Cross-Border Shopping in a Federalist Economy

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Abstract

The purpose of this paper is to consider an economy in which incorporates cross-border shopping and where the different levels of government worry about the well-being of their citizens. We assume a federalist economy with a central government and two regions with specific characteristics. Two kinds of externalities, horizontal and vertical, arise and we show the possibilities to internalise them. With the governments of symmetric regions behaving as Nash players they would optimally set their tax rate and replicate the unitary nation optimum. Finally, we show how the central government as a Stackelberg leader can adjust its fiscal instruments so that the tax externalities are also internalised.
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1 Introduction

Economic integration has played a fundamental role in the world economy. Trade has progressed throughout the second part of the 20th century. Blossoming free trade agreements have eliminated several tariff and non-tariff barriers between countries. Within such a context, differences in tax systems may hamper free trade in an, *a priory*, integrated economy. Hence, tax competition issues are of particular interest because, with integration, what is decided in one country with respect to its own tax system may induce corresponding decisions from other partner countries.

Normally, indirect tax competition generates horizontal tax externalities due to cross-border shopping and inter-regional transfers could be take into account. However, different taxes on the same base create vertical tax externalities between the central government and each regional government. Matching revenue grants, equalisation grant or inter-governmental transfer could internalise these externalities. It is known that a number of papers lead with these kinds of problems in federalist economies. However, economists typically consider these two kinds of externalities separately as if there was a simple structure of government. If we analyse a federation, at least two levels of government must be taken into account. By considering these two levels together then the two kinds of externalities could interact and some fiscal instruments be avoided making the fiscal structure much simpler.

A number of examples will illustrate our point. In MERCOSUR, for example, Brazil exports cigarettes to Paraguay while Brazilian consumers re-import their own cigarettes in order to save taxes. Similar cases of exporting and re-importing occur in NAFTA and in the European Community. This cross-border shopping behaviour is a consequence of different strategic tax settings associated with inefficient or even non-existent border controls.

Some issues in federalist economies that take into account the impact of cross-border shopping on other levels of government provide interesting cases study. In Canada, for example, where there are three levels of government (namely, the federal government, the provinces and the municipalities), the federal government levies an indirect tax of 7 percent on the final consumption (Good and Service Tax - GST), excise taxes and duties, and the 10 provinces (except Alberta) levy the Retail Sales Tax (RST) with tax rates between 6.5 and 12 percent on the selling price.[1] In 1995 the GST and the RST represented the equivalent of 7 and 6.5 percent of GDP, respectively.

[1] In fact Canada has five distinct sales tax system. An explanation for this can be seen in Bird and Gendron (1998).

In 1998-9, the GST represented 27.4 percent of the total revenue (without transfers) and 34 percent of the total tax revenue. The RST in some provinces represented in 1998 more than 50 percent of the tax revenue. The more than 5000 municipalities levy no indirect tax. As a matter of facts, changes in the tax base of the provincial level must affect the tax base of the federal one and vice-versa.

In Brazil, another federalist country with 27 states and 5561 municipal districts, under the indirect tax system the Industrial Production Tax (IPI) is levied on the origin principle by the federal level and the Tax on Circulation of Goods and Services Trade (ICMS) is levied on the origin principle by the regional level. The tax rate of IPI varies between zero and 330 percent (excise on cigarettes) but it has an average tax rate of 8 percent. The ICMS has a tax rate of between 6 and 16 percent depending of the level of essentiality of good and the municipal levels levy the Tax on Services (ISS) but they receive some share of the ICMS levied in their territories. In 1997 the IPI and the ICMS represented 2 and 7 percent of the GNP, respectively, and the federal and regional amount of tax represented 19 and 7.5 per cent of GNP. In conclusion, in Brazil the different levels of government compete between them through the same tax base of indirect tax.

There is already a well-known literature analysing tax competition between different regions or levels of government.[2] A sizeable number of papers on direct tax competition explore the possibility that horizontal tax competition almost results in underprovision of public goods, in which the regions reduce their tax rate below those values that are efficient from the viewpoint of the entire economy. The literature on indirect tax competition with cross-border shopping and Leviathan governments begins with Kanbur and Keen (1993), who consider two asymmetric economies with different sizes and analyse the partial equilibrium when the two regions behave as Nash players. They conclude that the tax revenue falls in the larger country and the per capita revenue increases in the smaller country. Also considering asymmetric regions Nielsen (2001) analyses the tax commodity competition with cross-border shopping. However, he introduces transportation costs on the commodities and analyses the possibility of inspection at the border. Wang (1999) extends Kanbur and Keen (1993) to the case in which one of the players behaves as Stackelberg leader. He shows that the tax rates of both regions are higher than their corresponding Nash tax rates and that reform through tax harmonisation or minimum tax rules harms the smaller region and benefits the larger region. Pieretti (1999) considers a tax com-

[2] Wilson (1999) reviews this literature.

petition between two regions with firms behaving imperfectly as duopolist and analyses the impact of tax competition on the price formation and consequently on the profits in each region. She concludes that tax competition leads the smallest jurisdiction to favour the lowest tax rate.

These papers consider population as being uniformly distributed in each region. Trandel (1994) considers two regions, one region more densely-populated than the other and concludes that, in the Nash equilibrium, the more densely populated region imposes a tax rate higher than the other. Ohsawa (1999) considers tax competition across more than two regions where sizes and positions of regions create differences in market power and consequently differences in the equilibrium tax rates and revenues.

