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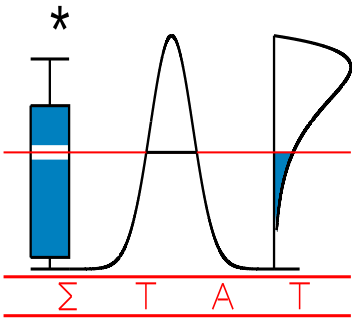

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How to Measure the Impact of Environmental Factors in a Nonparametric Production Model?

Luiza Bădin Cinzia Daraio Léopold Simar*

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Abstract:

The measurement of technical efficiency of decision making units is useful for making comparisons and informing managers and policy makers on existing differentials and potential improvements across a sample of analyzed units. The step further is to relate the obtained efficiency estimates to some external or environmental variables which may influence the production process, affect the performances and explain the efficiency differentials. Conditional efficiency measures (Daraio and Simar, 2005; 2007a), including conditional FDH, conditional DEA, conditional order$-m$ and conditional order$-\alpha$, have been recently introduced and became rapidly a useful tool to explore the impact of external-environmental factors on the performance of Decision Making Units in a nonparametric framework. In this paper, we show that analyzing these conditional efficiency scores we can disentangle the impact of these factors on the production process in its components: impact on the attainable set in the input $\times$ output space, and/or impact on the distribution of the inefficiency scores. We extend existing methodological tools to investigate these interrelationships, both from an individual and a global perspective. We emphasize the usefulness of regressing the conditional efficiencies on the explaining factors. The analysis of the residuals provides a measure of efficiency whitened from the main effect of the environmental factors. This allows to rank the firms according to their "managerial" efficiency, even when facing heterogeneous environmental conditions. Our approach is illustrated through simulated samples and with a real data set in the Banking industry.

Keywords: Data Envelopment Analysis (DEA), conditional efficiency measures, nonparametric frontiers, managerial efficiency, two-stage efficiency analysis, Banking industry

JEL Classification: C14, C40, C60, D20

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1 Introduction and Basic Notations

In productivity analysis, one is interested in the evaluation of the performances of firms to identify inefficient units where improvements could help to increase their profitability or to reduce their costs. Most of the efficiency analysis literature focused on the estimation of the production frontier, which provides the benchmark against which the economic producers are evaluated. Nevertheless, a very important component, that recent studies are more and more concerned with, is the explanation of efficiency differentials by including in the analysis exogenous variables or environmental factors, that cannot be controlled by the producer, but may influence the production process. From a managerial point of view, it is important to identify the "particularities" of the production process or the economic conditions that might be responsible for inefficiency as well as to detect and analyze possible influential factors that can determine changes in productivity patterns. The choice of the environmental variables has to be done on a case-by-case basis, by taking into account the economic field of application.

In this paper, we choose a nonparametric production model (Cazals et al., 2002 and Daraio and Simar, 2005) where the role of these environmental factors is explicitly introduced in a non-restrictive way. Then we will explain how in this framework, we can measure and infer about the impact of these factors on the production process. By doing so, we will develop previously introduced tools and suggest their extensions. In particular, we emphasize the usefulness of regressing the conditional efficiencies on the explaining factors. The analysis of the residuals provides a measure of efficiency whitened from the main effect of the environmental factors, allowing to rank the firms according to their "managerial" efficiency, even when facing heterogeneous environmental conditions.

We will first introduce the notations and the basic assumptions on the Data Generating Process (DGP) characterizing the production process in the presence of environmental factors. Let $X \in \mathbb{R}_+^p$ denote the vector of inputs and let $Y \in \mathbb{R}_+^q$ denote the vector of outputs. We consider a vector of environmental factors $Z \in \mathcal{Z} \subset \mathbb{R}^r$ that may influence the process and the productivity patterns. Firms transform quantities of inputs into outputs, but the environmental variables may affect this process. Let $(\Omega, \mathcal{A}, \mathbb{P})$ be the probability space on which the random variables are defined, we denote by $\mathcal{P}$ the support of the joint distribution of (X, Y, Z) and we denote a particular DGP by $P \in \mathbb{P}$.

A large part of the literature on this topic has been focused on the so-called 2-stage analysis, where typically, some first stage estimates of the efficiency of the firms are regressed in a second stage on these additional factors to investigate their effect on efficiency. Simar and Wilson (2007) clarified that these two stages approaches are restricted to models where these factors do not influence the shape of the production set (this is the "separability" condition detailed in the following). Banker and Natarajan (2008) suggest another model

where a two-stage approach is valid but the model heavily depends on quite restrictive and unrealistic assumptions on the production process, as described and commented in details in Simar and Wilson (2011b). If the 2-stage approach is validated by the appropriate test (see Daraio et al., 2010), one can indeed in a first stage estimate the efficiency scores of the units relative to the boundary of the unconditional attainable set in the inputs $\times$ outputs space and then regress, in a second stage, the obtained efficiencies on the environmental factors. We know that even if an appropriate model is used (Logit, Truncated Normal, Nonparametric truncated regression,...), the inference on the impact of Z on the efficiency measures has to be carefully conducted, using adapted bootstrap techniques (see Simar and Wilson, 2007 and 2011b for details).

The impact and influence of Z on the production process may be multiple and can be quite different from one application to the other. The effect of Z on the production may either affect the range of achievable values for the couples (X, Y), including the shape of the boundaries of the attainable set, or it may only affect the distribution of the inefficiencies inside a set with boundaries not depending on Z (only the probability of being more or less far from the efficient frontier may depend on Z) or it can affect both. Finally, the environmental factors Z may also be completely independent of (X, Y).

Daraio and Simar (2005) extending previous work of Cazals et al. (2002), provide a quite general and unrestricted framework to investigate the joint behavior of (X, Y, Z) from a productivity point of view. They consider a probability model that generates the variables (X, Y, Z) where the conditional distribution of (X, Y) given a particular value of Z will be of particular interest. This conditional process can be described by

$$H(x, y|z) = \text{Prob}(X \leq x, Y \geq y|Z = z), \tag{1.1}$$

or any equivalent variation of it (the joint conditional density function or the joint conditional cumulative distribution function, ...). The function $H(x, y|z)$ is simply the probability for a unit operating at level (x, y) to be dominated by firms facing the same environmental conditions z. Given that $Z = z$, the range of possible combinations of inputs $\times$ outputs, Ψ^z, is the support of $H(x, y|z)$:

$$\Psi^z = \{(x, y)|\, Z = z,\ x \text{ can produce } y\}, \tag{1.2}$$

If $H(x, y)$ denotes the unconditional probability of being dominated, we have

$$H(x, y) = \int_{\mathcal{Z}} H(x, y|z)\, f_Z(z)\, dz, \tag{1.3}$$

having support Ψ, the marginal (unconditional) attainable set defined as

$$\Psi = \{(x, y)|\, x \text{ can produce } y\} = \bigcup_{z \in \mathcal{Z}} \Psi^z. \tag{1.4}$$

Remember that the joint support of the variables (X, Y, Z) is denoted by $\mathcal{P}$. It is clear that, by construction, for all $z \in \mathcal{Z}$, $\Psi^z \subseteq \Psi$.

The "separability" condition, described in Simar and Wilson (2007) states that the support of (X, Y) is not dependent of Z, equivalently

$$\text{"Separability" condition:} \quad \Psi^z = \Psi, \quad \text{for all } z \in \mathcal{Z}. \tag{1.5}$$

In this latter case, the support of (X, Y, Z) can be written as $\mathcal{P} = \Psi \times \mathcal{Z}$, where $\times$ represents the cartesian product. As clearly illustrated by Figures 1 and 2 in Simar and Wilson (2011b), it is important to understand the implications of condition (1.5). If the condition is verified, the only potential remaining impact of the environmental factors on the production process may be on the distribution of the efficiencies. This justifies the use of 2-stage approaches as illustrated in Simar and Wilson (2007). If the condition (1.5) is not verified, the measure of the distance of a unit (x, y) to the boundary of Ψ, even if it can be well defined and estimated (see details below), has little economic interest, because it ignores the heterogeneity introduced by Z on the attainable sets of values for (X, Y).

Whether or not Ψ^z is independent of z is an empirical issue and Daraio et al. (2010) provide a statistical procedure to test this hypothesis. The test is a "global" test of separability since it tests the null hypothesis $\Psi^z = \Psi$, $\forall z \in \mathcal{Z}$ against its complement: $\exists z \in \mathcal{Z}$ such that $\Psi^z \neq \Psi$.

As described e.g. in Daraio and Simar (2007a), the two measures $H(x, y|z)$ and $H(x, y)$ allow to define conditional and marginal efficiency scores that can be estimated by nonparametric methods. The comparison of the conditional and marginal efficiency scores can be used to investigate the impact of Z on the production process. One of the objectives of this paper is to clarify what can be learned from the analysis of these conditional efficiency scores. We will also focus on the particular role of efficiency scores relative to partial order frontiers (order-m frontiers from Cazals et al., 2002 and order-α quantile type frontiers from Daouia and Simar, 2006), that not only provide robust versions of the efficient frontier, but also allow to investigate different aspects of the role of Z on the production process.

In this paper, we propose also a regression-type procedure allowing to make inference on the impact of Z on the conditional efficiency scores. Confidence intervals for the local impact of Z will be obtained by adapting the subsampling ideas from Simar and Wilson (2011a). The latter analysis can be seen as a 2-stage method, as described above, but with the great difference that here, the object regressed on Z (the conditional efficiency) is economically well defined. The unexplained part of the conditional efficiencies can then be interpreted as a measure of "managerial" efficiency allowing to rank the performance of firms facing different environmental conditions.

The paper is organized as follows. Section 2 reviews the basic definition of marginal and conditional efficiency scores, with respect to full frontier and also to more robust partial

frontiers. Then Section 3 explains how we can disentangle the impact of external factors on the production process (i.e. impact on the support of the production set and impact on the distribution of the inefficiency scores) by the analysis of conditional and unconditional efficiencies and by their comparison. In addition, we propose a flexible model to try to whiten the conditional efficiencies from the effect of Z, in order to derive a measure of "managerial" efficiency. Section 4, provides the various nonparametric estimates of the quantities of interest and offers useful guidelines to conduct inference, by using the bootstrap. We illustrate the procedure with simulated data set, and we apply the approach to a real data set in the banking sector in Section 5.2. Section 6 summarizes the main findings and concludes the paper.

2 Marginal and Conditional Efficiency Measures

2.1 Farrell Efficiency scores

The literature on efficiency analysis propose several ways for measuring the distance of a firm operating at the level (x_0, y_0) to the efficient boundary of the attainable set. In the lines of the pioneering work of Debreu (1950), Farrell (1957) and Shephard (1970), radial distances became very popular in the efficiency literature. They can be input or output oriented (maximal radial contraction of the inputs or maximal radial expansion of the outputs to reach the efficient boundary). Recently, Färe et al. (1985) introduced hyperbolic radial distances that avoid some of the ambiguity in choosing output or input orientation. In this case, input and output levels are adjusted simultaneously. These radial measures can be defined as follows:

$$\theta(x_0, y_0) = \inf\{\theta > 0 | (\theta x_0, y_0) \in \Psi\} \tag{2.1}$$

$$\lambda(x_0, y_0) = \sup\{\lambda > 0 | (x_0, \lambda y_0) \in \Psi\} \tag{2.2}$$

$$\gamma(x_0, y_0) = \sup\{\gamma > 0 | (\gamma^{-1} x_0, \gamma y_0) \in \Psi\}. \tag{2.3}$$

In this section, we limit the technical presentation with the output orientation, but it is easy to adapt the formulae to the input oriented and to the hyperbolic cases. From Cazals et al. (2002) and Daraio and Simar (2005), we know that under the assumption of free disposability of the inputs and of the outputs, these measures can be characterized by some appropriate probability function determined by $H(x, y)$. We have, for the marginal Farrell output measure of efficiency,

$$\lambda(x_0, y_0) = \sup\{\lambda > 0 | S_{Y|X}(\lambda y_0 | X \leq x_0) > 0\}, \tag{2.4}$$

where $S_{Y|X}(y_0|X \leq x_0) = \text{Prob}(Y \geq y_0|X \leq x_0) = \dfrac{H(x_0, y_0)}{H(x_0, 0)}$ is the (nonstandard) conditional survival function of Y, nonstandard because the condition is $X \leq x_0$ and not $X = x_0$.

