On the use of equispaced discrete distributions

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Abstract

The Kolmogorov distance is used to transform arithmetic severities into equispaced arithmetic severities in order to reduce the number of calculations when using algorithms like Panjer's formulae for compound distributions. An upper bound is given for the Kolmogorov distance between the true compound distribution and the transformed one. Advantages of the Kolmogorov distance and disandvantages of the total variation distance are discussed. When the bounds are too big, a Berry-Esseen result can be used. Then almost every case can be handled by the techniques described in this paper. Numerical examples show the interest of the methods.

Keywords

Recursive formula; compound distribution; infinite divisibility; equispaced discrete distribution; Kolmogorov distance; total variation distance; upper and lower bounds; Berry-Esseen bound.

1. Introduction

The total claims distribution of a risk is given by

$$S = X_1 + \cdots + X_N \tag{1}$$

where X denotes the severity of the claim distribution and N the claim frequency distribution. We suppose that the claim severity has a discrete probability function $f_X(x)$ and the claim frequency distribution has a probability function $p(n)$.
The evaluation of the probability distribution S is difficult since it needs the evaluation of convolutions of order n and an infinite summation :

$$f_S(x) = \sum_{n=0}^{\infty} p(n) f_X^{*^n}(x) \tag{2}$$

De Pril (1985) has given a recursive algorithm for the evaluation of the convolutions of arithmetic severities :

$$f_X^{*^n}(0) = f_X^n(0) \tag{3}$$

$$f_X^{*^n}(x) = \frac{1}{f_X(0)} \sum_{i=1}^{x} \left(\frac{n+1}{x} i - 1\right) f_X(i) f_X^{*^n}(x-i) \quad x \geq 1 \tag{4}$$

Under the hypothesis that the claim frequency distribution satisfies the recursion :

$$p(n) = p(n-1)\left(a + \frac{b}{n}\right) \qquad n \geq 1 \tag{5}$$

Panjer (1981) has given a recursive algorithm for the evaluation of the compound distribution :

$$f_S(0) = \Psi_N(\ln f_X(0)) \tag{6}$$

$$f_S(x) = \frac{1}{1 - a f_X(0)} \sum_{i=1}^{x} \left(a + b\frac{i}{x}\right) f_X(i) f_S(x-i) \quad x \geq 1 \tag{7}$$

where $\Psi_N(z) = \mathbb{E}e^{zN}$ denotes the moment generating function of N.
For both the recursions (4) and (7), the computing time will be significantly reduced if the claim severity is equispaced because in this case working on $0, h, 2h, \ldots$ is the same as working on $0, 1, 2, \ldots$.
Our goal is to replace the evaluation of

$$\mathbb{P}[S \leq a]$$

by the evaluation of

$$\mathbb{P}[S_{app} \leq a]$$

and to measure the difference between the two by the Kolmogorov distance.
The concepts discussed hereafter will be illustrated with the Poisson distribution. Of course the other laws satisfying (5) may be used as well as more complicated distributions satisfying more complicated recursions (see section 8).
The following hypothetical claim severity distribution will be used for illustration :

X	0	7	12	17	21	23	28	39	46	53	67
f_X	0.05	0.1	0.1	0.15	0.05	0.05	0.05	0.1	0.1	0.15	0.1

Table 1: Claim severity distribution

2. Techniques given in the litterature

Two methods are given in the litterature in order to construct discrete equispaced distributions.
Gerber and Jones (1976) propose a very intuitive rounding method. Suppose we want to transform the claim severity distribution X into an equispaced distribution on $x = 0, h, 2h, \ldots$
The probabilities are simply given by

$$f_{X_{app}}(0) = F_X(\frac{h}{2} - 0)$$

$$f_{X_{app}}(xh) = F_X(xh + \frac{h}{2} - 0) - F_X(xh - \frac{h}{2} - 0) \quad x = 1, 2, \ldots$$

Suppose we choose $h = 20$. We find the approached equispaced distribution (X_{app}) :