The models above consider a simple structure of government. If we analyse a federation, at least two levels of government must be taken into account. Keen (1998) initially analyses the vertical indirect tax competition between Leviathan federal and regional governments on the same indirect tax base. Without difficulties to reach information the federal level has identical regions which consist of a single consumer in each. Considering separability between private and public consumption he concludes that an increase of the tax rate at the federal level implies a higher tax rate at regional level. However, this result depends on a particular type of demand curve. Afterwards, with benevolent levels of government, the federal one can increase its tax rate leading to a higher tax rate in the region. Keen concludes that tax rates are too high when cross-border shopping does not exist and shows the existence of externalities due to excessively high tax rates in both levels of governments. To eliminate these externalities he suggests the introduction of inter-governmental transfers.

Hoyt (2001) extends the preceding approach by considering a federalist economy with two traded goods, n identical regions and one central government that can behave both *independently* and in a *coordinated* manner. Hoyt also considers two options to tax this economy: first, an economy where the two levels have an *identical* tax base and, second, where the region has a *limited* tax base (taxing only one commodity) but the central government can tax two commodities. In the first case the federal level underprovides the public good. However, with limited tax base both underprovision and overprovision of the public good are possible, that is, the central government can adjust its fiscal instruments according to the behaviour of regional governments. Hoyt also shows that the fiscal instruments of the central government can solve the vertical externalities that emerge by using matching grant and equalisation transfers as previously suggested by Dahlby (1996).

To summarise, almost all models consider either a Leviathan government and therefore analyse just the partial equilibrium of the impact on tax revenue due to cross-border shopping or benevolent government in federalist economies but with no cross-border shopping among different regions. The purpose of this paper is to combine the two approaches by considering an economy with different levels of governments in which we observe cross-border shopping. As indicated before, indirect tax competition generates two tax externalities in federalist countries: a horizontal tax externality arises due to cross-border shopping and, a vertical tax externality is created by the competition for tax bases between the central government and each regional government. Inter-regional transfers or even by increasing the number of regions in this federation could be considered in terms to internalise the horizontal tax externality. With the second kind of externality if the different levels of government consider matching revenue grant, equalisation grant or inter-governmental transfers the vertical tax externality could vanish (Hoyt, 2001). In this approach to allow simply that the different kind of externalities vanish themselves is not appropriate due to there is no interdependency between the two externalities in this economy with particular characteristics.

To analyse whether these different kind of externalities could be internalised, first we consider a unitary economy that assumes all fiscal responsibilities. These fiscal policies will serve as a benchmark for additional analyses. After that, we consider the regional and central governments' behaviour through their fiscal instruments to internalise these externalities. Our main result is that the central government and the regional one, under some specific characteristics of the individuals and regions, could internalise the externalities that arise basically by implementing matching grants.

The rest of the paper is organised as follows. In section 2 we analyse the behaviour of households and the unitary nation. Section 3 considers the structure of the federal system with different levels of government such as the relationship between them for trying to eliminate externalities that arise. In section 4 we conclude and draw some policy implications from our results.

2 The Model

We assume a federalist economy with a central government and two symmetric regions in relation to the number of household. These households are identical and mobile across regions. In this case they can travel to the other region to buy the commodity. There are two normal traded good in each economy. However, only one good could be bought in another region. We can think of a developing economy where it is difficult to observe individuals' income.[3] Firms maximise profits in competitive markets. The production function is characterised by constant returns to scale where one unit of labour produces one unit of traded or public good.

2.1 Consumers behaviour in a cross-border shopping economy

The consumers are distributed uniformly in each region and are identical among them except for their trade cost to the other region. The preference of consumers of region i $(i = a, b)$ is represented by a separable utility function

$$x_{1i}^{i} + v\left(x_{2i}^{i}\right) + b\left(g^{i}\right) + B\left(G\right)$$

where x_{ki}^{i} $(k = 1, 2)$ are the commodities consumed in region i by individuals that live in region i, g^{i} is the spending on goods publicly provided by region i and G is the public good supplied by central government[4]. We consider a well behaved utility function with $v(.), b(.)$ and $B(.)$ increasing and concave functions of their respective arguments. Consumers pay $q_{1i}^{i} = p_1$ and $q_{2i}^{i} = p_2 + \tau_{2i}^{i} + T$ for the commodity k $(k = 1, 2)$, that is, only the commodity 2 is taxed. p_k $(k = 1, 2)$ are the producer prices and we consider them as given and equal to one $(p_1 = p_2 = 1)$, τ_{2i}^{i} is the destination-based regional indirect tax on the commodity 2 paid for the individual that lives in region i and buys in this region and T is the federal indirect tax levied on commodity 2.

Households may buy the commodity 2 in one region or the other depending on the relative values of τ_{2i}^{i} and τ_{2i}^{j} and two cases can be considered. In the first case, if $\tau_{2i}^{i} \leq \tau_{2i}^{j}$ households will consider a budget constraint as

$$x_{1i}^{i} + x_{2i}^{i} \cdot \left(1 + t_{2i}^{i}\right) - wl^{i} \leq 0 \tag{1}$$

where $t_{2i}^{i} = \tau_{2i}^{i} + T$, w is the wage rate and l^{i} is the labour supplied by individuals in region i. Consumers behaviour can be described as

[3]In fact, *developing country* means that it could not observe, for example, the real income of workers or firms' profit to implement other kinds of taxation. So, this is just one artifice to justify the only commodity taxation in this economy.