If the firm is facing environmental factors $Z = z_0$, then Daraio and Simar (2005) define the conditional Farrell output measure of efficiency as

$$\begin{aligned} \lambda(x_0, y_0|z_0) &= \sup\{\lambda > 0|(x_0, \lambda y_0) \in \Psi^{z_0}\} && (2.5) \\ &= \sup\{\lambda > 0|S_{Y|X,Z}(\lambda y_0|X \leq x_0, Z = z_0) > 0\}, && (2.6) \end{aligned}$$

where $S_{Y|X,Z}(y_0|X \leq x_0, Z = z_0) = \text{Prob}(Y \geq y_0|X \leq x_0, Z = z_0) = \dfrac{H(x_0, y_0|z_0)}{H(x_0, 0|z_0)}$ is the conditional survival function of Y, here we condition on $X \leq x_0$ and $Z = z_0$. Since for all $z_0 \in \mathcal{Z}$, $\Psi^{z_0} \subseteq \Psi$, we have for all $(x_0, y_0, z_0) \in \mathcal{P}$ the relations $1 \leq \lambda(x_0, y_0|z_0) \leq \lambda(x_0, y_0)$.

Daraio et al. (2010) uses these two measures to conduct a global test of separability. In their approach, using unconditional and conditional efficiency measures, they propose to estimate (by using FDH or DEA techniques) a mean integrated square difference between the boundaries of $\mathcal{P}$ and $\Psi \times \mathcal{Z}$. This provide a test statistic whose sampling distribution is approximated by the bootstrap.

2.2 Partial order Frontiers

Partial frontiers, and the resulting partial efficiency scores, have been proposed to provide robust measures of efficiencies, robust to extreme data points or outliers (a survey and a detailed analysis of these approaches can be found in Daraio and Simar, 2007a). In our setup here, this remains true when we will use partial frontiers of extreme orders, as explained below. However, when using partial frontiers of lower order, we will see that we obtain useful complementary information on the impact of Z on the distribution of the inefficiencies inside the attainable set. To save space, we limit the presentation to the output oriented case and to the order-α quantile frontiers. The extension to other orientations (input and hyperbolic) is immediate. The case of the partial order-m frontier is described in Cazals et al. (2002) and in Daraio and Simar (2005), see also Daraio and Simar (2007a) for a general presentation and applications to real data.

Order-α quantile frontiers

Daouia and Simar (2007) define for any $\alpha \in (0, 1]$ the order-α output efficiency score as

$$\lambda_\alpha(x_0, y_0) = \sup\{\lambda > 0|S_{Y|X}(\lambda y_0|X \leq x_0) > 1 - \alpha\}. \quad (2.7)$$

We see that if $\alpha \to 1$, $\lambda_\alpha(x_0, y_0) \to \lambda(x_0, y_0)$. If $\lambda_\alpha(x_0, y_0) = 1$, the point (x_0, y_0) belongs to the order-α quantile frontier, meaning that only $(1 - \alpha) \times 100\%$ of the firms using less

resources than x_0, dominate the unit (x_0, y_0). A value $\lambda_\alpha(x_0, y_0) < 1$ indicates a firm producing more than the level determined by the order-α frontier at x_0.

By conditioning on $Z = z_0$, Daouia and Simar (2007) define similarly the conditional order-α output efficiency score of (x_0, y_0) as

$$\lambda_\alpha(x_0, y_0|z_0) = \sup\{\lambda > 0 | S_{Y|X,Z}(\lambda y_0|X \leq x_0, Z = z_0) > 1 - \alpha\}. \tag{2.8}$$

Again, if $\alpha \to 1$, $\lambda_\alpha(x_0, y_0|z_0) \to \lambda(x_0, y_0|z_0)$.

3 What do we learn by the analysis of Conditional and Unconditional efficiency scores?

3.1 Individual analysis

The individual efficiency scores $\lambda(x, y)$ and $\lambda(x, y|z)$ have their usual interpretation: they measure the radial feasible proportionate increase of output a unit operating at the level (x, y) should perform to reach the efficient boundary of Ψ and Ψ^z respectively. In case the environmental factor Z has an effect on this boundary, the first measure $\lambda(x, y)$ suffers from a lack of economic sounding, because, facing the external conditions z, this firm may not be able to reach the frontier of Ψ, that may be quite different from the one of Ψ^z. So, the conditional measure is more appropriate to evaluate the effort a firm has to perform to be considered as efficient. Note however, that ranking firms according to these conditional measures can always be done, but as far as managerial efficiency is concerned, this ranking is meaningless because firms face different operating conditions, and may be, some external conditions may be easier (or harder) to handle than others to reach the frontier. We will see below how to derive a measure of managerial efficiency allowing to rank the units even when they face different environmental conditions.

The analysis of the individual ratios may also be of interest: they allow to measure, for a unit (x, y), the local effect of Z on the reachable frontier, independently of the inherent inefficiency of the unit (x, y). Indeed, $R_O(x, y|z) = \lambda(x, y|z)/\lambda(x, y) \leq 1$ is the ratio of the radial distances of (x, y) to the two frontiers. The inherent level of inefficiency of the unit (x, y) has been cleaned off, in the following sense:

$$R_O(x, y|z) = \frac{\lambda(x, y|z)}{\lambda(x, y)} = \frac{||y||\lambda(x, y|z)}{||y||\lambda(x, y)} = \frac{||y_x^{\partial,z}||}{||y_x^{\partial}||} \tag{3.9}$$

where $||y||$ is the modulus (Euclidean norm) of y and y_x^{∂} and $y_x^{\partial,z}$ are the projections of (x, y) on the efficient frontiers (unconditional and conditional, respectively), along the ray y and orthogonally to x. Clearly $||y_x^{\partial,z}||$ and $||y_x^{\partial}||$ are both independent of the inherent inefficiency

of the unit (x, y). So, the ratio measures the shift of the frontier in the output direction, due to the particular value of z, along the ray y and for an input level x, whatever being the modulus of y.

This is even more easy to see if we consider the particular case of univariate y. To be specific, in this case, the efficient boundaries can be described by maximal production functions:

$$\varphi(x) = \sup\{y|S_{Y|X}(y|X \leq x) > 0\} \tag{3.10}$$

$$\varphi(x|z) = \sup\{y|S_{Y|X,Z}(y|X \leq x, Z = z) > 0\}. \tag{3.11}$$

Here we have $R_O(x, y|z) = \varphi(x|z)/\varphi(x) \leq 1$, and we note that $\varphi(x) = \sup_z \varphi(x|z)$. We observe that the ratio is indeed independent of the level of output y. So, to summarize, these ratios allow to investigate the local effect of Z of the attainable frontier itself, for a given x and a given output mix. Using efficiency scores is particularly useful when y is multidimensional.

The same can be said for the input orientation, where $R_I(x, y|z) = \theta(x, y|z)/\theta(x, y) \geq 1$. In the particular case where x is univariate, the efficient boundaries can be described by the minimal input functions:

$$\phi(y) = \inf\{x \in \mathbb{R}_+|F_{X|Y}(x|Y \geq y) > 0\} \tag{3.12}$$

$$\phi(y|z) = \inf\{x \in \mathbb{R}_+|F_{X|Y,Z}(x|Y \geq y, Z = z) > 0\}, \tag{3.13}$$

where the notation introduced here is unambiguous. In this case the ratio can be written as $R_I(x, y|z) = \phi(y|z)/\phi(y) \geq 1$, with the relation $\phi(y) = \inf_z \phi(y|z)$. The same analysis as the one described above, can be done, mutatis mutandis, for the input orientation. The top panel of Figure 9, reported in Appendix A, illustrates possible behaviors of these minimal input functions, conditional and unconditional, in the simple case of $p = q = r = 1$.

3.2 Global analysis

The first important global analysis required in this setup, is the one provided by Daraio et al. (2010), where the conditional and unconditional efficiency scores are used to build some test statistics to test the separability condition (1.5). This statistic is built by measuring, in some way, the difference between the two efficient boundaries. The bootstrap is then used to find critical values. To save space, we refer to Daraio et al.(2010) for the details.

Besides a global test of separability, the comparison of the individual ratios of conditional to unconditional scores as a function of Z may also be useful. However, this comparison may be misleading, when wrongly conducted. In this section, we clarify exactly what can be done and how to interpret the resulting pictures, extending the previous methodologies suggested in Daraio and Simar (2005, 2007a) to more general setups.

Indeed, Daraio and Simar have described how useful is the analysis of the ratios considered as a function of z. This allows to capture the marginal effect of Z on the frontier shifts, but this effect may change according the level of the inputs, when frontier output ratios are considered or according the level of the outputs, when frontier input ratios are analyzed.

This situation is explained and illustrated in details in Appendix A, for a simple univariate scenario in the input oriented framework. To summarize the Appendix, the interpretation of the ratios as a function of z only can always be done to explore the marginal effect of z on the frontier shifts, but the picture might be rather difficult to interpret when some dependence exists between Z and both the efficient input levels and the outputs Y. Therefore, in absence of any information, it is better to first analyze the behavior of the ratios $R_I(x, y|z)$ as a function of z, for fixed level of the outputs y (multivariate analysis), or as illustrated in the applications below, as a joint function of both y and z. Of course, if Y is independent of Z, or in a less restrictive way, under the assumption that the shape of the boundaries of $\mathcal{P}$ in the sections $Y = y$ (in the (X, Z) space) would not change with the level y (which formally defines what we call "partial separability"), the analysis of the ratios $R_I(x, y|z)$ as a function of z is largely simplified.

For the output orientation, and for the same reasons, the analysis of the ratios $R_O(x, y|z)$ as a function of z should first be conducted for fixed levels of the inputs X. Here, for given values of the inputs x, an increasing shape for $R_O(x, y|z)$ as a function of z, would correspond to a favorable effect of Z (higher values of Z allow to reach higher outputs, Z is acting as a freely available input) and the opposite for a decreasing shape (Z is acting as an undesirable output). Here again, under the additional assumption of partial separability, i.e. the shape of the boundaries of $\mathcal{P}$ in the sections $X = x$ (in the (Y, Z) space) would not change with the level x, the ratios $R_O(x, y|z)$ would have the same shape for all values of x, and so the analysis of the effect of Z on the efficient frontier, as a function of z only, would be simplified.

3.3 Full frontier or Partial frontier?

Partial frontiers are very popular nowadays, because they produce robust estimators of the efficient frontiers and of the efficiency scores, sharing nice statistical properties. Here robustness is with respect to outliers or extreme data points and we know that sometimes, outliers can mask the effect of Z on the production process (see Daraio and Simar, 2007a, section 5.4.1 for details). In our setup here, we will clarify what the partial scores and their corresponding ratios, e.g. $R_{O,\alpha}(x, y|z) = \lambda_\alpha(x, y|z)/\lambda_\alpha(x, y)$ can add to the analysis of the effect of Z on the production process. Here we will focus the presentation on the output orientation.

First, as already pointed above, it is important to remind that the ratios $R_O(x, y|z)$, when defined relative to the "full" frontiers, only bring information on potential differences

between the boundaries of Ψ and Ψ^z. They are not sensitive to changes in the distribution of inefficiencies. We have seen above that the measure $R_O(x,y|z) \leq 1$ for a fixed point (x,y) only depends on the relative position of the boundaries of Ψ and Ψ^z (in the radial direction given by y). This is no more true for partial frontiers: the values of $\lambda_\alpha(x,y)$ and $\lambda_\alpha(x,y|z)$ do not depend only on the boundary, they also depend on the effect of Z on the distribution of the output Y inside Ψ^z, conditionally to $X \leq x$. It is easy to see that the ratios $R_{O,\alpha}(x,y|z) = \lambda_\alpha(x,y|z)/\lambda_\alpha(x,y)$ could be either ≤ 1 or ≥ 1, depending on the actual effect of Z on the distribution of Y given $X \leq x$ (when conditionning on $Z = z$). We illustrate these facts in Appendix B, in the simple case of a univariate output y and a univariate z.

So, to summarize the analysis of the Appendix, we see that if α is near 1, the partial measures provide the same information as the full measure, but using more robust frontiers. Using "small" values of α, could be misleading, without having a clear picture on the separability condition. Under the latter, the analysis with small values of α (e.g. $\alpha = 0.5$: median frontier) provides complementary information on the effect of Z on the distribution of the inefficiencies (its median value when $\alpha = 0.5$). The same analysis could be done, mutatis mutandis, for the input orientation and for partial order-m frontiers.

