores. Also, in many industries, such as the food retailing sector in Europe and US, the market is concentrated both at the seller-side and the buyer-side. These facts can be conveniently accounted for by considering an upstream and downstream duopolistic structure where retailers are allowed to carry the two brands. Another interesting feature of a retail duopoly lies in that all distribution systems can be treated symmetrically since manufacturers may defect to another retailer, if profitable. With these assumptions in hand, the number of possible channel structures is larger than in previous work on this area, including among them the well-known exclusive and common distribution systems. We are then interested in whether these systems now arise at equilibrium and also study the strategic reasons why manufacturers may introduce intra-brand competition.

An interesting question addressed in the received literature on distribution systems has been whether manufacturers would prefer having a single common retailer rather than one separate each. These two structures, common and exclusive dealership, have been compared by Lin (1990) and O'Brien and Shaffer (1993) who show that the latter is chosen since manufacturer competition is dampened. In a setting with two manufacturers and only one retailer the central theme is whether a manufacturer's brand is excluded from the market. Bernheim and Whinston (1998) and O'Brien and Shaffer (1997) prove that vertical foreclosure is not an equilibrium. Under successive duopoly a key assumption is to permit retailers to defect to a rival manufacturer under both structures: manufacturers then distribute their products through the same retailer (Gabrielsen and Sørgard, 1999). We would not want to exogenously restrict the brand policy choice by manufacturers and thus the number of (possible) distribution systems. This leads us to contemplate the possibility of defection from the viewpoint of manufacturers. Moreover, to enrich the analysis on distribution systems we consider asymmetric product demands to grasp the effect of products with different equilibrium margins on the brand policy choice. Hence our analysis complements and generalizes earlier work on this area in that both manufacturers, of asymmetric and differentiated brands, choose how many retailers to employ, and that both retailers are allowed to carry both products. Consequently, the introduction of intra-brand competition becomes a strategic decision for manufacturers and may co-exist with inter-brand competition.[1]

The model we propose assumes two differentiated manufacturers which are asymmetric because products are differently valued by consumers, and two potential identical retailers who play a non-cooperative multi-stage game. Decisions at each stage are taken simultaneously and independently. In the first stage, manufacturers choose whether to employ retailer one, retailer two, both or none of them. That is, each manufacturer chooses its brand policy and both their choices will result in a particular distribution system. In the second stage, and given the inherited outcome of the first stage, manufacturers decide on transfer prices. Finally, and knowing transfer prices, retailers choose quantities. Thus, we develop a model where the existence of both inter and intra-brand competition is endogenously obtained.

Our analysis shows, in concordance with earlier work in the literature, that the well-known exclusive dealing structure arises when there is little product differentiation and low brand asymmetry. However, other equilibrium distribution systems emerge which depend on manufacturers' opportunity cost to introduce intra-brand competition, once there exists inter-brand competition. In particular, the manufacturer with the most profitable brand will employ two retailers, while the rival employs one, for large enough brand asymmetry and product differentiation. Under these conditions, that manufacturer finds it advantageous to withdraw from exclusive trading, despite that its perceived demand becomes more elastic. Furthermore, if the degree of product differentiation is large and brand asymmetry is either low or intermediate, then the manufacturer carrying the least profitable brand will also employ two retailers. We end up with a very much competitive situation where inter-brand competition coexists with intra-brand competition in both brands.[2]

[1]Rey-Stiglitz (1995) assume that manufacturers can hire several retailers in a perfect competition setting and compare this situation with one where each retailer is granted an exclusive territory. They endogenously obtain the separated structure assumed in Bonanno and Vickers (1988). Also note that Besanko and Perry (1994) allow spatially differentiated retailers to carry two brands under non-exclusive dealing. Yet the number of retailers is endogenously determined by free entry. If several exclusive dealers are hired then each manufacturer's retailers will be located alternately and hence there is no direct intra-brand competition.

[2]One way of analyzing market power both on buyers and sellers is to consider the incentives to mutual agreements by each manufacturer-retailer pairing (see Chang, 1992, and Dobson and Waterson, 1997). In contrast with us, each pair can agree on an exclusive trading contract which may encompass either distribution only or also cover purchasing. Regardless of one or another type of cooperation, Dobson and Waterson (1997) show that, with high levels of product and retailer differentiation, firms engage in non-exclusive trading contracts. For homogeneous

Finally, we compare the well-known common and exclusive distribution systems. Such a comparison follows naturally in our model from assuming away intra-brand competition. It is shown that, when products are differentiated, there is brand asymmetry and a manufacturer can threat to defect to the retailer chosen by the rival, exclusive dealing is not always the equilibrium. Common distribution is never. The remainder of the paper is organized as follows. The next Section presents the model. Several sub-sections offer the subgame perfect equilibria of the game. Then common and exclusive distribution systems are compared. Some brief concluding remarks close the paper.

2 The Model

We set up a three-stage non-cooperative game to study the equilibrium distribution structure that will arise by the strategic decisions of two manufacturers, M_i, $i = 1$, 2, and two retailers, R_k, $k = 1, 2$. The two manufacturers produce their own branded good under constant returns to scale and incur a common unit cost c. The retailers are supplied by the manufacturers at a constant unit price, the transfer price. Let w_i denote the transfer price set by manufacturer i. We assume that retailers are not differentiated in the sense that consumers get for brand i the same utility no matter which retailer k is selling the brand i to them.

In the first stage of the game, each manufacturer i chooses simultaneously and independently its brand policy: one element s_i from the set $S_i = \{0, 1, 2, 12\}$ where $s_i = 0$ denotes not to deal, $s_i = 1$ denotes that manufacturer i employs R_1 to distribute his brand, likewise for $s_i = 2$, and finally $s_i = 12$ means that manufacturer i will employ both retailers. Any pair of brand policies, (s_1, s_2), defines a *distribution system*. Although retailers are not differentiated, it is relevant which one of them is chosen. This allows us to distinguish distribution systems where a common retailer is employed from those where a different retailer is employed by each of the manufacturers. Then, sixteen different distribution systems may result from the manufacturers' strategic choice of retailers. In the second stage and having observed the outcome of the first stage, manufacturers choose simultaneously and independently the transfer prices to retailers. Finally, and given the inherited outcome of the previous two stages, each active retailer k selects the quantity for each branded good they are willing to market, denoting by q_{ik} the quantity of brand i that retailer k sells to consumers. Let Q_i stand for the total amount of brand i produced which is $Q_i = q_{i1} + q_{i2}$ when both retailers distribute it. The corresponding element(s) of that sum are set to zero, according to the manufacturers' choice of brand policy. As well as paying the transfers, the retailers incur common retailing costs at constant per unit level r which,

products, Chang (1992) finds that each manufacturer only supplies one retailer.

for the sake of the exposition and without loss of generality, are assumed to be zero. Also, it is assumed that neither manufacturers nor retailers can enforce a given distribution system by including clauses in the contract. In other words, the equilibrium distribution system is the result from the strategic interaction between manufacturers and is such that induces an equilibrium in the products market.

The retailers face a continuum of consumers of the same type. The representative consumer maximizes $U(Q_1, Q_2, y)$ subject to the budget constraint $I = y + p_1Q_1 + p_2Q_2$, where I is the income, y is the quantity of the numeraire commodity consumed and Q_i, p_i, $i = 1, 2$, are the quantity of the brand produced by manufacturer i and its market price, respectively. The function U is assumed to be separable, linear in the numeraire commodity and quadratic and strictly concave in the differentiated good: $U = y+a_1Q_1+a_2Q_2-[b(Q_1^2+Q_2^2)+2dQ_1Q_2]/2$, where $a_i,\ i = 1, 2,\ b$ and d are positive, $b > d$ and $a_ib - a_jd > 0$ for $i \neq j$. This utility function gives rise to a linear demand schedule, where inverse demands are given by,

$$\begin{aligned} p_1 &= a_1 - bQ_1 - dQ_2 \\ p_2 &= a_2 - bQ_2 - dQ_1 \end{aligned} \tag{1}$$

We assume, without loss of generality, that $a_1 \geq a_2$ meaning that the highest price (when quantities are set to zero) consumers are willing to pay for the good produced by M_1 is greater than or equal to that produced by M_2. Also, since $d > 0$ and $b > d$, own effects on prices are greater than cross effects. Note that the distance between b and d is measuring the degree of inter-brand rivalry, that is, how similar the brands are perceived by consumers. Then, brands 1 and 2 are imperfect substitutes and when d approaches b brands become closer substitutes, this meaning that inter-brand rivalry increases. Intra-brand rivalry, that is how similar the retailers' services are perceived by consumers to be when selling the same brand, is maximal. They are perfect substitutes: retailers are not differentiated.

As already mentioned, we look for the subgame perfect equilibria of this three-stage game. As usual, by backward induction, we begin by computing the third-stage Nash equilibrium quantities for each possible distribution scheme inherited from the second stage. Then, rolling back the game, we compute the subgame perfect Nash equilibrium choice of transfer prices by manufacturers. Finally, at the first stage, we find the subgame perfect Nash equilibrium choice of brand policies by manufacturers.

2.1 The retailers' decisions on quantities

Noting that in the first stage of the game, each manufacturer i decides simultaneously and independently the strategy $s_i \in S_i = \{0, 1, 2, 12\}$ there are 16 possible distribution systems. However, given the symmetry between retailers, just 10 different distribution system need to be analyzed. Remind that we denote each distribution system by the pair of strategies (s_1, s_2). In this subsection, we compute the third stage equilibrium quantities and profits for each distribution system. Given the first stage manufacturers brand policies s_1 and s_2, denote by $R_k(s_1, s_2)$ and $q_k(s_1, s_2)$, $k = 1, 2$ the retailer $k's$ *equilibrium* profits and total quantity, respectively.

For the sake of the exposition we present here the case where both retailers sell both brands, i.e. the distribution system $(12, 12)$. Retailers maximize,

$$\begin{aligned} \max_{q_{11},\, q_{21}} R_1(q_{11}, q_{12}, q_{21}, q_{22}) &= (a_1 - b(q_{11} + q_{12}) - d(q_{21} + q_{22}) - w_1)q_{11} \qquad (2) \\ &\quad +(a_2 - b(q_{21} + q_{22}) - d(q_{11} + q_{12}) - w_2)q_{21} \\ \max_{q_{12},\, q_{22}} R_2(q_{11}, q_{12}, q_{21}, q_{22}) &= (a_1 - b(q_{11} + q_{12}) - d(q_{21} + q_{22}) - w_1)q_{12} \qquad (3) \\ &\quad +(a_2 - b(q_{21} + q_{22}) - d(q_{11} + q_{12}) - w_2)q_{22} \end{aligned}$$

The equilibrium outputs come out as the solution to the four-equation system of first order conditions to obtain,

$$q_1(12, 12) = q_2(12, 12) \equiv q_{11}(12, 12) + q_{21}(12, 12) = q_{12}(12, 12) + q_{22}(12, 12) =$$

$$\begin{cases} 0 + \frac{(a_2 - w_2)}{3b} & \text{for } 0 < \frac{a_1 - w_1}{a_2 - w_2} < \frac{d}{b} \\ \frac{b(a_1 - w_1) - d(a_2 - w_2)}{3(b^2 - d^2)} + \frac{b(a_2 - w_2) - d(a_1 - w_1)}{3(b^2 - d^2)} & \text{for } \frac{d}{b} \leq \frac{a_1 - w_1}{a_2 - w_2} \leq \frac{b}{d} \\ \frac{(a_1 - w_1)}{3b} + 0 & \text{for } \frac{b}{d} < \frac{a_1 - w_1}{a_2 - w_2} \end{cases} \qquad (4)$$

