



# LMAT2120

2014-2015

# Galois theory and groups representations

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|-------------|-----------------|----|
| 5.0 credits | 45.0 h + 15.0 h | 2q |
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| Teacher(s) :        | Tignol Jean-Pierre ; Caprace Pierre-Emmanuel ;  |
| Language :          | Français  |
| Place of the course | Louvain-la-Neuve  |
| Inline resources:   | <p>&amp;lt;!--[cke_protected](C)%3C!%2D%2D%0A%20%2F*%20Font%20Definitions%20*%2F%0A%40font-face%0A%09%7Bfont-family%3A%22Cambria%20Math%22%3B%0A%09panose-1%3A2%204%205%203%205%204%206%203%202%204%3B%0A%09mso-font-charset%3A0%3B%0A%09mso-generic-font-family%3Aauto%3B%0A%09mso-font-pitch%3Avariable%3B%0A%09mso-font-signature%3A3%200%200%200%201%200%3B%7D%0A%40font-face%0A%09%7Bfont-family%3A%22%20E3%83%92%20E3%83%A9%20E3%82%AE%20E3%83%8E%20E8%A7%92%20E3%82%B4%20Pro%20W3%22%3B%0A%09mso-font-charset%3A0%3B%0A%09mso-generic-font-family%3Aroman%3B%0A%09mso-font-pitch%3Aauto%3B%0A%09mso-font-signature%3A0%200%200%200%200%200%3B%7D%0A%20%2F*%20Style%20Definitions%20*%2F%0A%09mso-normal%2C%20li.MsoNormal%2C%20div.MsoNormal%0A%09%7Bmso-style-unhide%3A%3B%0A%09mso-style-qformat%3Ayes%3B%0A%09mso-style-parent%3A%22%22%3B%0A%09margin%3A0cm%3B%0A%09margin-bottom%3A.0001pt%3B%0A%09mso-pagination%3Awidow-orphan%3B%0A%09font-size%3A12.0pt%3B%0A%09font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-fareast-font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-bidi-font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-ansi-language%3AEN-US%3B%0A%09mso-fareast-language%3AEN-US%3B%7D%0A%09mso-style-parent%3A%22%22%3B%0A%09margin%3A0cm%3B%0A%09margin-bottom%3A.0001pt%3B%0A%09mso-pagination%3Awidow-orphan%3B%0A%09font-size%3A12.0pt%3B%0A%09mso-bidi-font-size%3A10.0pt%3B%0A%09font-family%3AHelvetica%3B%0A%09mso-fareast-font-family%3A%22%20E3%83%92%20E3%83%A9%20E3%82%AE%20E3%83%8E%20E8%A7%92%20E3%82%B4%20Pro%20W3%22%3B%0A%09mso-bidi-font-family%3A%22Times%20New%20Roman%22%3B%0A%09color%3Ablack%3B%0A%09mso-ansi-language%3AFR%3B%7D%0A.MsoChpDefault%0A%09%7Bmso-style-type%3Aexport-only%3B%0A%09mso-default-props%3Ayes%3B%0A%09font-size%3A10.0pt%3B%0A%09mso-ansi-font-size%3A10.0pt%3B%0A%09mso-bidi-font-size%3A10.0pt%3B%7D%0A%40page%20WordSection1%0A%09%7Bsize%3A612.0pt%20792.0pt%3B%0A%09margin%3A70.85pt%2070.85pt%2070.85pt%2070.85pt%3B%0A%09mso-header-margin%3A36.0pt%3B%0A%09mso-footer-margin%3A36.0pt%3B%0A%09mso-paper-source%3A0%3B%7D%0Adiv.WordSection1%0A%09%7Bpage%3AWordSection1%3B%7D%0A%2D%2D%3E--&amp;gt;</p> <p>Website iCampus (&gt; <a href="http://icampus.uclouvain.be/">http://icampus.uclouvain.be/</a>).</p> |