3.4 Second-stage regression and Managerial efficiency

The idea of regressing efficiency scores on the environmental variables to estimate the average effect of Z on the efficiency is quite old. However, as pointed in Section 1 above, and in details in Simar and Wilson (2007, 2011b), this 2nd stage regression is meaningless, or at minimum difficult to interpret, when using the unconditional efficiency scores $\lambda(x,y)$. Indeed, if the separability assumption (1.5) is not verified, the unconditional efficiencies are relative to the boundary of Ψ defined by (1.4), which has no economic meaning for firms facing different environmental conditions, i.e. facing different attainable sets Ψ^z.[1]

It is therefore much more meaningful to analyze the average behavior of $\lambda(x,y|z)$ as a function of z, to capture the main effect of Z on these conditional measures.[2] It is clear that here too, the conditional measures $\lambda(x,y|Z=z)$ may vary with both x and z. However, here we want to capture the marginal effect of Z on the efficiency scores, so it is legitimate to analyze the regression $\mathbb{E}(\lambda(X,Y|Z)|Z=z)$ as a function of z. We suggest to use a flexible

[1] This is a relevant empirical issue due to the great number of papers that appeared in recent years. See *e.g.* Kao and Hwang, (2008); Chen et al. (2009a,b); Zha, and Liang (2010) just to cite a few of the most recent ones.

[2] As a matter of fact, since $\lambda(X,Y|Z=z)$ is a ratio of two output levels, and since an additive model will be suggested in (3.14), it might be more appropriate to perform this second stage regression on the $\log \lambda(X,Y|Z=z)$. This is an empirical issue and we come back to this in the empirical illustrations.

regression model defining $\mu(z) = \mathbb{E}\big(\lambda(X,Y|Z)|Z=z\big)$, and $\sigma^2(z) = \mathbb{V}\big(\lambda(X,Y|Z)|Z=z\big)$, we may write

$$\lambda(X,Y|Z=z) = \mu(z) + \sigma(z)\varepsilon, \tag{3.14}$$

where $\mathbb{E}(\varepsilon|Z=z) = 0$ and $\mathbb{V}(\varepsilon|Z=z) = 1$. In the next section we will briefly address the problem of estimating the functions $\mu(z)$ and $\sigma(z)$ in a nonparametric way, with some guidelines for a bootstrap procedure for getting confidence interval for $\mu(z)$, at any given value z. Whereas, $\mu(z)$ measures the average effect of z on the efficiency, $\sigma(z)$ provides additional information on the dispersion of the efficiency distribution as a function of z.

Another important result of the above approach is the analysis of the residuals. For a particular given unit (x,y,z), we can define the error term

$$\varepsilon = \frac{\lambda(x,y|z) - \mu(z)}{\sigma(z)}. \tag{3.15}$$

This can be viewed as the "unexplained" part of the conditional efficiency score. If Z is independent of ε in (3.14), this quantity can be interpreted as a "pure" managerial efficiency measure of the unit (x,y).[3] If Z is not completely independent of ε, still the quantity defined in (3.15) can be used as a proxy of the managerial efficiency, since it is the remaining part of the conditional efficiency after removing the location and scale effect due to Z. It is a kind of whitening the conditional efficiency scores, from the effects due to the environmental conditions Z. We can use these quantities, which are standardized (mean zero and variance one), to compare the firms between them: a large value of ε indicates a unit which has poor performance, even after eliminating the main effects of the environmental factors. A small (negative) value, on the contrary, indicates very good managerial performance of the firm (x,y,z). It allows to rank the firms facing different environmental conditions, because the main effects of these factors have been eliminated. Extreme (unexpected) values of ε would also warn for potential outliers.

The above analysis could also be performed by using partial efficiency scores, like $\lambda_\alpha(X,Y|Z=z)$. When α is near 1, this would provide a robust version of the above analysis. We will also remind below that the quality of the estimation is better when using partial efficiency measures (better rates of convergence).

[3]Our approach may be seen as a recent advanced and robust interpretation of the Leibenstein's (1966, 1979) X-inefficiency theory which has among its proximate causes those related to the performance of management (see also Leibenstein and Maital, 1992).

4 Nonparametric Estimator

4.1 Efficiency Estimators

Nonparametric estimators of the conditional and unconditional efficiency scores are very easy to obtain. We summarize the notations and properties here to what is needed for the rest of the paper (details can be found in Daraio and Simar, 2007a, or Simar and Wilson, 2008). We will denote $\mathcal{S}_n = \{(X_i, Y_i, Z_i)|\ i = 1, \ldots, n\}$ the sample of n iid observations on (X, Y, Z) generated in $\mathcal{P}$ according the DGP $P \in \mathbb{P}$. If we plug nonparametric estimators of $S_{Y|X}$ and $S_{Y|X,Z}$ in all the formulae above, we obtain very natural nonparametric estimators of the efficiencies. For the $S_{Y|X}$ we can use the empirical probabilities

$$\widehat{S}_{Y|X}(y_0|X \leq x_0) = \frac{1/n \sum_{i=1}^n \mathbb{1}(X_i \leq x_0, Y_i \geq y_0)}{1/n \sum_{i=1}^n \mathbb{1}(X_i \leq x_0)}, \tag{4.1}$$

where $\mathbb{1}(\cdot)$ is the indicator function. This provides the popular FDH estimator of $\lambda(x_0, y_0)$

$$\widehat{\lambda}(x_0, y_0) = \max_{\{i|X_i \leq x_0\}} \left\{ \min_{j=1,\ldots,q} \frac{Y_i^j}{y_0^j} \right\} \tag{4.2}$$

whose statistical properties are well known (see e.g. Simar and Wilson, 2008). To summarize, under mild regularity conditions:

$$n^{1/(p+q)} \left(\lambda(x_0, y_0) - \widehat{\lambda}(x_0, y_0) \right) \xrightarrow{\mathcal{L}} \text{Weibull}(\mu_0^{p+q}, p+q), \tag{4.3}$$

where μ_0 is a constant depending on the DGP $P \in \mathbb{P}$ that is described in Park et al. (2000). For the conditional (conditional to $Z = z_0$) some smoothing techniques are required. We have the estimator

$$\widehat{S}_{Y|X,Z}(y_0|X \leq x_0, Z = z_0) = \frac{1/n \sum_{i=1}^n \mathbb{1}(X_i \leq x_0, Y_i \geq y_0) K((z_0 - Z_i)/b)}{1/n \sum_{i=1}^n \mathbb{1}(X_i \leq x_0) K((z_0 - Z_i)/b)}, \tag{4.4}$$

where for simplicity, we wrote the expression for a univariate Z. Here $K(\cdot)$ is a kernel with compact support and $b > 0$ is the bandwidth. For the general multivariate case, see Daraio and Simar (2007a). In the general multivariate setup, an optimal bandwidth selection procedure has been suggested in Bădin et al. (2010), based on a least-squares cross validation technique. This leads to the conditional efficiency estimator

$$\widehat{\lambda}(x_0, y_0|z_0) = \max_{\{i|X_i \leq x_0, ||Z_i - z_0|| \leq b\}} \left\{ \min_{j=1,\ldots,q} \frac{Y_i^j}{y_0^j} \right\} \tag{4.5}$$

So, it appears that the estimation of the conditional efficiency score is a kind of "restricted" FDH program (restricted to data points having $||Z_i - z_0|| \leq b$). The statistical properties

of the estimators of the conditional measures have been determined in Jeong et al (2010). To summarize and roughly speaking, these estimators keep similar properties as the FDH estimator but with an "effective" sample size depending on the bandwidth: n is replaced by nb^r, where r is the dimension of Z. In practice since the optimal bandwidth has a size $n^{-1/(r+4)}$ (see Bădin et al., 2010 for details), this gives a rate of convergence for the conditional measures estimators of $n^{4/((r+4)(p+q))}$ in place of the better rate $n^{1/(p+q)}$ achieved by the FDH estimators. It is important to report these rates in order to derive below consistent bootstrap algorithms.

The nonparametric partial frontier efficiency estimates are obtained in a similar way, by plugging the estimators $\widehat{S}_{Y|X}$ and $\widehat{S}_{Y|X,Z}$ in the expressions defining the partial efficiency measures: algorithms have been proposed in Cazals et al. (2002), Daraio and Simar (2005, 2007a) for the order-m case and in Daouia and Simar (2006) and Daraio and Simar (2007a) for the order-α quantile case. Their statistical properties have been also established. Under mild regularity conditions, we have for instance

$$\sqrt{n}\left(\lambda_\alpha(x_0,y_0) - \widehat{\lambda}_\alpha(x_0,y_0)\right) \xrightarrow{\mathcal{L}} \mathcal{N}\big(0,\sigma^2(\alpha,x_0)\big), \tag{4.6}$$

where an expression for $\sigma^2(\alpha,x_0)$ is given in Daouia and Simar (2006). A similar result holds for the order-m case (see Cazals et al. 2002).

For the estimators of the conditional partial measures, we have similar results where the rate of convergence $\sqrt{n}$ deteriorates to $\sqrt{nb^r} = n^{2/(r+4)}$ when the optimal bandwidth of Bădin et al. (2010) described above is used.

Robust Estimators of the Full Frontier

As explained above, the partial frontiers have their particular usefulness providing less extreme surfaces to benchmark individual units and allowing to investigate the impact of Z on the distribution of the efficiencies. In particular for $m = 1$, the order-m frontier is not looking to an optimal behavior but rather to an average behavior of firms (the same is true for the order-α frontier with $\alpha = 0.50$).

But as pointed and illustrated in Daraio and Simar (2007a) it may happen that outliers or extreme data points can hide the real effect of the environmental factors. So, in this case, it is particularly useful to build robust estimators of the full frontier. This can be achieved by using partial order frontier with extreme orders.

Indeed, if we let $\alpha = \alpha(n) \to 1$ (or $m = m(n) \to \infty$) when $n \to \infty$ fast enough (see Cazals et al., 2002 and Daouia and Simar, 2006, for details), the respective partial frontier estimators will converge to the full frontier sharing the same properties as the FDH estimator (with the same limiting Weibull distribution). But for finite n (as we use in practice), $\alpha(n)$ will be less than 1 (and $m(n)$ will be less than infinity) and so the corresponding estimate

of the full frontier will not envelop all the data points being more robust and resistant to outliers and extreme values than the standard envelopment estimators like FDH or DEA.

Simar (2003) has suggested some data driven techniques to select reasonable values of α and m by analyzing the proportion of data points remaining outside the corresponding partial frontiers over a grid of values of the orders. This allows to detect potential outliers. Daouia and Gijbels (2009) provide a theoretical background for the comparison of both partial frontiers in terms of their robustness properties; see also Daouia and Gijbels (2010), where a theoretical rule is given to select the appropriate order of the partial frontiers for obtaining robust estimators of the full frontier in the presence of outliers.

Estimation of the Ratios $R_O(x, y|z)$ and $R_{O,\alpha}(x, y|z)$

Consistent estimators of the ratios are directly obtained by plugging the nonparametric estimators derived above in the corresponding formulae. So we have

$$\widehat{R}_O(x, y|z) = \frac{\widehat{\lambda}(x, y|z)}{\widehat{\lambda}(x, y)}. \tag{4.7}$$

For any given point (x, y, z), it is easy to prove that $\widehat{R}_O(x, y|z)$ is a consistent estimator of $R_O(x, y|z)$, sharing the worst rate of convergence of its components, i.e. the numerator. The limiting distribution of the error is rather complicated, but it can be shown (see Daraio et al., 2010, for details) that

$$n^{\kappa}\left(\widehat{R}_O(x, y|z) - R_O(x, y|z)\right) \xrightarrow{\mathcal{L}} Q_P^z(\cdot), \tag{4.8}$$

where the rate of convergence was given above, $\kappa = 4/((r + 4)(p + q))$ and where Q_P^z is a nondegenerate distribution (i.e. it is not a Dirac distribution with mass 1 at one single value) depending on the current value of z and on characteristics of the DGP P.