X_{app}	0	20	40	60
$f_{X_{app}}$	0.15	0.40	0.20	0.25

Table 2: Approached distribution; Rounding, $h = 20$

The rounding method has the disadvantage that the approached distribution does not conserve any moment of the true distribution.
Gerber (1982) propose a method that matches some moments : the local moment matching method (LMM).
Suppose for example we want to match two moments. Let the interval $(x_k, x_k + 2h]$. The masses m_0^k, m_1^k, m_2^k are associated to the points x_k, x_{k+h}, x_{k+2h} in order to preserve the first two moments :

$$\sum_{j=0}^{2}(x_k + jh)^r m_j^k = \int_{x_k}^{x_k+2h} x^r dF_X(x) \quad r = 0, 1, 2 \tag{8}$$

Using the Lagrange formula, Gerber (1982) gives the solution of the system (8) :

$$m_j^k = \int_{x_k}^{x_k+2h} \prod_{i \neq j} \frac{x - x_k - ih}{(j-i)h} dF_X(x) \quad j = 0, 1, 2$$

In our case the integral is a sum.
The method might be applied in order to match more than two moments.
By taking $h = 20$, we find

m_0^0	m_1^0	m_2^0	m_0^1	m_1^1	m_2^1
0.1318	0.4389	0.0793	0.0836	0.2704	−0.0040

Table 3: Intermediate masses; LMM

and thus the following approached distribution :

X	0	20	40	60	80
$f_{X_{app}}$	0.1318	0.4389	0.1629	0.2704	−0.0040

Table 4: Approached distribution; LMM, $h = 20$

This example shows an important disadvantage of the local moment matching method : the probabilities can take negative values.

With $h = 17$, we find

X	0	17	34	51	68
$f_{X_{app}}$	0.0998	0.4268	0.0921	0.3009	0.0804

Table 5: Approached distribution; LMM, $h = 17$

3. The Kolmogorov distance

The methods presented in the previous section are essentially local and they present some important disadvantages. In the sequel we will propose a global method consisting in the minimization of a distance between the true distribution and the approached distribution. Having choosen the equispaced points of the approached distribution, we minimize the distance in order to find the probabilities associated with the equispaced points.
The Kolmogorov distance between two random variables X and Y is given by

$$d_K(X, Y) = \max_x |F_X(x) - F_Y(x)| \tag{9}$$

where $F_X(x)$ denotes the cumulative distribution function of X.
This distance is used for the Kolmogorov-Smirnov adjustment test between two distributions. A small d_K is in favour of the null hypothesis that the two distributions are the same while a large value of d_K is in favour of the alternative hypothesis that the two distributions are different.
Suppose the same claim severity distribution as in the previous section. We want to approximate it by a distribution with masses at $0, 20, 40, 60, 80$ i.e. such that $h = 20$. The problem is to find $f_{X_{app}}(0), \cdots, f_{X_{app}}(80)$ in order to minimize $d_K(X, X_{app})$. This is easily done numerically.
We find

X_{app}	0	20	40	60	80
$f_{X_{app}}$	0.2250	0.2976	0.2976	0.0899	0.0899

$$d_K(X, X_{app}) = 0.175$$

Table 6: Approached distribution ; Kolmogorov, $h = 20$

Other approached distributions might be obtained with the same Kolmogorov distance. This suggests that we might minimize d_K under the constraint that the first moment of X is preserved.

We then find

X_{app}	0	20	40	60	80
$f_{X_{app}}$	0.225	0.2960	0.2604	0.1312	0.0874

$$d_K(X, X_{app}) = 0.175$$

Table 7: Approached distribution ; Kolmogorov, $h = 20$, first moment constrained

If the minimization is proceeded with the constraint of the first two moments, we find

X_{app}	0	20	40	60	80
$f_{X_{app}}$	0.1833	0.2500	0.4206	0.1156	0.0305

$$d_K(X, X_{app}) = 0.2167$$

Table 8: Approached distribution ; Kolmogorov, $h = 20$, first two moments constrained

We see that in this case the Kolmogorov distance is higher with two moments constrained than with only one. The cost of matching several moments is such that the the d_K is higher. The preceding considerations suggest the following algorithm when using the Kolmogorov distance to construct an equispaced distribution :

