IMPACT OF DEPENDENCE AMONG MULTIPLE CLAIMS IN A SINGLE LOSS

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Abstract

In the collective risk model, the aggregate claim amount for the portfolio is denoted by $S = X_1 + X_2 + ... + X_N$ where X_i, $i \geq 1$, is the amount of loss resulting from the ith accident and N is the total number of accidents incurred by the insurance company during a certain reference period (e.g. one year). Suppose that the amount of a loss is the sum of the claims related to the different coverages offered by a policy. These claims are most often correlated. The present paper aims to obtain bounds on the cumulative distribution function of S. These bounds can be derived when the marginal distributions of the claim amounts are specified or when only partial information is available.

Keywords: Stochastic Dominance, Dependence, Collective Risk Model.

1 Introduction

Various processes in casualty insurance involve correlated random variables. As an example, consider a travel insurance contract, including the following coverages: medical costs, repatrial costs, a lump sum in case of death, an indemnity in case of disablement (proportional to the degree of disability), loss of luggage, and different travel assistances. Some of the claims under these coverages are clearly positively correlated (medical costs and disablement payments, for instance) while others are rather negatively correlated, or even mutually exclusive (death and disability payments). In car insurance, a typical contract includes two or more coverages such as mechanical damage, bodily injuries and even lawyer's fees. In such a case, a single accident possibly produces claims related to the different coverages: the mechanical damage repairment cost, the payment for medical and hospitalization fees in case of bodily injuries and the payment of the advocate in a court battle. Recently, Frees and Valdez (1998) and Klugman and Parsa (1999) considered the loss (pure) and allocated loss adjustment expenses on a single accident. The studies conducted by these authors suggest a strong relationship between losses and expenses.

Our purpose here is to carry on with these works by quantifying the impact of correlations among the multiple claims relating to a single loss. To be specific, let S be the aggregate claim amount relating to a given insurance portfolio during a fixed period of time (e.g. one year); it can be written as

$$S = \sum_{i=1}^{N} X_i, \tag{1.1}$$

where N represents the number of accidents and X_i, $i \geq 1$, the ith loss amount; N being independent of the X_i's. Further, each loss leads to a fixed number m of claims, i.e. each X_i is decomposed as

$$X_i = X_i^{(1)} + X_i^{(2)} + ... + X_i^{(m)}, \ \ i \geq 1, \tag{1.2}$$

where $X_i^{(j)}$ is jth claim on the ith loss (with the understanding that $X_i^{(j)} = 0$ means no claim). For a fixed i, the random variables $X_i^{(j)}$, $j = 1, 2, ..., m$, are clearly dependent (since they result from the same event), but the correlation structure is mostly unknown to the actuary. Henceforth, the cumulative distribution function of $X_i^{(j)}$ is denoted as $F_{X^{(j)}}$, $j = 1, 2, \ldots, m$, $i \geq 1$. The random vectors $(X_i^{(1)}, X_i^{(2)}, \ldots, X_i^{(m)})$, $i \geq 1$, taking up all the claims resulting from the ith loss, are assumed to be independent and identically distributed, with unknown common joint cumulative distribution function $F_{(X^{(1)}, X^{(2)}, \ldots, X^{(m)})}$. The portfolio is thus assumed to be homogeneous. Finally, F_X denotes the common cumulative distribution function of the X_i's defined in (1.2).

In summary, we consider the classical collective risk model in which the losses are mutually independent but decompose each one as a sum of dependent components. In the model (1.1)-(1.2), we aim to derive bounds on the cumulative distribution function of S, F_S say, in two situations:

1. the marginals $F_{X^{(j)}}$, $j = 1, 2, \ldots, m$, are specified;

2. the marginals $F_{X^{(j)}}$, $j = 1, 2, \ldots, m$, are unknown but their first few moments (either the mean and variance or the mean, variance and skewness) and upper bound (if any) are given.

In order to obtain bounds on F_S, it suffices in fact to bound F_X with two extremal cumulative distribution functions derived with the help of the method presented in Denuit, Genest and Marceau (1999). When only partial information about the $X_i^{(j)}$'s is available, we first use the method of Kaas and Goovaerts (1985) to get bounds on the $F_{X^{(j)}}$'s and then we proceed similarly.

