Another Look at the American Electrical Utility Data

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Abstract

The American electric utility data, which are frequently analyzed in the context of frontier models, can be explained by a linear model without inefficiencies. The observed maximum likelihood for this linear model is very mildly smaller than the maximum likelihood for more flexible stochastic frontier models and the the log-likelihood-ratio statistic for an approximate χ^2 test of the simple least squares model against normal-exponential and normal-gamma stochastic frontier models is far from significant.
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1 Introduction

The data on 123 American electric utilities presented by Christensen and Greene (1976) and revisited by Greene (1990) have become a popular example for the study of frontier models. In the first part of this paper, a simple analysis of the American utility data using graphical methods and ordinary least squares is conducted. This analysis suggests that a simple linear model with symmetric errors is sufficient to explain the data and that there is no evidence of a frontier-inefficiency structure in the data. The simple least squares results are contrasted with results obtained for a selection of stochastic frontier models and it is shown that these models do not achieve significantly better explanations of the data.

If a normal-gamma stochastic frontier model is imposed on data such as for the American electrical utilities the corresponding estimation problem is poorly conditioned and needs to be treated very carefully. Its likelihood is a product of difficult integrals which require numerical integration. An algorithm for solving these integrals in the case of the American utility data is presented in Section 5 of this paper. This algorithm is used to show that some of the numerical results reported by Greene (1990) on the American utility data are incorrect. Corrected results are reported. Even if the normal-gamma model is treated very carefully, a maximum likelihood estimate is hard to find. The likelihood continues to increase with decreasing shape parameter α, even if the latter is as small as 0.25. Even the highest achieved log-likelihood does not much exceed the corresponding log-likelihood of the simple least squares model. Therefore, also a normal-gamma stochastic frontier model does not lead to a significantly improved explanation of the data.

The paper is concluded by a discussion.

2 The Data and the Model

Data consisting of records for 123 American utility companies (Table 4) was presented by Christensen and Greene (1976) and re-analyzed by Greene (1990) and van den Broeck, Koop, Osiewalski and Steel (1993). The variables are: the abbreviation of the name of the utility company (id), the cost (C), the output (Q), the price of labor (P_l), the price of fuel (P_f), and the price of capital (P_k). From these variables, the response variable y and the explanatory variables x_1, x_2, x_3, and x_4 are constructed as follows; $y = -\log(C/P_f)$, $x_1 = \log Q$, $x_2 = (\log Q)^2$, $x_3 = \log(P_l/P_f)$, and $x_4 = \log(P_k/P_f)$.

In those variables, Greene (1990) suggested a simplified translog model of the form

$$Y_i = \beta_0 + \beta_1 x_{1;i} + \beta_2 x_{2;i} + \beta_3 x_{3;i} + \beta_4 x_{4;i} + \epsilon_i. \tag{1}$$

If there are no inefficiencies, the error term could be modeled as Gaussian or symmetric, thus leading to ordinary least squares as a fitting method. If instead a composed error stochastic frontier model is assumed, ϵ_i can be modeled as consisting of two summands, ν_i and z_i, which represent stochastic fluctuations of the frontier and firm specific inefficiencies (Meeusen and van den Broeck, 1977; Aigner, Lovell and Schmidt, 1977).

For the latter, it is popular to presume that the fluctuations ν_i are independent and normally distributed with zero mean and a variance σ^2, that the inefficiencies z_i are independent and gamma distributed with scale parameter λ and shape parameter α, and that the fluctuations are independent of the inefficiencies (Stevenson, 1980; Beckers and Hammond, 1987; Greene, 1990; van den Broeck et al., 1993).