The partial measures will benefit from the better rate of convergence of the individual efficiency estimators. We have

$$\widehat{R}_{O,\alpha}(x, y|z) = \frac{\widehat{\lambda}_\alpha(x, y|z)}{\widehat{\lambda}_\alpha(x, y)}. \tag{4.9}$$

and the asymptotic distribution of the error of estimation follows

$$n^{\gamma}\left(\widehat{R}_{O,\alpha}(x, y|z) - R_{O,\alpha}(x, y|z)\right) \xrightarrow{\mathcal{L}} Q_P^{z,\alpha}(\cdot), \tag{4.10}$$

where here the rate is $\gamma = 2/(r + 4)$ and where $Q_P^{z,\alpha}$ is another nondegenerate limiting distribution.

Note that the unit (x, y, z) of interest can be any point in $\mathcal{P}$, even if in practice we will be interested to estimate these ratios at the observed data points (X_i, Y_i, Z_i).

Since the limiting distributions are unknown, the bootstrap is the only available route to draw inference on these individual ratios. Here we can directly apply the subsampling procedure described in Simar and Wilson (2011a) to derive confidence intervals for $R_O(x,y|z)$ (and for the partial correspondents). To save place we refer simply to the algorithms described in Simar and Wilson's paper (section 4.2), the adaptation being straightforward.

Just to avoid misunderstandings, we give a sketch of the subsampling algorithm:

[1] First we compute from the sample $\mathcal{S}_n = \{(X_i, Y_i, Z_i)|\ i = 1, \ldots, n\}$ the efficiency score $\widehat{\lambda}(x,y)$ and its conditional version $\widehat{\lambda}(x,y|z)$. By doing so, we compute $b_{n,z}$ the optimal bandwidth for the conditional survival function at z (we do this by using the Bădin et al. (2010) approach). We compute the ratio $\widehat{R}_O(x,y|z)$.

[2] For a given value of $m < n$, we will repeat the next steps [2.1] to [2.2] L times, for $\ell = 1, \ldots, L$, where L is large enough (say, $L = 2000$).

[2.1] Draw a random sample $\mathcal{S}^*_{m,\ell} = \{(X_i^{*,\ell}, Y_i^{*,\ell}, Z_i^{*,\ell})|\ i = 1, \ldots, m\}$ without replacement from $\mathcal{S}_n$.

[2.2] We compute the ratio $\widehat{R}^{*,\ell}_O(x,y|z)$ by the same techniques as in [1]. Note that here we have to rescale the corresponding bandwidth at the appropriate size. So we will use the bandwidths $b_{m,z} = (n/m)^{1/(r+4)} b_{n,z}$ for computing the conditional scores with the bootstrap sample $\mathcal{S}^*_{m,\ell}$.[4]

[3] From the collection of L values $\widehat{R}^{*,\ell}_O(x,y|z)$ with $\widehat{R}_O(x,y|z)$, we build the confidence interval obtained for this particular value of m.

[4] Select the appropriate value of the subsample size m, which will correspond to a value where the results show low volatility with respect to m (see Simar and Wilson, 2011a).

4.2 2nd stage regression of the conditional efficiency scores

To save place we only present the full frontier case, where we want to estimate $\mu(z)$ and $\sigma(z)$ in model (3.14) by using basic tools from the nonparametric econometrics literature. Our presentation is for continuous Z.[5]

[4] As pointed in Jeong and Simar (2006), if the point of interest (x,y,z) is rather extreme with respect to the cloud of data $\mathcal{S}_n$, some of the FDH estimators (conditional and unconditional) may be undefined when computed relative to a bootstrap sample $\mathcal{S}^*_m$. This is a small sample issue and this event should disappear asymptotically. In this case, as Jeong and Simar recommend, we define the estimators as being equal to 1. This does not alter the asymptotic consistency of the bootstrap.

[5] For more details on how to handle discrete variables Z in a similar framework, see Bădin and Daraio (2011) and the references therein.

Several flexible nonparametric estimators could be provided, when working with the full frontier efficiency scores. We know indeed that, by definition, $\lambda(X, Y|Z = z) \geq 1$ with probability one. So, we could for instance estimate in a first step $\mu(z)$, by local constant methods (Pagan and Ullah, 1999) or local exponential smoothing (see Ziegelmann, 2002) on the values $\lambda(X, Y|Z = z) - 1$. The estimation of the variance function $\sigma^2(z)$ is rather standard (see Fan and Gijbels, 1996, Fan and Yao, 1998, Pagan and Ullah, 1999 and the references therein) and is obtained by regressing the squares of the residuals obtained from the first step, on Z. Here again, local constant or local exponential methods can be used. Once $\hat{\mu}(z)$ and $\hat{\sigma}^2(z)$ are obtained, we can compute the residuals by applying (3.15).[6]

Whatever being the selected nonparametric estimators, they share typically similar asymptotic properties, with the same rate of convergence. As pointed in Simar and Wilson (2007), the main statistical issue in this 2nd-stage regression, comes from the fact that we do not have observations of $\lambda(X_i, Y_i|Z = z)$, neither observations $\lambda(X_i, Y_i|Z_i)$ because the lambda's are unknown. What we only have is the set of the n estimators $\widehat{\lambda}(X_i, Y_i|Z_i)$, obtained from the sample $\mathcal{S}_n$. So we have a sample of n pairs $\big(Z_i, \widehat{\lambda}(X_i, Y_i|Z_i)\big)$, $i = 1, \ldots, n$ from which we will estimate $\mu(z)$ and $\sigma(z)$, by one of the method described above. For the regression, the resulting estimator will be written $\hat{\mu}_n(z)$. All these techniques involve smoothing (localizing) techniques requiring the selection of bandwidths h_z, for the Z variables. Bandwidths h_z with appropriate size (i.e. $h_z = c\, n^{-1/(r+4)}$) can be obtained by least-squares cross validation criterion (see e.g. Li and Racine, 2007 for details).

If the true, but unavailable independent $\lambda(X_i, Y_i|Z_i)$ would be used as dependent variables in the regression, standard tools would provide an estimator $\tilde{\mu}_n(z)$ with standard properties, i.e., as $n \to \infty$ and $h_z \to 0$ with $nh_z^r \to \infty$

$$\sqrt{nh_z^r}\Big(\tilde{\mu}_n(z) - \mu(z) - h_z^2 B^z\Big) \xrightarrow{\mathcal{L}} \mathcal{N}(0, V^z), \tag{4.11}$$

where the bias B^z and the variance V^z are bounded constants depending on the model and on the chosen estimator. Balancing between bias and variance, the optimal bandwidth should indeed be of the order $h_z = c\, n^{-1/(r+4)}$, providing the asymptotic result

$$n^{2/(r+4)}\big(\tilde{\mu}_n(z) - \mu(z)\big) \xrightarrow{\mathcal{L}} \mathcal{N}(B^z, V^z), \tag{4.12}$$

When replacing $\lambda(X_i, Y_i|Z_i)$ by $\widehat{\lambda}(X_i, Y_i|Z_i)$, which are estimators with n^{κ} rate of convergence, we obtain the estimator $\hat{\mu}_n(z)$. By using the same arguments as in Kneip et al. (2011), it can be shown that it is a consistent estimator of $\mu(z)$ but at the rate of convergence $n^{4/((r+4)(p+q))}$, which is lower as soon as $p + q > 2$. Asymptotic behavior of the error

[6] As pointed above, in some applications, the additive model (3.14) could be more appropriate for the $\log \lambda(X, Y|Z = z)$. In a particular application, it is important to use some standard checks to see if the hypothesis of the independence between ε and Z, is more reasonable or not in the log scale (see the empirical illustrations below).

$\big(\hat{\mu}_n(z)-\mu(z)\big)$ should also be available along the recent theoretical results developed in Kneip et al. (2011), allowing to develop bootstrap algorithms and derive confidence intervals for $\mu(z)$, but this is rather technical and out of the scope of this paper.

5 Numerical Illustrations

5.1 Simulated Examples

To illustrate how the procedure can work in practice, we first introduce some simulated examples, because there we know what we expect to find. We will use, as simulated scenario, an example inspired from Simar and Wilson (2011b) where we see clearly the 2 different ways an environmental factor can influence the production process. We analyze the three following different DGPs:

$$Y = g(X)e^{-U} \tag{5.1}$$

$$Y^* = g(X)e^{-U|Z-2|} \tag{5.2}$$

$$Y^{**} = g(X)(1+|Z-2|/2)^{1/2}\, e^{-U}, \tag{5.3}$$

where $g(X) = [1-(X-1)^2]^{1/2}$ with $X \sim U(0,1)$ and $Z \sim U(0,4)$. Finally $U \geq 0$ with $U \sim \mathcal{N}^+(0,\sigma_U^2)$ and we choose for the illustration $\sigma_U^2 = 0.05$.

In the first DGP1 (5.1), Z has no effect on the production process (Z is independent of (X,Y)). In the DGP2 (5.2), we have the "separability" condition $\Psi^z \equiv \Psi$, $\forall z$ but Z influences the distribution of the inefficiencies (higher probability of being inefficient when $|Z-2|$ increases). In the last DGP3 (5.3), the effect of Z is only on the boundary of the attainable (X,Y), violating the "separability" condition, the shift (increasing the level of the attainable frontier) is multiplicative and more important when $|Z-2|$ increases.

Analysis of the ratios $\widehat{R}_O(x,y|z)$

We first investigate how the ratios $\widehat{R}_O(x,y|z)$ can inform us on the potential shifts of the frontier due to the environmental factor Z. We look at $\widehat{R}_O(x,y|z)$ as a function of Z and of X. A summary of the results for the case $n = 200$ is displayed in Figure 1 (the case $n = 100$ gave similar plots with more sampling noise). DGP1 corresponds to the top panels, then DGP2 is in the middle and DGP3 in the bottom panels. Since it is not easy to "see" a 3-dimensional pictures without rotating the pictures (that most of the softwares allow to do on the screen), we display at the right panels the 2 marginal views from the X-side (marginal effect of X) and from the Z-side (marginal effect of Z). The results are as we expected from the comments of Section 3.2 taking into account for the fact that we use estimates with low rates of convergence ($n^{4/((r+4)(p+q))} = n^{2/5}$) in place of the true values. For the 3 DGPs, the

cloud of points are flat from the perspective of X, because in the 3 cases, the effect of Z on the efficient frontier, if any, is independent of X. For the two first DGPs the "separability" condition is verified, the cloud is really flat with respect to the 2 dimensions. For the DGP3, the U-shape with respect to Z that appears in both figures is exactly what we expected form (5.3). So, to conclude, the pictures of these ratios as functions of x and z are clearly informative. In our examples here, a marginal analysis of the effect of Z on the shifts of the efficient frontiers would also provide meaningful interpretations.

The analysis of the partial ratios, $\widehat{R}_{O,\alpha}(x, y|z)$, with α not far from 1, would provide the same pictures (we do not have outliers here). However, it is interesting to look at these ratios for smaller values of α to detect potential effects of Z on the distribution of the inefficiencies. This is done in Figure 2, for the median value ($\alpha = 0.5$). Here again the results confirm mostly what we expected (again, spurious unexpected behaviors could come from the fact that we use estimates with low rates of convergence). For DGP1, top panel, the cloud of points is flat: no effect of Z on the efficiencies. In the case of DGP2, where we have the separability, we see some curvature in the z direction and flat behavior in the x direction; indeed, the marginal and conditional frontiers have the same support but the distribution of the inefficiencies is changing with the value of z. Near the center ($z = 2$) the sampling variations of $\widehat{R}_{O,\alpha}(x, y|z)$ are near 1, and for larger values of $|z - 2|$ we have more values smaller than 1. For DGP3, we reproduce for the median, the U-shaped effect of the shift of the frontier we have observed by looking to the bottom panel of Figure 1.