It is worth mentioning that the bounds on S derived in the present work are the best-possible bounds in the classical sense of stochastic dominance. Bounds on S in the stop-loss order can also be obtained with the aid of the results of Dhaene and Goovaerts (1996) – when the marginals are specified – and of Hürlimann (1998) – when the mean and variance of the marginals are known, but also the covariances.

Note that the model investigated in this paper can be reinterpreted in the context of group life insurance, where $X_i^{(1)}, X_i^{(2)}, \ldots, X_i^{(m)}$ are the claim amounts relating to the ith group of individuals (of size m). For instance, if $m = 2$, $X_i^{(1)}$ and $X_i^{(2)}$ can be regarded as the amount at risk in case of a husband and his wife deaths, respectively. This is precisely the model studied by Dhaene and Goovaerts (1997).

The paper is organized as follows. In Section 2, we briefly present the method of Denuit *et al.* (1999) to obtain bounds on F_X when the marginals are specified. Using the results of Kaas and Goovaerts (1985), this method is adapted in Section 3 to the case where only partial information on the marginals is available. Then, in Section 4, we deduce bounds on F_S. Finally, the concluding Section 5 is devoted to numerical examples.

2 Stochastic bounds on a single loss: known marginals

In this section we construct bounds on the common cumulative distribution function F_X of the X_i's defined as in (1.2). In Denuit *et al.* (1999, Proposition 2), it was shown that there exists $F_{\min}$ and $F_{\max}$ such that

$$F_{\min}(s) \leq F_X(s) \leq F_{\max}(s), \text{ for all } s \geq 0, \tag{2.1}$$

with, for $s \in I\!R$,

$$F_{\min}(s) = \sup_{(x_1, x_2, \ldots, x_m) \in \Sigma(s)} \max \left\{ \sum_{j=1}^{m} P[X_i^{(j)} < x_j] - (m-1), 0 \right\},$$

and

$$F_{\max}(s) = \inf_{(x_1, x_2, \ldots, x_m) \in \Sigma(s)} \min \left\{ \sum_{j=1}^{m} F_{X^{(j)}}(x_j), 1 \right\},$$

where

$$\Sigma(s) = \{(x_1, x_2, \ldots, x_m) \in I\!R^m | x_1 + x_2 + \ldots + x_m = s\}.$$

Note that $F_{\max}$ is a *bona fide* cumulative distribution function, whereas $F_{\min}$ is the left-continuous version of some cumulative distribution function. The bounds in (2.1) are the best-possible bounds on F_X in the full information case when no assumption is made in regards to the correlation structure between $X_i^{(1)}, X_i^{(2)}, \ldots, X_i^{(m)}$, $i \geq 1$.

Explicit expressions for $F_{\min}$ and $F_{\max}$ involved in (2.1) are available when the $X_i^{(j)}$'s have e.g. common Exponential, Pareto or Uniform distributions. In general, closed form expressions cannot be obtained and one must resort to numerical evaluation. For more details, see Denuit *et al.* (1999).

Now, assume we have at our disposal some partial knowledge of the dependence existing between the $X_i^{(j)}$'s for a fixed i, namely that there exists a multivariate cumulative distribution function H satisfying

$$H(x_1, x_2, \cdots, x_m) \leq F_{(X^{(1)}, X^{(2)}, \ldots, X^{(m)})}(x_1, x_2, \ldots, x_m) \tag{2.2}$$

for all $x_1, x_2, \ldots, x_m \in \mathbb{R}$, and a joint decumulative distribution function $\overline{G}$ such that

$$P[X_1^{(1)} > x_1, X_1^{(2)} > x_2, \ldots, X_1^{(m)} > x_m] \geq \overline{G}(x_1, x_2, \ldots, x_m) \tag{2.3}$$

for all $x_1, x_2, \ldots, x_m \in \mathbb{R}$. Note that $\overline{G}$ has not to correspond to the decumulative distribution function associated to H. The cumulative distribution function H and the decumulative distribution function $\overline{G}$ may be anything whatsoever. In practice, copula models provide a good tool to select appropriate candidates for H and $\overline{G}$; see, e.g., Wang (1998, Part II).