3 Exploratory Graphical Analysis

Before discussing specific models, we begin with a simple exploratory analysis of the data. Figure 1 consists of plots of the response y versus the three explanatory variables x_1, x_3, and x_4. The upper left plot shows that the response $y = -\log(C/P_f)$ is clearly related to the explanatory variable $x_1 = \log Q$. In the right half of the picture, where there are most of the data points, the relationship is close to linear, while in the left half, one encounters negative curvature and somewhat higher variability. There are no apparent relationships between the response and the either of the variables $x_3 = \log(P_k/P_f)$ and $x_4 = \log(P_l/P_f)$. At first glance there is no evidence suggesting a stochastic frontier and inefficiencies. If a frontier were assumed, the corresponding technical inefficiencies would be small. This first inspection suggests that a linear model without inefficiencies of the $-log(C/P_f)$ as a function of $\log Q$, $(\log Q)^2$, and possibly $\log(P_l/P_f)$ and $\log(P_k/P_f)$ may be sufficient to explain the data.

4 Fit of a Linear Model Without Inefficiencies

A fit of the above formulated model without inefficiencies, i.e., of the model

$$Y_i = \mathbf{x}_i'\boldsymbol{\beta} + \nu_i, \quad \nu_i \overset{iid}{\sim} \mathcal{N}(0, \sigma^2), \tag{2}$$

where $\mathbf{x}_i = (1, x_{1;i}, x_{2;i}, x_{3:i}, x_{4;i})'$ and $\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4)'$, can be obtained via ordinary least squares. The resulting parameter estimates and their standard errors are as shown in Table 1. This table also contains the results obtained by several other authors using the same data, the same model of the frontier, but imposing different frontier-inefficiency structures. These other methods are corrected ordinary least squares based on a normal-gamma model (Greene, 1990), half-normal inefficiencies (Aigner et al., 1977), truncated-normal inefficiencies (Stevenson, 1980), and exponential inefficiencies, all with respect to a normal stochastic frontier. We refer to these models as COLS, ALS, STE, and NE. The values for the half-normal and truncated-normal models are taken from Greene (1990). We find that the above listed stochastic frontier models do not achieve much better log-likelihoods than the ordinary least squares model while introducing more parameters.

Note that our values for the parameters β_0, λ and α of the corrected ordinary least squares model (COLS) are different from the ones in Greene (1990). We believe that the values in Greene (1990) are incorrect and list our intermediate calculations for the empirical moments in Table 2. Note also that in our formulation the signs of the coefficients β_0, β_1, ..., β_4 are reversed from Greene (1990) since we use $y = -log(C/P_f)$ as the response.

For the ordinary least squares model, the coefficients corresponding to $x_1 = \log Q$ and $x_2 = (\log Q)^2$ are strongly significant. It appears that also $x_3 = \log(P_l/P_f)$ is linked to the response $y = \log(C/P_f)$ whereas the last predictor, $x_4 = \log(P_k/P_f)$ has no significant influence.

In order to assess the quality of the fit over the entire data range we divide the data into four partitions as shown in Figure 2. For each partition, we plot y versus x_1 and superimpose the fit obtained when x_3 and x_4 are kept at their mean values in the partition. Figure 3 shows the fits and the data in each partition, Figure 4 the histograms of the corresponding residuals and one histogram of all residuals combined. The combinded histogram is superimposed by a smooth density estimate obtained by the method of Kooperberg and Stone (1991). The diagnostic plots of the fits and the histograms suggest that the model fits well over the whole range of the data and that there is little reason to suspect a non-symmetric error distribution.

If the data had been generated by a composed-error frontier model with appreciable inefficiencies and if an ordinary least squares model was fit to these data, one would expect a skewed distribution of the residuals with a heavy left tail. Moreover, one would expect that the above mentioned composed-error frontier models would achieve much better log-likelihoods than the ordinary least squares model. Our analyses do not reveal any such features. The log-likelihoods are very similar for all selected models and the empirical skewness and kurtosis of the residuals under the ordinary least squares model are -0.0711 and 0.5907 suggesting longer tails than displayed by a normal probability distribution but no skewness.