[4]I am grateful to an anonymous referee for suggesting this utility specification and therefore simplifying further interpretations.

$$\begin{array}{lll} \max\limits_{x_k^i} & x_{1i}^i + v\left(x_{2i}^i\right) + b\left(g^i\right) + B\left(G\right) & \\ s.t. & x_{1i}^i + x_{2i}^i \cdot \left(1 + t_{2i}^i\right) - wl^i \leq 0 & \text{(CP)} \\ & x_{ki}^i \geq 0, t_{2i}^i, g^i, G = \text{constants} & \end{array}$$

where the first order conditions give us the demand functions $x_{ki}^i\left(q_{ki}^i\right), k = 1, 2$. These functions into the utility function gives us the indirect utility function $V^i\left(1 + \tau_{2i}^i + T\right) + b\left(g^i\right) + B\left(G\right)$ and Roy's identity gives us $V_{\tau_{2i}^i}^i = -x_{2i}^i$.

In the second case, if $\tau_{2i}^i \geq \tau_{2i}^j$ households that live in region i may buy in the other region j and pay $q_{2i}^j = 1 + \tau_{2i}^j + T$ where τ_{2i}^j is region j's indirect tax rate on the commodity 2 bought by individual that lives in region i and buys the commodity in region j. In this case, the individual is forced to pay in addition a travel cost d to reach the store in region j.[5]

We consider that the households that live into region i are differentiated by their exogenously given distance d^i from the border. This distance d^i is distributed according to a continuous distribution function $N\left(d^i\right)$ with the positive density $n\left(d^i\right), d^i \in \left[0, \overline{d}^i\right] \subseteq R_0^+$ with $n\left(d^i\right)$ being the number of households at each d^i. We denote the size of the overall population that resides in region i by $n^i = \int_0^{\overline{d}^i} n\left(d^i\right) \mathrm{d}d^i$ and throughout the paper it is normalised to unity. Similar interpretation to the distribution of households that live in region j.

Hence a typical cross-shopper considers a budget constraint as

$$x_{1i}^i + x_{2i}^j \cdot \left(1 + \tau_{2i}^j + T\right) - wl^i + d^i \leq 0$$

and will behave according the following program:

$$\begin{array}{lll} \max\limits_{x_k^i} & x_{1i}^i + v\left(x_{2i}^j\right) + b\left(g^i\right) + B\left(G\right) & \text{(CP2)} \\ s.t & x_{1i}^i + x_{2i}^j \cdot \left(1 + t_{2i}^j\right) - wl^i + d^i \leq 0 & \\ & \tau_{2i}^j, T, g^i, G = \text{constants} & \end{array}$$

[5]Mintz and Tulkens (1986) and Haufler (1996, 1998) consider a convex transportation cost in $x_{ki}^j, k = 1, 2$. This assumption is crucial to show the existence of equilibrium in those economies. Here, we can think in a fixed travel cost just to buy in another region and independent of the amount of commodity.

where the first order condition jointly with the budget constraint gives us the demand functions x^i_{1i} and $x^j_{2i}\left(q^j_{2i}, d^i\right)$ with $q^j_{2i} = 1 + \tau^j_{2i} + T$, and therefore the indirect utility function $\widehat{V}_i\left(1 + \tau^j_{2i} + T, d^i\right) + b\left(g^i\right) + B\left(G\right)$. Applying the envelope theorem yields: $\widehat{V}^i_{d^i} = -U^i_{\widehat{x}^i_{2i}}\left(\frac{1}{1+\tau^j_{2i}+T}\right) = -1 < 0$ and $\widehat{V}^i_{\tau^j_{2i}} = -x^j_{2i} < 0$.[6] In particular the specification of the utility function ensures that the marginal utility of income is simply unity and the same regardless of whether the individual cross-border shops or not. This characteristic leads to no income effect on the commodity purchased in the other region. Also, the additivity in $b\left(g^i\right)$ and $B\left(G\right)$ implies that the demand for the taxed good is independent of public expenditure and, consequently, the public good provision does not affect the decision to cross-border.[7] Moreover, this additivity leads to the no spillover effect or interregional externality in a public good and then the strategy of a regional government does not depend on the strategy of the other region or even of the central one. These characteristics are crucial to the main results. The following Table would clarify the definitions of demand:

Cross-Border Shopper		Purchases (***upper indices***)	
		Region i	Region j
Residence (***lower indices***)	Region i	$x^i_{2i}\left(p+\tau^i_{2i}+T\right)$	$x^j_{2i}\left(p+\tau^j_{2i}+T,d^i\right)$
	Region j	$x^i_{2i}\left(p+\tau^i_{2j}+T,d^j\right)$	$x^j_{2j}\left(p+\tau^j_{2j}+T\right)$

Table:Cross-Border Shopper

The consumers of region i with travel cost d^i to the border can get

$$V^i\left(1+\tau^i_{2i}+T,0\right) \gtreqless \widehat{V}^i\left(1+\tau^j_{2i}+T,d^i\right) \quad (2)$$

from buying from their own region i or the other region j. Observe that when $\tau^i_{2i} \geq \tau^j_{2i}$ there exists a unique location at which consumers are indifferent

[6] Henceforth, the hat means that the expression is related to a cross-shopper.

