Workshop in Category Theory and Algebraic Topology

at the Université catholique de Louvain, in collaboration with the École polytechnique fédérale de Lausanne

Thursday 10th September 2015

9:15 Registration

9:45 Opening

10:00 **S. Lack**: "Hochschild cohomology, lax descent, and paracyclic structure"

11:00 Coffee

11:30 **J. Kock**: "From quasitoposes to infinity quasitoposes" 12:30 **J. Vercruysse**: "Reconstruction of Hopfish algebras"

13:00 Lunch

15:00 **B. Chorny**: "Relative homotopy theory"

15:30 **P.-A. Jacqmin**: "Bicategories of fractions for groupoids in monadic categories"

16:00 Coffee

16:30 **T. Leinster**: "The categorical origins of entropy"

17:30 End

18:00 Apéro dînatoire

Friday 11th September 2015

9:30 **S. Mantovani**: "Torsion theories for crossed modules"

10:30 Coffee

11:00 **T. Van der Linden**: "Homotopy in semi-abelian categories: an overview"

12:00 V. Even: "Central extensions and closure operators in the category of quandles"

12:30 Lunch

14:00 M. Rovelli: "A looping-delooping adjunction for topological spaces"

14:30 M. Kedziorek: "Accessible model structures"

15:00 Coffee

15:30 E. Riehl: "Model independent infinity-category theory in the homotopy 2-category"

16:30 **D. Zaganidis**: "The universal 2-category containing *n*-composable monad morphisms and its associated nerve"

17:00 End

19:00 Conference dinner

Saturday 12th September 2015

9:30 I. Moerdijk: "Around Quillen's Theorem B"

10:30 Coffee

11:00 K. Werndli: "A cellular version of Blakers-Massey"

11:30 **P. Boavida**: "Classifying spaces of homotopy sheaves"

12:00 **F. Muro**: "How many units may an A_{∞} -algebra have?"

13:00 End

Workshop in Category Theory and Algebraic Topology

registered participants

Miradain Atontsa Nguemo Matija Bašić Pier Giorgio Basile Pedro Boavida Mitchell Buckley Alexander Campbell José Carrasquel Giovanni Caviglia Georgios Charalambous Dimitri Chikhladze Boris Chorny Alan Cigoli Alex Corner Thomas William Coyne Thibault Defourneau

Ivo Dell'Ambrogio Mathieu Duckerts-Antoine Fatima Zahra El Khamouri Tony Elmendorf Valérian Even Timmy Fieremans Aron Fischer

Marino Gran Manfred Hartl

Sina Hazratpour Magnus Hellstrøm-Finnsen

Kathryn Hess Bellwald Pierre-Alain Jacqmin Gabriel Kadjo Magdalena Kedziorek Joachim Kock Stephen Lack Tom Leinster

Guillermo Andrés López Cafaggi Fernando Lucatelli Nunes

Jeroen Maes Sandra Mantovani Giuseppe Metere Ieke Moerdijk Fernando Muro Stephen Nand-Lal Izael do Nascimento Nikitas Nikandros Joost Nuiten

Sophie Raynor Emily Riehl Martina Rovelli Deividas Sabonis

Oriol Raventós

Jérôme Scherer Martina Scolamiero Mark Sioen

Paul Arnaud Songhafouo Tsopmene

Tim Van der Linden Karen Van Opdenbosch Joost Vercruysse Enrico Vitale Kay Werndli

Ryan Wissett Abdoulkader Yacouba Barma

Dimitri Zaganidis

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Workshop in Category Theory and Algebraic Topology

abstracts

Pedro Boavida de Brito (UCL)

Classifying spaces of homotopy sheaves

I will describe how one can associate a classifying space to a homotopy sheaf on the category of manifolds. This involves a universal procedure of turning a homotopy sheaf into a homotopy invariant one and it gives a homotopical strengthening of a result of Madsen-Weiss which states that the set of homotopy classes of maps from a manifold X to the classifying space of a sheaf F is in one-to-one correspondence with the set of concordance classes of sections over X. Examples abound.

Joint work with Daniel Berwick-Evans and Dmitri Pavlov.

Boris Chorny (Haifa)

Relative homotopy theory

Characterization of presheaf categories is a classical problem in category theory. Simple criteria determining if a category is equivalent to a category of presheaves were found by M. Bunge in her Ph.D. thesis.