5 Estimation of A Normal-Gamma Frontier Model

In this section, a normal-gamma frontier model is imposed on the data, as it is done in (Greene, 1990). One can now attempt to estimate the eight parameters β_0,...,β_4, α, λ, and σ^2 by maximum likelihood. This estimation is not trivial, since computing the likelihood requires the evaluation of the difficult numerical integrals

$$p(y_i \mid \lambda, \alpha, \sigma^2) = \int_0^\infty \frac{\exp\left[-\frac{(y_i - \mathbf{x}_i'\boldsymbol{\beta} + z)^2}{2\sigma^2}\right]}{\sqrt{2\pi\sigma^2}} \frac{z^{(\alpha-1)}\exp\left(-\frac{z}{\lambda}\right)}{\lambda^\alpha \Gamma(\alpha)} dz.$$

where $\mathbf{x}_i = (1, x_{i1}, \ldots, x_{i4})$ and $\boldsymbol{\beta} = (\beta_0, \ldots, \beta_4)'$. These integrals are improper, since the domain for z extends to ∞. If $\alpha < 1$, they are also improper at $z = 0$. For different values of y_i, the shape of the integrants varies considerably. Therefore, it is difficult to define a common grid or integration interval in z.

To deal with those difficulties, one can either use adaptive numerical integration or one can seek a transformation $z = g(\zeta)$, such that the integrant transformed to the ζ scale has a more stable shape for different values of y_i. Here, in order to preserve as much simplicity as possible, we pursue the second route. We use the trapezoidal rule over a fixed interval for a suitable transformation of z.

A natural candidate for such a transformation is $z = \log(\zeta)$, leading to an integral over $(-\infty, \infty)$. However, since α can be very small, the mass of the integral can be very far to the left of zero and it is difficult to find a common integration region which works for all y_i. After some experimentation $g(\zeta) = \zeta^3$ was found to provide a sufficient stabilization of the shape of the integrant. Moreover, the interval $(10^{-7}, 2.5)$ was found to cover the interesting portion of all integrants, except when α is very close to zero. If α is very close to zero, and if ζ approaches zero, the integrant tends to infinity so quickly that a non-negligible portion of the integral is contained in the interval $(0, 10^{-7})$. Fortunately, in this region, the integrant can be approximated by

$$\frac{\exp\left[-\frac{(y_i - \mathbf{x}_i'\boldsymbol{\beta})^2}{2\sigma^2}\right]}{\sqrt{2\pi\sigma^2}} \frac{\zeta^{(3\alpha-1)}}{\lambda^\alpha \Gamma(\alpha)}$$

which integrates to

$$\frac{\exp\left[-\frac{(y_i - \mathbf{x}_i'\boldsymbol{\beta})^2}{2\sigma^2}\right]}{\sqrt{2\pi\sigma^2}} \frac{(10^{-7})^{(3\alpha-1)}}{\lambda^\alpha \Gamma(\alpha)}$$

over the interval $(0, 10^{-7})$. This allows an adjustment of the of the overall integral. An algorithm to evaluate the log-likelihood, which follows the above steps, is included in the appendix.

Note that a modification had to be used to calculate the likelihood for the corrected ordinary least squares model listed in Table 1. Here, $\alpha = 0.00168$ is too close to zero. After some further experimentation, the transformation $g(\zeta) = \zeta^{10}$ was found to produce satisfactory results. In addition the number of intervals in the integration was increased from 350 to 2000 for these calculations. These modifications were also used to find the overall maximum likelihood estimate (Section 6).

6 Application to the American Electric Utility Data

Greene (1990) uses the American utility data (Christensen and Greene, 1976) as an example for maximum likelihood estimation of the parameters of the normal-gamma frontier. He compares these estimates with the ones obtained with various other methods and observes that the proportion of variation explained by the technical inefficiencies is only half as large under the normal-gamma model than under the alternatives.

In particular, he also observes that the maximum value of the log-likelihood is 112.72 as compared to approximately 66 for the other models. If we use the above described algorithm to evaluate the log-likelihood at Greene's estimates

$$\boldsymbol{\beta} = (7.810, -0.4728, -0.02581, -0.2910, -0.02527)',$$

$$\alpha = 2.45,\ \lambda = 0.04684,\ \sigma^2 = 0.01827,$$

we find a value of only 62.47. Table 3 shows our maximum likelihood estimates and their corresponding log-likelihood values for several fixed values of α, and the overall maximum at $\hat{\alpha} \approx 0.20$ which we were able to obtain. Here, to avoid numerical difficulties, the following strategy was employed. For values of $\alpha > 1/3$, the calculation of the likelihood was done using the algorithm. Below this threshold, a stronger transformation, $g(\zeta) = \zeta^{10}$, was used. Note that there are directions near the overall maximum of the log-likelihood in which the log-likelihood is very flat limiting the precision with which we determined the maximum likelihood estimate to about two significant digits.