| Prerequisites :     | <p>&amp; mp;lt;!--[cke_protected](C)%3C!%2D%2D%0A%20%2F*%20Font%20Definitions%20*%2F%0A%40font-face%0A%09%7Bfont-family%3A%22Cambria%20Math%22%3B%0A%09panose-1%3A2%204%205%203%205%204%206%203%202%204%3B%0A%09mso-font-charset%3A0%3B%0A%09mso-generic-font-family%3Aauto%3B%0A%09mso-font-pitch%3Avariable%3B%0A%09mso-font-signature%3A3%200%200%200%201%200%3B%7D%0A%40font-face%0A%09%7Bfont-family%3A%22%20E3%83%92%20E3%83%A9%20E3%82%AE%20E3%83%8E%20E8%A7%92%20E3%82%B4%20Pro%20W3%22%3B%0A%09mso-font-charset%3A0%3B%0A%09mso-generic-font-family%3Aroman%3B%0A%09mso-font-pitch%3Aauto%3B%0A%09mso-font-signature%3A0%200%200%200%200%200%3B%7D%0A%20%2F*%20Style%20Definitions%20*%2F%0A%09mso-normal%2C%20li.MsoNormal%2C%20div.MsoNormal%0A%09%7Bmso-style-unhide%3A%3B%0A%09mso-style-qformat%3Ayes%3B%0A%09mso-style-parent%3A%22%22%3B%0A%09margin%3A0cm%3B%0A%09margin-bottom%3A.0001pt%3B%0A%09mso-pagination%3Awidow-orphan%3B%0A%09font-size%3A12.0pt%3B%0A%09font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-fareast-font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-bidi-font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-ansi-language%3AEN-US%3B%0A%09mso-fareast-language%3AEN-US%3B%7D%0A%09mso-style-parent%3A%22%22%3B%0A%09margin%3A0cm%3B%0A%09margin-bottom%3A.0001pt%3B%0A%09mso-pagination%3Awidow-orphan%3B%0A%09font-size%3A12.0pt%3B%0A%09font-family%3A%22Times%20New%20Roman%22%3B%0A%09color%3Ablack%3B%0A%09mso-ansi-language%3AFR%3B%7D%0A.MsoChpDefault%0A%09%7Bmso-style-type%3Aexport-only%3B%0A%09mso-default-props%3Ayes%3B%0A%09font-size%3A10.0pt%3B%0A%09mso-ansi-font-size%3A10.0pt%3B%0A%09mso-bidi-font-size%3A10.0pt%3B%7D%0A%40page%20WordSection1%0A%09%7Bsize%3A612.0pt%20792.0pt%3B%0A%09margin%3A70.85pt%2070.85pt%2070.85pt%2070.85pt%3B%0A%09mso-header-margin%3A36.0pt%3B%0A%09mso-footer-margin%3A36.0pt%3B%0A%09mso-paper-source%3A0%3B%7D%0Adiv.WordSection1%0A%09%7Bpage%3AWordSection1%3B%7D%0A%2D%2D%3E--&amp; mp;gt;</p> <p>LMAT1131 - linear algebra, or equivalent course.<br/>                 LMAT1231 - multilinear algebra and group theory, or equivalent course.<br/>                 LMAT1331 - commutative algebra, or equivalent course.</p>  |

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| <p>Main themes :</p> | <p>&amp; mp;lt;!--(cke_protected){C}%3C!%2D%2D%0A%20%2F%*20Font%20Definitions%20*%2F%0A%40font-face%0A%09%7Bfont-family%3A%22Cambria%20Math%22%3B%0A%09panose-1%3A2%204%205%203%205%204%206%203%202%204%3B%0A%09mso-font-charset%3A0%3B%0A%09mso-generic-font-family%3Aauto%3B%0A%09mso-font-pitch%3Avariable%3B%0A%09mso-font-signature%3A3%200%200%200%201%200%3B%7D%0A%40font-face%0A%09%7Bfont-family%3A%22E3%83%92%E3%83%A9%E3%82%AE%E3%83%8E%E8%A7%92%E3%82%B4%20Pro%20W3%22%3B%0A%09mso-font-charset%3A0%3B%0A%09mso-generic-font-family%3Aroman%3B%0A%09mso-font-pitch%3Aauto%3B%0A%09mso-font-signature%3A0%200%200%200%200%200%3B%7D%0A%20%2F%*20Style%20Definitions%20*%2F%0A%09mso-normal%2C%20li.MsoNormal%2C%20div.MsoNormal%0A%09%7Bmso-style-unhide%3Ano%3B%0A%09mso-style-qformat%3Ayes%3B%0A%09mso-style-parent%3A%22%22%3B%0A%09margin%3A0cm%3B%0A%09margin-bottom%3A.0001pt%3B%0A%09mso-pagination%3Awidow-orphan%3B%0A%09font-size%3A12.0pt%3B%0A%09font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-fareast-font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-bidi-font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-ansi-language%3AEN-US%3B%0A%09mso-fareast-language%3AEN-US%3B%7D%0A%09mso-style-name