Retailers' prices and profits are,

$$p_1(12, 12) = \begin{cases} \emptyset & \text{for } 0 < \frac{a_1 - w_1}{a_2 - w_2} < \frac{d}{b} \\ \frac{a_1 + 2w_1}{3} & \text{for } \frac{d}{b} \leq \frac{a_1 - w_1}{a_2 - w_2} \end{cases} \qquad (5)$$

$$p_2(12, 12) = \begin{cases} \frac{a_2 + 2w_2}{3} & \text{for } 0 < \frac{a_1 - w_1}{a_2 - w_2} \leq \frac{b}{d} \\ \emptyset & \text{for } \frac{b}{d} < \frac{a_1 - w_1}{a_2 - w_2} \end{cases} \qquad (6)$$

$$R_1(12, 12) = R_2(12, 12) = \begin{cases} \frac{(a_2 - w_2)^2}{9b} & \text{for } 0 < \frac{a_1 - w_1}{a_2 - w_2} < \frac{d}{b} \\ \frac{b(a_1 - w_1)^2 + b(a_2 - w_2)^2 - 2d(a_1 - w_1)(a_2 - w_2)}{9(b^2 - d^2)} & \text{for } \frac{d}{b} \leq \frac{a_1 - w_1}{a_2 - w_2} \leq \frac{b}{d} \\ \frac{(a_1 - w_1)^2}{9b} & \text{for } \frac{b}{d} < \frac{a_1 - w_1}{a_2 - w_2} \end{cases} \qquad (7)$$

Where $\emptyset$ stands for undefined. The equilibrium quantities are a function of earlier choices, w_1 and w_2, and the brand policies. As is clear from above the quantities corresponding to a particular distribution system may become negative depending on the size of $(a_1 - w_1)/(a_2 - w_2)$, *the relative per unit profitability of brands for retailers*, and thus corner solutions must be taken into account. For example, when $\frac{a_1 - w_1}{a_2 - w_2} < \frac{d}{b}$ both $q_{11}(12, 12)$ and $q_{12}(12, 12)$ become negative and, therefore, retailers do not sell any amount of brand one. They will sell the amount of brand two corresponding to a situation where there is only intra-brand competition in this brand. Table 1 reports just the interior solutions for the third stage equilibrium quantities and retailers' gross profits. Remark that whenever two distribution systems are reported, the equilibrium quantities and gross profits follow by a simple exchange of subindexes. The detailed computations for the remainder of the distribution systems are relegated to Appendix A.

[Insert Table 1 about here]

2.2 The manufacturers' decision on transfer prices

For any given brand policies s_1 and s_2 chosen in the first stage, manufacturers decide independently and simultaneously the pair of transfer prices ($w_1(s_1, s_2)$, $w_2(s_1, s_2)$) that maximize their profits. Thus we have to calculate the equilibrium transfer prices for the ten different subgames. For the sake of the exposition we present here the subgame perfect equilibrium of the second stage of the game corresponding to the first stage brand policies $s_1 = s_2 = 12$.

By subgame perfection the manufacturers' profit function must reflect the retailers' choice of quantities, which depends on the range of $\bar{w} = \frac{a_1 - w_1}{a_2 - w_2}$. For a given w_2, manufacturer one maximizes

$$M_1(w_1, w_2) = \begin{cases} 0 & \text{for } w_1 \text{ such that } \bar{w} \in (0, \frac{d}{b}) \\ \frac{2(w_1 - c)(b(a_1 - w_1) - d(a_2 - w_2))}{3(b^2 - d^2)} & \text{for } w_1 \text{ such that } \bar{w} \in [\frac{d}{b}, \frac{b}{d}] \\ \frac{2}{3b}(w_1 - c)(a_1 - w_1) & \text{for } w_1 \text{ such that } \bar{w} \in (\frac{b}{d}, \infty) \end{cases} \quad (8)$$

Similarly, for a given w_1, manufacturer two maximizes

$$M_2(w_1, w_2) = \begin{cases} 0 & \text{for } w_2 \text{ such that } \bar{w} \in (\frac{b}{d}, \infty) \\ \frac{2(w_2 - c)(b(a_2 - w_2) - d(a_1 - w_1))}{3(b^2 - d^2)} & \text{for } w_2 \text{ such that } \bar{w} \in [\frac{d}{b}, \frac{b}{d}] \\ \frac{2}{3b}(w_2 - c)(a_2 - w_2) & \text{for } w_2 \text{ such that } \bar{w} \in (0, \frac{d}{b}) \end{cases} \quad (9)$$

where c is the common per unit cost of production with $w_i \geq c$. An inspection of the profit functions above shows that, for a given transfer price of the rival, there exist two local maxima for each manufacturer. The global maximum depends on the ratio $C = \frac{a_1 - c}{a_2 - c}$ which can be interpreted as *the relative per unit profitability* *of brands for the manufacturers* and it is greater than or equal to one. When $C \in [1, \frac{2b^2-d^2}{db}]$ the equilibrium pair $(w_1(12,12), w_2(12,12)) = (\hat{w}_1, \hat{w}_2)$, the interior solution, implies that $\bar{w} \in [\frac{d}{b}, \frac{b}{d}]$. As C increases manufacturer one finds it profitable to deviate from the above equilibrium. The new equilibrium will imply either manufacturer one setting a limit transfer price $\tilde{w}_1(\hat{w}_2)$ or the monopoly transfer price w_1^m. Both these cases involve that $\bar{w} \in (\frac{b}{d}, \infty)$. Notice that there is no analogous reasoning for manufacturer two given brand asymmetry. Marginal cost c is the lowest transfer price that can be set by manufacturer two. Knowing this, manufacturer one sets a transfer price such that retailers will not demand any amount of the rival brand. This is part b) of the result below which is similar to a standard result in auction theory, where the winning manufacturer's bid is dictated by the characteristics of the second bidder. Further, when brand asymmetry is sufficiently marked, manufacturer one may behave as an upstream monopolist on setting the equilibrium transfer price (part c). The next Result summarizes the above discussion.

Result 1 *The subgame perfect equilibrium pair of transfer prices for the (12,12) subgame is equal to either:*
a) (interior solution) $w_i(12,12) = \hat{w}_i = \frac{(2b^2-d^2)a_i - bda_j + b(2b+d)c}{4b^2-d^2}$ $i,j = 1,2 \quad i \neq j$ *when* $C \in [1, \frac{2b^2-d^2}{bd}]$, *or*
b) (exclusionary behaviour by M_1*)* $w_1(12,12) = \tilde{w}_1 = a_1 - \frac{b(a_2-c)}{d}$, $w_2(12,12) = c$ *when* $C \in (\frac{2b^2-d^2}{bd}, \frac{2b}{d}]$, *or*
c) (natural foreclosure by M_1*)* $w_1(12,12) = w_1^m = \frac{a_1+c}{2}$, $w_2(12,12) = c$ *when* $C > \frac{2b}{d}$.

Proof. : See Appendix B.

A similar analysis is undertaken for each of the remaining first stage brand policies chosen by manufacturers. We wish to focus on those situations where there is neither exclusionary behaviour nor natural foreclosure by manufacturer one. Table 2 displays the interior equilibrium transfer prices for each of the brand policies. Therefore, we assume that brand asymmetry is low enough to ensure that the *whole* set of brand policies can be chosen with positive payoffs for manufacturer two, that is $C \in [1, \frac{2b^2-d^2}{bd}]$. This follows from taking the restriction on C which ensures that the smallest numerator in $w_2(s_1, s_2) - c$ is positive, i.e. $w_2(12,12) - c > 0$. Otherwise, although M_2 would choose to employ retailers, the rival manufacturer would find it profitable to set a transfer price such that retailers indeed would not sell a positive amount of brand two. Table 3 shows the first-stage payoff matrix with the interior equilibrium manufacturers' payoffs, $M_i(s_1, s_2)$, where the top row in each cell corresponds to manufacturer one.

[Insert Tables 2 and 3 about here]

It is interesting to note the following:

$$\begin{aligned} w_1(1,0) &= w_1(2,0) = w_1(12,0) > w_1(2,1) = w_1(1,2) > w_1(12,1) = \\ w_1(12,2) &> w_1(1,12) = w_1(2,12) > w_1(1,1) = w_1(2,2) = w_1(12,12) \end{aligned}$$

$$\begin{aligned} w_2(0,1) &= w_2(0,2) = w_2(0,12) > w_2(2,1) = w_2(1,2) > w_2(1,12) = \\ w_2(2,12) &> w_2(12,1) = w_2(12,2) > w_2(1,1) = w_2(2,2) = w_2(12,12) \end{aligned}$$

Each manufacturer sets the highest equilibrium transfer price when it is an upstream monopoly regardless it hires either one or two retailers. The subgame perfect equilibrium transfer price decreases first with the appearance of inter-brand competition. Then, and given that there is inter-brand competition, it decreases first with the introduction of only own intra-brand competition and later with the introduction of only rival's intra-brand competition. The equilibrium transfer price is lowest when both manufacturers introduce intra-brand competition. Naturally, the same ordering applies for manufacturers' margins. Since the equilibrium own elasticity of the demand perceived by the manufacturers, in absolute terms, can be written as $|\epsilon_{q_i w_i}(s_1, s_2)| = \frac{w_i(s_1,s_2)}{(w_i(s_1,s_2)-c)}$, it is straightforward to conclude that the lower the equilibrium transfer price the higher the elasticity. As shall shortly be seen, and under some conditions, manufacturers will choose the brand policy involving the highest competition intensity.

Finally note that, as is well known, delegating sales to independent retailers generates an inefficiency when linear transfer prices are used. Double marginalization ($p > w > c$) pushes retailers to demand lower amounts of the intermediate good - in our case to sell less output. Further computations indicate that such a distortion is minimized when a manufacturer each hires two retailers. Given that total surplus is increasing with output, total surplus is highest when there is intra-brand competition in both brands.

2.3 The manufacturers' choice of brand policies

In the first stage of the game, each manufacturer i chooses simultaneously and independently the strategy $s_i \in S_i$. We introduce the following useful terminology to ease comparison with earlier work on distribution systems. In particular, the term *"distribution"* has to do with the (number of) retailers employed by manufacturers, whereas the word *"purchasing"* is used to refer to the (number of) brands that retailers wish to sell. The possible combination of strategies gives rise to the following distribution schemes:

- *Non-exclusive distribution and exclusive purchasing* (Figure 1a): both retailers distribute one of the manufacturer's branded product whereas the rival manufacturer is not present in the market. It refers to the above mentioned brand policies $(12, 0)$ and $(0, 12)$. This distribution system gives rise to a homogeneous product duopoly in the downstream market and then, only intra-brand rivalry appears in the market.

- *Duopoly exclusive distribution and purchasing* (Figure 1b): each retailer purchases only one brand and each manufacturer uses just one retailer. It refers to the brand policies $(1, 2)$ and $(2, 1)$. This distribution system gives rise to a differentiated duopoly in the downstream market, there is only inter-brand rivalry, and it has been usually called exclusive dealing by papers in the literature.

- *Non-exclusive distribution and purchasing* (Figure 1c)*:* the two retailers purchase both brands. It relates to the distribution system $(12, 12)$. Therefore, both retailers are multi-product dealers. Furthermore, each retailer faces for each brand inter-brand rivalry by both the other brand he is selling and the one sold by the competing retailer. Also, he faces intra-brand rivalry by the same brand sold by his competitor dealer.

- *Exclusive distribution and non-exclusive purchasing* (Figure 2a): a single retailer distributes both manufacturers' brands, as in the distribution systems $(1, 1)$ and $(2, 2)$. The retailer behaves as a multi-product monopoly and thus there is only inter-brand rivalry. This brand policy has been usually referred to as common agency or common distribution system by related papers in the literature.

- *Monopoly exclusive distribution and purchasing* (Figure 2b): it corresponds with schemes $(1, 0)$, $(2, 0)$ and $(0, 1), (0, 2)$ where only one retailer and one manufacturer are active in the market. Note that this distribution system is what has been termed as a successive monopoly, either in brand 1 or in brand 2.