1. Choose a set of equispaced points
2. m=0
3. Minimize $d_K(X, X_{app})$ with m moments constrained
4. Minimize $d_K(X, X_{app})$ with $m + 1$ moments constrained
5. If $d_K(m) = d_K(m + 1)$ then $m = m + 1$ and go to 4
   else keep the approached distribution given by the minimization of $d_K(m)$.
6. Note that the necessity of keeping n moments may overrule the rule 4
7. If the distance cannot be accepted then go back to 1 and change the equispaced points

Clearly the minimum may be achieved by more than one constrained distribution. This is not a problem in practice because the bounds we will derive for the distance between the compound distributions will depend only on the Kolmogorov distance between the distributions of X and X_{app}.

4. The effect of the approximation on the compound distribution

Let us now have a look at the Kolmogorov distance between the compound distribution ($f_S(x)$) and the approached compound distribution ($f_{S_{app}}(x)$).
Suppose that N is Poisson distributed $N \sim Po(\lambda = 0.1)$.
The following distances have been calculated .

	$d_K(X, X_{app})$	$d_K(S, S_{app})$
$h = 10; 4$ moments	0.125	0.0115
$h = 10; 5$ moments	0.125	0.0115
$h = 10; 6$ moments	0.1273	0.0117
$h = 17; LMM$	0.1696	0.0157
$h = 17; 2$ moments	0.1395	0.0131
$h = 20;$ *Rounding*	0.25	0.0228
$h = 20; 1$ moment	0.175	0.0161
$h = 20; 2$ moments	0.2167	0.0202
$h = 20; 3$ moments	0.2311	0.0212
$h = 25; 1$ moments	0.225	0.0207
$h = 25; 2$ moments	0.2646	0.0242

Table 9: Kolmogorov distances with $\lambda = 0.1$

We see that the $d_K(X, X_{app})$ increases with h. This is not a surprise because, at the limit $h = 0$, we would trivially have $d_K(X, X_{app}) = 0$.
Concerning $d_K(S, S_{app})$, the results are excellent. In each case, it is less than 10% of $d_K(X, X_{app})$.
Of course the parameters of the claim frequency distribution must play a role. In our example, there is only one parameter : λ. Intuitively, we expect the $d_K(S, S_{app})$ to grow with λ. The following calculations show it is not necessarily the case.
We choose $h = 10$ and 5 moments constrained ($d_K(X, X_{app}) = 0.125$). We find

λ	$d_K(S, S_{app})$
0.05	0.005992
0.10	0.011488
0.20	0.021113
1	0.053471
2	0.048496
3	0.036778
4	0.027632
5	0.023072
20	0.010845

Table 10: Kolmogorov distances; $h = 10$, first five moments constrained

As this example shows, the $d_K(S, S_{app})$ grows until around $\lambda = 1$ and then decreases. In this example, the true distribution X and the approached distribution X_{app} are so closed that high order convolutions of the distributions remain very close. This explains why the Kolmogorov distance between the compound distributions decreases when λ becomes very large.
Of course this is not the general rule. The following example shows that the Kolmogorov distance between the compound distributions increases with λ when the approximation between X and X_{app} is bad :

X	0	2	4
f_X	0.4	0.2	0.4

X_{app}	0	3
$f_{X_{app}}$	0.3	0.7

$$d_K(X, X_{app}) = 0.4$$

λ	$d_K(S, S_{app})$
0.1	0.037062
1	0.185621
10	0.126143
50	0.147345
100	0.180262
500	0.344425
1000	0.464542

Table 11: Kolmogorov distances; pathological example

5. Upper and lower bounds for $d_K(S, S_{app})$

We are going to use the following lemma (see De Pril and Dhaene (1992) for a proof) :

$$\text{If } d_K(X, Y) \leq a \text{ then } d_K(X^{*^n}, Y^{*^n}) \leq na$$

With the analogous formula of (2) for the cdf's, we easily find an upper bound for the Kolmogorov distance between the true distribution and the approached distribution.