When (2.2) and (2.3) hold, we are in a position to use the following result, due to Denuit *et al.* (1999, Proposition 5): the inequalities

$$\sup_{(x_1, x_2, \cdots, x_m) \in \Sigma(s)} H(x_1, x_2, \ldots, x_m) \leq F_X(s) \leq 1 - \sup_{(x_1, x_2, \ldots, x_m) \in \Sigma(s)} \overline{H}^*(x_1, x_2, \cdots, x_m) \tag{2.4}$$

hold for all $s \in \mathbb{R}$. For instance, when (2.2) is satisfied with

$$H(x_1, x_2, \cdots, x_m) = \prod_{j=1}^{m} F_{X^{(j)}}(x_j), \;\; x_1, x_2, \ldots, x_m \in \mathbb{R}, \tag{2.5}$$

then the vectors $(X_i^{(1)}, X_i^{(2)}, \ldots, X_i^{(m)})$ are said to be positively lower orthant dependent; when (2.3) is fulfilled with for $\overline{G}$

$$\overline{G}(x_1, x_2, \cdots, x_m) = \prod_{j=1}^{m} (1 - F_{X^{(j)}}(x_j)), \;\; x_1, x_2, \ldots, x_m \in \mathbb{R}, \tag{2.6}$$

then the $(X_i^{(1)}, X_i^{(2)}, \ldots, X_i^{(m)})$'s are said to be posively upper orthant dependent, and finally, when (2.2) and (2.3) are simultaneously verified with H and $\overline{G}$ given in (2.5) and (2.6), the $(X_i^{(1)}, X_i^{(2)}, \ldots, X_i^{(m)})$'s are said to be positively orthant dependent (POD, in short). For more details about POD, the interested reader is referred e.g. to Szekli (1995, pp. 144-145).

Value of s	$M^{(\mu,\sigma)}(s)$	$W^{(\mu,\sigma)}(s) - M^{(\mu,\sigma)}(s)$
$0 < s < \mu$	0	$\frac{\sigma^2}{(s-\mu)^2+\sigma^2}$
$\mu < s < \frac{\delta_2}{\mu}$	$\frac{s-\mu}{s}$	$\frac{\mu}{s}$
$s > \frac{\delta_2}{\mu}$	$\frac{(s-\mu)^2}{(s-\mu)^2+\sigma^2}$	$\frac{\sigma^2}{(s-\mu)^2+\sigma^2}$

Table 3.1: Extremal distributions in (3.1), two moments known, infinite spectrum.

Value of s	$M^{(\mu,\sigma,b)}(s)$	$W^{(\mu,\sigma,b)}(s) - M^{(\mu,\sigma,b)}(s)$
$0 < s < \mu - \frac{\sigma^2}{b-\mu}$	0	$\frac{\sigma^2}{(s-\mu)^2+\sigma^2}$
$\mu - \frac{\sigma^2}{b-\mu} < s < \frac{\delta_2}{\mu}$	$\frac{\sigma^2+(\mu-b)(\mu-s)}{sb}$	$\frac{\sigma^2+\mu(\mu-b)}{s(s-b)}$
$s > \frac{\delta_2}{\mu}$	$\frac{(s-\mu)^2}{(s-\mu)^2+\sigma^2}$	$\frac{\sigma^2}{(s-\mu)^2+\sigma^2}$

Table 3.2: Extremal distributions in (3.2), two moments known, finite spectrum.

3 Stochastic bounds on a single loss: unknown marginals

We now examine the construction of bounds on F_X when only the support and the first few moments of the marginals $F_{X^{(j)}}$, $j = 1, 2, \ldots, m$, are known. We first derive the best stochastic upper and lower bounds on the $X_i^{(j)}$'s. For that purpose, we use the following results of Kaas and Goovaerts (1985).

Consider a non-negative random variable Y for which only the mean μ, and the standard deviation σ are known. Then, there exists two cumulative distribution functions, $M^{(\mu,\sigma)}$ and $W^{(\mu,\sigma)}$, say, such that

$$M^{(\mu,\sigma)}(s) \leq F_Y(s) \leq W^{(\mu,\sigma)}(s) \tag{3.1}$$

is verified for all $s \geq 0$. Explicit expressions for the extremal distributions in (3.1) are provided in Table 3.1 (Table 2 in Kaas and Goovaerts (1985)), where δ_2 stands for the second moment of Y, i.e. $\delta_2 = EY^2$.

When it is further known that there exists an upper bound b for Y, i.e. $P[Y \leq b] = 1$, the extremal distributions in (3.1) can be refined as

$$M^{(\mu,\sigma,b)}(s) \leq F_Y(s) \leq W^{(\mu,\sigma,b)}(s) \tag{3.2}$$

which is verified for all $s \geq 0$. Explicit expressions for these extremal distributions are provided in Table 3.2 (Table 1 in Kaas and Goovaerts (1985)).