For the range of α we investigated, the differences in the maximized log-likelihood are small. Twice the log-likelihood of the ordinary least squares solution is 131.36 versus 134.18 for $\alpha = 1$, the exponential case, and 135.26 for $\alpha = 0.25$, and 135.28 at the maximum we were able to find. Twice the log-likelihood ratio is 2.82 between the exponential case and the ordinary least squares case and 3.92 between the case at $\hat{\alpha}$ and the ordinary least squares case. Neither is by any means significant if a chi-square distribution with one and two degrees of freedom is used to approximate the respective distributions of twice the log-likelihood ratio.

A little caution is needed a this point. Strictly speaking, the ordinary least squares case does not occur in the interior of the parameter space of the normal-gamma model or the normal-exponential model. Therefore, the chi-square approximation to twice the log-likelihood does not follow from the standard derivation. However, the ordinary least squares case can be approximated in many ways from inside the parameter domain of the normal-gamma model. For any fixed value α including $\alpha = 1$ the ordinary least squares model is approached if the scale parameter λ tends to zero. Moreover, for any fixed variance $\alpha\lambda^2$ of the gamma component, the ordinary least squares case is approached if the shape parameter α tends to infinity. Indeed, one finds that the maximized likelihood in these approximating regions is very close to the maximum of the likelihood under the ordinary least squares case.

7 Discussion

Our study serves to alert the user of frontier-inefficiency models of the pitfalls of their mechanical application without careful consideration of simpler alternatives. We recommend to begin frontier-inefficiency analyses by a basic inspection of the data. In this inspection it should be noted whether there is clear evidence of a frontier-inefficiency structure and whether the intended model appears reasonable. If there is no obvious frontier-inefficiency structure, a simple model without inefficiencies should be tried first. It can later serve to benchmark more sophisticated models. If, subsequently, a frontier-inefficiency models are tried but do not achieve significant improvements over the simple model, estimates of the frontier and of inefficiencies are of doubtful value.

In normal-gamma or normal-exponential frontier models, the ordinary least squares model occurs as a limiting case when the scale parameter of the gamma or exponential distribution approaches zero or when the shape parameter of the gamma distribution becomes large. It is therefore easily forgotten as a real competitor to the frontier-inefficiency model, although it can provide a much simpler interpretation of the data.

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9 Appendix

9.1 Splus Code to Compute the Likelihood

```
"Loglik"<-
function(theta, data)
{
        x <- data$x
        y <- data$y
        beta <- theta[1:5]
        alpha <- theta[6]
        lambda <- theta[7]
        sig2 <- theta[8]
        y0 <- c(y - x %*% beta)
        z0 <- c( - y0 - sig2/lambda)
        nobs <- nrow(x)
        l.integ <- logintz(z0, alpha, sqrt(sig2))
        return(sum(
                        l.integ +
                        sig2/2/lambda^2 +
                        y0/lambda -
                        alpha * log(lambda) -
                        lgamma(alpha)
                ))
}
"logintz"<-
function(z0, alpha, sig, nint = 350, start = 1e-7, end = 2.5)
{
        nobs <- length(z0)
        index <- z0/sig < -10
        z1 <- seq(start^(1/3), end^(1/3), length = nint + 1)
        dh <- diff(z1)
        z <- matrix(z1^3, nrow = nobs, ncol = nint + 1, byrow = T)
        tt <-  - ((z - z0)/sig)^2/2 -
                log(sig) + (alpha - 1/3) * log(z) - 0.5 * log(2 * pi)
        tt <- exp(tt)
        frontend <- exp( - (z0/sig)^2/2 - log(sig) +
                        alpha * log(start) - log(alpha) -
                        0.5 * log(2 * pi))
        log.integ <- log(1.5 *
                        (tt[, -1] + tt[,  - (nint + 1)]) %*% dh +
                        frontend)
        return(log.integ)
}
```