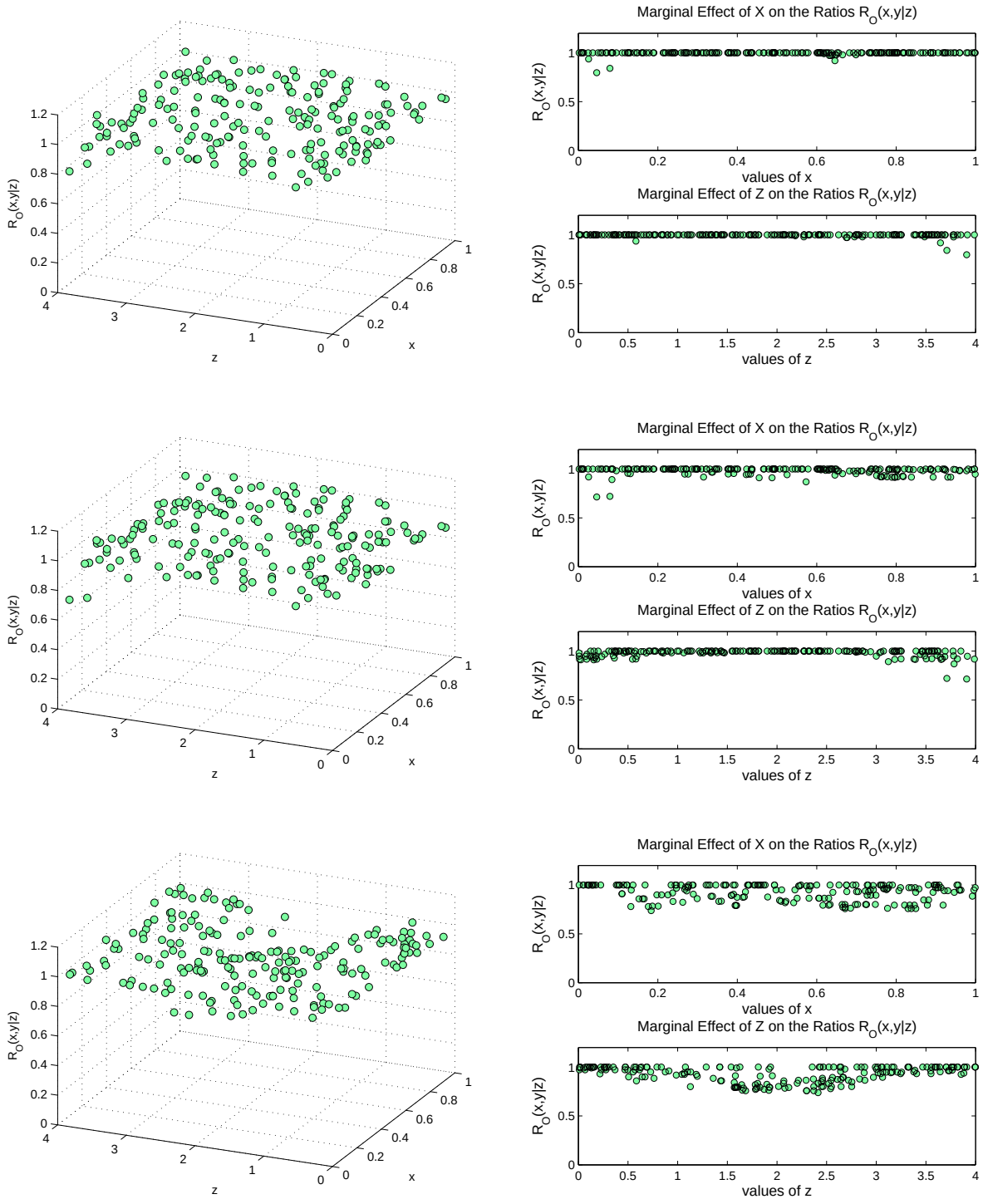

Figure 1: *Effect of X and Z on the ratios $\widehat{R}_O(x, y|z)$. From top to bottom: DGP1, DGP2 and DGP3. Here $n = 200$ and the circles are the estimated ratios.*

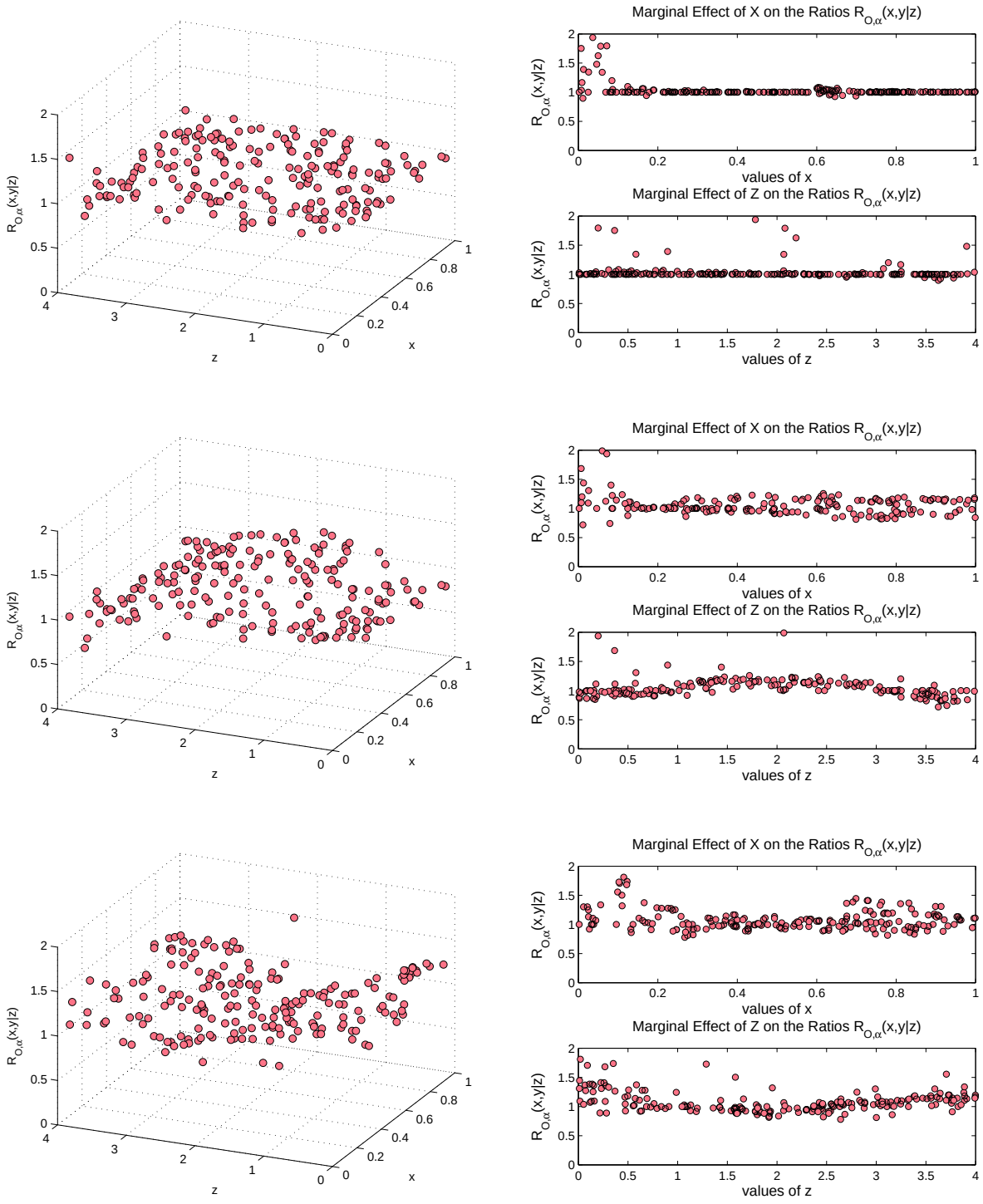

Figure 2: *Effect of X and Z on the ratios $\widehat{R}_{O,\alpha}(x,y|z)$, with $\alpha = 0.5$. From top to bottom: DGP1, DGP2 and DGP3. Here $n = 200$ and the circles are the estimated ratios.*

2nd stage regression

The results of the analysis of the second stage regression of $\log \widehat{\lambda}(x, y|z)$ on z are summarized in Figure 3. The analysis done with $\widehat{\lambda}(x, y|z)$ gave very similar results. For each DGPs, we have on the left, the results of the nonparametric regression for $\mu(z)$ and $\sigma(z)$, in the middle the histogram of the resulting residuals, that can be interpreted as managerial efficiency and on the right, the clouds of n points $(Z_i, \hat{\varepsilon}_i)$ to check if some pattern is still apparent after whitening the effect of Z on the conditional efficiencies.

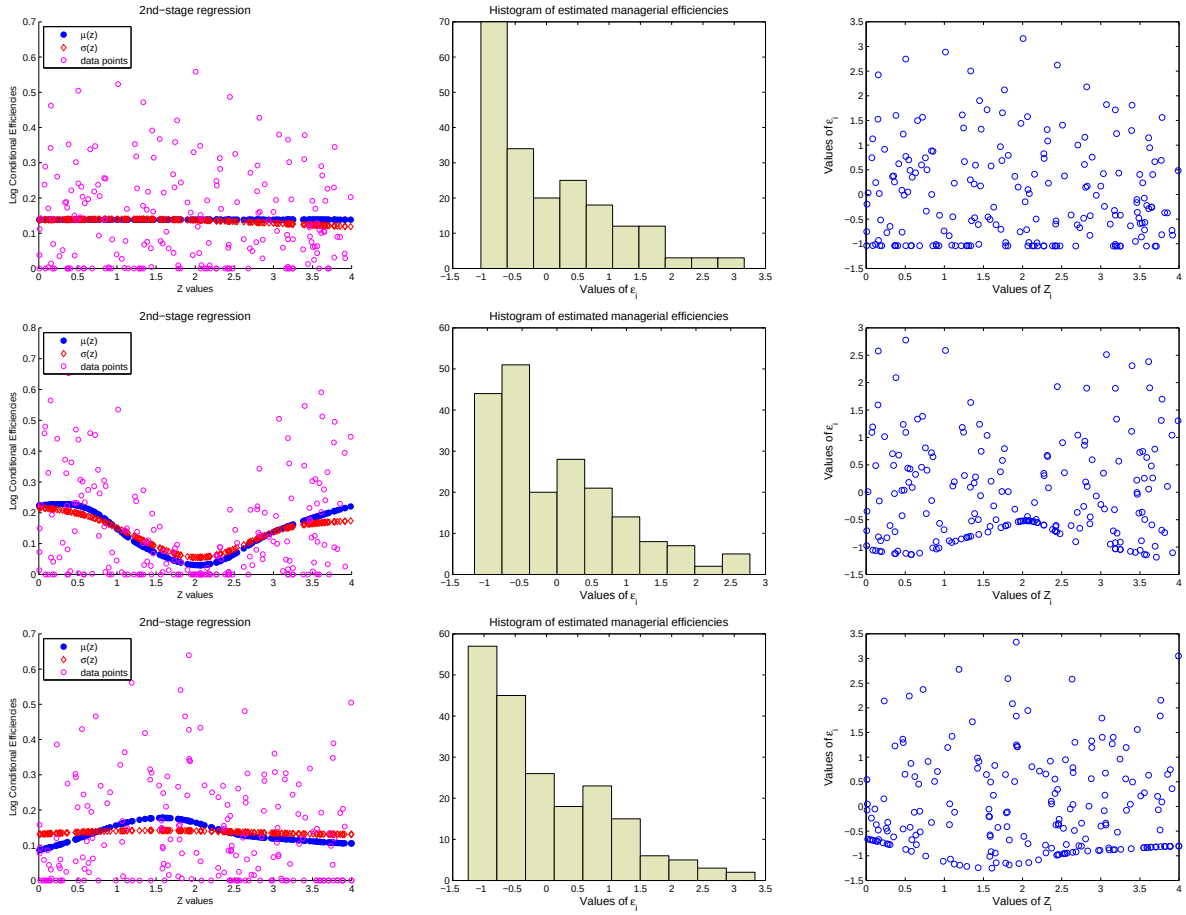

Figure 3: *Results of second stage regression. From left to right: location and scale estimates of* $\log \widehat{\lambda}(x, y|z)$ *as a function of* z*, histograms of managerial efficiencies, scatter plot of* $\hat{\varepsilon}_i$ *against* Z_i*. From top to bottom: DGP1, DGP2 and DGP3. Here* $n = 200$ *and the circles are the estimated ratios.*

Again, the results are as we expected. We do not see any effect of z for DGP1, for DGP2, we observe a visible U-shaped effect for both $\mu(z)$ and $\sigma(z)$, confirming that the

distribution of the inefficiencies varies with z. For DGP3, a small spurious effect (due to sampling uncertainties), but still, roughly a stable $\mu(z)$ and a constant $\sigma(z)$, as it should (the distribution of the inefficiencies does not depend on z). Note that the histograms of the managerial efficiencies $\hat{\varepsilon}_i$, recover in the 3 cases the shape of the half-normal distribution that has been simulated for U. The correlation between U_i and $\hat{\varepsilon}_i$ are quite high in each case: 0.94, 0.87, 0.84 for DGP1, DGP2 and DGP3, respectively. So, it seems legitimate and meaningful to use these residuals to rank the firms according their managerial inefficiencies. The scatter plots between $\hat{\varepsilon}_i$ and Z_i do not show particular structure, the correlation between the two variables are indeed very low: -0.05, 0.02, 0.01 for DGP1, DGP2 and DGP3, respectively.