- *Mixed schemes* (Figure 2c): one retailer distributes both brands whereas the other sells only one of the brands. Therefore, one of the retailers is a non-exclusive dealer while the other is an exclusive dealer. It embodies the $(1, 12)$, $(12, 1)$, $(2, 12)$ and $(12, 2)$ distribution systems.

[Insert Figures 1a, 1b, 1c and 2a, 2b, 2c about here]

The next Proposition is the central result of the paper. Our non-cooperative three-stage game unveils a number of equilibrium distribution systems which do not necessarily entail exclusivity relationships. Product differentiation and brand asymmetry provide incentives for manufacturers to introduce intra-brand competition.

Proposition 1 *The equilibrium distribution system is either:*
i) unique and it is:
a) Non-exclusive distribution and purchasing whenever either
a.1) $\frac{d}{b} \in (0,\ 0.682]$ *and* $C \in [1,\ r_d)$ *or*
a.2) $\frac{d}{b} \in (0.682,\ 0.805]$ *and* $C \in [r_C,\ r_d)$
b) The mixed schemes (12,1) and (12,2) whenever either
b.1) $\frac{d}{b} \in (0,\ 0.805]$ *and* $C \in [r_d,\ \frac{2b^2-d^2}{bd}]$ *or*
b.2) $\frac{d}{b} \in (0.805,\ 0.907]$ *and* $C \in [r_C,\ \frac{2b^2-d^2}{bd}]$
c) Duopoly exclusive distribution and purchasing whenever either
c.1) $\frac{d}{b} \in (0.805,\ 0.907]$ *and* $C \in [r_d,\ r_C)$ *or*
c.2) $\frac{d}{b} \in (0.907,\ 0.909]$ *and* $C \in [r_d,\ \frac{2b^2-d^2}{bd}]$ *or*
c.3) $\frac{d}{b} \in (0.909,\ 1]$ *and* $C \in [1,\ \frac{2b^2-d^2}{bd}]$
ii) or multiple: Non-exclusive distribution and purchasing and Duopoly exclusive distribution and purchasing whenever either $\frac{d}{b} \in (0.682,\ 0.805]$ *and* $C \in [1, r_C)$ *or* $\frac{d}{b} \in (0.805,\ 0.909]$ *and* $C \in [1, r_d)$.

Proof: See Appendix B.

Corollary 1 *The standard common dealership (exclusive distribution and non-exclusive purchasing in our terminology) is never chosen by manufacturers.*

Figure 3 below illustrates the equilibria presented in the Proposition and may be a useful tool to see the intuition of the results. It displays the first-stage equilibria as a function both of the degree of product differentiation ($\frac{d}{b}$) and the relative per unit profitability ratio for manufacturers (C). A variety of equilibria are obtained depending on manufacturers' strategic incentive to introduce intra-brand competition once there exists inter-brand competition. Consider the duopoly exclusive distribution and purchasing system. Now, the manufacturer with the better brand compares the gain in market share and profits from using two retailers with the loss associated with intra-brand competition. That is, it evaluates the difference $M_1(12,1) - M_1(2,1)$, the opportunity cost associated with the introduction of intra-brand competition. This difference gives rise to r_C in Figure 3. Manufacturer one will employ two retailers when, for a given $\frac{d}{b}$, the degree of brand asymmetry lies above r_C . Manufacturer two's strategic decision is linked with the presence of intra-brand competition in the rival brand. Analytically, it has to do with the difference $M_2(12, 12) - M_2(12, 1)$, the opportunity cost of introducing intra-brand competition for manufacturer two. Such a difference is depicted r_d in Figure 3, and manufacturer two will employ two retailers when, for a given $\frac{d}{b}$, the degree of brand asymmetry lies below r_d. The reader can find the precise expressions of r_d and r_C in Appendix B.

Thus, a structure with inter and intra-brand competition in both brands is obtained when both these opportunity costs are positive. When a manufacturer introduces intra-brand competition there are two effects. A competition effect which implies a reduction in its own transfer price and margin, and an output expansion effect. The latter effect dominates the former despite the fact that its perceived demand becomes more elastic with its brand policy choice. We will end up with the duopoly non-exclusive distribution and purchasing structure when the degree of product differentiation is large and brand asymmetry is low (i.a.1) or sufficiently large and intermediate, respectively (i.a.2).[3]

Mixed distribution systems show up when only manufacturer one's opportunity cost of introducing intra-brand competition is positive. That is, for manufacturer two the competition effect either dominates the output expansion effect or reinforces it when brands are very asymmetric. Under these conditions, the manufacturer with the most profitable brand decides to employ two retailers rather than one, while manufacturer two sticks to one retailer. This occurs for sufficiently large brand asymmetry and a sufficient degree of product differentiation.

If the opportunity cost of neither manufacturer is positive, then there are no incentives to introduce intra-brand competition and hence each manufacturer decides to use one separate retailer. Our analysis has therefore shown that the manufacturers' equilibrium brand policy never entails the choice of just one common retailer - the standard common agency in the received literature. This is because of the possibility of defection which becomes crucial when there is brand asymmetry. The manufacturer with the most profitable brand prefers a common retailer for sufficient product differentiation and brand asymmetry. However, the manufacturer with the least profitable brand has an incentive not to match the rival's choice and is better off by using a different retailer. Thus, when the degree of product differentiation is low and the profitability of brands does not differ much, the well-known exclusive dealership arises as an equilibrium brand policy: in Figure 3 the area above r_d and below r_C.

[3]Dobson and Waterson (1997) show $(12, 12)$ to be an equilibrium for high levels of product and retailer differentiation. They adopt a cooperative approach, and assume price competition and two-part tariff contracts.

Exclusive dealing is more likely to arise the more homogeneous products are and the lower the brand asymmetry. Though in a much different setting, this qualitative result coincides with earlier findings in the literature - as in Lin (1990), O'Brien and Shaffer (1993), Rey and Stiglitz (1995), Chang (1992). Nevertheless, our analysis puts forward that there are other equilibrium distribution systems and it points out the relevance of brand asymmetry, the choice of the number of retailers and manufacturers' possibility of defection. As shown by Dobson and Waterson (1997), retailer differentiation is also an important feature in deriving distribution systems without exclusivity relationships.

Finally, multiple equilibria arise when only manufacturer two has an incentive to introduce intra-brand competition. If one of the manufacturers employs one retailer then the rival's best response is to employ one different retailer. This gives rise to either $(1,2)$ or $(2,1)$. On the other hand, if one of them employs two retailers then the rival matches that decision, thus giving rise to distribution system $(12,12)$. There is a multiplicity of equilibria in the intersection of the areas below r_C and r_d.

2.3.1 Common distribution versus exclusive dealing revisited

Two distribution systems have drawn the attention of researchers in the field, exclusive and common dealership. They combine the tradeoff between the internalization of competition and collusive retail pricing. Whether one system prevails upon the other depends, among other things, on the degree of product differentiation, the type of contract and on the nature of competition. These systems correspond, in our terminology, to the $(1,2),(2,1)$ and $(1,1),(2,2)$ systems, respectively. Here, the choice between these well-known structures can be presented as a particular case from the previous general model, i.e. from assuming away intra-brand competition. The next Proposition offers the results when strategies $s_1 = s_2 = 12$ are not allowed.

Proposition 2 *Consider that the* $manufacturers'$ *set of strategies is restricted to* $s_1 \in \bar{S}_1 = \{0,1,2\}$ *and* $s_2 \in \bar{S}_2 = \{0,1,2\}$*. Then exclusive dealing is the only* $Nash$ *equilibrium distribution system when:*
either $\frac{d}{b} \in (0,\ 0.708]$ *and* $C \in [1,\ r_A]$,
or $\frac{d}{b} \in (0.708,\ 1]$ *and* $C \in [1,\ \frac{2b^2-d^2}{bd}]$.
Otherwise there is no Nash equilibrium brand policy in pure strategies, i.e. for $\frac{d}{b} \in (0,\ 0.708]$ *and* $C \in [\ r_A, \frac{2b^2-d^2}{bd}]$.

Proof: See Appendix B.

Typically, exclusive and common dealership have been treated asymmetrically in that, the two-retailer assumption has only been considered under exclusive dealing and not under common dealership. Therefore, when one common retailer is employed, a manufacturer cannot defect to the other retailer if it pays him to do so. Our result above crucially depends on the two-retailer assumption for both structures which permits to consider such a defection.[4] The intuition behind Proposition 2 can be cast in terms of the opportunity cost for manufacturer one of using separate retailers rather than the same retailer. Formally, the difference $M_1(2,1) - M_1(1,1)$. Remark that manufacturer two is always better off with exclusive retailers since $M_2(1,2) - M_2(1,1) > 0$ given the upper bound $\frac{2b^2-d^2}{bd}$ on the ratio C.

Rather naturally, in the absence of brand asymmetry and without the possibility of defection, both manufacturers earn higher profits using exclusive dealing regardless of the degree of product differentiation. This is precisely the result in Lin (1990) and O'Brien and Shaffer (1993). Nevertheless, we show that when products are differentiated, brands are not equally profitable and a manufacturer can threat to defect to the retailer chosen by the rival, exclusive dealing is not always the equilibrium.

When the degree of product differentiation is important the trade-off between separate exclusive retailers and a common retailer depends on how asymmetric brands are. If brand asymmetry is low, then manufacturer one is better off with exclusive dealing, i.e. $M_1(2,1) - M_1(1,1) > 0$. However, if brand asymmetry is sufficiently large, manufacturer one prefers a common retailer, i.e. $M_1(2,1) - M_1(1,1) < 0$. In the former case, the dampening-of-competition effect is at work. In the latter case, note that a common retailer will sell a greater amount of manufacturer one's brand relative to the rival's due to the internalization of inter-brand competition. As brands are more asymmetric, and provided that the perceived demands of the manufacturers are rendered more elastic under common dealership, manufacturer one's best response is to choose the same retailer as manufacturer two (that is, matches him). On the other hand, manufacturer two wishes to defect to another retailer (that is, does not want to match him). As a result there is no Nash equilibrium in pure strategies much as in the matching pennies game.

3 Concluding Remarks

We have proposed a non-cooperative game in order to emphasize the strategic rationale in shaping the distribution systems. Compared with the received lit-

[4]Gabrielsen and Sorgard (1999) is an exception of symmetric treatment. They let retailers defect to a rival manufacturer, that is, (full) foreclosure is allowed under both types of distribution systems. In contrast with earlier literature, these authors show that manufacturers prefer common rather than exclusive dealership.

erature, we have moved one step further by letting manufacturers select which retailer(s) will market their respective brand. This, together with retailers possibly being multi-product dealers, enlarges the set of distribution systems. Whether manufacturers employ two retailers rather than one reflects the tradeoff between two conflicting effects, there is an output increase but more competition is established. High levels of product differentiation and not too large brand asymmetry are enough to incentive manufacturers introduce intra-brand competition. This is a very competitive outcome yielding the highest total surplus. However, the well-known exclusive dealing system shows up for little product differentiation and low brand asymmetry.

It is worth emphasizing that, if any type of exclusivity relationship ever occurs, it is the equilibrium outcome of a non-cooperative game in which neither manufacturers nor retailers may impose any vertical clauses. The mere fact that these outcomes be indistinguishable from those derived from a cooperative approach or when vertical clauses are employed abound on competition authorities' worries towards the effects of vertical agreements and restraints. On the basis of our findings, we would stand for a case by case approach.