%3A%22Corps%20A%22%3B%0A%09mso-style-update%3Aauto%3B%0A%09mso-style-unhide%3Ano%3B%0A%09mso-style-parent%3A%22%22%3B%0A%09margin%3A0cm%3B%0A%09margin-bottom%3A.0001pt%3B%0A%09mso-pagination%3Awidow-orphan%3B%0A%09font-size%3A12.0pt%3B%0A%09mso-bidi-font-size%3A10.0pt%3B%0A%09font-family%3AHelvetica%3B%0A%09mso-fareast-font-family%3A%22E3%83%92%E3%83%A9%E3%82%AE%E3%83%8E%E8%A7%92%E3%82%B4%20Pro%20W3%22%3B%0A%09mso-bidi-font-family%3A%22Times%20New%20Roman%22%3B%0A%09color%3Ablack%3B%0A%09mso-ansi-language%3AFR%3B%7D%0A.MsoChpDefault%0A%09%7Bmso-style-type%3Aexport-only%3B%0A%09mso-default-props%3Ayes%3B%0A%09font-size%3A10.0pt%3B%0A%09mso-ansi-font-size%3A10.0pt%3B%0A%09mso-bidi-font-size%3A10.0pt%3B%7D%0A%40page%20WordSection1%0A%09%7Bsize%3A612.0pt%20792.0pt%3B%0A%09margin%3A70.85pt%2070.85pt%2070.85pt%2070.85pt%3B%0A%09mso-header-margin%3A36.0pt%3B%0A%09mso-footer-margin%3A36.0pt%3B%0A%09mso-paper-source%3A0%3B%7D%0Adiv.WordSection1%0A%09%7Bpage%3AWordSection1%3B%7D%0A%2D%2D%3E--&amp; mp;gt;</p> <p>Galois theory : field extensions and their automorphisms ; translation of field extension properties into properties of the associated groups and application to some classical problems (solubility of equations by radicals and ruler and compass constructions).<br/>Group representations : character of a linear representation ; group algebras and induced representations.</p> |
| <p>Aims :</p>        | <p>&amp; mp;lt;!--(cke_protected){C}%3C!%2D%2D%0A%20%2F%*20Font%20Definitions%20*%2F%0A%40font-face%0A%09%7Bfont-family%3A%22Cambria%20Math%22%3B%0A%09panose-1%3A2%204%205%203%205%204%206%203%202%204%3B%0A%09mso-font-charset%3A0%3B%0A%09mso-generic-font-family%3Aauto%3B%0A%09mso-font-pitch%3Avariable%3B%0A%09mso-font-signature%3A3%200%200%200%201%200%3B%7D%0A%40font-face%0A%09%7Bfont-family%3A%22E3%83%92%E3%83%A9%E3%82%AE%E3%83%8E%E8%A7%92%E3%82%B4%20Pro%20W3%22%3B%0A%09mso-font-charset%3A0%3B%0A%09mso-generic-font-family%3Aroman%3B%0A%09mso-font-pitch%3Aauto%3B%0A%09mso-font-signature%3A0%200%200%200%200%200%3B%7D%0A%20%2F%*20Style%20Definitions%20*%2F%0A%09mso-normal%2C%20li.MsoNormal%2C%20div.MsoNormal%0A%09%7Bmso-style-unhide%3Ano%3B%0A%09mso-style-qformat%3Ayes%3B%0A%09mso-style-parent%3A%22%22%3B%0A%09margin%3A0cm%3B%0A%09margin-bottom%3A.0001pt%3B%0A%09mso-pagination%3Awidow-orphan%3B%0A%09font-size%3A12.0pt%3B%0A%09font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-fareast-font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-bidi-font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-ansi-language%3AEN-US%3B%0A%09mso-fareast-language%3AEN-US%3B%7D%0A%09mso-style-name%3A%22Corps%20A%22%3B%0A%09mso-style-unhide%3Ano%3B%0A%09mso-style-parent%3A%22%22%3B%0A%09margin%3A0cm%3B%0A%09margin-bottom%3A.0001pt%3B%0A%09mso-pagination%3Awidow-orphan%3B%0A%09font-size%3A12.0pt%3B%0A%09mso-bidi-font-size%3A10.0pt%3B%0A%09mso-bidi-font-family%3AHelvetica%3B%0A%09mso-fareast-font-family%3A%22E3%83%92%E3%83%A9%E3%82%AE%E3%83%8E%E8%A7%92%E3%82%B4%20Pro%20W3%22%3B%0A%09mso-bidi-font-family%3A%22Times%20New%20Roman%22%3B%0A%09color%3Ablack%3B%0A%09mso-ansi-language%3AFR%3B%7D%0A%09mso-ansi-font-size%3A10.0pt%3B%0A%09mso-bidi-font-size%3A10.0pt%3B%7D%0A%40page%20WordSection1%0A%09%7Bsize%3A612.0pt%20792.0pt%3B%0A%09margin%3A70.85pt%2070.85pt%2070.85pt%2070.85pt%3B%0A%09mso-header-margin%3A36.0pt%3B%0A%09mso-footer-margin%3A36.0pt%3B%0A%09mso-paper-source%3A0%3B%7D%0Adiv.WordSection1%0A%09%7Bpage%3AWordSection1%3B%7D%0A%2D%2D%3E--&amp; mp;gt;</p> <p>Contribution