[7] Alternatively, the general case of non-separability could be treated. However, this extension to the model goes beyond the scope of the present discussion.

in terms of utility level between the two regions.[8] Hence, there exists a cut-point $\widehat{d^i}\left(\tau_{2i}^i + T, \tau_{2i}^j + T\right)$ for which (2) holds as equality. Observe also that in this case comparative statics can easily be derived as

$$\widehat{d^i}_{\tau_{2i}^i} = \frac{V_{\tau_{2i}^i}^i}{\widehat{V}_{d^i}^i} = x_{2i}^i > 0, \ \widehat{d^i}_{\tau_{2i}^j} = -\frac{\widehat{V}_{\tau_{2i}^j}^i}{\widehat{V}_{d^i}^i} = -x_{2i}^j < 0. \tag{3}$$

These two results are rather intuitive: critical distance $\widehat{d^i}$ increases (decreases) the relative tax rate increases (decreases). Figure 1 resumes the influence of travel costs in this economy. When $\tau_{2i}^i \geq \tau_{2i}^j$, hence, the critical distance $\widehat{d^i} \geq 0$ and $\widehat{d^j} = 0$.

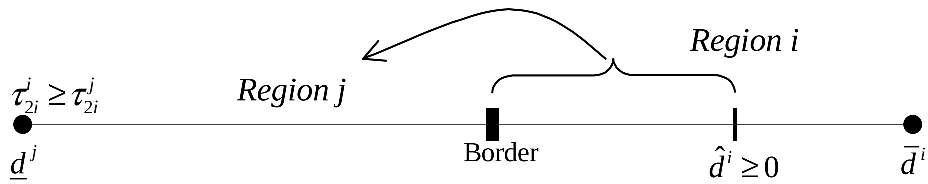

Figure 1: Cross-Border Shopping

Then, individuals in i with distance $d^i < \widehat{d^i}$ from the border (i.e., located to the left of the cut-point $\widehat{d^i}$) will prefer to buy in region j.[9] Otherwise, individuals that are located to the right of their cut-point $\left(d^i > \widehat{d^i}\right)$ will buy in their own region i. Since that $\tau_{2i}^i \leq \tau_{2i}^j$ the critical distance $\widehat{d^i} = 0$ and $\widehat{d^j} \leq 0$ with similar interpretation as before. In $\widehat{d^i}$ or $\widehat{d^j}$ individuals are indifferent to buy in their own region or abroad.

2.2 Unitary Nation

Consider now a central government that provides a public good to the individuals in each region. It behaves uniformly across regions and finances the public good provision by taxing the commodity. In this subsection, to simplify the notations, let us consider $t^i = \tau_{2i}^i + T_2$. Suppose initially that the central government behaves as a unitary nation.[10] Therefore, due to

[8] This kind of comparison is not usual in the cross-border shopping models which instead consider just comparisons in prices to decide where to buy.

[9] Observe that d is used as a travel cost as a distance. So fixed travel cost is proportional to distance.

[10] This strategy is traditional in tax competition literature. These results will be useful to compare with the federalist system described thereafter.

symmetric regions it could consider $t^i = t^j = t$ and $g^i = g^j = g$ to choose the fiscal instruments (t, g, G) by

$$\max_{t,g,G} \sum_{i=a,b} \left[V^i (1+t) + b\left(g^i\right) + B(G)\right] \qquad \text{(UNP)}$$
$$s.t. \sum_{i=a,b} g^i + G = t \sum_{i=a,b} x_{2i}^i (1+t).$$

The first order conditions give us[11]

$$(t) \qquad \sum_{i=a,b} \frac{\partial V^i}{\partial t} +$$
$$\psi \sum_{i=a,b} \left(x_i(\cdot, 0) + t \frac{\partial x_i(\cdot, 0)}{\partial t} \right) = 0 \qquad (4)$$
$$(g) \qquad b' - \psi = 0 \qquad (5)$$
$$(G) \qquad B'(G) - \psi = 0 \qquad (6)$$

for $i = a, b$ where the prime means the first derivative and $b' = b'(g)$. Considering that $\frac{\partial V^i}{\partial t} = -x_2^i$ and (5) into (4) we get the necessary conditions for the second best provision of the local public good by the unitary government as

$$b' = \frac{x_2(\cdot)}{x_2(\cdot) + t\frac{\partial x_2(\cdot)}{\partial t}} \qquad (7)$$

since everything is identical between the two regions. Equation (7) gives us the modified Samuelson condition for the regional provision of public goods by the unitary level and the right hand side of this equation is the *marginal social cost of public fund* $(MSCPF)$ of unitary nation providing also the regional public good. The modified Samuelson condition for the federal provision good is given by the introduction of (4) into (6) and therefore we obtain it as

$$B'(G) = \frac{x_2(\cdot)}{x_2(\cdot) + t\frac{\partial x_2(\cdot)}{\partial t}}. \qquad (8)$$

Observe that the equations (7) and (8) together with the budget constraint characterise the fiscal optimum of unitary government denoted by (t^*, g^*, G^*).

[11] Normally spatial models consider inelastic demand and, therefore, non-distortionary indirect taxes with efficient public good provision.

3 The Structure of the Federal System

In this section we consider a federal jurisdiction with two regions and cross-border shopping. Henceforth, the length of each region is normalised to unity, $\overline{d}^i = 1$, and the border between two regions is represented by the point 0.[12] The interaction between different levels of government is defined by a two-stage game in which, in the first stage, the federal government takes action as a Stackelberg leader and consequently defines its fiscal instruments before the regions themselves. In the second stage, regions as Nash players simultaneously choose their fiscal instruments taking the fiscal policies of both the central government and the other region as given. We solve this game by backward induction, considering the second stage first.