When the skewness γ of Y is also known, tighter bounds $M^{(\mu,\sigma,\gamma)}$ and $W^{(\mu,\sigma,\gamma)}$, say, can be found such that

$$M^{(\mu,\sigma,\gamma)}(s) \leq F_Y(s) \leq W^{(\mu,\sigma,\gamma)}(s) \tag{3.3}$$

is verified for all $s \geq 0$. Explicit expressions for the extremal distributions in (3.3) are provided in Table 3.3 (Table 4 in Kaas and Goovaerts (1985)), where the following symbols

Value of s	$M^{(\mu,\sigma,\gamma)}(s)$	$W^{(\mu,\sigma,\gamma)}(s) - M^{(\mu,\sigma,\gamma)}(s)$
$0 < s < \alpha_-$	0	$\frac{\mu-\beta_2(s)}{s-\beta_2(s)}$
$\alpha_- < s < \frac{\delta_2}{\mu}$	$\frac{\sigma^2+(\mu-s)(\mu-\beta_1(s))}{s\beta_1(s)}$	$\frac{\sigma^2+(\mu-\beta_1(s))\mu}{s(s-\beta_1(s))}$
$\frac{\delta_2}{\mu} < s < \alpha_+$	$\frac{\mu-s}{\beta_2(s)-s}$	$\frac{\mu-\beta_2(s)}{s-\beta_2(s)}$
$s > \alpha_+$	$\frac{\sigma^2+(\mu-s)(\mu-\beta_1(s))}{s\beta_1(s)} + \frac{\sigma^2+(\mu-s)\mu}{(\beta_1(s)-s)\beta_1(s)}$	$\frac{\sigma^2+(\mu-\beta_1(s))\mu}{s(s-\beta_1(s))}$

Table 3.3: Extremal distributions in (3.3), three moments known, infinite spectrum.

Value of s	$M^{(\mu,\sigma,\gamma,b)}(s)$	$W^{(\mu,\sigma,\gamma,b)}(s) - M^{(\mu,\sigma,\gamma,b)}(s)$
$0 < s < \alpha_-$	0	$\frac{\sigma^2+(\mu-\beta_2^*(s))(\mu-b)}{(s-\beta_2^*(s))(s-b)}$
$\alpha_- < s < \zeta$	$\frac{\sigma^2+(\mu-s)(\mu-\beta_1(s))}{s\beta_1(s)}$	$\frac{\sigma^2+(\mu-\beta_1(s))\mu}{s(s-\beta_1(s))}$
$\zeta < s < \alpha_+$	$\frac{\sigma^2+(\mu-s)(\mu-b)}{(\beta_2^*(s)-s)(\beta_2^*(s)-b)}$	$\frac{\sigma^2+(\mu-\beta_2^*(s))(\mu-b)}{(s-\beta_2^*(s))(s-b)}$
$s > \alpha_+$	$\frac{\sigma^2+(\mu-s)(\mu-\beta_1(s))}{s\beta_1(s)} + \frac{\sigma^2+(\mu-s)\mu}{(\beta_1(s)-s)\beta_1(s)}$	$\frac{\sigma^2+(\mu-\beta_1(s))\mu}{s(s-\beta_1(s))}$

Table 3.4: Extremal distributions in (3.4), three moments known, finite spectrum.

are used: δ_3 stands for third moment of Y, i.e. $\delta_3 = EY^3$,

$$\beta_1(s) = \frac{\gamma + 3\sigma^2 + \mu^3 - s\delta_2}{\delta_2 - s\mu}, \quad \beta_2(s) = \frac{\delta_2 - \mu s}{\mu - s},$$

and

$$\alpha_\pm = \frac{\delta_3 - \mu\delta_2 \pm \sqrt{(\delta_3 - \mu\delta_2)^2 - 4\sigma^2(\mu\delta_3 - \delta_2^2)}}{2\sigma^2}.$$

As in (3.2), when it is further known that there exists an upper bound b for Y, the extremal distributions in (3.3) can be refined as

$$M^{(\mu,\sigma,\gamma,b)}(s) \leq F_Y(s) \leq W^{(\mu,\sigma,\gamma,b)}(s) \tag{3.4}$$

which is verified for all $s \geq 0$. Explicit expressions for these extremal distributions are provided in Table 3.4 (Table 3 in Kaas and Goovaerts (1985)), where the following symbols are used:

$$\zeta = \frac{\gamma + 3\sigma^2 + \mu^3 - b\delta_2}{\delta_2 - b\mu} \text{ and } \beta_2^*(s) = \frac{\gamma + 3\sigma^2 + \mu^3 - (b+s)\delta_2 + bs\mu}{\delta_2 - (b+s)\mu + bs}.$$