We have restricted attention to linear contracts instead of two-part tariff contracts since we wish to focus on the strategic effect of the introduction of intra-brand competition in a context of multi-product dealership. When considering two-part tariff contracts along with intra-brand competition, it happens that each manufacturer would have two instruments to control for competition intensity (the transfer price and whether employing one or two retailers), and one instrument for retailer profit extraction (the up-front fixed fee). With two instruments for the same purpose, one of them turns out to be redundant (see e.g. Rysman, 2001). The assumption of linear contracts is a way to avoid such a redundancy while not distorting the understanding of the strategic reasons why manufacturers introduce intra-brand competition.[5]

Following most of the literature, we have conceded manufacturers a leading position in their trading relationships with retailers. However, the rise in retailer concentration together with the scarcity of shelf space advises one to consider more retailer decision power. Thus a natural question to ask is whether the equilibrium distribution systems here obtained coincide with those resulting when it

[5]Some related work in the literature considers the choice of the number of downstream firms by upstream firms with assuming two-part tariff contracts and no multi-product dealership. The conclusions are that, regardless that products are homogeneous (Rysman, 2001) or differentiated (Saggi and Vettas, 1999), neither upstream firm chooses to introduce intra-brand competition, i.e. each upstream firm deals with one separate downstream firm. The intuition, as already indicated, is that the two-part tariff contract and the choosing of the number of downstream firms are interchangeable instruments to maximize upstream firms payoffs.

is retailers who decide how many brands to sell. It has been shown by Moner-Colonques et al. (2001) that retailers always prefer to carry both brands. Therefore, we conclude that a) there is a conflict of interests between manufacturers and retailers unless brands are sufficiently differentiated and brands are rather equally profitable, and that b) there are equilibrium distribution systems other than those contemplated by the existing literature. The results here obtained encourage us to pursue future research along these lines.

References

[1] **Bernheim, B.D. and M.D. Whinston (1998)**, "Exclusive Dealing", *Journal of Political Economy*, 106, 64-103.

[2] **Besanko, D. and M.K. Perry (1994)**, "Exclusive Dealing in a Spatial Model of Retail Competition", *International Journal of Industrial Organization*, 12, 297-329.

[3] **Bonanno, G. and J. Vickers (1988)**, "Vertical Separation", *Journal of Industrial Economics*, 36, 257-265.

[4] **Chang, M-H. (1992)**, "Exclusive Dealing Contracts in a Successive Duopoly with Side Payments", *Southern Economic Journal*, October, 180-193.

[5] **Dobson, P.W. and M. Waterson (1997)**, "Exclusive Trading Contracts in Successive Differentiated Duopoly", *Southern Economic Journal*, 63 (2), 361-377.

[6] **Gabrielsen, T.S. and L. Sørgard (1999)**, "Exclusive versus Common Dealership", *Southern Economic Journal*, 66, 353-366.

[7] **Lin, Y.J. (1990)**, "The Dampening-of-Competition Effect of Exclusive Dealing", *Journal of Industrial Economics*, 39, 209-223.

[8] **Moner-Colonques, R., J.J. Sempere-Monerris and A. Urbano (2001)**, "The Retailers' Choice of Distribution Systems", I.V.I.E. WP-AD 2001-01.

[9] **O'Brien, D. and G. Schaffer (1993)**, "On the Dampening-of-Competition Effects of Exclusive Dealing", *Journal of Industrial Economics*, 41, 215-221.

[10] **O'Brien, D. and G. Schaffer (1997)**, "Nonlinear Supply Contracts, Exclusive Dealing, and Equilibrium Market Foreclosure", *Journal of Economics and Management Strategy*, 6, 755-785.

[11] **Rey, P. and J. Stiglitz (1995)**, "The Role of Exclusive Territories in Producers' Competition", *Rand Journal of Economics*, 26, 431-51.

[12] **Rysman, M. (2001)**, "How Many Franchises in a Market?", *International Journal of Industrial Organization*, 19, 519-542.

[13] **Saggi, K. and N. Vettas (1999)**, "On Intrabrand and Interbrand Competition: The Strategic Role of Fees and Royalties", C.E.P.R. Discussion Paper 2110.

1 Appendix A: the third stage equilibrium outcomes.

a) **The** $(1,0)$, $(2,0)$, $(0,1)$ **and** $(0,2)$ **distribution systems.**

In any of these cases there is only one brand and one retailer present in the market. The distribution system $(i,0)$ means that M_1 is distributing its brand through R_i, $i = 1,2$, while M_2 is out of the market; similarly for $(0,i)$. We analyze for example the subgame $(1,0)$. R_1 chooses q_{11} to maximize,

$$R_1(q_{11}) = (a_1 - bq_{11} - w_1)q_{11}$$

the equilibrium output, price and retailer's profits are

$$q_1(1,0) \equiv q_{11}(1,0) = \tfrac{a_1-w_1}{2b} \quad p_1(1,0) = \tfrac{a_1+w_1}{2} \quad R_1(1,0) = \tfrac{(a_1-w_1)^2}{4b}$$

while for R_2 it is obvious that it distributes zero and gets zero profits. The restriction on the parameter space to get interior (nonnegative) output equilibria for subgames $(1,0)$, $(2,0)$, $(0,1)$ and $(0,2)$ is that $(a_i - w_i) > 0$ for $i = 1,\ 2$.

b) **The** $(1,1)$ **and** $(2,2)$ **distribution systems.**

Under these distribution systems, both manufacturers have chosen the same retailer which is a multi-product monopolist retailer. Take for example the case $(1,1)$, R_1 chooses q_{11} q_{21} to maximize

$$R_1(q_{11}, q_{21}) = (a_1 - bq_{11} - dq_{21} - w_i)q_{11} + (a_2 - bq_{21} - dq_{11} - w_2)q_{21}$$

which results in the following equilibrium outcomes

$$q_1(1,1) = q_{11}(1,1) + q_{21}(1,1) =$$

$$= \begin{cases} 0 + \frac{(a_2-w_2)}{2b} & \text{for } 0 < \frac{a_1-w_1}{a_2-w_2} < \frac{d}{b} \\ \frac{b(a_1-w_1)-d(a_2-w_2)}{2(b^2-d^2)} + \frac{b(a_2-w_2)-d(a_1-w_1)}{2(b^2-d^2)} & \text{for } \frac{d}{b} \leq \frac{a_1-w_1}{a_2-w_2} \leq \frac{b}{d} \\ \frac{(a_1-w_1)}{2b} + 0 & \text{for } \frac{b}{d} < \frac{a_1-w_1}{a_2-w_2} \end{cases}$$

Retailers' prices and profits are,

$$p_1(1,1) = \tfrac{a_1+w_1}{2};\ p_2(1,1) = \tfrac{a_2+w_2}{2} \text{ for } \tfrac{a_1-w_1}{a_2-w_2} > 0$$

$$R_1(1,1) = \begin{cases} \frac{(a_2-w_2)^2}{4b} & \text{for } 0 < \frac{a_1-w_1}{a_2-w_2} < \frac{d}{b} \\ \frac{b(a_1-w_1)^2+b(a_2-w_2)^2-2d(a_1-w_1)(a_2-w_2)}{4(b^2-d^2)} & \text{for } \frac{d}{b} \leq \frac{a_1-w_1}{a_2-w_2} \leq \frac{b}{d} \\ \frac{(a_1-w_1)^2}{4b} & \text{for } \frac{b}{d} < \frac{a_1-w_1}{a_2-w_2} \end{cases}$$

In this case, to get interior equilibrium outputs, we need to restrict the relative per unit profitability ratio to the next interval

$$\frac{d}{b} \leq \frac{a_1 - w_1}{a_2 - w_2} \leq \frac{b}{d}$$

Note that whenever $0 < \frac{a_1-w_1}{a_2-w_2} < \frac{d}{b}$, the retailer does not sell brand one and sells the amount of brand 2 equal to that in under distribution system $(0,1)$. By the same token, for $\frac{a_1-w_1}{a_2-w_2} > \frac{b}{d}$ we have $q_{21}(1,1) = 0$ and the retailer sells an amount of brand 1 $q_{11}(1,1) = q_{11}(1,0)$.

c) **The** $(12,0)$ **and** $(0,12)$ **distribution systems.**

In both cases we have a homogenous duopoly in the downstream market. One of the manufacturers employs both retailers to distribute its brand while the other manufacturer's brand is not sold in the market. Take as an example the case of $(12,0)$. Retailers one and two maximize profits choosing q_{11}, and q_{12}, respectively.

$$
\begin{aligned}
R_1(q_{11},q_{12}) &= (a_1 - b(q_{11}+q_{12}) - w_1)q_{11} \\
R_2(q_{11},q_{12}) &= (a_1 - b(q_{12}+q_{11}) - w_1)q_{12}
\end{aligned}
$$

The equilibrium quantities are obtained by solving the two-equation system of first order conditions for q_{11} and q_{12}. These are:

$$
\begin{aligned}
q_1(12,0) &\equiv q_{11}(12,0) = \frac{a_1-w_1}{3b} \\
q_2(12,0) &\equiv q_{12}(12,0) = \frac{a_1-w_1}{3b} \\
p_1(12,0) &= \frac{a_1+2w_1}{3} \\
R_1(12,0) &= R_2(12,0) = \frac{(a_1-w_1)^2}{9b}
\end{aligned}
$$

with the same restriction as in the case presented in the first place.

d) **The** $(1,2)$ **and** $(2,1)$ **distribution systems.**

Here, each manufacturer uses one retailer and a different one from the retailer employed by the other manufacturer. Therefore, there is a differentiated duopoly in the downstream market. Consider the case $(1,2)$. Each retailer maximizes profits by choosing quantities q_{11} and q_{22}.

$$
\begin{aligned}
R_1(q_{11},q_{22}) &= (a_1 - bq_{11} - dq_{22} - w_1)q_{11} \\
R_2(q_{11},q_{22}) &= (a_2 - bq_{22} - dq_{11} - w_2)q_{22}
\end{aligned}
$$

We obtain the following equilibrium outcomes

$$
q_1(1,2) \equiv q_{11}(1,2) = \begin{cases} 0 & \text{for } 0 < \frac{a_1-w_1}{a_2-w_2} < \frac{d}{2b} \\ \frac{2b(a_1-w_1)-d(a_2-w_2)}{4b^2-d^2} & \text{for } \frac{d}{2b} \le \frac{a_1-w_1}{a_2-w_2} \le \frac{2b}{d} \\ \frac{(a_1-w_1)}{2b} & \text{for } \frac{2b}{d} < \frac{a_1-w_1}{a_2-w_2} \end{cases}
$$

$$
q_2(1,2) \equiv q_{22}(1,2) = \begin{cases} \frac{(a_2-w_2)}{2b} & \text{for } 0 < \frac{a_1-w_1}{a_2-w_2} < \frac{d}{2b} \\ \frac{2b(a_2-w_2)-d(a_1-w_1)}{4b^2-d^2} & \text{for } \frac{d}{2b} \le \frac{a_1-w_1}{a_2-w_2} \le \frac{2b}{d} \\ 0 & \text{for } \frac{2b}{d} < \frac{a_1-w_1}{a_2-w_2} \end{cases}
$$