9.2 The American Electrical Utility Data

id	C	Q	P_l	P_k	P_f
CPU	0.2130	8	6869.47	64.945	18.0000
MP	3.0427	869	8372.96	68.227	21.0670
UPP	9.4059	1412	7960.90	40.692	41.5300
MCP	0.7606	65	8791.89	41.243	28.5390
BH	2.2587	295	8218.40	71.940	39.2000
CPS	1.3422	183	5063.49	74.430	35.5100
NE	0.6159	50	9204.24	90.470	32.0700
MPS	0.4887	14	5438.89	86.110	34.1500
NWPS	1.1474	90	7189.67	79.101	21.5030
PPL	7.5492	2969	8183.34	80.657	9.0000
CK	2.0532	374	7884.94	82.485	26.3014
AMP	0.6363	67	6696.50	58.258	25.4000
LSDP	3.1504	378	7895.43	60.277	42.4683
MoPS	10.3136	1886	6833.93	67.680	25.6000
MDU	5.8488	1025	7093.32	68.227	22.2793
UGI	4.5050	467	8410.34	76.300	26.8500
SJLP	5.5971	938	7703.26	77.197	25.4000
BHPL	3.7255	643	7332.13	64.506	21.1619
IS	6.0065	1328	6680.77	51.817	25.2319
TGE	12.7830	2632	8469.84	70.295	30.9710
MGE	6.6214	856	8033.72	67.680	39.5812
CLP	9.6429	2689	6364.40	78.440	23.4889
SEP	8.6852	1627	7912.40	79.220	31.6801
OTP	8.6372	1090	8507.92	39.127	33.9000
CMP	11.2419	2258	6892.61	81.096	29.1000
EDE	6.6221	1500	7588.70	82.840	21.3951
EPE	10.9665	2506	7412.36	57.240	27.4207
IPO	13.6270	2437	7846.06	65.565	30.6712
SIGE	7.4395	1293	6821.09	78.225	21.4305
IPS	9.7843	1627	8355.75	76.140	31.3374
PSCN	20.8671	3965	8403.59	74.480	33.1992
CHGE	19.7092	2682	9484.85	81.750	50.4516
IIGE	14.0431	2001	8611.53	74.025	30.6446
WTU	10.1902	2764	6338.00	66.232	21.1970
IELP	16.1764	2487	8088.45	66.622	32.6186
SCEG	42.2514	7320	5879.51	92.063	39.2104
WPS	22.5612	3571	7297.71	78.225	41.5951
NIPS	33.0175	6837	7310.15	69.795	28.4405
RGE	18.8963	2020	10806.20	79.570	46.9000
IPL	13.2679	2445	8764.75	76.140	28.3648
WPL	21.5454	3981	8186.05	75.082	35.2049
TE	35.5303	6770	7798.26	67.570	29.8250
ACE	29.8011	4187	7996.44	74.120	47.4257
UIC	30.8773	5643	10182.49	61.040	27.8498
APS	24.3565	6793	6336.88	70.295	18.5909
KPL	17.4802	4148	7536.89	74.025	24.5873
KGE	19.9008	5785	7969.55	71.910	22.2448
LPL	28.7861	9660	6686.73	79.542	20.2630
CPL	27.0832	7896	7119.96	74.513	20.1100
MPL	22.4421	5648	8954.12	78.440	25.9160
SDGE	30.2067	5286	7084.10	73.325	38.3384
PSO	30.1678	9602	7054.18	59.977	20.2010
DPL	32.5840	7930	7119.01	48.997	22.8380
FP	47.3864	9530	7624.57	83.880	31.5825
CIPS	24.2903	5316	9759.83	74.025	27.8380
IPL	31.2922	7484	8063.73	67.680	23.5267
OGE	31.9884	10149	6437.92	73.140	18.5343
TPL	41.9016	12954	6460.64	62.330	21.7550
GSU	58.1154	17875	6288.41	73.395	20.6191
NYSEG	40.5281	6891	6769.55	74.120	35.9651
KCPL	37.0666	6754	10177.89	77.197	25.6208