Now, let $\mu^{(j)}$, $\sigma^{(j)}$ and $\gamma^{(j)}$ be the mean, the standard deviation and the skewness corresponding to $F_{X^{(j)}}$, $j = 1, 2, \ldots, m$. With Tables 3.1 and 3.3, we can find the best upper and lower bounds on $F_{X^{(j)}}$, $j = 1, 2, \ldots, m$, namely

$$M_j(s) \leq F_{X^{(j)}}(s) \leq W_j(s), \text{ for } j = 1, 2, \ldots, m, \tag{3.5}$$

where M_j (resp. W_j) stands for either $M^{(\mu^{(j)},\sigma^{(j)})}$ or $M^{(\mu^{(j)},\sigma^{(j)},\gamma^{(j)})}$ (resp. $W^{(\mu^{(j)},\sigma^{(j)})}$ or $W^{(\mu^{(j)},\sigma^{(j)},\gamma^{(j)})}$). When there exist b_j such that $F_{X^{(j)}}(b_j) = 1$, $j = 1, 2, \ldots, m$, then (3.5) becomes

$$M_j^{(b_j)}(s) \leq F_{X^{(j)}}(s) \leq W_j^{(b_j)}(s), \text{ for } j = 1, \ldots, m. \tag{3.6}$$

where $M_j^{(b_j)}$ (resp. $W_j^{(b_j)}$) stands for either $M^{(\mu^{(j)},\sigma^{(j)},b_j)}$ or $M^{(\mu^{(j)},\sigma^{(j)},\gamma^{(j)},b_j)}$ (resp. $W^{(\mu^{(j)},\sigma^{(j)},b_j)}$ (resp. $W^{(\mu^{(j)},\sigma^{(j)},\gamma^{(j)},b_j)}$), $j = 1, 2, \ldots, m$. Then,

$$\widetilde{F}_{\min}(s) \leq F_X(s) \leq \widetilde{F}_{\max}(s), \text{ for all } s \geq 0, \tag{3.7}$$

with, for $s \in I\!R$,

$$\widetilde{F}_{\min}(s) = \sup_{(x_1,x_2,\ldots,x_m)\in\Sigma(s)} \max\left\{\sum_{j=1}^{m} \lim_{n\to\infty} M_j\left(x_j - \frac{1}{n}\right) - (m-1), 0\right\},$$

and

$$\widetilde{F}_{\max}(s) = \inf_{(x_1,x_2,\ldots,x_m)\in\Sigma(s)} \min\left\{\sum_{j=1}^{m} W_j(x_j), 1\right\}.$$

Of course, when there exist b_j such that $F_{X^{(j)}}(b_j) = 1$, $j = 1, ..., m$, M_j and W_j in the expressions of $\widetilde{F}_{\min}$ and $\widetilde{F}_{\max}$ are replaced by $M_j^{(b_j)}$ and $W_j^{(b_j)}$, respectively. If the $(X_i^{(1)}, X_i^{(2)}, \ldots, X_i^{(m)})$'s are POD, then (3.7) is valid with the following improved bounds:

$$\widetilde{F}_{\min}(s) = \sup_{(x_1,x_2,\ldots,x_m)\in\Sigma(s)} \prod_{j=1}^{m} M_j(x_j)$$

and

$$\widetilde{F}_{\max}(s) = 1 - \sup_{(x_1,x_2,\ldots,x_m)\in\Sigma(s)} \prod_{j=1}^{m} (1 - W_j(x_j)).$$

4 Stochastic bounds on the total amount of loss

If N is independent of the X_i's, the cumulative distribution function of the total amount of loss S defined in (1.1) can be expressed as

$$F_S(s) = \sum_{n=0}^{\infty} P[N = n] F_X^{*n}(s), \; s \geq 0,$$

where F_X^{*n} is the n-fold convolution of F_X. The random variable N is usually a Poisson, Negative Binomial or Poisson-Inverse Gaussian random variable (see e.g. Panjer and Willmot (1992)).