A similar question in homotopy theory was treated by W.G. Dwyer and D. Kan. They introduced the concept of a *small* subcategory $\mathfrak O$ of *orbits* in a simplicial category $\mathfrak M$, such that $\mathfrak M$ equipped with a *set* of orbits carries a model structure Quillen equivalent to the simplicial presheaves $\mathcal P(\mathfrak O)$ with the projective model structure

In this talk we will present a generalization of Dwyer-Kan theorem. We will show that a category $\mathfrak M$ equipped with a locally small *class* of orbits is Quillen equivalent to the category of *relative* simplicial presheaves $\mathcal P(\mathfrak M, \mathfrak O)$.

As an application we show that the equivariant model structure on the category of diagrams of spaces, introduced by E. Dror Farjoun, is Quillen equivalent to the category of *relative* simplicial presheaves, even though the orbit category is no longer small, extending similar result about Bredon homotopy theories. It also extends our previous result with Dwyer that the category of maps of spaces with the equivariant model structure and the contravariant functors from spaces to spaces are Quillen equivalent.

Valérian Even (UCL) Central extensions and closure operators in the category of quandles

The aim of this talk is to present some recent results concerning two adjunctions: the first one between the category Qnd of quandles and its subcategory Qnd* of trivial quandles, and the second one between the category of quandles and its Mal'tsev subcategory of abelian symmetric quandles. We will show that these adjunctions are admissible in the sense of categorical Galois theory thanks to some results about the permutability of two different classes of congruences in the category of quandles [5,1]. We will then give an algebraic description of the corresponding central extensions [6], which for the first adjunction turn out to correspond [3] to the quandle coverings investigated in [2]. We will also examine closure operators for subobjects in the category of quandles. The regular closure operator and the pullback closure operator both corresponding to the reflector from Qnd to Qnd* coincide [4], and we will give an algebraic description of this closure operator.

Joint work with M. Gran and A. Montoli.

- [1] D. Bourn, A structural aspect of the category of quandles, Cahiers LMPA, n. 513 (2014) Univ. Littoral 34 pp.
- [2] M. Eisermann, Quandle Coverings and their Galois Correspondence, Fund. Math. 225 (2014) 103-167.
- [3] V. Even, A Galois-Theoretic Approach to the Covering Theory of Quandles, Appl. Categ. Structures, 22(5-6) (2014) 817-832.
- [4] V. Even, M. Gran, Closure operators in the category of quandles, to appear in Topology Appl.
- [5] V. Even, M. Gran, On factorization systems for surjective quandle homomorphisms, J. Knot Theory Ramifications, 23, 11, 1450060 (2014).
- [6] G. Janelidze, G. M. Kelly, Galois theory and a general notion of central extension, J. Pure Appl. Algebra, 97 (1994) 135-161.

Pierre-Alain Jacqmin (UCL) Bicategories of fractions for groupoids in monadic categories

The bicategory of fractions [2] of the 2-category of internal groupoids and internal functors in groups with respect to weak equivalences (i.e. functors which are internally full, faithful and essentially surjective [1]) has an easy description: one has just to replace internal functors by monoidal functors [3]. The aim of this talk is to present a generalization of this result from groups to any monadic category over a regular category \mathcal{C} , assuming that the axiom of choice holds in \mathcal{C} . For \mathbb{T} a monad on \mathcal{C} , the bicategory of fractions of $\operatorname{Grpd}(\mathcal{C}^{\mathbb{T}})$ with respect to weak equivalences is now obtained replacing internal functors by what we call \mathbb{T} -monoidal functors.

Joint work with Enrico Vitale.

- [1] M. Bunge, R. Paré, Stacks and equivalence of indexed categories, *Cahiers de Topologie et Géométrie Différentielle Catégorique* 20 (1979) 373–399.
- [2] D. Pronk, Etendues and stacks as bicategories of fractions, Compositio Mathematica 102 (1996) 243–303.
- [3] E.M. Vitale, Bipullbacks and calculus of fractions, *Cahiers de Topologie et Géométrie Différentielle Catégorique* 51 (2010) 83–113.