id	C	Q	P_l	P_k	P_f
SWPS	25.1686	6779	7826.93	74.200	20.2790
TES	45.1827	12936	8320.06	65.760	22.0330
LIL	55.1764	10855	8061.96	71.490	31.7601
IPO	41.1798	9275	8657.53	76.140	24.5804
CAPL	80.3593	16311	7282.61	81.550	40.9692
BGE	68.4800	12542	8142.84	80.385	35.7882
PEP	97.3859	13846	7786.37	88.540	44.1571
BOED	52.7634	9145	10373.50	81.750	35.8083
ARPL	38.8472	10057	6035.95	81.578	25.8240
PSI	48.1125	11114	8413.86	69.975	22.5536
WIEP	79.0705	12706	9282.51	70.853	37.2477
VIEP	111.8863	23217	6873.73	83.880	33.3944
PWPL	77.8849	13702	7113.79	70.850	34.9616
HULP	87.1015	27708	6378.23	63.600	20.3000
DUL	57.7267	10004	6472.86	76.300	28.0959
COP	90.7168	17280	9191.47	72.967	36.8816
PSEG	183.2315	27118	9914.36	78.480	41.7578
SCE	169.2354	38343	9117.16	65.992	31.5897
NIM	76.2528	11667	10436.32	80.660	46.0701
PHE	134.2283	19445	9829.32	67.580	38.8027
DUPC	168.3777	34212	5683.83	80.385	40.5286
PAGE	125.3356	24001	8047.35	74.372	33.0932
DEED	191.5625	30958	9810.10	69.541	36.3076
COED	240.5137	29613	9312.93	81.750	41.8872
CVPS	0.1304	4	6009.70	92.650	33.1990
CIU	0.7293	60	7972.71	75.464	33.7070
MAE	1.7705	153	10963.90	57.612	46.1600
HEL	2.2367	198	7046.50	64.945	38.0000
FIGE	2.5593	243	7000.81	81.750	33.4360
NMES	2.0358	617	9547.72	81.750	18.4900
SPP	7.6236	1340	6438.42	79.220	38.1483
CTEU	11.1091	1961	6964.81	82.485	28.2634
PSNM	9.4674	2233	6717.18	59.957	22.6933
NEPO	13.8262	2582	8334.89	77.531	34.1811
OR	17.2895	2763	8058.91	80.660	45.6636
KEU	16.1704	2863	7509.37	81.550	30.2374
HAE	24.3975	3490	8477.23	75.420	38.0000
TOE	27.2443	4568	9619.76	76.140	35.6645
CSO	29.4870	5292	8176.33	76.140	22.8374
DPL	35.1091	5699	7988.48	80.370	42.6351
LOGE	25.4814	5702	6947.11	78.225	24.2596
CIGE	42.4117	8650	7146.67	72.967	27.8693
CLEI	72.0355	12724	8037.84	74.025	37.5174
FLPL	146.9890	25147	9900.76	75.725	33.4742
NEE	51.7415	10361	9578.63	68.016	28.1423
NEGE	21.5587	3886	9538.68	63.569	30.8894
EUA	14.2710	1901	9822.64	69.411	36.0000
NSP	66.1032	11837	8709.43	75.379	31.3321
UEC	113.2555	22522	9500.78	76.732	25.0289
ALLP	90.3718	21956	7954.47	83.338	22.9115
SCO	240.4858	53918	6068.87	78.380	31.1954
AEP	277.2965	72247	7419.92	56.301	25.0914
GPU	107.9776	18455	6690.23	76.300	32.9654
COED	269.7728	46870	9761.38	69.541	33.1999
NOUT	79.6207	16508	9404.97	78.044	42.2086
PSCO	33.8814	7382	7512.72	72.362	25.9001
MPL	14.9529	2325	8568.23	53.890	38.9161
DEPL	33.9733	5708	10024.20	78.102	42.1660
OEDO	78.7028	17132	8160.80	78.899	25.5378
UPL	19.4391	4560	8558.37	76.464	23.7777
SWEP	20.8836	6259	6697.02	69.764	21.2920
NOPS	21.7792	6746	9419.27	49.778	20.1000

Table 4: Data for a selection of American utilities. The columns are id: abbreviation of the utility, C: cost, Q: output, P_l: price of labor, P_k: price of capital, and P_f: price of fuel. Source: Christensen and Greene (1976) and Greene (1990).