Retailers' prices and profits are,

$$p_1(1,2) = \begin{cases} \emptyset & \text{for } 0 < \frac{a_1-w_1}{a_2-w_2} < \frac{d}{2b} \\ \frac{2b^2a_1+(2b^2-d^2)w_1-bd(a_2-w_2)}{4b^2-d^2} & \text{for } \frac{d}{2b} \leq \frac{a_1-w_1}{a_2-w_2} \leq \frac{2b}{d} \\ \frac{a_1+w_1}{2} & \text{for } \frac{2b}{d} < \frac{a_1-w_1}{a_2-w_2} \end{cases}$$

$$p_2(1,2) = \begin{cases} \frac{a_2+w_2}{2} & \text{for } 0 < \frac{a_1-w_1}{a_2-w_2} < \frac{d}{2b} \\ \frac{2b^2a_2+(2b^2-d^2)w_2-bd(a_1-w_1)}{4b^2-d^2} & \text{for } \frac{d}{2b} \leq \frac{a_1-w_1}{a_2-w_2} \leq \frac{2b}{d} \\ \emptyset & \text{for } \frac{2b}{d} < \frac{a_1-w_1}{a_2-w_2} \end{cases}$$

$$R_1(1,2) = \begin{cases} 0 & \text{for } 0 < \frac{a_1-w_1}{a_2-w_2} < \frac{d}{2b} \\ \frac{b[2b(a_1-w_1)-d(a_2-w_2)]^2}{(4b^2-d^2)^2} & \text{for } \frac{d}{2b} \leq \frac{a_1-w_1}{a_2-w_2} \leq \frac{2b}{d} \\ \frac{(a_1-w_1)^2}{4b} & \text{for } \frac{2b}{d} < \frac{a_1-w_1}{a_2-w_2} \end{cases}$$

$$R_2(1,2) = \begin{cases} \frac{(a_2-w_2)^2}{4b} & \text{for } 0 < \frac{a_1-w_1}{a_2-w_2} < \frac{d}{2b} \\ \frac{b[2b(a_2-w_2)-d(a_1-w_1)]^2}{(4b^2-d^2)^2} & \text{for } \frac{d}{2b} \leq \frac{a_1-w_1}{a_2-w_2} \leq \frac{2b}{d} \\ 0 & \text{for } \frac{2b}{d} < \frac{a_1-w_1}{a_2-w_2} \end{cases}$$

where the restriction in order to get interior equilibrium outputs becomes:

$$\frac{d}{2b} < \frac{a_1-w_1}{a_2-w_2} < \frac{2b}{d}$$

e) **The** $(12,1)$, $(12,2)$ **and** $(1,12)$, $(2,12)$ **distribution systems.**

These are asymmetric cases where one of the manufacturers employs both retailers, while the other employs just one. Then, we have a multi-product retailer facing a single-product one. Take as an example the distribution system $(12,1)$. Each retailer maximizes,

$$\begin{aligned} \max_{q_{11},q_{21}} R_1(q_{11},q_{12},q_{21}) &= (a_1 - b(q_{11}+q_{12}) - dq_{21} - w_1)q_{11} \\ &\quad +(a_2 - bq_{21} - d(q_{11}+q_{12}) - w_2)q_{21} \\ \max_{q_{12}} R_2(q_{11},q_{12},q_{21}) &= (a_1 - b(q_{11}+q_{12}) - dq_{21} - w_1)q_{12} \end{aligned}$$

The equilibrium outputs come as the solution to the three-equation system of first order conditions. for q_{11}, q_{12}, and q_{21}. We obtain

$q_1(12,1) \equiv q_{11}(12,1) + q_{21}(12,1) =$

$$\begin{cases} 0 + \frac{2b(a_2-w_2)-d(a_1-w_1)}{4b^2-d^2} & \text{for } 0 < \frac{a_1-w_1}{a_2-w_2} < \frac{3bd}{2b^2+d^2} \\ \frac{(2b^2+d^2)(a_1-w_1)-3bd(a_2-w_2)}{6b(b^2-d^2)} + \frac{b(a_2-w_2)-d(a_1-w_1)}{2(b^2-d^2)} & \text{for } \frac{3bd}{2b^2+d^2} \leq \frac{a_1-w_1}{a_2-w_2} \leq \frac{b}{d} \\ \frac{(a_1-w_1)}{3b} + 0 & \text{for } \frac{b}{d} < \frac{a_1-w_1}{a_2-w_2} \end{cases}$$

$$q_2(12,1) \equiv q_{12}(12,1) \begin{cases} \frac{2b(a_1-w_1)-d(a_2-w_2)}{4b^2-d^2} & \text{for } 0 < \frac{a_1-w_1}{a_2-w_2} < \frac{3bd}{2b^2+d^2} \\ \frac{(a_1-w_1)}{3b} & \text{for } \frac{3bd}{2b^2+d^2} \leq \frac{a_1-w_1}{a_2-w_2} \end{cases}$$

Retailers' prices and profits are,

$$p_1(12,1) = \begin{cases} \frac{2b^2a_1+(2b^2-d^2)w_1-bd(a_2-w_2)}{4b^2-d^2} & \text{for } 0 < \frac{a_1-w_1}{a_2-w_2} < \frac{3bd}{2b^2+d^2} \\ \frac{a_1+2w_1}{3} & \text{for } \frac{3bd}{2b^2+d^2} \leq \frac{a_1-w_1}{a_2-w_2} \end{cases}$$

$$p_2(12,1) = \begin{cases} \frac{2b^2a_2+(2b^2-d^2)w_2-bd(a_1-w_1)}{4b^2-d^2} & \text{for } 0 < \frac{a_1-w_1}{a_2-w_2} < \frac{3bd}{2b^2+d^2} \\ \frac{3b(a_2+w_2)-d(a_1-w_1)}{6b} & \text{for } \frac{3bd}{2b^2+d^2} \leq \frac{a_1-w_1}{a_2-w_2} \leq \frac{b}{d} \\ \emptyset & \text{for } \frac{b}{d} < \frac{a_1-w_1}{a_2-w_2} \end{cases}$$

$$R_1(12,1) = \begin{cases} \frac{b[2b(a_2-w_2)-d(a_1-w_1)]^2}{(4b^2-d^2)^2} & \text{for } 0 < \frac{a_1-w_1}{a_2-w_2} < \frac{3bd}{2b^2+d^2} \\ \frac{(4b^2+5d^2)(a_1-w_1)^2+9b^2(a_2-w_2)^2-18bd(a_1-w_1)(a_2-w_2)}{36b(b^2-d^2)} & \text{for } \frac{3bd}{2b^2+d^2} \leq \frac{a_1-w_1}{a_2-w_2} \leq \frac{b}{d} \\ \frac{(a_1-w_1)^2}{9b} & \text{for } \frac{b}{d} < \frac{a_1-w_1}{a_2-w_2} \end{cases}$$

$$R_2(12,1) = \begin{cases} \frac{b[2b(a_1-w_1)-d(a_2-w_2)]^2}{(4b^2-d^2)^2} & \text{for } 0 < \frac{a_1-w_1}{a_2-w_2} < \frac{3bd}{2b^2+d^2} \\ \frac{(a_1-w_1)^2}{9b} & \text{for } \frac{3bd}{2b^2+d^2} \leq \frac{a_1-w_1}{a_2-w_2} \end{cases}$$

where it is clear from above that the restriction to ensure that $q_{11}(12,1) = q_{12}(1,12)$ are nonnegative is,

$$\frac{3bd}{2b^2+d^2} < \frac{a_1-w_1}{a_2-w_2}$$

while, by a similar analysis as above, the restriction applying for $q_{21}(12,2) = q_{22}(2,12)$ is,

$$\frac{a_1-w_1}{a_2-w_2} < \frac{2b^2+d^2}{3bd}$$

2 Appendix B: proofs.

Proof of Result 1.

The strategy of the proof is to find whether and when manufacturer M_1 has an incentive to deviate from the equilibrium in which both manufacturers set interior transfer prices. Fix w_2 equal to $\hat{w}_2 = \frac{\left(2b^2-d^2\right)a_2 - bda_1 + b(2b+d)c}{4b^2-d^2}$. Given that, we may write the corresponding profit function of M_1 as $M_1(w_1, \hat{w}_2)$. Let $\Pi'(w_1, \hat{w}_2) = \frac{2(w_1-c)(b(a_1-w_1)-d(a_2-\hat{w}_2))}{3(b^2-d^2)}$ and let $\Pi''(w_1, \hat{w}_2) = \frac{2}{3b}(w_1 - c)(a_1 - w_1)$. These expressions are concave in w_1. The unconstrained equilibrium transfer price w_1^* for $\Pi'(w_1, \hat{w}_2)$ is $\hat{w}_1 = \frac{\left(2b^2-d^2\right)a_1 - bda_2 + b(2b+d)c}{4b^2-d^2}$; the unconstrained equilibrium transfer price w_1^* for $\Pi''(w_1, \hat{w}_2)$ is the monopoly transfer price $w_1^m = \frac{a_1+c}{2}$; the limit transfer price is obtained from the intersection of $\Pi'(w_1, \hat{w}_2)$ and $\Pi''(w_1, \hat{w}_2)$, $\tilde{w}_1(\hat{w}_2) = a_1 - \frac{b(a_2-\hat{w}_2)}{d}$. These equilibrium transfer prices must be ranked in order to find any profitable deviation.

It turns out that: a) for $\frac{a_1-c}{a_2-\hat{w}_2}$ belonging to $[1, \frac{2b^2-d^2}{bd}]$, $w_1^m > \hat{w}_1 > \tilde{w}_1(\hat{w}_2)$; b) for $\frac{a_1-c}{a_2-\hat{w}_2}$ belonging to $[\frac{2b^2-d^2}{bd}, \frac{2b}{d}]$, $w_1^m > \tilde{w}_1(\hat{w}_2) > \hat{w}_1$ and c) for $\frac{a_1-c}{a_2-\hat{w}_2}$ greater than$\frac{2b}{d}$, $\tilde{w}_1(\hat{w}_2) > w_1^m > \hat{w}_1$. Note that when manufacturer M_1 sets a transfer price smaller than or equal to the limit transfer price, the rival manufacturer is foreclosed from the market, manufacturer M_1 relevant profit branch is the one at the bottom in equation (8) . Thus, in case a), the maximum of $\Pi''(w_1, \hat{w}_2)$ subject to $w_1 \in [c, \tilde{w}_1(\hat{w}_2)]$ is $\tilde{w}_1(\hat{w}_2)$, while the maximum of $\Pi'(w_1, \hat{w}_2)$ subject to $w_1 > \tilde{w}_1(\hat{w}_2)$ is $\hat{w}_1$, and noting that $\Pi'(\tilde{w}_1(\hat{w}_2), \hat{w}_2) = \Pi''(\tilde{w}_1(\hat{w}_2), \hat{w}_2)$ we conclude that the global maximum is $\hat{w}_1$ and manufacturer M_1 has no incentive to deviate from the interior solution.

In case b), the maximum of $\Pi''(w_1, \hat{w}_2)$ subject to $w_1 \in [c, \tilde{w}_1(\hat{w}_2)]$ is $\tilde{w}_1(\hat{w}_2)$, while the maximum of $\Pi'(w_1, \hat{w}_2)$ subject to $w_1 > \tilde{w}_1(\hat{w}_2)$ is $\tilde{w}_1(\hat{w}_2)$. We conclude that the global maximum is $\tilde{w}_1(\hat{w}_2)$ and manufacturer M_1 has an incentive to deviate.

Finally, in case c), the maximum of $\Pi''(w_1, \hat{w}_2)$ subject to $w_1 \in [c, \tilde{w}_1(\hat{w}_2)]$ is w_1^m, while the maximum of $\Pi'(w_1, \hat{w}_2)$ subject to $w_1 > \tilde{w}_1(\hat{w}_2)$ is $\tilde{w}_1(\hat{w}_2)$. Noting that $\Pi'(\tilde{w}_1(\hat{w}_2), \hat{w}_2) = \Pi''(\tilde{w}_1(\hat{w}_2), \hat{w}_2)$ we conclude that the global maximum is w_1^m and manufacturer M_1 has an incentive to deviate.

Whenever manufacturer M_1 has an incentive to deviate, manufacturer M_2 may lower the transfer price w_2 in order to remain in the market. The lowest w_2 it can fix is $w_2 = c$. Substituting in cases b) and c) above $\hat{w}_2$ for $w_2 = c$, leads to the intervals for $C = \frac{a_1-c}{a_2-c}$ stated in the result. Q.E.D.

Proof of Proposition 1.

We compute the Nash equilibrium in pure strategies in the payoff matrix given by Table 2. The strategy of the proof consists of constructing the best-response function for each manufacturer when the rival hires either one retailer or two retailers. These best responses are presente in four lemmatas, the combination of which yields the equilibria reported in the proposition.

Five different payoffs for each player need to be considered. For the sake of the proof we will make use the following:
i) $C = \frac{a_1-c}{a_2-c}$ is assumed to belong to the interval $[1, \frac{2b^2-d^2}{db}]$ and,
ii) $\frac{d}{b} \in (0,1)$ since $b > d > 0$.