3.1 Regional Government

The government of region i also supplies a public good g^i and finances it by taxing the private good and, eventually, receiving matching grant m^i from the central government. Therefore, the region i behaves as a Nash competitor by choosing fiscal instruments $\left(\tau_{2i}^i, g^i\right)$ to maximise the utility of its residents by considering its budget constraint. We have two cases to distinguish here. The first is $\tau_{2i}^i \geq \tau_{2i}^j$. Then, the balanced budget constraint of i's region government is

$$g^i = \int_{\widehat{d}^i}^{\overline{d}^i} \left[\tau_{2i}^i \left(1 + m^i\right) x_{2i}^i \left(1 + \tau_{2i}^i + T\right)\right] \, \mathrm{d}d^i$$

where the $\widehat{d}^i \left(\tau_{2i}^i + T, \tau_{2i}^j + T\right)$ is the share of population of region i that travels to buy in the other region and m^i is the matching grant on the region tax rate. Observe that for each dollar of a regional tax revenue, the central government brings m^i dollar. It can be of either sign and if $m^i < 0$, this implies that the central government taxes the region $i\prime s$ government.[13] Hence, the region $i's$ problem is:

[12]Other papers have assumed two regions lie on the interval $[-1, 1]$ (Kanbur and Keen, 1993) or $[0, 1]$ (Trandel, 1994). Note that if we consider our interval in absolute terms our analyse converges to the Kanbur and Keen's approach.

[13]We could think in more realistic forms of matching grants, e.g., directly linked to the level of local public good provision. However, the symmetry of regions leads us to consider the implementation of this instrument for efficiency reasons essentially. Therefore, its form does not matter for the derivation of our main results.

$$
\begin{aligned}
\max_{\tau^i_{2i},\ g^i} \quad & \int_{\widehat{d^i}}^{1} \left[V^i\left(1+\tau^i_{2i}+T\right)+b\left(g^i\right)+B\left(G\right)\right]\ \mathrm{d}d^i \\
& + \int_0^{\widehat{d^i}} \left[\widehat{V}_i\left(1+\tau^j_{2i}+T,d^i\right)+b\left(g^i\right)+B\left(G\right)\right]\ \mathrm{d}d^i \quad (RGP) \\
s.t. \quad & g^i=\tau^i_{2i}\left(1+m^i\right)\int_{\widehat{d^i}}^{1} x^i_{2i}\left(1+\tau^i_{2i}+T\right)\ \mathrm{d}d^i
\end{aligned}
$$

Observe that the region i's social welfare function has two parts. The first one is the utility function of the individuals that buy at home and second part is the utility function of the individuals that buy abroad. The first order conditions give us

$$
\begin{aligned}
\left(\tau^i_{2i}\right) \quad & \left(1-\widehat{d^i}\right)\frac{\partial V^i}{\partial \tau_i}+ \\
& \psi^i\left(1+m^i\right)\left[\left(1-\widehat{d^i}\right)\left(x^i_{2i}+\tau^i_{2i}\frac{\partial x^i_{2i}}{\partial \tau^i_{2i}}\right)-\frac{\partial \widehat{d^i}}{\partial \tau^i_{2i}}\tau^i_{2i}x^i_{2i}\right]=0 \quad (9) \\
\left(g^i\right) \quad & b^{i\prime}-\psi^i=0 \quad (10)
\end{aligned}
$$

and consequently, the optimal second best fiscal instruments $\left(\tau^{i*}_{2i},g^i\right)$ when $\tau^i_{2i}\geq\tau^j_{2i}$ and otherwise $\tau^{i*}_{2i}=\tau^j_{2i}$ due to symmetry.

The second case is the one where $\tau^i_{2i}\leq\tau^j_{2i}$. Hence, region $i's$ problem is:

$$
\begin{aligned}
\max_{\tau^i_{2i},\ g^i} \quad & V^i\left(1+\tau^i_{2i}+T\right)+b\left(g^i\right)+B\left(G\right) \\
s.t. \quad & g^i=\tau^i_{2i}\left(1+m^i\right)\left[\int_{\widehat{d^i}}^{1}\left[x^i_{2i}\left(1+\tau^i_{2i}+T\right)\ \mathrm{d}d^i+\right.\right. \quad \text{(RGP2)} \\
& \left.\int_{\widehat{d^j}(\cdot,\cdot)}^{0} x^i_{2j}\left(1+\tau^i_{2j}+T,d^i\right)\ \mathrm{d}d^i\right].
\end{aligned}
$$

Observe that the budget constraint of the regional government is composed by two sources. The first one is the tax revenue reached by the individuals living in region i and the second source is the tax revenue on purchases of individuals living in region j with purchases in region i. The first order conditions give us

$$
\begin{aligned}
(\tau_{2i}^{i}) \qquad & \frac{\partial V^{i}}{\partial \tau_{2i}^{i}}+\psi^{i}\left(1+m^{i}\right)\left[x_{2i}^{i}+\right. \\
& \int_{\widehat{d^{j}}(\cdot,\cdot)}^{0} x_{2j}^{i}\left(\cdot,d^{i}\right)\,\mathrm{d}d^{i}+\tau_{2i}^{i}\frac{\partial x_{2i}^{i}}{\partial \tau_{2i}^{i}}+ \qquad (11)\\
& \left.\tau_{2i}^{i}\frac{\partial x_{2j}^{i}}{\partial \tau_{2i}^{i}}\int_{\widehat{d^{j}}(\cdot,\cdot)}^{0} x_{2j}^{i}\left(\cdot,d^{i}\right)\,\mathrm{d}d^{i}-\tau_{2i}^{i}x_{2j}^{i}\frac{\partial \widehat{d^{i}}}{\partial \tau_{2i}^{i}}\right]=0 \\
(g^{i}) \qquad & b_{i}'-\psi^{i}=0 \qquad (12)
\end{aligned}
$$

and lead also to $\left(\tau_{2i}^{i*},g^{i}\right)$ when $\tau_{2i}^{i}\geq\tau_{2i}^{j}$ and otherwise $\tau_{2i}^{i*}=\tau_{2i}^{j}$ due to the fact that we have symmetric regions in this economy.