Magdalena Kedziorek (EPFL)

Accessible model structures

In this talk I will introduce a class of accessible model structures on locally presentable categories, which includes, but is more general than, combinatorial model structures. An accessible model structure is particularly good if one wants to left or right induce it along an adjunction — by a theorem of Burke and Garner [1] the induced weak factorization systems always exist, so one needs to check only a compatibility condition. If it holds then the resulting model structure is again accessible.

One example of an accessible model structure is the Hurewicz model structure on Ch_R , which can be induced to many categories of interest, like algebras, coalgebras, comodules, comodule algebras, coring comodules and bialgebras [2]. I will discuss ideas behind some of the proofs for induced model structures and give specific examples.

Joint work with Kathryn Hess, Emily Riehl and Brooke Shipley.

- [1] J. Bourke, R. Garner, Algebraic weak factorisation systems I: accessible AWFS, arXiv:1412.6559
- [2] K. Hess, M. Kedziorek, E. Riehl, B. Shipley, Accessible model categories, in preparation

Joachim Kock (Barcelona) From

From quasitoposes to infinity quasitoposes

I will explain how the general construction of (Grothendieck) quasitoposes as categories of separated objects can be generalised, in the infinity setting, to be made relative to certain stable factorisation systems, the motivating examples being the (n-connected, n-truncated) factorisation systems, (the classical case being n = -1, the (epi, mono) factorisation system).

Joint work with David Gepner, arXiv:1208.1749.

Steve Lack (Macquarie) Hochschild cohomology, lax descent, and paracyclic structure

Thanks to the universal property of the simplex category Δ , every comonad gives rise to a(n augmented) simplicial set, and one can then linearize and normalize to obtain a chain complex. Many homology theories arise in this way.

On the other hand cyclic homology involves a simplicial object together with an action of the cyclic group in each degree, subject to various conditions. The resulting structure is called a cyclic object, although it turns out that more general things called paracyclic objects can also be used. The fundamental example of a cyclic object is the Hochschild complex of a ring, where in each degree n we have the n-fold tensor power of the ring, and the cyclic group acts on this by cyclically permuting the factors.

Generalizing the classical construction of a simplicial object from a comonad, Böhm and Stefan have shown how to construct paracyclic structure from a pair of comonads with a distributive law between them, along with certain further ingredients.

I shall describe the universal nature of this construction of paracyclic structure, using a 2-categorical version of Hochschild cohomology in which the construction becomes the cap product.

Joint work with Richard Garner and Paul Slevin.

Tom Leinster (Edinburgh)

The categorical origins of entropy

Entropy is fundamental to many parts of mathematics (such as dynamical systems, information theory and probability theory) as well as many branches of applied science, but it is not often considered by the kind of people who attend workshops on category theory and algebraic topology. However, I will show that the concept of Shannon entropy is present in the heartlands of pure mathematics, whether we like it or not.

Specifically, I will describe a categorical machine which, when fed as input the concepts of topological simplex and real number, produces as output the concept of Shannon entropy. The most important component of this machine is the notion of "internal algebra" in an algebra for an operad (generalizing the notion of monoid in a monoidal category). The resulting characterization of Shannon entropy can be stripped completely of its categorical garb, to obtain a simple and entirely elementary characterization. This last theorem is joint work with John Baez and Tobias Fritz (arXiv:1106.1791).

Sandra Mantovani (Milano)

Torsion Theories for Crossed Modules

Following [3], in a category C with pullbacks and pushouts, we consider a full replete subcategory C_0 , both reflective and coreflective in C, as the class of null objects in C. Accordingly, we define constant morphisms, kernels and cokernels (with respect to C_0). We can then extend the well known notion of torsion theory $(\mathcal{T}, \mathcal{F})$ given in a pointed category in [4] (previously introduced in [2] for the abelian case) by using exact sequences (with respect to C_0). This general context includes factorization systems $(\mathcal{E}, \mathcal{M})$ in C as torsion theories in the category of arrows Arr(C) with respect to its subcategory C. In particular, in the category C0 of crossed modules in a semi-abelian category C1, if we consider as null objects the trivial crossed modules C1 modules C2 with respect to its subcategory C3. We will see how this torsion class, whose corresponding torsion free class is given by normal subobjects. We will see how this torsion theory is related to the (pointed) torsion theory in C3 studied in [1] given by the same torsion free class of normal subobjects, but with abelian objects as corresponding torsion class.