5.2 Efficiency in the Banking Sector

Simar and Wilson (2007) includes an empirical example based on Aly et al. (1990) using data on 6.955 US Commercial Banks observed at the end of the 4th quarter, 2002.[7] They run a truncated regression on the input oriented DEA estimates of efficiency in a second stage (as suggested in Aly et al., 1990). Daraio et al. (2010) used the same data set to test the "separability" condition which was rejected at any reasonable level, indicating that any two-stage procedure is meaningless for this dataset. This was a global test; we will here proceed to a local analysis and trying to detect the size and direction of the detected effect.

The original data set contains 3 inputs (purchased funds, core deposits and labor) and 4 outputs (consumer loans, business loans, real estate loans, and securities held) for banks. Aly et al.1990 considered 2 continuous environmental factors, the size of the banks Z_1, and a measure of the diversity of the services proposed by the banks Z_2 (see Aly. et al., 1990, for details) and one binary variable indicating if the banks belong or not to a Metropolitan Statistical Area (MSA). We will use, as in Simar and Wilson (2007), a measure of the size of the banks by the log of the total assets, rather than the total deposit as in Aly et al. For simplifying the presentation, we will illustrate our procedure with a subsample of 322 Banks (also used in Simar and Wilson, 2007).

Some prior exploratory data analysis indicates that the 3 inputs are highly correlated among themselves and the same is true for the 4 outputs. So, due the dimensionality of the problem (3 inputs, 4 outputs, and 3 environmental factors) with the limited sample used here (322 units), we first reduce the dimension in the input $\times$ output space by using the methodology suggested in Daraio and Simar (2007a).

Since the radial measures are scale invariant, we divide each inputs and outputs by their mean (to be "unit" free) and replace the 3 scaled inputs by their best (non-centered) linear combination (we use here a kind of non-centered PCA, as explained in details in Daraio

[7] We would like to thank Paul W. Wilson who provided us this data set.

and Simar, 2007a), and we check that we did not loose much information by doing so, and that the resulting univariate input factor is highly correlated with the 3 original inputs. We follow the same procedure with the 4 outputs. The results are

$$\begin{aligned} IF &= 0.5707X_1 + 0.5731X_2 + 0.5881X_3, \\ OF &= 0.4851Y_1 + 0.4875Y_2 + 0.5095Y_3 + 0.5172Y_4, \end{aligned}$$

indicating that both the input and the output factor are a kind of average of the scaled inputs and outputs respectively (the weights are equal). We obtain the following correlations $\hat{\rho}_{IF,X_j} = (0.972,\ 0.971,\ 0.996)$ for $j = 1, 2, 3$ and IF explains 96% of total inertia of the original data (X_1, X_2, X_3). We obtain similar results when reducing the dimension in the output space: $\hat{\rho}_{OF,Y_j} = (0.924,\ 0.938,\ 0.975,\ 0.990)$ for $j = 1, \ldots, 4$, and OF explains 92% of total inertia of the original data $(Y_1, \ldots, Y_4)$. Hence we can conclude that we do not loose much information by this dimension reduction and the factors IF and OF are good representatives of the input and output activities of the Banks.

Remember that with the full data set and with all the original variables, Daraio et al. (2010) rejected the null hypothesis of global separability. We will here illustrate in our simplified version of the examples what we can learn by the methodology we proposed in this paper.

We first investigate what could be the effect of $Z_1 =$ SIZE on the production process. It is clear that in this example Z_1 is highly correlated with Y (the linear correlation is 0.57 but the Spearman rank correlation is 0.97). Figure 4 shows the ratios as function of Y and Z_1. Here it is the input orientation: $\widehat{R}_I(X_i, Y_i)|Z_i)$ and for the partial frontiers $\widehat{R}_{I,\alpha}(X_i, Y_i)|Z_i)$ with $\alpha = 0.95$, to see if some extreme data points could hide some effect and with $\alpha = 0.5$ to investigate the effect on the middle of the distribution of the inefficiencies. Without being able to rotate the 3d figures on the left panels, we have an idea on what happens complementing the left picture by the two marginal views. It is not clear from the full frontier ratios if Y has some effect on the frontier levels, but looking to the picture for the extreme quantile 0.95, it is more clear. For Z_1 it is also clear for the 3 pictures that Z_1 has a negative (unfavorable) effect on the frontier levels. When $\alpha = 0.5$, the effect is also visible, confirming the effect of the shift of the frontier. This short descriptive analysis also confirms that the separability condition for Z_1 seems unrealistic.

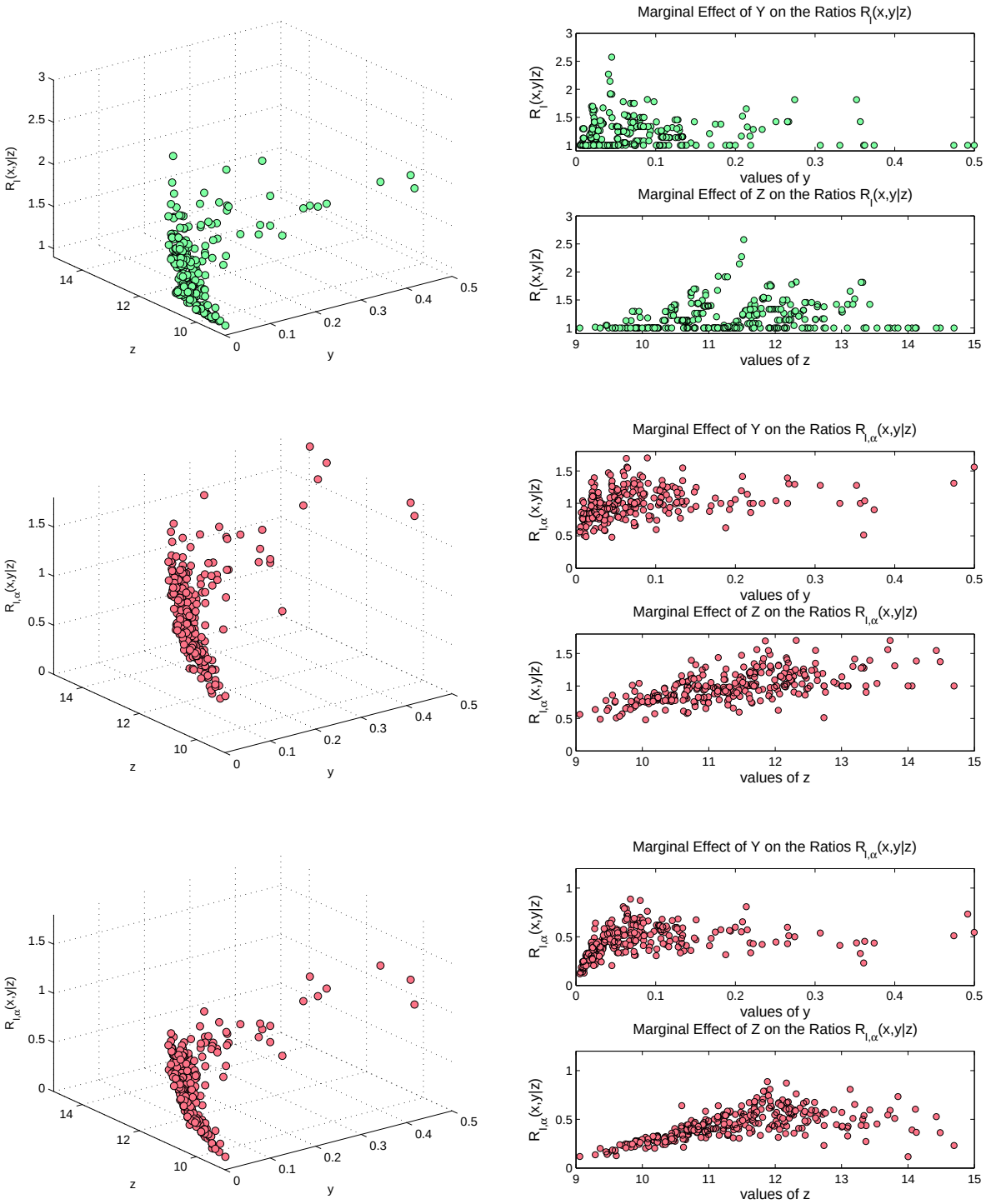

Figure 4: *Effect of* Y *and* Z_1 = SIZE *on the ratios* $\widehat{R}_I(X_i, Y_i)|Z_i)$ *(top panel) and* $\widehat{R}_{I,\alpha}(X_i, Y_i)|Z_i)$ *(middel panel* $\alpha = 0.95$ *and bottom panel* $\alpha = 0.5$*).*

Figure 5 below show the results of the analysis for the full-frontier conditional efficiencies as a function of Z_1 (the analysis was done on the *log*s, but the picture in original units is very similar). The regression line $\mu(z)$ has a global shape not far from the horizontal line, indicating that the effect of Z_1 on the distribution of the efficiencies is rather low. This is confirmed by the shape of $\sigma(z)$ rather constant. The managerial efficiencies have a reasonable shape (typically not far from an halfnormal). The effect of Z_1 on the conditional efficiency scores has been nicely whitened: the Pearson linear correlation between Z_i and $\hat{\varepsilon}_i$ is -0.009 (Pearson) and the Spearman rank correlation is -0.008. So the ranking of the banks according $\hat{\varepsilon}_i$ is cleaned from the effect of the size variable Z_1. Note that the resulting ranking is different from the ranking obtained by the marginal FDH scores $\widehat{\lambda}(X_i, Y_i)$, but since the effect of Z_1 is not a "big" effect, there remains some correlation (the correlation between the two rankings is 0.64).

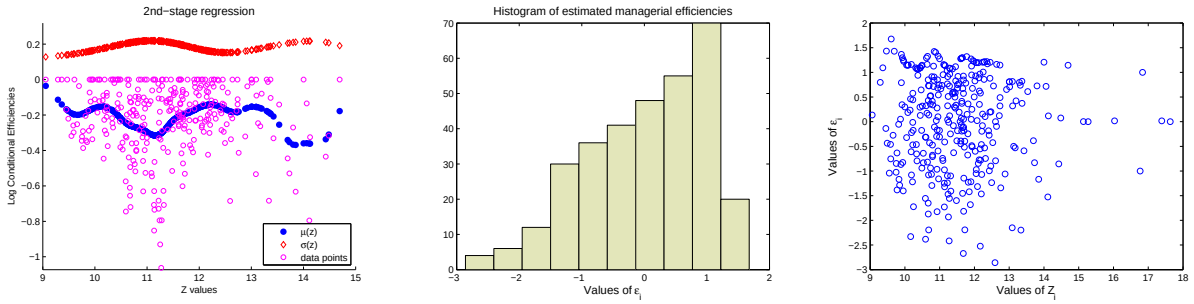

Figure 5: *Effect of* $Z_1 =$ SIZE *on conditional efficiencies* $\log \widehat{\lambda}(x, y|z)$*, histograms of managerial efficiencies, scatter plot of* $\hat{\varepsilon}_i$ *against* Z_1*.*

We did the same univariate exercise to investigate the marginal effect of the variable Z_2 (DIVERSE: a measure of the diversity of the products of the Banks). Figures 6 and 7 display the results. We summarize very shortly the conclusions and let the reader complete the analysis. The effect of Z_2 seems quite small (see top panel of Figure 6). Z_2 is not responsible for the rejection of the separability condition. However, we can see a small effect on the partial ratios $\widehat{R}_{I,\alpha}(X_i, Y_i)|Z_i)$, indicating a small effect on the distribution of the efficiencies, but a favorable one: banks having more diversity seems to have a distribution of their efficiency slightly more concentrated near the effcient frontier. This is confirmed by looking to Figure 7, where $\mu(z)$ is slightly increasing, combined with a slightly decreasing $\sigma(z)$. The effect of Z_2 on the conditional efficiencies has been well removed, the correlation between $\hat{\varepsilon}_i$ and Z_i is 0.06 (the right panel of Figure 7 does not show any clear remaining pattern).