$$\begin{aligned} d_K(S,S_{app}) &= \max_x |F_S(x) - F_{S_{app}}(x)| \\ &= \max_x \left| \sum_{n=0}^{\infty} p(n)(F_X^{*^n}(x) - F_{X_{app}}^{*^n}(x)) \right| \\ &\le \sum_{n=0}^{\infty} p(n) \max_x |F_X^{*^n}(x) - F_{X_{app}}^{*^n}(x)| \qquad (10) \\ &= \sum_{n=0}^{\infty} p(n) d_K(X^{*^n}, X_{app}^{*^n}) \qquad (11) \\ &\le \sum_{n=0}^{\infty} np(n) d_K(X, X_{app}) \qquad (12) \\ &= \mathbb{E}N d_K(X, X_{app}) \qquad (13) \end{aligned}$$

A trivial lower bound is given by

$$d_K(S,S_{app}) \ge |\Psi_N(1 - f_X(0)) - \Psi_N(1 - f_{X_{app}}(0))| \qquad (14)$$

Obviously $d_K(X^{*^n}, X_{app}^{*^n})$ is always ≤ 1. Therefore the upper bound can be sharpened :

$$\begin{aligned} d_K(S,S_{app}) &\le \mathbb{E}N d_K(X, X_{app}) \\ &\quad - \sum_{n=2}^{\infty} p(n) \max(0, n d_K(X, X_{app}) - 1) \qquad (15) \end{aligned}$$

Let us take the example of table 10 :

λ	$d_K(S, S_{app})$	lower bound	upper bound	sharpened upper bound
0.1	0.011488	0.000712	0.0125	0.0125
0.2	0.021113	0.001297	0.0250	0.025
1	0.053471	0.003042	0.125	0.124999

Table 12: Comparison between the Kolmogorov distance and the bounds; $h = 10$

Even when λ (or $\mathbb{E}N$ in a more general setting) is far from 0, a precise bound can be obtained. There will be a cost for this operation : the calculation of some convolutions of X and X_{app}. Define $\Gamma_{\mathbb{E}N,a,b} = \{\lfloor \mathbb{E}N \rfloor - a; \cdots ; \lfloor \mathbb{E}N \rfloor + b\}$
We have

$$\begin{aligned} d_K(S,S_{app}) &\le \max_x \left| \sum_{j \in \Gamma_{\mathbb{E},a,b}} p(j)(F_X^{*^j}(x) - F_{X_{app}}^{*^j}(x)) \right| \\ &\quad + \sum_{j \in (\mathbb{Z}^+ \setminus \Gamma_{\mathbb{E},a,b})} p(j) \min(1, j d_K(X, X_{app})) \end{aligned}$$

$d_K(S, S_{app})$	0.053471
twice sharpened upper bound $\Gamma = \{1, 2\}$	0.086197
twice sharpened upper bound $\Gamma = \{1, 3\}$	0.063503
twice sharpened upper bound $\Gamma = \{1, 4\}$	0.055846

Table 13: Comparison between the Kolmogorov distance and the bounds; $h = 10$, $\lambda = 1$

6. The distance of total variation

Whenever you want to compare two integer random variables X and Y, the total variation distance between the two variables is defined by

$$d_{TV}(X, Y) = \sup_{A \subseteq Z^+} |P(X \in A) - P(Y \in A)| \tag{16}$$

seems adequate. For example, this distance is very interesting when comparing the Poisson-Binomial and the Poisson distributions (see Barbour, Holst and Janson(1992)).
It is easy to show that the definition of the total variation distance is equivalent to

$$d_{TV}(X, Y) = \frac{1}{2} \sum_{m=0}^{\infty} |P(X = m) - P(Y = m)| \tag{17}$$

and so, it is immediate to deduce that

$$d_K \leq d_{TV}$$

In the present study, the total variation distance is uninteresting because it couldn't discriminate equispaced distributions. The reason for this fact is that only a few points share a positive probability for both the claim severity distribution and the approximate one.
This explains why we prefer to use the Kolmogorov distance than the total variation distance even if, at first side, it seems less general.