1) In order to construct the best response function for M_1, suppose that:

A) M_2 chooses $s_2 = 1$ (respectively, $s_2 = 2$).
The best response for M_1 follows from ranking the next expressions:

$$M_1(1,1) = M_1(2,2) = \frac{b((2b^2-d^2)(a_1-c)-bd(a_2-c))^2}{2(b^2-d^2)(4b^2-d^2)^2}$$

$$M_1(2,1) = M_1(1,2) = \frac{2b((8b^2-d^2)(a_1-c)-2bd(a_2-c))^2}{(4b^2-d^2)(16b^2-d^2)^2}$$

$$M_1(12,1) = M_1(12,2) = \frac{(4b^2-d^2)((8b^2-5d^2)(a_1-c)-3bd(a_2-c))^2}{6b(b^2-d^2)(16b^2-7d^2)^2}$$

Note that the strategy $s_1 = 0$ is always dominated. We start by comparing $M_1(2,1)$ and $M_1(1,1)$ (which is equivalent to comparing $M_1(1,2)$ and $M_1(2,2)$).

The difference $M_1(2,1) - M_1(1,1)$ defines a concave quadratic function in C. Since the roots of the quadratic function set the range of C for which the function is either positive or negative and since it is assumed that the range of C is bounded, the strategy of the proof amounts to ranking the roots and the boundaries of $[1, \frac{2b^2-d^2}{db}]$. Let r_A be the upper root of that function (the lower root, whenever it exists, is the same expression as the upper one up to a negative sign before the term with the square root),

$$r_A = \frac{b[256b^6 + 32b^4d^2 - 70b^2d^4 + 7d^6 + 2(16b^2-d^2)(4b^2+d^2)\sqrt{(4b^2-d^2)(b^2-d^2)}]}{d(384b^6 - 204b^4d^2 + 48b^2d^4 - 3d^6)}$$

It is easily proven that r_A is greater than one since $b > d$ and it is smaller than $\frac{2b^2-d^2}{db}$ if $\frac{d}{b} < 0.708$. Likewise, the lower root is smaller than one since $b > d$. Therefore, it follows that

$$\begin{aligned} M_1(2,1) &\geq M_1(1,1) \text{ if } \begin{cases} \text{either } \frac{d}{b} \in (0, 0.708] & \text{and } C \in [1, r_A] \\ \text{or } \frac{d}{b} \in (0.708, 1) & \text{and } C \in [1, \frac{2b^2-d^2}{db}] \end{cases} \\ & \text{while,} \\ M_1(1,1) &\geq M_1(2,1) \text{ if } \frac{d}{b} \in (0, 0.708] \text{ and } C \in [r_A, \frac{2b^2-d^2}{db}]. \end{aligned}$$

By the same token, the difference $M_1(12,1) - M_1(1,1)$ defines a convex quadratic function in C. Since the upper root of that equation,

$r_B = \frac{bd[3d^4(14b^2-5d^2)+2b(16b^2-7d^2)(4b^2-d^2)\sqrt{3(4b^2-d^2)}]}{1024b^8-1408b^6d^2+756b^4d^4-208b^2d^6+25d^8}$, is always smaller than one, we conclude that:

$$M_1(12,1) \geq M_1(1,1) \text{ if } \frac{d}{b} \in (0,1) \text{ and } C \in [1, \frac{2b^2-d^2}{db}].$$

Finally, the difference $M_1(12,1)-M_1(2,1)$ defines a convex quadratic function in C. It is easily proven that the lower root is always smaller than one. The upper one, denoted by r_C,

$$\begin{aligned} r_C \;=\;& \frac{3bd(16384b^{10} - 8192b^8d^2 - 4288b^6d^4 + 2360b^4d^6 - 184b^2d^8 - 5d^{10})}{65536b^{12} - 73728b^{10}d^2 + 36096b^8d^4 - 12992b^6d^6 + 3780b^4d^8 - 492b^2d^{10} + 25d^{12}} \\ &+ \frac{2b^2d(4b^2-d^2)(16b^2-d^2)(16b^2-7d^2)(8b^2+7d^2)\sqrt{3(b^2-d^2)}}{65536b^{12} - 73728b^{10}d^2 + 36096b^8d^4 - 12992b^6d^6 + 3780b^4d^8 - 492b^2d^{10} + 25d^{12}} \end{aligned}$$

is increasing with the ratio $\frac{d}{b}$, and satisfies the following:

$$\begin{array}{ll} 0 < r_C \leq 1 & \text{if } \frac{d}{b} \in (0, 0.682] \\ 1 < r_C \leq \frac{2b^2-d^2}{bd} & \text{if } \frac{d}{b} \in (0.682, 0.907] \\ \frac{2b^2-d^2}{bd} < r_C & \text{if } \frac{d}{b} \in (0.907, 1) \end{array}$$

Therefore, it is true that:

$$M_1(12,1) \geq M_1(2,1) \text{ if } \begin{cases} \text{either } \frac{d}{b} \in (0, 0.682] & \text{and } C \in [1, \frac{2b^2-d^2}{db}] \\ \text{or } \frac{d}{b} \in (0.682, 0.907] & \text{and } C \in [r_C, \frac{2b^2-d^2}{db}] \end{cases}$$

while,

$$M_1(2,1) \geq M_1(12,1) \text{ if } \begin{cases} \text{either } \frac{d}{b} \in (0.682, 0.907] & \text{and } C \in [1, r_C] \\ \text{or } \frac{d}{b} \in (0.907, 1) & \text{and } C \in [1, \frac{2b^2-d^2}{db}] \end{cases}$$

The above discussion in summarized in the following lemma:

Lemma 1 *The best response to $s_2 = 1$ (respectively, to $s_2 = 2$) is,*
a) $s_1 = 12$ if
either $\frac{d}{b} \in (0, 0.682]$ and $C \in [1, \frac{2b^2-d^2}{db}]$,
or $\frac{d}{b} \in (0.682, 0.907]$ and $C \in [r_C, \frac{2b^2-d^2}{db}]$.
b) $s_1 = 2$ (respectively, $s_1 = 1$) if
either $\frac{d}{b} \in (0.682, 0.907]$ and $C \in [1, r_C]$,
or $\frac{d}{b} \in (0.907, 1)$ and $C \in [1, \frac{2b^2-d^2}{db}]$.

B) M_2 chooses $s_2 = 12$.
The best response for M_1 follows from comparing:

$M_1(1,12) = M_1(2,12) = \frac{(b(8b^2-5d^2)(a_1-c)-d(4b^2-d^2)(a_2-c))^2}{2b(b^2-d^2)(16b^2-7d^2)^2}$

$M_1(12,12) = \frac{2b((2b^2-d^2)(a_1-c)-bd(a_2-c))^2}{3(b^2-d^2)(4b^2-d^2)^2}$

The difference $M_1(12,12) - M_1(1,12)$ (or equivalently $M_1(12,12) - M_1(2,12)$) defines a convex quadratic function in C. Let r_D denote the upper root of that equation,

$$r_D = \frac{d[512b^8 - 704b^6d^2 + 280b^4d^4 + 8b^2d^6 - 15d^8 + 2d^2(64b^6 - 108b^4d^2 + 51b^2d^4 - 7d^6)\sqrt{3}]}{b(1024b^8 - 2304b^6d^2 + 2080b^4d^4 - 840b^2d^6 + 121d^8)}$$

This root is smaller or equal than one if $\frac{d}{b} \in (0,\, 0.909]$ while it belongs to the interval $[1, \frac{2b^2-d^2}{db}]$ when $\frac{d}{b} \in (0.909,\, 1)$. Then, the next lemma is stated:

Lemma 2 *The best response to* $s_2 = 12$ *is*
a) $s_1 = 12$ *if*
either $\frac{d}{b} \in (0, 0.909]$ *and* $C \in [1, \frac{2b^2-d^2}{db}]$,
or $\frac{d}{b} \in (0.909, 1]$ *and* $C \in [r_D, \frac{2b^2-d^2}{db}]$.
b) $s_1 = 1$ *and* $s_1 = 2$ *if* $\frac{d}{b} \in (0.909, 1]$ *and* $C \in [1, r_D]$.

2)The next step is to obtain the best response function for M_2, suppose that:

A) M_1 chooses either $s_1 = 1$ (respectively $s_1 = 2$).
To compute the best response for M_2 the relevant payoffs are:

$M_2(1,1) = M_2(2,2) = \frac{b((2b^2-d^2)(a_2-c)-bd(a_1-c))^2}{2(b^2-d^2)(4b^2-d^2)^2}$

$M_2(1,2) = M_2(2,1) = \frac{2b((8b^2-d^2)(a_2-c)-2bd(a_1-c))^2}{(4b^2-d^2)(16b^2-d^2)^2}$

$M_2(1,12) = M_2(2,12) = \frac{(4b^2-d^2)((8b^2-5d^2)(a_2-c)-3bd(a_1-c))^2}{6b(b^2-d^2)(16b^2-7d^2)^2}$

The difference $M_2(1,2) - M_2(1,1)$ (equivalently $M_2(2,1) - M_2(2,2)$) defines a concave quadratic function in C. It is easily proven that the upper root of that equation,
$r_a = \frac{256b^6+32b^4d^2-70b^2d^2+7d^6+2(16b^2-d^2)(4b^2+d^2)\sqrt{(4b^2-d^2)(b^2-d^2)]}}{3bd(64b^4+16b^2d^2-5d^4)}$ is greater than $\frac{2b^2-d^2}{db}$ and that the lower root is smaller than one since $b > d$. It follows that

$$M_2(1,2) = M_2(2,1) \geq M_2(1,1) = M_2(2,2) \text{ if } \frac{d}{b} \in (0,1) \text{ and } C \in [1, \frac{2b^2-d^2}{db}].$$

Similarly, the difference $M_2(1,12)-M_2(1,1)$ (equivalently $M_2(2,12)-M_2(2,2)$) defines a concave quadratic function in C. Since the upper root of that equation, $r_b = \frac{-3d^4(14b^2-5d^2)+2b(16b^2-7d^2)(4b^2-d^2)\sqrt{3(4b^2-d^2)}}{bd(192b^4-48b^2d^2-9d^4)}$, is always greater than $\frac{2b^2-d^2}{db}$ and the lower one is smaller than one, we conclude that:

$$M_2(1,12) = M_2(2,12) \geq M_2(1,1) = M_2(2,2) \text{ if } \frac{d}{b} \in (0,1) \text{ and } C \in [1, \frac{2b^2-d^2}{db}]$$

Finally, the difference $M_2(1,12) - M_1(1,2)$ (equivalently $M_2(2,12) - M_2(2,1)$) defines a convex quadratic function in C. It is easily proven that the upper root is always greater than $\frac{2b^2-d^2}{db}$. The lower one, denoted by r_c,

$$\begin{aligned} r_c &= \frac{3(16384b^{10} - 8192b^8d^2 - 4288b^6d^4 + 2360b^4d^6 - 184b^2d^8 - 5d^{10})}{3bd(8192b^8 - 2784b^4d^4 + 664b^2d^6 + 3d^8)} \\ &\quad - \frac{2b(4b^2-d^2)(16b^2-d^2)(16b^2-7d^2)(8b^2+7d^2)\sqrt{3(b^2-d^2)}}{3bd(8192b^8 - 2784b^4d^4 + 664b^2d^6 + 3d^8)} \end{aligned}$$

is decreasing with the ratio $\frac{d}{b}$, and satisfies the following:

$$\begin{array}{ll} 1 < r_c < \frac{2b^2-d^2}{bd} & \text{if } \frac{d}{b} \in (0, 0.682] \\ 0 < r_c < 1 & \text{if } \frac{d}{b} \in (0.682, 1] \end{array}$$

Therefore, it follows that:

$$\begin{aligned} M_2(1,2) &\geq M_2(1,12) \text{ if } \begin{cases} \text{either } \frac{d}{b} \in (0,0.682] & \text{and } C \in [r_c, \frac{2b^2-d^2}{db}] \\ \text{or } \frac{d}{b} \in (0.682, 1] & \text{and } C \in [1, \frac{2b^2-d^2}{db}] \end{cases} \\ & \text{while,} \\ M_2(1,12) &\geq M_2(1,2) \text{ if } \frac{d}{b} \in (0,0.682] \text{ and } C \in [1, r_c] \end{aligned}$$

The following lemma summarizes the above analysis:

Lemma 3 *The best response to $s_1 = 1$ (respectively, to $s_1 = 2$) is,*
a) $s_2 = 12$ if $\frac{d}{b} \in (0, 0.682]$ and $C \in [1, r_c]$
b) $s_2 = 2$ (respectively, $s_2 = 1$) if
either $\frac{d}{b} \in (0, 0.682]$ and $C \in [r_c, \frac{2b^2-d^2}{db}]$,
or $\frac{d}{b} \in (0.682, 1]$ and $C \in [1, \frac{2b^2-d^2}{db}]$.