It is well-known that the stochastic dominance is closed under compounding (see e.g. Shaked and Shanthikumar (1994), Goovaerts *et al.* (1990) or Rolski *et al.* (1999)). Therefore, the stochastic bounds found in the two previous sections lead to bounds on the cumulative distribution function F_S in both cases (namely, "full" and "partial" information about the marginals).

Firstly, from (2.1), F_S is constrained by

$$F_{S_{\min}}(s) \leq F_S(s) \leq F_{S_{\max}}(s), \text{ for all } s \geq 0, \tag{4.1}$$

with, for $s \in I\!R$,

$$F_{S_{\min}}(s) = \sum_{n=0}^{\infty} P[N=n] F_{\min}^{*n}(s) \text{ and } F_{S_{\max}}(s) = \sum_{n=0}^{\infty} P[N=n] F_{\max}^{*n}(s).$$

Improved bounds are similarly deduced from (2.4) when additional information about the structure of dependence is available. The fast Fourier transform or Panjer's algorithm (see Klugman *et al.* (1998) or Rolski *et al.* (1999)) can be used to approximate numerically $F_{S_{\min}}$ and $F_{S_{\max}}$.

On the other hand, when only the support, the mean, the variance and the skewness of the $X_i^{(j)}$'s are known, bounds on F_S are deduced from (3.7):

$$\widetilde{F}_{S_{\min}}(s) \leq F_S(s) \leq \widetilde{F}_{S_{\max}}(s), \text{ for all } s \geq 0, \tag{4.2}$$

with, for $s \in I\!R$,

$$\widetilde{F}_{S_{\min}}(s) = \sum_{n=0}^{\infty} P[N=n] \widetilde{F}_{\min}^{*n}(s) \text{ and } \widetilde{F}_{S_{\max}}(s) = \sum_{n=0}^{\infty} P[N=n] \widetilde{F}_{\max}^{*n}(s).$$

5 Numerical illustrations

In order to illustrate the method proposed in the preceeding sections, we have considered $m = 2$, $X_1^{(1)}$ distributed according to the Exponential distribution with mean 1 and $X_1^{(2)}$ according to a Pareto with parameters 4 and 3, i.e.

$$P[X_1^{(2)} \leq x] = 1 - \left(\frac{3}{3+x}\right)^4, \quad x \geq 0.$$

In Figure 1, we plotted the cumulative distribution function of $X_1^{(1)}$ together with its two-moment and three-moment approximations M_1 and W_1 (for $\mu^{(1)} = 1$, $\sigma^{(1)} = 1$ and $\gamma^{(1)} = 3$). Figure 2 is the analog for $X_1^{(2)}$, with $\mu^{(2)} = 1$, $\sigma^{(2)} = \sqrt{2}$ and $\gamma^{(2)} = 23$. It can be seen that the knowledge of the first three moments determines a satisfactory band for the cumulative distribution function of $X_1^{(1)}$ and $X_1^{(2)}$. In Figure 3, the bounds on the unknown F_X are depicted, together with those based on the three-moment approximation. Note that the graph consists of step functions because of the numerical method used; see Denuit *et al.* (1999) for more details. Figure 4 gives the bounds on F_X when the random couple $(X_1^{(1)}, X_1^{(2)})$ is POD. Finally, Figures 5 and 6 show the bounds on F_S (without and with the POD assumption). We considered for N a Poisson random variable with mean 1. Note that there is now a probability mass in 0 equal to $1/e$.