7. Berry-Esseen bound for the Normal approximation

In the preceding sections, we saw that whenever $\mathbb{E}N$ is high, the upper bound (15) is useless, even if, in practice, the approximation based on the Kolmogorov distance remains a good one. This fact, not really surprising, can be confirmed by a Berry-Esseen result.
Michel (1993) gives a Berry-Esseen bound for the normal approximation of a compound Poisson distribution.
Let S a compound Poisson distribution. Assume the the skewness of S is finite. Then

$$\sup_{t \in \mathbb{R}} \left| \mathbb{P}\left(\frac{S - \mathbb{E}S}{\sigma_S} \right) - \Phi(t) \right| \leq 0,8\gamma_S$$

where $\Phi(t)$ *is the cumulative distribution of a standard normal distribution and*

$\gamma_S = \frac{\mathbb{E}S^3 - 3\mathbb{E}S\mathbb{E}S^2 + 2(\mathbb{E}S)^3}{\sigma_S^3}$ *is the skewness of S*

For our numerical example, whenever N is a Poisson random variable with mean λ, we obtain the following upper bound for the difference between the distribution of S and the normal approximation :

λ	bound
$0,1$	$3,4965$
1	$1,1057$
10	$0,3496$
100	$0,1105$
1000	$0,0349$

Table 14: Berry-Esseen bound in function of λ

The result is not surprising : whenever the mean of the counting distribution is low the approximation by the normal distribution is bad but in that case the method described in the previous section can be applied. Whenever this mean is high, the central limit approximation is good. Nevertheless, the techniques described in the previous sections remain interesting if one wants an approximation of S by a discrete random variable S_{app}.
Assume the respective skewness are γ and γ_{app}. If X is a $N(\mathbb{E}S, \mathbb{V}arS)$ random variable, then by the Berry-Esseen result of Michel and the triangular inequality, we obtain

$$\sup_{t\in\mathbb{R}} |\mathbb{P}(S < t) - \mathbb{P}(S_{app} < t)| \leq 0,8\gamma + 0,8\gamma_{app} + \sup_{t\in\mathbb{R}} |\mathbb{P}[X < t] - \mathbb{P}[Y < t]| \tag{18}$$

where $X \sim N(\mathbb{E}S; \mathbb{V}arS)$
$Y \sim N(\mathbb{E}S_{app}; \mathbb{V}arS_{app})$
In particular, when S_{app} is constructed in such a way that the first two moments of X and Y are the same, (18) becomes

$$\sup_{t\in\mathbb{R}} |\mathbb{P}(S < t) - \mathbb{P}(S_{app} < t)| \leq 0,8\gamma + 0,8\gamma_{app}$$

Remarks :

1) Whenever the mean of N is very large, a technical problem arises : the initialization of the recursion gives an underflow. This problem has been handled by Panjer and Wang (1995) but the technique they propose can not be controlled if one wants an arithmetic distribution. However if a normal approximation is sufficient, then it is very well controlled by Michel's (1993) result.

2) Michel (1993) gives also a non-uniform result under the same hypothesis :

$$\left|\mathbb{P}\left(\frac{S - \mathbb{E}S}{\sigma_S}\right) - \Phi(t)\right| \leq \frac{30,6}{1 + |t|^3}\gamma_S$$

Unluckily this result is more interesting than the uniform bound only if $|t| \geq 3,34$, i.e. in the very far tails of the distribution. This is the reason why we concentrate only on the uniform result. Of course the non-uniform result of Michel is interesting to use when approximating quantities like stop-loss premiums for example.