Considering Roy's identity again, with some manipulations of the first order conditions (9) and (10), we can obtain

$$
b^{i\prime}=\frac{x_{2i}^{i}}{\left(1+m^{i}\right)\left(x_{2i}^{i}+\underbrace{\tau_{2i}^{i}\frac{\partial x_{2i}^{i}}{\partial \tau_{2i}^{i}}}_{(a)}-\underbrace{\tau_{2i}^{i}x_{2i}^{i}\frac{\partial \widehat{d^{i}}}{\partial \tau_{2i}^{i}}}_{(b)}\right)}. \qquad (13)
$$

The left-hand side (LHS) of equation (13) is the marginal value of public funds and it shows the positive impact of a higher tax rate through the provision of a public good. The right-hand side (RHS) of equation (13) is the marginal cost of public funds ($MCPF$), that is, the marginal loss that the consumer suffers due to one unit of dollar needed to finance the public good by the regional government. The denominator of the RHS shows: (a) the vertical externality that arises due to the tax competition between different levels of government and (b) the horizontal tax competition due to cross-border shopping. Similar results and interpretations could be implemented to the region j in order to get the optimal second best fiscal instruments $\left(\tau_{2j}^{j*},g_{j}^{*}\right)$.

Given that both regions are identical we look only for a subgame perfect equilibria where the federal government chooses in the first stage $m^{i*}=m^{j*}=m^{*}$.[14] In the stage two both regions choose the same tax rate and the same public good provision, $\tau_{2i}^{i*}=\tau_{2j}^{j*}=\tau^{*},g^{i*}=g^{j*}=g^{*}$ (which is a symmetric Nash equilibrium of the stage two). Therefore, no cross-border shopping occurs in equilibrium, that is, $\widehat{d^{i}}(\cdot,\cdot)=0$. Intuitively, the reaction function that arises from RGP and $RGP2$ is continuous since we

[14] We look for an equilibrium where the federal government will not discriminate, at equilibrium, between regions because they are symmetric regions.

have symmetric regions and individuals homogeneously distributed. Since the marginal cost (RHS) and the marginal benefit (LHS) are equalised in absolute terms, we observe no discontinuous jumps in the payoffs. This can be seen straightforwardly for the region i and identical arguments hold for region j. Using these particular characteristics at least one Nash equilibrium must exist and $\tau_{2i}^{i*} = \tau_{2j}^{j*} = \tau^*$ and $g^{i*} = g^{j*} = g^*$ must be a solution.

We can compare the result provided in (13) with one of unitary nation problem given in (7) to analyse the kind of externality that arises due to independent levels of government in a federalist economy. In the regional government problem the region has power to tax and it behaves as a Nash player by considering the central government's instruments and the fiscal instruments of the other region as given. In this economy we continue to verify the vertical externality that arises due to the tax competition between different levels of government and we also verify the horizontal tax competition due to the effect of the cross-border shopping. The commodity tax competition literature shows that horizontal tax competition leads to the underprovision of a public good and vertical tax competition to the overprovision of a public good. Furthermore, the optimum behaviour of two levels of government may imply that the externalities cancel each other out. To internalise these externalities the region i's government may replicate the unitary nation. We can rewrite (13) in a way that makes it directly comparable to the unitary nation optimum condition given in (7). To that end, let

$$b' = \frac{x_2}{x_2 + (\tau + T)\frac{\partial x_2}{\partial \tau} + m^*\left[x_2 + \tau\frac{\partial x_2}{\partial \tau} - \tau x_2 \frac{\partial \widehat{d}}{\partial \tau}\right] - T\frac{\partial x_2}{\partial \tau} - \tau x_2 \frac{\partial \widehat{d}}{\partial \tau}}. \tag{14}$$

To replicate the unitary result we need to satisfy the difference between (7) and (14) by equating it to zero. Hence,

$$m^*\left[x_2 + \tau\frac{\partial x_2}{\partial \tau} - \tau x_2 \frac{\partial \widehat{d}}{\partial \tau}\right] - T\frac{\partial x_2}{\partial \tau} - \tau x_2 \frac{\partial \widehat{d}}{\partial \tau} = 0 \tag{15}$$

where the second term of the LHS is the change of the federal tax revenue (vertical tax externality) and the third term is the loss of the regions' net tax revenue due to the cross-border shopping (horizontal tax externality). More explicitly, due to symmetric regions at the symmetric equilibrium the term (b) in (13)becomes $\tau_{2i}^{j} x_{2i}^{i} \frac{\partial \widehat{d}^{i}}{\partial \tau_{2i}^{j}}$, that is, the externality due to the change in tax revenue in the other region.

As suggested by Dahlby (1996) we may introduce matching grant $m^i = m^j = m^*$ to internalise these tax externalities. In doing so we can establish this matching grant as

$$m^* = \frac{\overbrace{T\frac{\partial x_2}{\partial \tau}}^{(a)} + \overbrace{\tau x_2 \frac{\partial \widehat{d}}{\partial \tau}}^{(b)}}{x_2 + \tau \frac{\partial x_2}{\partial \tau} - \tau x_2 \frac{\partial \widehat{d}}{\partial \tau}} \tag{16}$$

and of course we internalise the externalities. Remember that the sign of m^* defines the direction of this matching grant and we can observe that it will depend basically on a) the impact of the regional tax rate on the federal tax revenue (vertical fiscal externality) and consequently on the price elasticity of demand for x_2, and b) the impact of cross-border shopping on the regional tax revenue (horizontal fiscal externality) that is the impact of the distribution of population and the travel costs on the tax revenue. As (16) suggests and remembering (3), the greater the impact of cross-border shopping on the region tax revenue, the lesser the matching grant needed. However, by considering the two effects, if the former dominates the latter then $m^* < 0$ and so the regions transfer funds to the federal level.