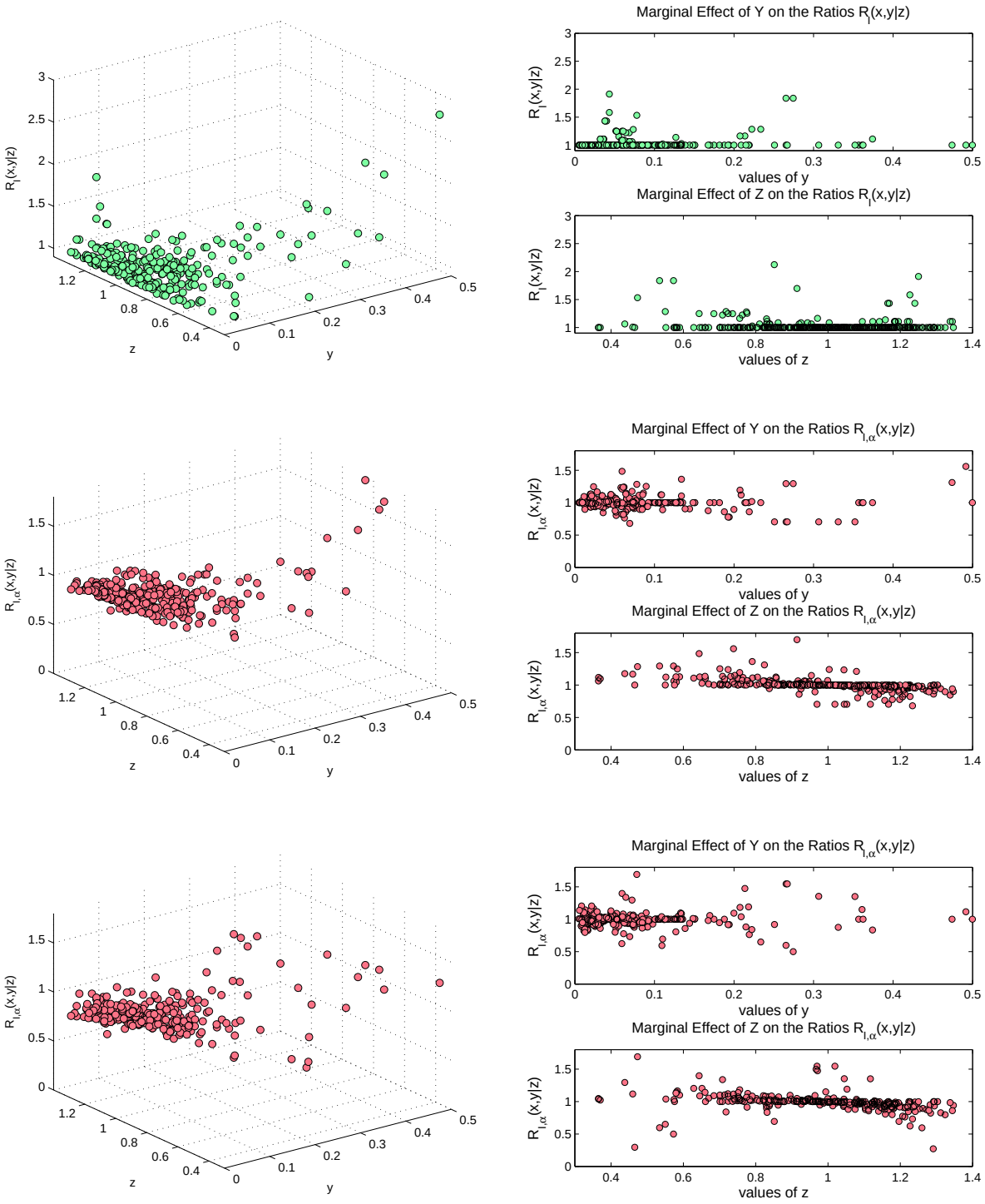

Figure 6: *Effect of* Y *and* Z_2 = DIVERSE *on the ratios* $\widehat{R}_I(X_i, Y_i)|Z_i)$ *(top panel) and* $\widehat{R}_{I,\alpha}(X_i, Y_i)|Z_i)$ *(middle panel* $\alpha = 0.95$ *and bottom panel* $\alpha = 0.5$*)*

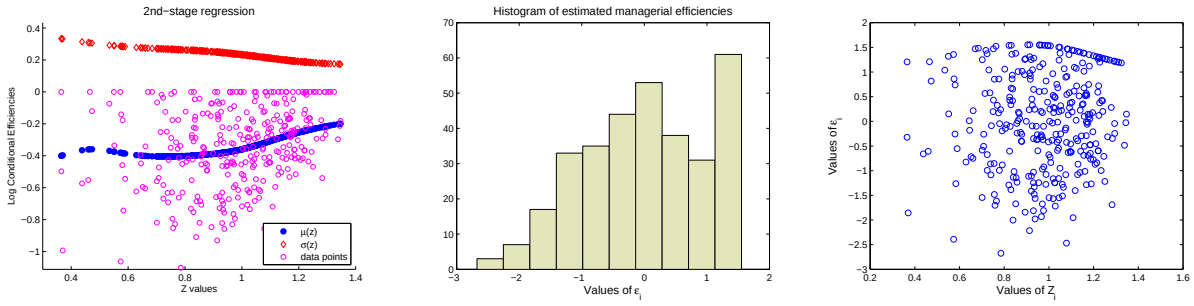

Figure 7: *Effect of* $Z_2 =$ DIVERSE *on conditional efficiencies* $\log \widehat{\lambda}(x, y|z)$*, histograms of managerial efficiencies, scatter plot of* $\hat{\varepsilon}_i$ *against* Z_2*.*

The bivariate analysis would consist of using the location-scale regression model with $Z = (Z_1, Z_2)$. Pictures to see the joint effect of Z on the frontier levels for fixed levels of the outputs are difficult to display (4 dimensions), but we know that the assumption of separability was rejected in Daraio et al. (2011). The analysis of the conditional efficiency scores as a function of Z is similar to what as been done above for one dimension. Figure 8 displays the results for the surfaces $\mu(z)$ and $\sigma(z)$. To summarize, we confirm typically the 2 marginal analysis done above, and we do not see any interaction between Z_1 and Z_2 in the effect on the conditional efficiencies. The resulting managerial estimates have correlation -0.0461 and -0.0339 with Z_1 and Z_2 respectively, so most of the effects of Z has been removed by our location scale model. The right panel of Figure 8 shows the resulting histogram of these residuals, the shape looks very similar to those obtained by the marginal analysis above, but we have as expected more mass near the efficient frontier, because we explain the conditional efficiencies by 2 environmental factors here.

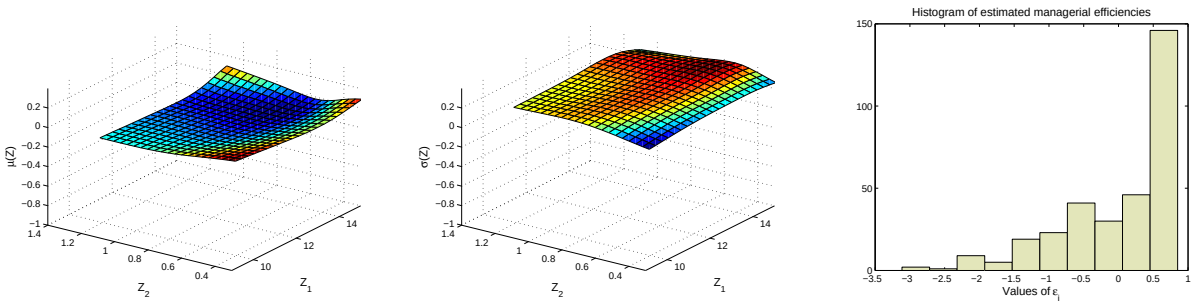

Figure 8: *Effect of* $Z = (Z_1, Z_2)$ *on conditional efficiencies* $\log \widehat{\theta}(x, y|z)$ *(left panel* $\mu(z)$*, middel panel* $\sigma(z)$*) and the histogram of estimated managerial efficiencies.*

In the case of the discrete $Z_3 = \text{MSA}$, the analysis could be performed separately on the 2 groups, or Z_3 could be introduced in the models above by using appropriate kernels (see Bădin and Daraio, 2011, for details). The procedure would go along the same lines as the one illustrated above. Of course increasing the number of variables will give estimators with less precision and the descriptive tools presented above are limited to pictures in 3 dimensions.

6 Conclusions

This paper has formalized in a nonparametric model of production the role of environmental variables Z by introducing these external factors in a non-restrictive way.
The paper clarifies what can be learned by analyzing the conditional efficiency measures and proposes a general approach to measure and infer about the impact of these factors on the production process.
By using conditional efficiency measures we can indeed measure the impact of external factors on the attainable set in the input-output space, and/or we can investigate the impact of the external factors on the distribution of inefficiency scores.
We extend existing methodological tools to explore these interrelationships, both from individual and global perspectives.
We emphasize the usefulness of regressing, in a second stage, the conditional efficiencies on the explaining factors. We suggest a flexible model that eliminates the location and the scale effect of Z on the efficiencies. The analysis of the residuals provides a measure of efficiency whitened from the main effects of the environmental factors. This allows to rank the firms according their "managerial" efficiency, even when facing heterogeneous environmental conditions.
The procedure is illustrated through simulated samples and with a real data set on US commercial banks.
The paper stresses the importance of three trails for future research: how to test the partial separability condition, which would simplify the analysis; how to test the independence between the error term and the environmental factors in the second stage regression; establish the asymptotic properties of the second stage regression. The two latter should be interrelated.

A Appendix: Effect of Z on the Frontier Levels

We illustrate the basic ideas for the input orientation and in a simple scenario where $p = q = r = 1$. In this case, we can describe the (input) efficient frontier and its conditional version by the functions $\phi(y)$ and $\phi(y|z)$. Suppose that when $Y = y_1$, Z is favorable to the production process (it acts like a free disposal input), the frontier $\phi(y_1|z)$ is displayed in dashed line in the top panel of Figure 9, we see also where is the marginal input-frontier $\phi(y_1)$. Suppose that when $Y = y_2$, Z is favorable till a level z_a and then unfavorable (acting like an undesired output), the conditional frontier $\phi(y_2|z)$ is represented by the solid line in the figure. Finally, suppose that when $Y = y_3$, Z is unfavorable, the conditional frontier $\phi(y_3|z)$ is then displayed as the dotted line in the figure. We see that for all the cases, $R_I(x, y|z) \geq 1$ but the shape of the ratios as a fucntion of z can be different according the values of Y (see the bottom panels for the 3 different levels of Y). In the case illustrated here, the analysis of these ratios $R_I(x, y|z)$ as a function of z only would be problematic, so in general, without additional assumptions, it is better to carry the analysis for fixed levels of the outputs Y.

Of course, the example illustrated in Figure 9 is rather extreme, and in many situations, the interactions between Z and Y on the frontier levels will be less complicated. In particular, if Y is independent of Z, or in a less restrictive way, under the assumption that the shape of the boundaries of $\mathcal{P}$ in the sections $Y = y$ (in the (X, Z) space) would not change with the level y, the conditional frontiers in the top panel of Figure 9 would be "parallel", so that the ratios $R_I(x, y|z)$ would have the same shape when considered as a function of z for all values of Y. For instance, this would be the case if Z would act as a free disposal input for all the values of Y (Panel I in the bottom panels of Figure 9). In the lines of Simar and Wilson (2007), this corresponds to an assumption of "partial" separability, which was implicitly assumed, and illustrated, in Daraio and Simar (2005, 2007a). In this case, the analysis of the effect of Z on the efficient frontier is largely simplified. Testing this partial separability assumption remains an open issue for future work, in the numerical illustrations below we provide some descriptive tools to investigate this issue.