- [1] Bourn, D. and Gran, M., Torsion theories in homological categories, J. Algebra 305 (2006) 18-47.
- [2] Dickson, S. E., A torsion theory for Abelian categories, Trans. Amer. Math. Soc. 121 (1966) 223–235.
- [3] Grandis, M., Janelidze, G. and Márki, L., Non-pointed exactness, radicals, closure operators, J. Aust. Math. Soc. 94 (2013) 348–361.
- [4] Janelidze, G. and Tholen, W., Characterization of torsion theories in general categories, Categories in algebra, geometry and mathematical physics, 249256, Contemp. Math., 431, Amer. Math. Soc., Providence, RI, 2007

Ieke Moerdijk (Nijmegen)

Around Quillen's Theorem B

For a functor $F:D\to C$ between small categories, Quillen's Theorem B provides a tool for identifying the homotopy fibre of the induced map $BD\to BC$ between classifying spaces. I will state an extension of this theorem to simplicial categories, and show how this extension can be applied to obtain many classical results (e.g. about the classifying space of the monoid of self-equivalences of a space, or complex Bott periodicity) as well a new results (such as a version of the univalence theorem for universal fibrations).

- [1] D. Quillen, Higher K-Theory I, 1974
- [2] B. Harris, Bott periodicity via simplicial spaces, 1980
- [3] I. Moerdijk, Bisimplicial sets and the group-completion theorem, 1989

Jesper M. Møller (Copenhagen)

Equivariant Euler characteristics

Let \mathcal{C} be a finite category and A a group acting on it. Under appropriate assumptions the equivariant Euler characteristics $\chi_r(\mathcal{C}, A)$ are defined for all $r \geq 0$. The significance of these invariants are unclear in general. However, in the very special case described below they seem to carry some rather interesting information.

Let G be a finite group, p a prime number. Then $\mathcal{C}=\mathcal{S}_G^{p+*}$, the Brown poset of nontrivial p-subgroups of G, is a G-poset under the conjugation action. The Quillen conjecture, $|G|\tilde{\chi}_0(\mathcal{S}_G^{p+*},G)=1\iff O_p(G)\neq 1$, Webb's theorem, $\tilde{\chi}_1(\mathcal{S}_G^{p+*},G)=0$, and the Knörr-Robinson formulation of Alperin's Weight Conjecture, $\tilde{\chi}_2(\mathcal{S}_G^{p+*},G)=z_p(G)$, are all statements about equivariant Euler characteristics. There is at present no interpretation of the higher equivariant Euler characteristic $\chi_r(\mathcal{S}_G^{p+*},G)$ for $r\geq 3$.

Equivariant Euler characteristics are defined in terms of Euler characteristics of centralizer subcategories. In my talk I'd like to present results from my preprint [1] containing an analysis of centralizer subcategories mainly of subgroup categories associated to finite groups.

[1] J.M. Møller, Euler characteristics of centralizer subcategories, arXiv:1502.01317.

Fernando Muro (Sevilla)

How many units may an A-infinity algebra have?

A-infinity algebras are homotopical counterparts of associative algebras. They arose in Stasheff's 1961 thesis [1] in the topological and in the differential graded categories. Since then, *A*-infinity algebras have permeated many fields of mathematics and physics.

We all know that an associative algebra may have at most one unit. What about *A*-infinity algebras? In a homotopical context we are not interested in counting units. We will rather define a sensible space of units for an *A*-infinity algebra and show that it is always contractible for a wide range of base model categories, including chain complexes, groupoids, and spaces. For this, we will make extensive use of the homotopy theory of non-symmetric operads.

[1] Stasheff, James Dillon *Homotopy associativity of H-spaces. I, II.* Trans. Amer. Math. Soc. 108 (1963), 275-292; ibid. 108 1963 293–312.

Emily Riehl (Johns Hopkins) *Model-independent* ∞ -category theory in the homotopy 2-category Quillen's model category axioms provide a well-behaved homotopy category, spanned by the fibrant-co-fibrant objects, in which the poorly behaved notion of weak equivalence is equated with a better behaved notion of homotopy equivalence. For many of the model categories presenting the homotopy theories of models of $(\infty, 1)$ -categories, their homotopy categories can be categorified, defining a homotopy 2-category with certain properties. We explain how the basic category theory of ∞ -categories, objects of some homotopy 2-category, can be developed in a model independent and to a large extent model invariant fashion by working internally to the homotopy 2-category and its associated ∞ -cosmos.