3.2 Federal Government

In this first stage of the game the federal government also supplies a public good G and finances it by taxing the private good and, eventually, by receiving matching grant $m^k, k = i, j.$ from regions. However, it behaves as a Stackelberg leader by taking action before regional governments and therefore able to anticipate their behaviour. Then, the tax rate of the follower is $\tau_{2i}^i \left(T, m^i, m^j\right)$. Considering also that $\tau_{2i}^i \geq \tau_{2i}^j$, the central government solves the following problem:

$$\begin{aligned}
\max_{T,G,m^i,m^j} \quad & \int_{\widehat{d^i}}^1 \left[V^i \left(1 + \tau_{2i}^i + T\right) + b\left(g^i\right) + B\left(G\right)\right] \mathrm{d}d^i \\
& + \int_0^{\widehat{d^i}} \left[\widehat{V}^i \left(1 + x_{2i}^j + T, d^i\right) + b\left(g^i\right) + B\left(G\right)\right] \mathrm{d}d^i \\
& + \left[V^j \left(1 + \tau_{2j}^j + T\right) + b\left(g^j\right) + B\left(G\right)\right] \qquad (FGP) \\
s.t. \quad G = & \int_{\widehat{d^i}}^1 x_{2i}^i \left(1 + \tau_{2i}^i + T\right) \left(T - \tau_{2i}^i m^i\right) \mathrm{d}d^i
\end{aligned}$$

$$+\left(T-\tau^{j}_{2j}m^{j}\right)\left[x^{j}_{2j}\left(1+\tau^{j}_{2j}+T\right)+\int_{0}^{\widehat{d^{i}}}x^{j}_{2i}\left(1+\tau^{i}_{2j}+T,d^{i}\right)\mathrm{d}d^{i}\right].$$

Observe that the federal government worries with the individuals that live in region i and buys in the same region (first term of the social welfare function), with the individuals that lives in region i but buys in the region j (second term) and with individuals that live in region j and buy in the same region (third term). The same interpretation to the three parts in the LHS of the central government's budget constraint given above. This problem has a solution since we have four equations [(7),(8) , the budget constraint of the federal government stated above and $\tau^{i}_{2i}\left(T,m^{i},m^{j}\right)$] and four variables (G,T,m^{i},m^{j}). Observe, as shown in the Appendix, that the solution requires simultaneously that

$$\left(1+m^{i}\right)\tau^{i}_{2i}x^{i}_{2i}\frac{\partial\widehat{d^{i}}}{\partial\tau^{i}_{2i}}=0 \tag{17}$$

and

$$m^{i}\left(x^{i}_{2i}+\tau^{i}_{2i}\frac{\partial x^{i}_{2i}}{\partial\tau^{i}_{2i}}\right)-T\frac{\partial x^{i}_{2i}}{\partial\tau^{i}_{2i}}=0 \tag{18}$$

and consequently $B'(G)=\sum_{i}b'\left(g^{i}\right)$. Observe also that (17) and (18) lead to the optimal matching grant

$$m^{i*}=\frac{T\frac{\partial x^{i}_{2i}}{\partial\tau^{i}_{2i}}+\tau^{i}_{2i}x^{i}_{2i}\frac{\partial\widehat{d^{i}}}{\partial\tau^{i}_{2i}}}{x^{i}_{2i}+\tau^{i}_{2i}\frac{\partial x^{i}_{2i}}{\partial\tau^{i}_{2i}}-\tau^{i}_{2i}x^{i}_{2i}\frac{\partial\widehat{d^{i}}}{\partial\tau^{i}_{2i}}} \tag{19}$$

that is just (16). Due to the symmetry of regions similar interpretations to the region j. Remember that both regions are identical and we look for an equilibrium where the federal government will not discriminate, at equilibrium. So, we look only for a subgame perfect equilibrium where the federal government chooses $m^{i*}=m^{j*}=m^{*}$. In conclusion, we can derive the following proposition:

Proposition 1 *Under specific characteristics of the utility function and symmetric regions the federal and regional governments could replicate the second-best unitary nation results by implementing only matching grants.*

Specific conditions are necessary to obtain this result. The first one is the quasi-linearity on the commodity one of the utility function. This commodity absorbs all income effect and consequently the consumers' revenue do not change independently whether they cross-border or not to shop the commodity two. Second, the separability of the utility function into g and G simplify the analysis. Third, the symmetry of regions leads to $m^{*i} = m^{*j} = m^*$ and thus the federal government does not discriminate between regions for implementing the matching grants.

In conclusion the federal government, by using its fiscal policy instruments optimally, will replicate the second-best unitary nation results. This is straightforward once the federal level (as a first mover) will observe the regions' fiscal instruments and will set its own fiscal instruments in such a way that the vertical and horizontal tax externalities will be internalised. Observe that this optimal matching grant leads to the same unitary result given in (7) , that is, the regional government also internalises the externalities when this matching grant is implemented. In contrast, in Hoyt (2001) tax policy must serve for *adjusting the levels of both services and minimizing* [but no fully internalise] *the fiscal externality.*

The question here is whether the federal government needs to be a Stackelberg leader to internalise the externalities. Observe that (16) and (19) do not include the reaction for the regional taxes with respect to changes in the federal taxes. This implies that the solution could be obtained in Nash or Stackelberg equilibrium. This is so because different levels of benevolent government take action in the same direction. Hence, when the central government maximises the social welfare function it takes into account that the regional government also maximises in part this social welfare function (*envelope theorem*).