B) M_1 chooses $s_1 = 12$.
The best response for M_2 follows from comparing of the next expressions:

$$M_2(12,1) = M_2(12,2) = \frac{(4b^2-d^2)((8b^2-5d^2)(a_2-c)-3bd(a_1-c))^2}{6b(b^2-d^2)(16b^2-7d^2)^2}$$
$$M_2(12,12) = \frac{2b((2b^2-d^2)(a_2-c)-bd(a_1-c))^2}{3(b^2-d^2)(4b^2-d^2)^2}$$

The difference $M_2(12,12) - M_2(12,1)$ (equivalently $M_2(12,12) - M_2(12,2)$) defines a convex quadratic function in C. Let r_d be the lower root of that equation,

$$r_d = \frac{b(512b^8 - 704b^6d^2 + 280b^4d^4 + 8b^2d^6 - 15d^8) - 2d(64b^6 - 108b^4d^2 + 51b^2d^4 - 7d^6)\sqrt{3}}{d(256b^8 - 128b^6d^2 - 92b^4d^4 + 48b^2d^6 - 3d^8)}$$

This root is smaller than one if $\frac{d}{b} \in (0.909,\ 1)$ while it belongs to the interval $(1, \frac{2b^2-d^2}{bd})$ when $\frac{d}{b} \in (0,\ 0.909)$. The upper root is always greater than $\frac{2b^2-d^2}{bd}$. Then, the next lemma states M_2's best response to $s_1 = 12$.

Lemma 4 *The best response to* $s_1 = 12$ *is*
a) $s_2 = 1$ *and* $s_2 = 2$ *if*
either $\frac{d}{b} \in (0, 0.909]$ *and* $C \in [r_d, \frac{2b^2-d^2}{db}]$,
or $\frac{d}{b} \in (0.909, 1]$ *and* $C \in [1, \frac{2b^2-d^2}{db}]$.
b) $s_1 = 12$ *if* $\frac{d}{b} \in (0, 0.909]$ *and* $C \in [1, r_d]$.

Taking into account the above lemmatas the range of $\frac{d}{b}$ is partitioned into four intervals: i) $\frac{d}{b} \in (0, 0.682]$; ii) $\frac{d}{b} \in (0.682, 0.907]$; iii) $\frac{d}{b} \in (0.907,\ 0.909]$ and iv) $\frac{d}{b} \in (0.909,\ 1)$. Now, we establish the Nash equilibrium for each of these intervals as a function of C.

1)Let us assume that $\frac{d}{b} \in (0, 0.682]$.

By Lemmatas 1 and 2, the best response of M_1 to $s_2 = 1$, $s_2 = 2$ and $s_2 = 12$ is $s_1 = 12$ for all $C \in [\ 1, \frac{2b^2-d^2}{bd}]$. That is, $s_1 = 12$ is a dominant strategy.

By Lemma 3 the best response of M_2 to $s_1 = 1$ (respectively, to $s_1 = 2$) is either $s_2 = 12$ if $C \in [1, r_c]$ or $s_2 = 2$ (respectively, $s_2 = 1$) if $C \in [r_c, \frac{2b^2-d^2}{bd}]$. By Lemma 4 the best response of M_2 to $s_1 = 12$ is either $s_2 = 12$ if $C \in [1, r_d]$ or $s_2 = 1$ and $s_2 = 2$ if $C \in [r_d, \frac{2b^2-d^2}{bd}]$. Furthermore, it can be proven that $r_c < r_d$ for $\frac{d}{b} \in (0, 0.682]$. Therefore we conclude that:

1.1) for $\frac{d}{b} \in (0, 0.682]$ and $C \in [1, r_c]$ the Nash Equilibrium is the pair of strategies $(12, 12)$.

1.2) for $\frac{d}{b} \in (0, 0.682]$ and $C \in [r_c, r_d]$ the Nash Equilibrium is also $(12, 12)$.

1.3) for $\frac{d}{b} \in (0, 0.682]$ and $C \in [r_d, \frac{2b^2-d^2}{bd}]$ the Nash Equilibria are both $(12, 1)$ and $(12, 2)$.

2)Let us assume that $\frac{d}{b} \in (0.682, 0907]$.

Both r_d and r_C appear as relevant in this interval. Remind that r_d is decreasing with $\frac{d}{b}$ in the interval $(0.682, 0.907]$ and it equals one for $\frac{d}{b} = 0.909$, while r_C is increasing with $\frac{d}{b}$ in the same interval, being equal to one for $\frac{d}{b} = 0.682$ and equal to $\frac{2b^2-d^2}{bd}(= 1.298)$ for $\frac{d}{b} = 0.907$. Then it is easy to find that r_d and r_C cross each other at $\frac{d}{b} = 0.805$, and therefore, we analyze the cases $\frac{d}{b} \in (0.682, 0.805]$ and $\frac{d}{b} \in (0.805, 0907]$ separately.

2.1) For $\frac{d}{b} \in (0.682, 0.805]$ it is the case that $r_C < r_d$ and we conclude that:

2.1.a) for $\frac{d}{b} \in (0.682, 0.805]$ and $C \in [1, r_C]$ the Nash Equilibria are $(12, 12)$, $(1, 2)$ and $(2, 1)$.

2.1.b) for $\frac{d}{b} \in (0.682, 0.805]$ and $C \in [r_C, r_d]$ the Nash Equilibrium is $(12, 12)$.

2.1.c) for $\frac{d}{b} \in (0.682, 0.805]$ and $C \in [r_d, \frac{2b^2-d^2}{bd}]$ the Nash Equilibria are both $(12, 1)$ and $(12, 2)$.

2.2) For $\frac{d}{b} \in (0.805, 0.907]$ it is the case that $r_d < r_C$. Then, we conclude that:

2.2.a) for $\frac{d}{b} \in (0.805, 0.907]$ and $C \in [1, r_d]$ the Nash Equilibria are $(12, 12)$, $(1, 2)$ and $(2, 1)$.

2.2.b) for $\frac{d}{b} \in (0.805, 0.907]$ and $C \in [r_d, r_C]$ the Nash Equilibria are $(1, 2)$ and $(2, 1)$.

ii.2.c) for $\frac{d}{b} \in (0.805, 0.907]$ and $C \in [r_C, \frac{2b^2-d^2}{bd}]$ the Nash Equilibria are both $(12, 1)$ and $(12, 2)$.

3) Let us assume that $\frac{d}{b} \in (0.907, 0.909]$.

Two different conclusions are reached depending on the size of C.

3.1) for $\frac{d}{b} \in (0.907, 0.909]$ and $C \in [1, r_d]$ the Nash Equilibria are $(12, 12)$, $(1, 2)$ and $(2, 1)$.

3.2)for $\frac{d}{b} \in (0.907, 0.909]$ and $C \in [r_d, \frac{2b^2-d^2}{bd}]$ the Nash Equilibria are $(1, 2)$ and $(2, 1)$.

4) Let us assume that $\frac{d}{b} \in (0.909, 1)$.

Following the lemmatas above we conclude that for $\frac{d}{b} \in (0.909, 1)$ and $C \in [1, \frac{2b^2-d^2}{bd}]$ the Nash Equilibria are $(1, 2)$ and $(2, 1)$.

The different Nash equilibria presented above are shown in Proposition 1 noting the difference between a unique equilibrium system at equilibrium and a multiplicity of them. Q.E.D.

Proof of Proposition 2.

The proof of this proposition is based on some information derived in the above proof, both on the sign of $M_1(2,1) - M_1(1,1)$ and $M_2(1,2) - M_2(1,1)$.

We know that the difference $M_2(1,2) - M_2(1,1)$ (equivalently, $M_2(2,1) - M_2(2,2)$) is nonnegative for all $\frac{d}{b} \in (0,1)$ and $C \in [1, \frac{2b^2-d^2}{db}]$. M_2 always prefers exclusive dealing. Whether an equilibrium in pure strategies will depend on M_1's best response. The difference $M_1(2,1) - M_1(1,1)$ (equivalently, $M_1(1,2) - M_1(2,2)$) is nonnegative either if $\frac{d}{b} \in (0, 0.708]$ and $C \in [1, r_A]$ or if $\frac{d}{b} \in (0.708, 1)$ and $C \in [1, \frac{2b^2-d^2}{db}]$, being negative otherwise. The conditions for $M_1(2,1) - M_1(1,1)$ to be nonnegative give rise to the first statement in the proposition. And the condition for $M_1(2,1) - M_1(1,1)$ to be negative give rise to the second statement. Q.E.D.

Distribution Scheme	Quantities	Profits
(1,0) and (0,1)	$q_1(1,0) = \frac{a_1-w_1}{2b}$ $q_2(1,0) = 0$	$R_1(1,0) = \frac{(a_1-w_1)^2}{4b}$ $R_2(1,0) = 0$
(2,0) and (0,2)	$q_1(2,0) = \frac{a_2-w_2}{2b}$ $q_2(2,0) = 0$	$R_1(2,0) = \frac{(a_2-w_2)^2}{4b}$ $R_2(2,0) = 0$
(12,0) and (0,12)	$q_1(12,0) = \frac{(a_1-w_1+a_2-w_2)}{2(b+d)}$ $q_2(12,0) = 0$	$R_1(12,0) = \frac{b((a_1-w_1)^2+(a_2-w_2)^2)-2d(a_1-w_1)(a_2-w_2)}{4(b^2-d^2)}$ $R_2(12,0) = 0$
(1,1)	$q_1(1,1) = q_2(1,1) = \frac{a_1-w_1}{3b}$	$R_1(1,1) = R_2(1,1) = \frac{(a_1-w_1)^2}{9b}$
(2,2)	$q_1(2,2) = q_2(2,2) = \frac{a_2-w_2}{3b}$	$R_1(2,2) = R_2(2,2) = \frac{(a_2-w_2)^2}{9b}$
(12,12)	$q_1(12,12) = q_2(12,12) = \frac{(a_1-w_1+a_2-w_2)}{3(b+d)}$	$R_1(12,12) = R_2(12,12) = \frac{b((a_1-w_1)^2+(a_2-w_2)^2)-2d(a_1-w_1)(a_2-w_2)}{9(b^2-d^2)}$
(1,2) and (2,1)	$q_1(1,2) = \frac{2b(a_1-w_1)-d(a_2-w_2)}{4b^2-d^2}$ $q_2(1,2) = \frac{2b(a_2-w_2)-d(a_1-w_1)}{4b^2-d^2}$	$R_1(1,2) = \frac{b(2b(a_1-w_1)-d(a_2-w_2))^2}{(4b^2-d^2)^2}$ $R_2(1,2) = \frac{b(2b(a_2-w_2)-d(a_1-w_1))^2}{(4b^2-d^2)^2}$
(12,1) and (1,12)	$q_1(12,1) = \frac{(2b-d)(a_1-w_1)+3b(a_2-w_2)}{6b(b+d)}$ $q_2(12,1) = \frac{(a_1-w_1)}{3b}$	$R_1(12,1) = \frac{(4b^2+5d^2)(a_1-w_1)^2+9b^2(a_2-w_2)^2-18bd(a_1-w_1)(a_2-w_2)}{36b(b^2-d^2)}$ $R_2(12,1) = \frac{(a_1-w_1)^2}{9b}$
(12,2) and (2,12)	$q_1(12,2) = \frac{(2b-d)(a_2-w_2)+3b(a_1-w_1)}{6b(b+d)}$ $q_2(12,2) = \frac{(a_2-w_2)}{3b}$	$R_1(12,2) = \frac{9b^2(a_1-w_1)^2+(4b^2+5d^2)(a_2-w_2)^2-18bd(a_1-w_1)(a_2-w_2)}{36b(b^2-d^2)}$ $R_2(12,2) = \frac{(a_2-w_2)^2}{9b}$
(0,0)	$q_1(0,0) = q_2(0,0) = 0$	$R_1(0,0) = R_2(0,0) = 0$

Table 1: The Third Stage Nash Equilibrium Quantities and Retailers' Profits for Each Distribution System.