Joint work with Dominic Verity.

- [1] E. Riehl and D. Verity, Fibrations and Yoneda's lemma in an ∞-cosmos (2015), 1–75, arXiv:1506.05500.
- [2] E. Riehl and D. Verity, *Kan extensions and the calculus of modules for* ∞ -categories, in preparation.

Martina Rovelli (EPFL) A looping-delooping adjunction for topological spaces

In [1], Farjoun and Hess introduced *twisted homotopical categories*, a framework for monoidal categories that come with a looping-delooping adjunction, in which a formal theory of bundles is available. This recovers twisted cartesian products for simplicial sets, and twisted tensor products for chain complexes. Although much of this kind of structure was inspired by classical constructions and results holding for topological spaces, it does not not seem possible to construct a full twisted homotopical structure that recovers principal bundles in spaces. However, we provide a *weak twisted homotopical structure*, by showing that the relation between the loop space functor that Milnor introduced in [2] and the classifying space functor is a sort of adjunction between pointed spaces and topological groups. The argument leads to a classification of principal bundles over a fixed space, as a dual version of the well-known classification of bundles with a fixed group. Such a result clarifies the deep relation that exists between the theory of bundles, the classifying space construction and the loop space, which are very important in topological *K*-theory, group cohomology and homotopy theory.

[1] Farjoun, and Hess, Normal and conormal maps in homotopy theory, Homology Homotopy Appl. Vol. 14, Number 1 (2012), 79–112.

[2] Milnor, Construction of universal bundles, I, Annals of Mathematics, Second Series, Vol. 63, Number 2 (1956), 272–284.

Tim Van der Linden (UCL) Homotopy in semi-abelian categories: an overview

In the first part of the talk we give an overview of some major differences between abelian and semi-abelian categories from a homotopical-algebraic point of view. Many constructions which are common in the abelian context become impossible to carry out once the hom-sets lose their additive structure. One way to deal with this problem is to use simplicial techniques [2,4], but perhaps this is not the only solution.

In the second part we focus on joint work-in-progress with Mathieu Duckerts-Antoine towards homotopy of maps between objects of a given semi-abelian category—a non-additive version of the homotopy theory introduced in [1, 3].

Joint work with Mathieu Duckerts-Antoine.

- [1] B. Eckmann, Homotopie et dualité, Colloq. Topologie Algébrique, Louvain (1956), 41–53.
- [2] T. Everaert and T. Van der Linden, Baer invariants in semi-abelian categories II: Homology, Theory Appl. Categ. 12 (2004), no. 4, 195–224.
- [3] P. J. Hilton, Homotopy theory of modules and duality, Proc. Mexico Sympos. (1958), 273–281.
- [4] T. Van der Linden, Simplicial homotopy in semi-abelian categories, J. K-Theory 4 (2009), no. 2, 379–390.

Joost Vercruysse (Bruxelles)

Reconstruction of Hopfish algebras

Bialgebras are algebras that also have a coalgebra structure satisfying an appropriate compatibility condition; Hopf algebras are bialgebras possessing an antipode. It is well-known that bialgebras (resp. Hopf algebras) over a commutative base ring k can be characterised as those k-algebras whose category of modules is monoidal (resp. closed monoidal) such that the forgetful functor to k-modules is strict monoidal (and resp. preserves internal Homs). Motivated by examples arising from non-commutative geometry (especially related to the so-called "quantum torus"), Tang, Weinstein and Zhu introduced a generalization of Hopf algebras which they termed "Hopfish algebras"; roughly in their work the coalgebra and antipode structure morphisms are replaced by bimodules.

Although the equivalent notion of a bialgebra in this new setting is quite obvious, giving a suitable definition of the antipode (that is, defining a Hopf-like object) is more tricky. We advocate another definition than the one which was proposed initially by Tang, Weinstein and Zhu, and show that our notion is invariant under arbitrary Morita equivalence and an appropriate category of representations becomes rigid, which allows an internal characterization and a reconstruction theorem for Hopfish algebras.

Joint work with Kenny De Commer.

[1] X. Tang, A. Weinstein, C. Zhu, Hopfish algebras, Pacific J. Math. 231 (2007), 193–216.