of the course to learning outcomes in the Master in Mathematics programme. By the end of this activity, students will have made progress in:</p>   |

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|                                    | <p>- mastering the disciplinary knowledge and basic transferable skills whose acquisition began in the Bachelor programme They will have expanded their basic disciplinary knowledge and skills, notably in</p> <ul style="list-style-type: none"> <li>-- recognizing the fundamental concepts of important current mathematical theories ;</li> <li>-- establishing the main connections between these theories, analysing them and explaining them through the use of examples.</li> </ul> <p>- showing evidence of abstract thinking and of a critical spirit :</p> <ul style="list-style-type: none"> <li>-- recognizing the fundamental concepts of important current mathematical theories ;</li> <li>-- identifying the unifying aspects of different situations and experiences ;</li> <li>-- arguing within the context of the axiomatic method ;</li> <li>-- constructing and drawing up a proof independently, clearly, and rigorously.</li> </ul> <p>Learning outcomes specific to the course.</p> <p>By the end of this activity, students should be able to use the methods of abstract algebra to analyse questions that display a high degree of symmetry and those in which the rationality domain plays an important role, such as the solvability of equations by radicals or ruler and compass constructions. Historical aspects will also be discussed. Particular emphasis will be set on techniques that use the representation of symmetry groups by linear operators.</p> <p><i>The contribution of this Teaching Unit to the development and command of the skills and learning outcomes of the programme(s) can be accessed at the end of this sheet, in the section entitled "Programmes/courses offering this Teaching Unit".</i></p>  |
| <p><b>Evaluation methods :</b></p> | <p>Assessment is by written examination. The examination tests knowledge and understanding of fundamental concepts, examples and results, ability to construct a coherent argument, and mastery of the techniques of proof introduced during the course.</p>  |
| <p><b>Teaching methods :</b></p>   | <p>The course is taught through lectures and practical exercises. In the practical exercise sessions, students will be asked to make suggestions and formulate ideas in order to further the course on the basis of their prior knowledge.</p>  |