Distribution System	Transfer Prices
(1,0), (2,0)	$w_1(1,0) = w_1(2,0) = \frac{a_1+c}{2}$ $w_2(1,0) = w_2(2,0) = \emptyset$
(0,1), (0,2)	$w_1(0,1) = w_1(0,2) = \emptyset$ $w_2(0,1) = w_2(0,2) = \frac{a_2+c}{2}$
(12,0)	$w_1(12,0) = \frac{a_1+c}{2}$ $w_2(12,0) = \emptyset$
(0,12)	$w_1(0,12) = \emptyset$ $w_2(0,12) = \frac{a_2+c}{2}$
(12,1), (12,2)	$w_1(12,1) = w_1(12,2) = \frac{(8b^2-5d^2)a_1-3bda_2+(8b^2+3bd-2d^2)c}{16b^2-7d^2}$ $w_2(12,1) = w_2(12,2) = \frac{b(8b^2-5d^2)a_2-d(4b^2-d^2)a_1+(8b^3+4b^2d-2bd^2-d^3)c}{b(16b^2-7d^2)}$
(1,12), (2,12)	$w_1(1,12) = w_1(2,12) = \frac{b(8b^2-5d^2)a_1-d(4b^2-d^2)a_2+(8b^3+4b^2d-2bd^2-d^3)c}{b(16b^2-7d^2)}$ $w_2(1,12) = w_2(2,12) = \frac{(8b^2-5d^2)a_2-3bda_1+(8b^2+3bd-2d^2)c}{16b^2-7d^2}$
(1,1), (2,2)	$w_1(1,1) = w_1(2,2) = \frac{(2b^2-d^2)a_1-bda_2+b(2b+d)c}{4b^2-d^2}$ $w_2(1,1) = w_2(2,2) = \frac{(2b^2-d^2)a_2-bda_1+b(2b+d)c}{4b^2-d^2}$
(1,2), (2,1)	$w_1(1,2) = w_1(2,1) = \frac{(8b^2-d^2)a_1-2bda_2+2b(4b+d)c}{16b^2-d^2}$ $w_2(1,2) = w_2(2,1) = \frac{(8b^2-d^2)a_2-2bda_1+2b(4b+d)c}{16b^2-d^2}$
(12,12)	$w_1(12,12) = \frac{\left(2b^2-d^2\right)a_1-bda_2+b(2b+d)c}{4b^2-d^2}$ $w_2(12,12) = \frac{(2b^2-d^2)a_2-bda_1+b(2b+d)c}{4b^2-d^2}$
(0,0)	$w_1(0,0) = w_1(0,0) = w_2(0,0) = w_2(0,0) = \emptyset$

Table 2: Interior Equilibrium Transfer Prices.

		M_2			
		1	2	12	0
	1	$\begin{pmatrix} \frac{b((2b^2-d^2)(a_1-c)-bd(a_2-c))^2}{2(b^2-d^2)(4b^2-d^2)^2} \\ \frac{b((2b^2-d^2)(a_2-c)-bd(a_1-c))^2}{2(b^2-d^2)(4b^2-d^2)^2} \end{pmatrix}$	$\begin{pmatrix} \frac{2b((8b^2-d^2)(a_1-c)-2bd(a_2-c))^2}{(4b^2-d^2)(16b^2-d^2)^2} \\ \frac{2b((8b^2-d^2)(a_2-c)-2bd(a_1-c))^2}{(4b^2-d^2)(16b^2-d^2)^2} \end{pmatrix}$	$\begin{pmatrix} \frac{(b(8b^2-5d^2)(a_1-c)-d(4b^2-d^2)(a_2-c))^2}{2b(b^2-d^2)(16b^2-7d^2)^2} \\ \frac{(4b^2-d^2)((8b^2-5d^2)(a_2-c)-3bd(a_1-c))^2}{6b(b^2-d^2)(16b^2-7d^2)^2} \end{pmatrix}$	$\begin{pmatrix} \frac{(a_1-c)^2}{8b} \\ 0 \end{pmatrix}$
M_1	2	$\begin{pmatrix} \frac{2b((8b^2-d^2)(a_1-c)-2bd(a_2-c))^2}{(4b^2-d^2)(16b^2-d^2)^2} \\ \frac{2b((8b^2-d^2)(a_2-c)-2bd(a_1-c))^2}{(4b^2-d^2)(16b^2-d^2)^2} \end{pmatrix}$	$\begin{pmatrix} \frac{b((2b^2-d^2)(a_1-c)-bd(a_2-c))^2}{2(b^2-d^2)(4b^2-d^2)^2} \\ \frac{b((2b^2-d^2)(a_2-c)-bd(a_1-c))^2}{2(b^2-d^2)(4b^2-d^2)^2} \end{pmatrix}$	$\begin{pmatrix} \frac{(b(8b^2-5d^2)(a_1-c)-d(4b^2-d^2)(a_2-c))^2}{2b(b^2-d^2)(16b^2-7d^2)^2} \\ \frac{(4b^2-d^2)((8b^2-5d^2)(a_2-c)-3bd(a_1-c))^2}{6b(b^2-d^2)(16b^2-7d^2)^2} \end{pmatrix}$	$\begin{pmatrix} \frac{(a_1-c)^2}{8b} \\ 0 \end{pmatrix}$
	12	$\begin{pmatrix} \frac{(4b^2-d^2)((8b^2-5d^2)(a_1-c)-3bd(a_2-c))^2}{6b(b^2-d^2)(16b^2-7d^2)^2} \\ \frac{(b(8b^2-5d^2)(a_2-c)-d(4b^2-d^2)(a_1-c))^2}{2b(b^2-d^2)(16b^2-7d^2)^2} \end{pmatrix}$	$\begin{pmatrix} \frac{(4b^2-d^2)((8b^2-5d^2)(a_1-c)-3bd(a_2-c))^2}{6b(b^2-d^2)(16b^2-7d^2)^2} \\ \frac{(b(8b^2-5d^2)(a_2-c)-d(4b^2-d^2)(a_1-c))^2}{2b(b^2-d^2)(16b^2-7d^2)^2} \end{pmatrix}$	$\begin{pmatrix} \frac{2b((2b^2-d^2)(a_1-c)-bd(a_2-c))^2}{3(b^2-d^2)(4b^2-d^2)^2} \\ \frac{2b((2b^2-d^2)(a_2-c)-bd(a_1-c))^2}{3(b^2-d^2)(4b^2-d^2)^2} \end{pmatrix}$	$\begin{pmatrix} \frac{(a_1-c)^2}{6b} \\ 0 \end{pmatrix}$
	0	$\begin{pmatrix} 0 \\ \frac{(a_2-c)^2}{8b} \end{pmatrix}$	$\begin{pmatrix} 0 \\ \frac{(a_2-c)^2}{8b} \end{pmatrix}$	$\begin{pmatrix} 0 \\ \frac{(a_2-c)^2}{6b} \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Table 3: The First Stage Payoff Matrix.

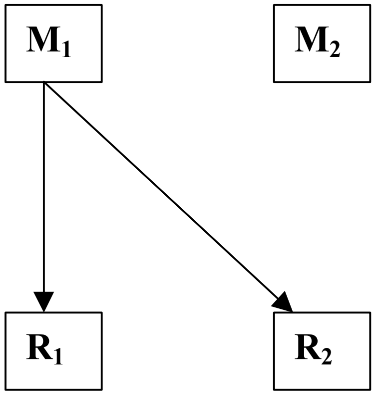

Figure 1a: Non-Exclusive Distribution and Exclusive Purchasing

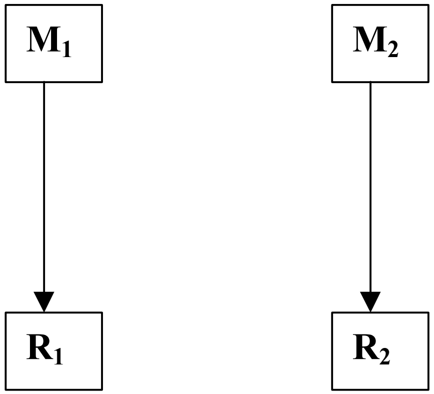

Figure 1b: Duopoly Exclusive Distribution and Purchasing

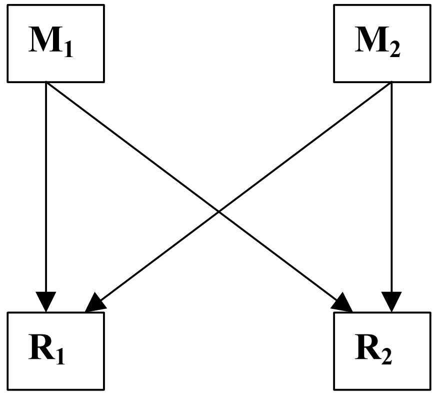

Figure 1c: Non-Exclusive Distribution and Purchasing

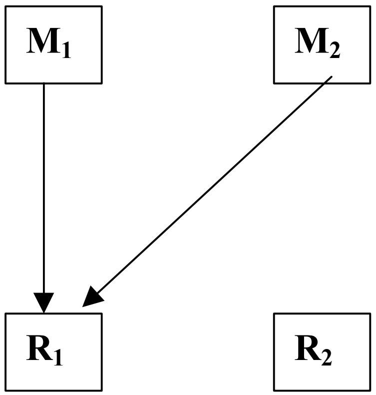

Figure 2a: Exclusive Distribution and Non-Exclusive Purchasing

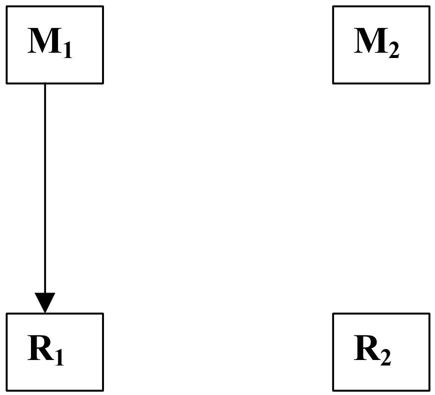

Figure 2b: Monopoly Exclusive Distribution and Purchasing

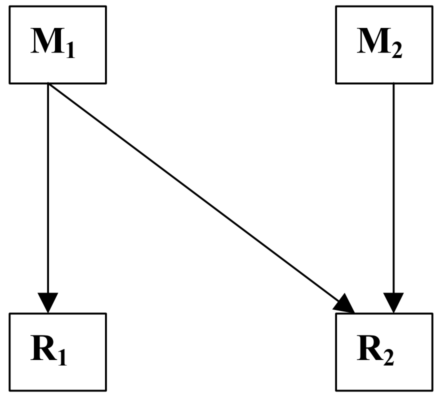

Figure 2c: Mixed Schemes

Figure 3: Equilibrium Brand Policies in the d/b and C space.