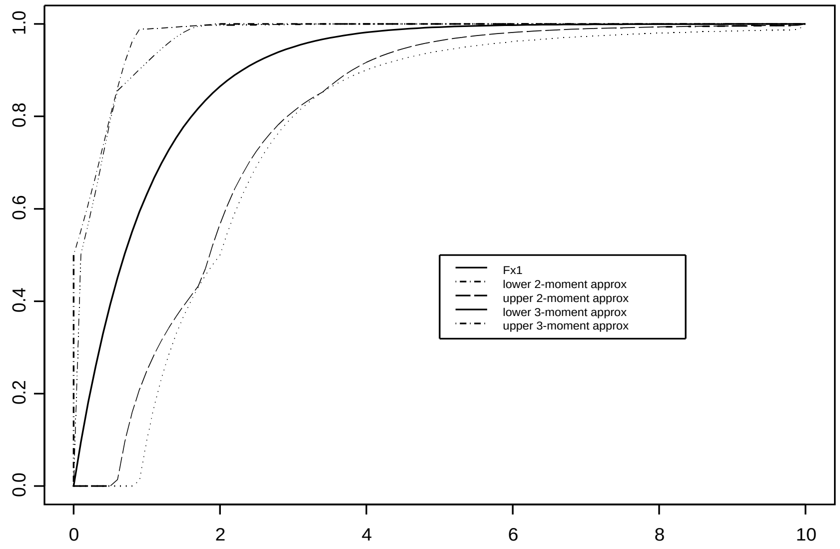

Figure 1: *Graph of* $F_{X^{(1)}}$ *together with* $M_1^{(1,1)}$, $M_1^{(1,1,3)}$, $W_1^{(1,1)}$ *and* $W_1^{(1,1,3)}$.

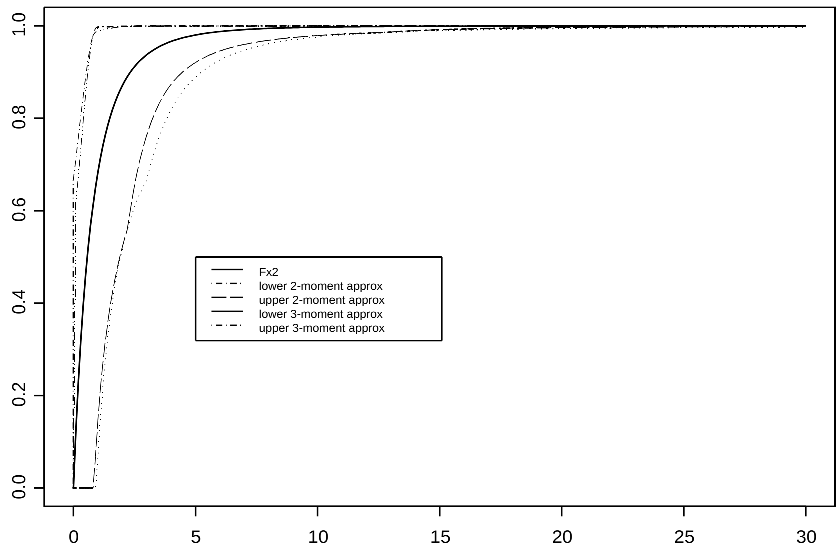

Figure 2: *Graph of* $F_{X^{(2)}}$ *together with* $M_1^{(1,1.4142)}$, $M_1^{(1,1.4142,23)}$, $W_1^{(1,1.4142)}$ *and* $W_1^{(1,1.4142,23)}$.