| <p><b>Content :</b></p>            | <p>&amp; mp;lt;!--{cke_protected}(C)%3C!%2D%2D%0A%20%2F%*%20Font%20Definitions%20*%2F%0A%40font-face%0A%09%7Bfont-family%3A%22Cambria%20Math%22%3B%0A%09panose-1%3A2%204%205%203%205%204%206%203%202%204%3B%0A%09mso-font-charset%3A0%3B%0A%09mso-generic-font-family%3Aauto%3B%0A%09mso-font-pitch%3Avariable%3B%0A%09mso-font-signature%3A3%200%200%200%201%200%3B%7D%0A%40font-face%0A%09%7Bfont-family%3A%22%22E3%83%92%E3%83%A9%E3%82%AE%E3%83%8E%E8%A7%92%E3%82%B4%20Pro%20W3%22%3B%0A%09mso-font-charset%3A0%3B%0A%09mso-generic-font-family%3Aroman%3B%0A%09mso-font-pitch%3Aauto%3B%0A%09mso-font-signature%3A0%200%200%200%200%200%3B%7D%0A%20%2F%*%20Style%20Definitions%20*%2F%0A%09mso-normal%2C%20div.MsoNormal%0A%09%7Bmso-style-unhide%3A%20%3B%0A%09mso-style-qformat%3Ayes%3B%0A%09mso-style-parent%3A%22%22%3B%0A%09margin%3A0cm%3B%0A%09margin-bottom%3A.0001pt%3B%0A%09mso-pagination%3Awidow-orphan%3B%0A%09font-size%3A12.0pt%3B%0A%09font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-fareast-font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-bidi-font-family%3A%22Times%20New%20Roman%22%3B%0A%09mso-ansi-language%3AEN-US%3B%0A%09mso-fareast-language%3AEN-US%3B%7D%0A%09mso-style-name%3A%22Corps%20A%22%3B%0A%09mso-style-update%3Aauto%3B%0A%09mso-style-unhide%3A%20%3B%0A%09mso-style-parent%3A%22%22%3B%0A%09margin%3A0cm%3B%0A%09margin-bottom%3A.0001pt%3B%0A%09mso-pagination%3Awidow-orphan%3B%0A%09font-size%3A12.0pt%3B%0A%09mso-bidi-font-size%3A10.0pt%3B%0A%09font-family%3AHelvetica%3B%0A%09mso-fareast-font-family%3A%22E3%83%92%E3%83%A9%E3%82%AE%E3%83%8E%E8%A7%92%E3%82%B4%20Pro%20W3%22%3B%0A%09mso-bidi-font-family%3A%22Times%20New%20Roman%22%3B%0A%09color%3Ablack%3B%0A%09mso-ansi-language%3AFR%3B%7D%0A.MsoChpDefault%0A%09%7Bmso-style-type%3Aexport-only%3B%0A%09mso-default-props%3Ayes%3B%0A%09font-size%3A10.0pt%3B%0A%09mso-ansi-font-size%3A10.0pt%3B%0A%09mso-bidi-font-size%3A10.0pt%3B%7D%0A%40page%20WordSection1%0A%09%7Bsize%3A612.0pt%20792.0pt%3B%0A%09margin%3A70.85pt%2070.85pt%2070.85pt%2070.85pt%3B%0A%09mso-header-margin%3A36.0pt%3B%0A%09mso-footer-margin%3A36.0pt%3B%0A%09mso-paper-source%3A0%3B%7D%0Adiv.WordSection1%0A%09%7Bpage%3AWordSection1%3B%7D%0A%2D%2D%3E--&amp; mp;gt;</p> <p>The course presents the basic notions of Galois theory and the theory of linear representations of finite groups. The following topics are discussed in the course of the lectures :</p> <ul style="list-style-type: none"> <li>- Algebraic field extensions, minimal polynomials ;</li> <li>- Galois correspondence between intermediate extensions of a Galois extension and subgroups of its Galois group ;</li> <li>- Solvability of equations by radicals and ruler and compass constructions ;</li> <li>- Decomposition of linear representations into direct sums of irreducible representations ;</li> <li>- Character theory ;</li> <li>- Group algebras and induced representations.</li> </ul> |

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| <p>Cycle and year of study :</p>    | <p><a href="#">&gt; Master [60] in Mathematics</a><br/> <a href="#">&gt; Master [120] in Mathematics</a></p>   |
| <p>Faculty or entity in charge:</p> | <p>MATH</p>  |