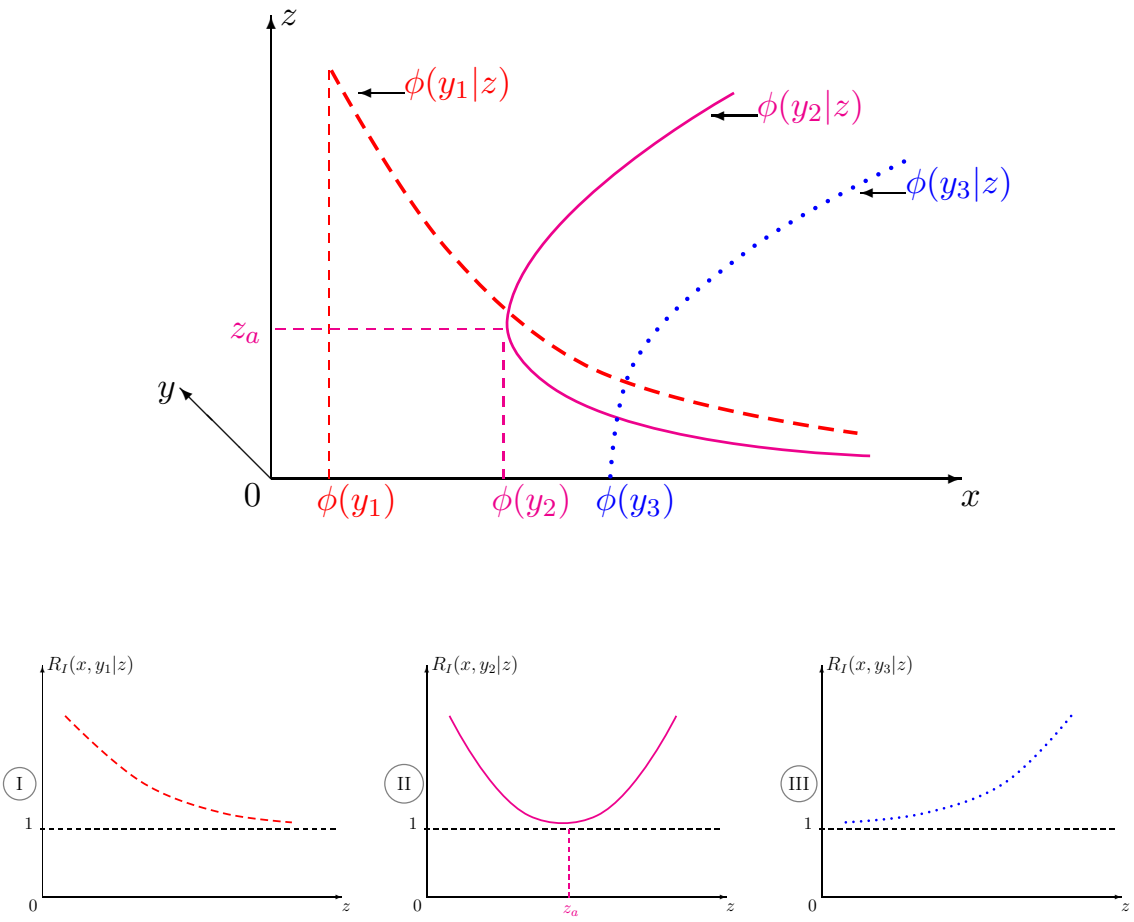

Figure 9: *Various scenarios for interpreting the effect of Z. Top panel, the minimal input frontiers, in the coordinates (x, z) for different levels of Y. Bottom panels, the corresponding ratios $R_I(x, y|z)$ as a function of z.*

B Appendix: Complementarity of Full Frontier and Partial Frontier Measures

In Figure 10 we illustrate the basic ideas of Section 3.3 for the output orientation, for the particular case of a univariate output, for a fixed level of input x_0, and for a fixed value z_0. The figure displays the conditional distributions $F(y|X \leq x_0)$ and $F(y|X \leq x_0, Z = z_0)$, for various scenarios, along with the upper boundary of their support and their α-quantiles. Remember that in this univariate output case, $R_O(x_0, y_0|z_0) = \varphi(x_0|z_0)/\varphi(x_0)$, with a similar expression for $R_{O,\alpha}(x_0, y_0|z_0)$. In the left panels, where the separability condition is verified (see panel II and III), the conditional and unconditional distributions of the inefficiencies are different, but share the same support: this results in ratios $R_O(x_0, y_0|z_0) = 1$ (we see indeed that $\varphi(x_0|z_0) \equiv \varphi(x_0)$).

Second, the information carried by the ratios $R_{0,\alpha}(x, y|z)$, when defined relative to the "partial" frontiers, is multiple. Suppose that $\Psi^z = \Psi$ and so $R_O(x, y|z) = 1$ for all points (x, y, z) (left panels in Figure 10). Then, if the distribution of the inefficiencies is affected by Z, the quantiles of $S_{Y|X,Z}$ will be different from those of $S_{Y|X}$. Therefore for all $(x, y) \in \Psi^z$, the ratios $R_{O,\alpha}(x, y|z)$ will be affected. Note that in this case ($\Psi^z = \Psi$), the changes can go in two directions for the partial parameter: if the distribution of the inefficiency is more spread in the direction of less efficient behavior (as in panel II), we observe $\varphi_\alpha(x_0|z_0) < \varphi\alpha(x_0)$ giving $R_{O,\alpha}(x_0, y_0|z_0) < 1$. On the contrary, if z_0 provides a favorable environment to efficient behavior of the firms (without affecting the upper boundary), the distribution of Y will be more concentrated near the efficient boundary when $Z = z_0$ (as in panel III), we have $\varphi_\alpha(x_0|z_0) > \varphi_\alpha(x_0)$ giving $R_{O,\alpha}(x_0, y_0|z_0) > 1$. This is of course less probable to happen when $\alpha \to 1$. That is the reason why the global test of "separability" of Daraio et al. (2010) uses test statistics only based on the full measures of efficiency and not on the partial efficiency scores, unless α is not far from one.

Third, if there is a shift on the frontier $\Psi^z \subset \Psi$, it is much more difficult to interpret the ratios $R_{O,\alpha}(x, y|z)$. It is clear that a shift of the boundary will be transferred to the partial frontier, at least for large values of α, near 1, but this effect can either be increased or compensated by a simultaneous change of the distribution of the inefficiencies from the unconditional to the conditional one. So, in the case of a shift of the boundary (see the right panels of Figure 10), we could observe $R_{O,\alpha}(x_0, y_0|z_0)$ less, equal or greater than 1. We illustrate 3 cases in Figure 10. We see that in panel IV, the shift of $\varphi_\alpha(x_0|z_0)$ with respect to $\varphi_\alpha(x_0)$ is the same as the shift of $\varphi(x_0|z_0)$ with respect to $\varphi(x_0)$, giving here $R_{O,\alpha}(x_0, y_0|z_0) < R_O(x_0, y_0|z_0) < 1$. In panel V, we have more spread toward inefficiencies when conditioning on z_0, the shift of the quantile of the conditional distribution is much more important so $R_{O,\alpha}(x_0, y_0|z_0) \ll R_O(x_0, y_0|z_0) < 1$. But we could observe, as in panel

VI, a different behavior when for a given z_0 it is more probable to reach the efficient frontier $\varphi(x_0|z_0)$ implying that we could obtain for some quantiles $R_{O,\alpha}(x_0, y_0|z_0) > R_O(x_0, y_0|z_0)$. So even if $R_O(x_0, y_0|z_0) < 1$ we could have in extreme cases $R_{O,\alpha}(x_0, y_0|z_0) \geq 1$ (as in panel VI of Figure 10).

So, to summarize the second and third points above, if $\Psi^z = \Psi$, the ratios $R_{O,\alpha}(x, y|z)$ are useful to shed light on the local impact of Z on the shape of the distribution of the inefficiencies. But it does not allow to detect, when considered alone, a local shift of the boundary of the support of (X, Y). Unless $\alpha \to 1$, because in this case, the partial frontier can serve as a robust estimator of the full frontier (see in the next section).

In any cases, these partial measures bring useful complementary information of the relative position of the quantiles of $S_{Y|X,Z}$ with respect to those of $S_{Y|X}$. It will therefore be useful to provide some detailed analysis of the ratios $R_O(x, y|z), R_{O,\alpha_1}(x, y|z), \ldots, R_{O,\alpha_k}(x, y|z)$, as described in Section 3.1, for a grid of selected values for α like, $0.99, 0.95, 0.90; \ldots, 0.50$. The latter case $\alpha = 0.50$ is providing for instance, a picture on the impact of z on the median of the inefficiency distribution.

The same would be true for the order-m partial ratios $R_{O,m}(x, y|z)$ where the particular case $m = 1$ would allow to investigate the effect of Z on the average frontier. Here, the choice of large values of m would provide the same information as for the full frontier case.

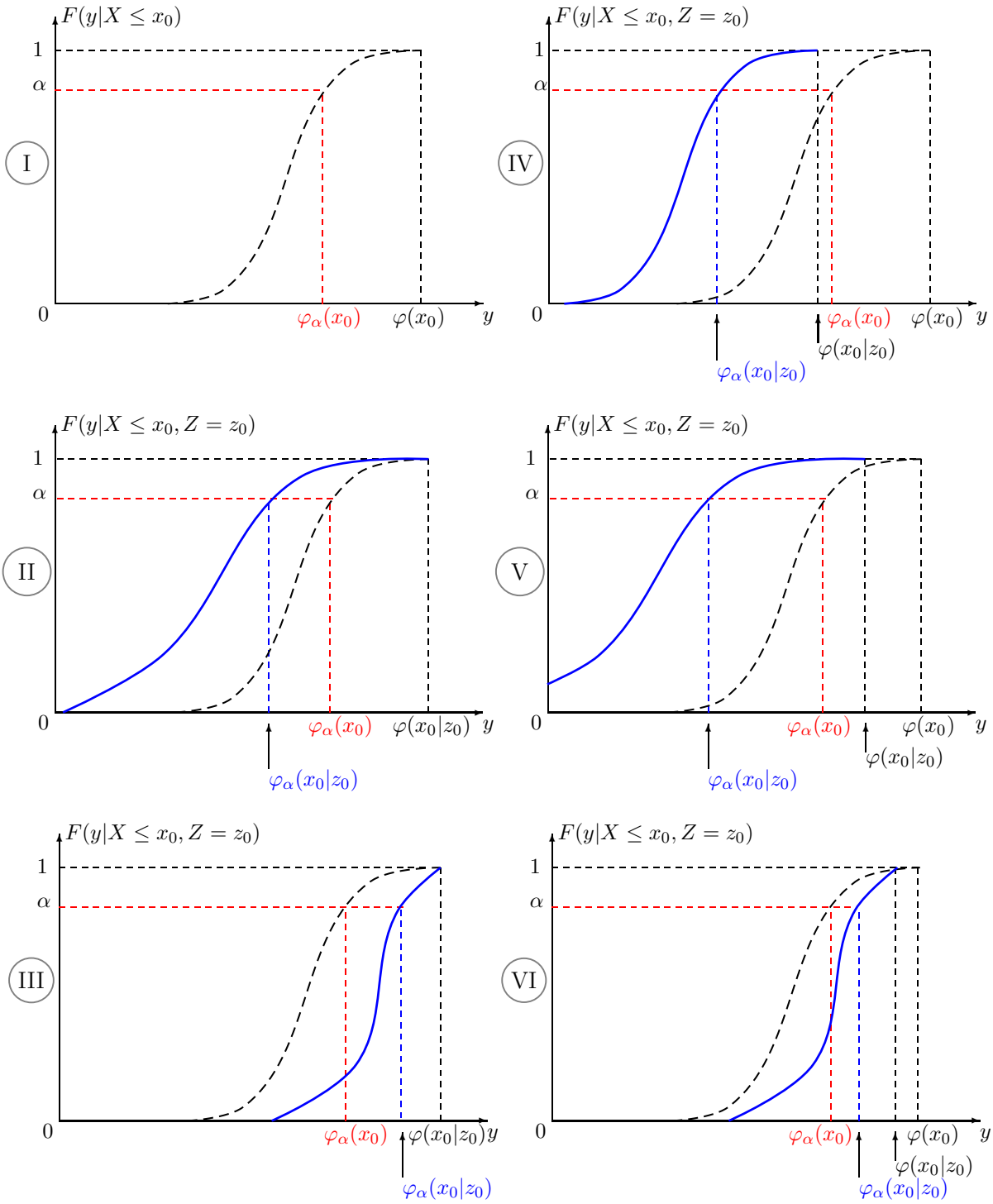

Figure 10: *Various scenarios for $F(y|X \leq x_0)$ and $F(y|X \leq x_0, Z = z_0)$. In the left panels the "separability" condition is verified at (x_0, z_0), while on the right panels, this condition is violated. In all the 6 panels, the dashed black line represents $F(y|X \leq x_0)$, with upper boundary of support $\varphi(x_0)$ and the solid blue line is $F(y|X \leq x_0, Z = z_0)$, with upper boundary of support $\varphi(x_0|z_0)$.*

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