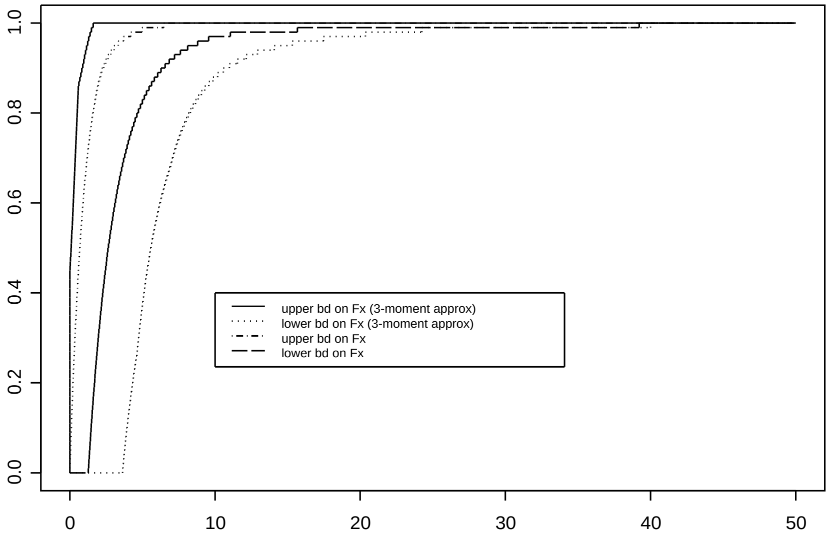

Figure 3: *Bounds on F_X.*

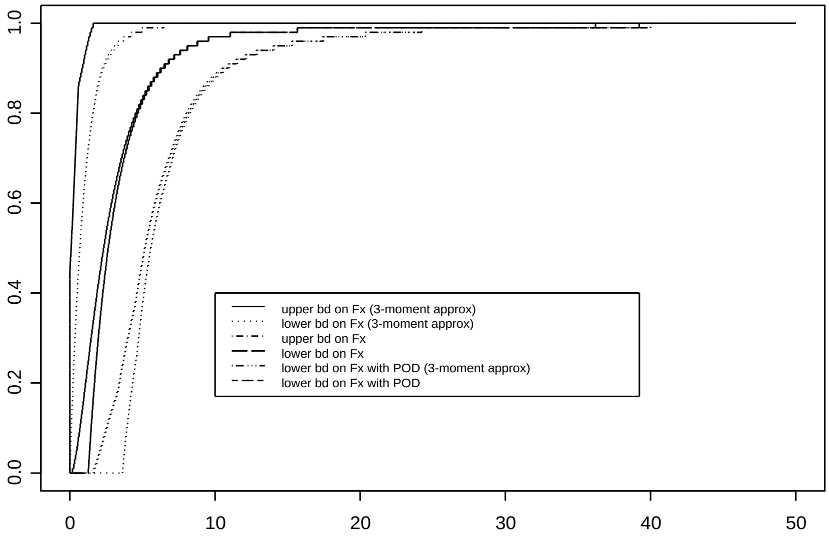

Figure 4: *Bounds on F_X in case of POD.*

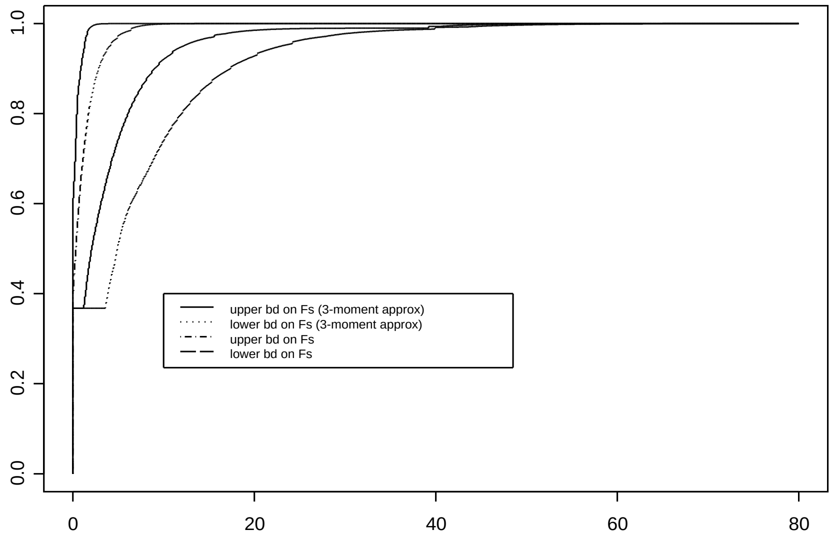

Figure 5: *Bounds on F_S.*

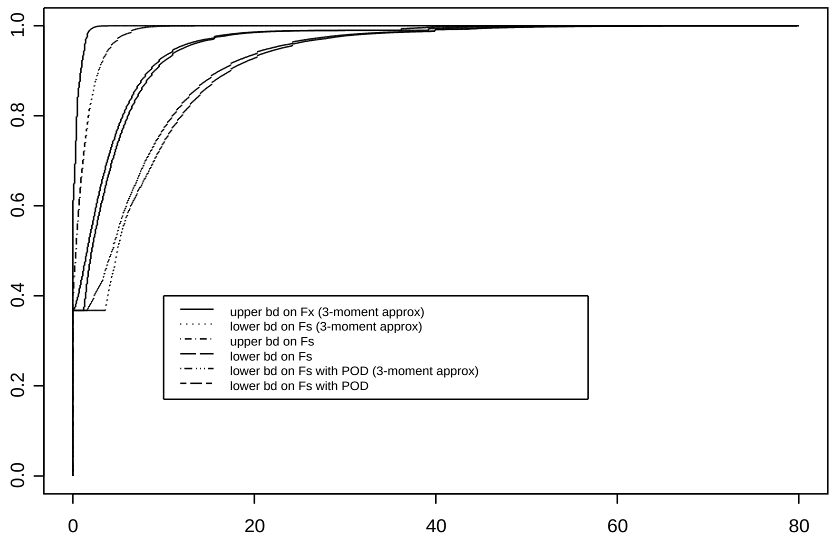

Figure 6: *Bounds on F_S in case of POD.*

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