4 Concluding Remarks

This paper provides a model with cross-border shopping and both federal and regional levels of government. Empirical evidence suggests that two kinds of externalities are at play in such a context. The first, the horizontal tax externality, arises due to the mobility of agents. The second, the vertical tax externality, is created by the competition for tax bases between the federal government and each regional government. We show that only matching grant could internalise these two kinds of externalities avoiding the use of other kinds of fiscal instruments. Of course, the specification of the model and its symmetry allows one *to kill two birds with one stone* since we have no income effect on the tradable commodity and consequently, the unitary marginal utility of income is determined independently whether the individual cross-border shops or not. Moreover, the symmetry of the model leads to the no differentiation of the regions and consequently a simple fiscal instrument internalises simultaneously both tax externalities.

The model analysed here has its limitations. First, we consider only homogeneous technologies between regions. It may be that asymmetric technologies could lead to different abilities and consequently different levels of demand such that the analysis leads to the other results. Second, regions behave as Nash players. In reality regions and even different countries bargain over transfer sharing or tax revenue. Finally, since we consider symmetric regions as a consequence optimum solution to one region is also an optimum for the other. Considering asymmetric regions would create a further constraint to the federal government, which would have to equalise the utilities across regions with different provision of the public good. To implement that, even the unitary nation should choose different tax rates for each region and this new characteristic would add complexity to our model. From a regional perspective, asymmetric regions in terms of size may be interesting: for a small region cross-border shopping may be more important than for a large region. The counteracting forces between taxes and distances would have different implication for regions. Furthermore, the technical problem of non-existence of equilibrium with asymmetric regions could arise due to the discontinuous jumps in the payoffs induced by cross-border shopping.[15] The present model can be extended to the cases above, but leaves these analyses for future research.

APPENDIX

Derivation of (17), (18) *and* (19) : Using the characteristics of symmetric regions and the envelope theorem with some manipulations the first order conditions of the FGP give us:

$$(T) \quad : \quad B'(G)\left[\sum_i \left(1+m^i\right)x_{2i}^i + \sum_i G_{\tau_{2i}^i}\left(1+\frac{\partial \tau_{2i}^i}{\partial T}\right)\right] -$$

$$\sum_i b'\left(g^i\right)\left[\left(1+m^i\right)x_{2i}^i + \left(1+m^i\right)\tau_{2i}^i x_{2i}^i \frac{\partial \widehat{d^i}}{\partial \tau_{2i}^i}\frac{\partial \tau_{2i}^i}{\partial T}\right] = 0,$$

$$(m^i) \quad : \quad B'(G)\left[\tau_{2i}^i x_{2i}^i - \sum_i \frac{\partial \tau_{2i}^i}{\partial m^i}\left[m^i\left(x_{2i}^i + \tau_{2i}^i\frac{\partial x_{2i}^i}{\partial \tau_{2i}^i}\right) - T\frac{\partial x_{2i}^i}{\partial \tau_{2i}^i}\right]\right] -$$

[15] See Mintz and Tulkens (1986), Kanbur and Keen (1993) and Haufler (1998) for an approach on the possibility of non-existence of equilibrium.

$$\sum_i b'\left(g^i\right)\left[\tau_{2i}^i x_{2i}^i - \left(1+m^i\right)\tau_{2i}^i x_{2i}^i \frac{\partial \widehat{d^i}}{\partial \tau_{2i}^i}\frac{\partial \tau_{2i}^i}{\partial m^i}\right] = 0, i = a, b.$$

In our particular case of symmetric regions which everything is identical between them we will obtain

$$B'(G)\left[\left(1+\frac{\partial \tau_{2i}^i}{\partial T}\right)\left(T\frac{\partial x_{2i}^i}{\partial \tau_{2i}^i} - m^i\left(x_{2i}^i + \tau_{2i}^i\frac{\partial x_{2i}^i}{\partial \tau_{2i}^i}\right)\right) + \left(1+m^i\right)x_{2i}^i\right] =$$

$$\sum_i b'\left(g^i\right)\left[\left(1+m^i\right)x_{2i}^i - \left(1+m^i\right)\tau_{2i}^i x_{2i}^i \frac{\partial \widehat{d}_i}{\partial \tau_{2i}^i}\left(1+\frac{\partial \tau_{2i}^i}{\partial T}\right)\right],$$

$$B'(G)\left[\tau_{2i}^i x_{2i}^i + 2\frac{\partial \tau_{2i}^i}{\partial m^i}\left(m^i\left(x_{2i}^i + \tau_{2i}^i\frac{\partial x_{2i}^i}{\partial \tau_{2i}^i}\right) - T\frac{\partial x_{2i}^i}{\partial \tau_{2i}^i}\right)\right] =$$

$$\sum_i b'\left(g^i\right)\left[\tau_{2i}^i x_{2i}^i - 2\left(1+m^i\right)\tau_{2i}^i x_{2i}^i \frac{\partial \widehat{d^i}}{\partial \tau_{2i}^i}\frac{\partial \tau_{2i}^i}{\partial m^i}\right], i = a, b.$$

Quite clearly, the solution one gets from this set of equations leads to (17) and (18) and consequently to (19).

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