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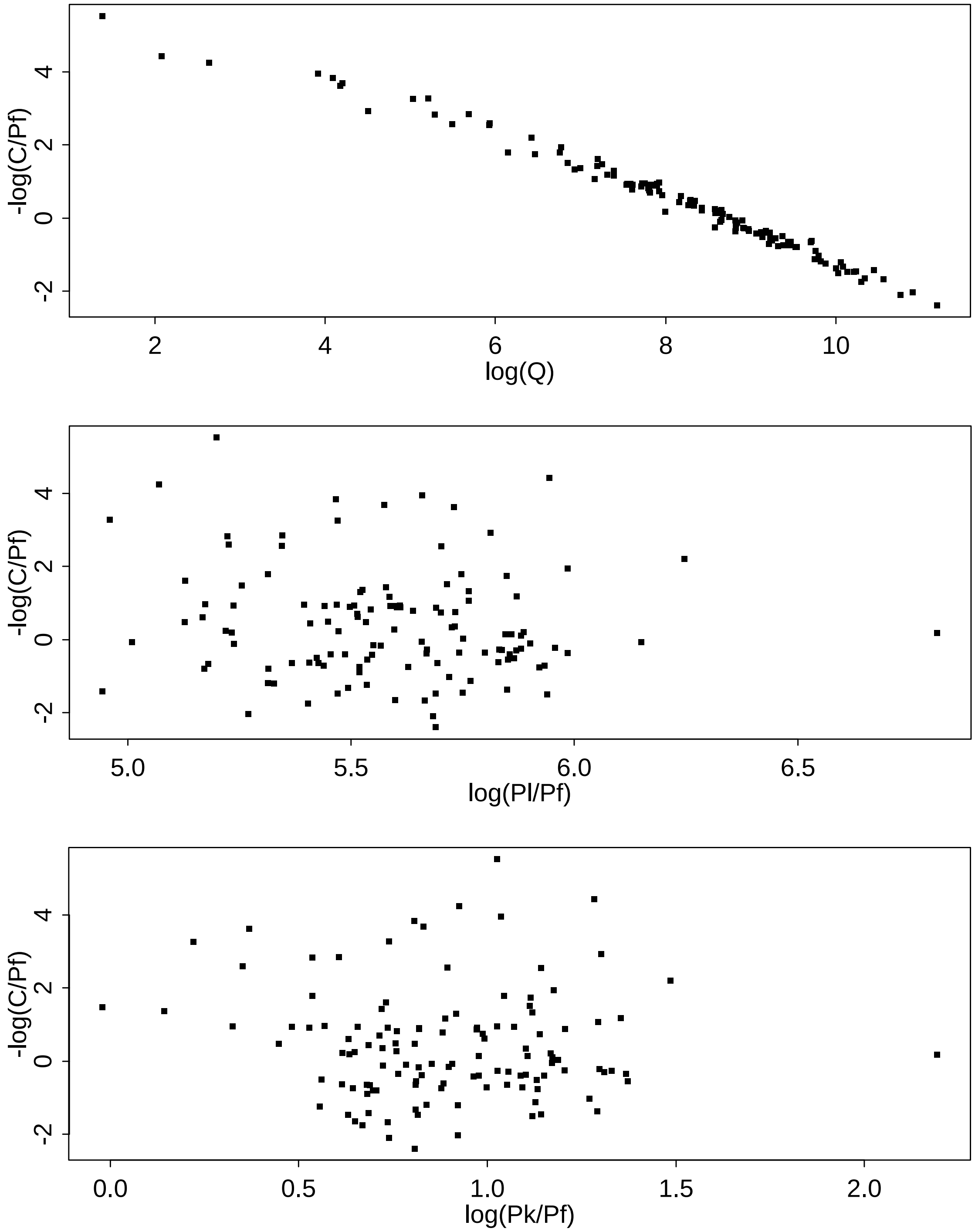

Figure 1: Top: $-\log(C/P_f)$ versus $\log Q$; middle: $-\log(C/P_f)$ versus $\log(P_l/P_f)$; bottom: $-\log(C/P_f)$ versus $\log(P_k/P_f)$.