8. Extension

The proof of Michel's(1993) result is given for the case where N, the number of components of the random sum, is a Poisson random variable but as the proof shows, the crucial point for the extension of the Berry-Esseen result to the random case is the fact that the distribution of S is infinitely divisible. Consequently, N has to be infinitely divisible.
In fact, by Feller (1971), there is a logical equivalence between infinite divisibility on integers and Compound Poisson distributions.
Then the result of Michel (1993) is immediately extended to every infinite divisible distribution on integers.
As an example of more general distribution for N, we take the Hofmann's family of distributions (see Hofmann (1955) and Kestemont and Paris (1985) for a discussion) whose probability generating function is

$$\Psi_N(z) = e^{-\frac{p}{c(1-a)}((1+c(1-z))^{1-a}-1)} \tag{19}$$

This distribution depends on three parameters : p, c and a. The parameter a distinguishes the distributions and presents the particular cases : Poisson ($a = 0$), Poisson Inverse Gaussian ($a = 1/2$), Binomial Negative ($a = 1$), Polya-Aeppli ($a = 2$) and Neymann Type A ($a \to \infty$, $c \to \infty$, $ac \to b$).
The mean of the distribution is p and has for maximum likelihood estimator the experimental mean.
This distribution is also known as the Generalized Poisson Pascal distribution (see Panjer and Willmot (1992)) but we prefer the Hofmann's presentation which has a better parametrization and natural extensions (see Walhin and Paris (1998)).
The Hofmann distribution $Ho(p, c, a)$ is itself a compound Poisson distribution and you can write

$$N = \xi_1 + \cdots + \xi_L$$

where L is Poisson distributed
ξ is such that $\frac{\mathbb{P}[\xi=n]}{\mathbb{P}[\xi=n-1]} = a + \frac{b}{n} \quad n \geq 2$
Then the generalization of Panjer's recursion (Sundt and Jewell (1981)) can be used when ξ is a counting distribution. The aggregate claims distribution can then be evaluated by a double application of Panjer's recursion because

$$\begin{aligned} V &= X_1 + \cdots + X_\xi \\ S &= V_1 + \cdots + V_L \end{aligned}$$

(see Panjer and Willmot (1992) for a general statement and Walhin and Paris (1998) for details concerning the Hofmann's distribution).
As N is infinitely divisible (see (19)), the results described in the previous section will apply for this general counting distribution.
The characteristics of the distribution are

$$\begin{aligned} S &= X_1 + \cdots + X_N \\ N &\sim Ho(p, c, a) \end{aligned}$$

$$
\begin{aligned}
\mu_1 &= \mathbb{E}X \\
\mu_2 &= \mathbb{E}X^2 \\
\mu_3 &= \mathbb{E}X^3 \\
\mathbb{E}S &= \mu_1 p \\
\mathbb{V}ar S &= (ca\mu_1^2 + \mu_2)p \\
\gamma_S &= \frac{ac^2\mu_1^3 + a^2c^2\mu_1^3 + 3ac\mu_1\mu_2 + \mu_3}{(ac\mu_1^2 + \mu_2)^{\frac{3}{2}}} p^{-\frac{1}{2}}
\end{aligned}
$$

9. Conclusion

We have seen how a global method can give an equispaced approached distribution of a general discrete distribution. The minimization of the Kolmogorov distance does not present the disadvantages of the methods presented in the litterature. The problem of matching moments between both distributions can be solved.
When the mean of the frequency claim distribution is low, which is typically the case in automobile insurance, the knowledge of the Kolmogorov distance between both the severity distributions is sufficient to give a sharp upper bound on the Kolmogorov distance between both the compound distributions.
When the mean of the counting distribution is high, there exists an interesting Berry-Esseen bound for the normal approximation when N is infinitely divisible. If one wants to keep the discrete approximate distribution, the Berry-Esseen bound may be used twice in order to give a bound on the Kolmogorov distance between the true and approximate compound distributions.
When the mean of the claim frequency distribution is not high enough for using the Berry-Esseen result, the bounds can be sharpened at the price of calculating some high order convolutions of both the true and approximate severity distributions. Then every almost situation can be handled, in term of controlling the error induced, by the techniques described in this paper. Note that the two practical situations, events with very low frequency and very large portfolios are very well controlled.
Our experience shows that even when the mean of the frequency claim distribution is medium, the Kolmogorov distance between both compound distributions remains a fraction of the Kolmogorov distance between both the claim severity distributions if the approximation is good. Note that the properties of the Kolmogorov distance between the compound distributions can also be used in the case of a bad knowledge of the claim severity distribution.

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