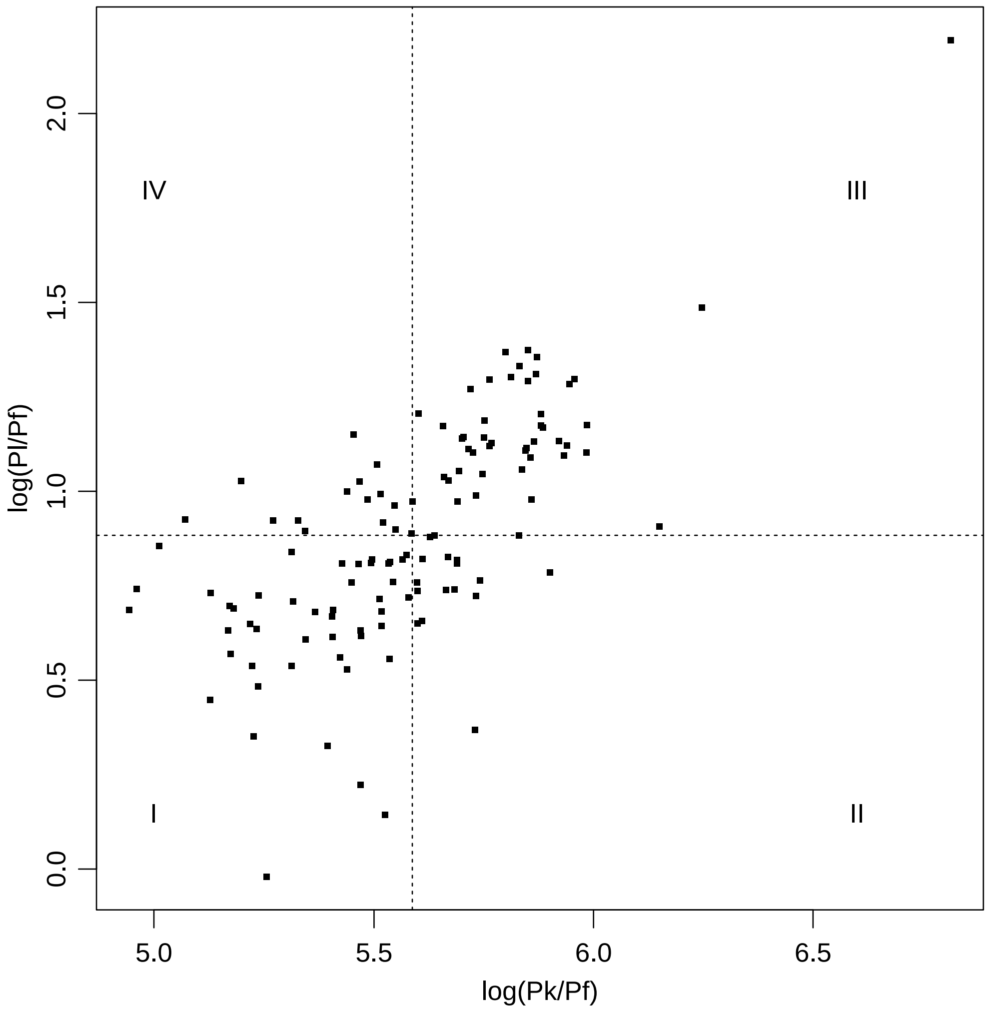

Figure 2: Plot of $\log(P_l/P_f)$ versus $\log(P_k/P_f)$. Dashed lines: medians of $\log(P_l/P_f)$ and $\log(P_k/P_f)$; roman numerals are used to label the four quadrants. They correspond to the partitions introduced in Section 4.

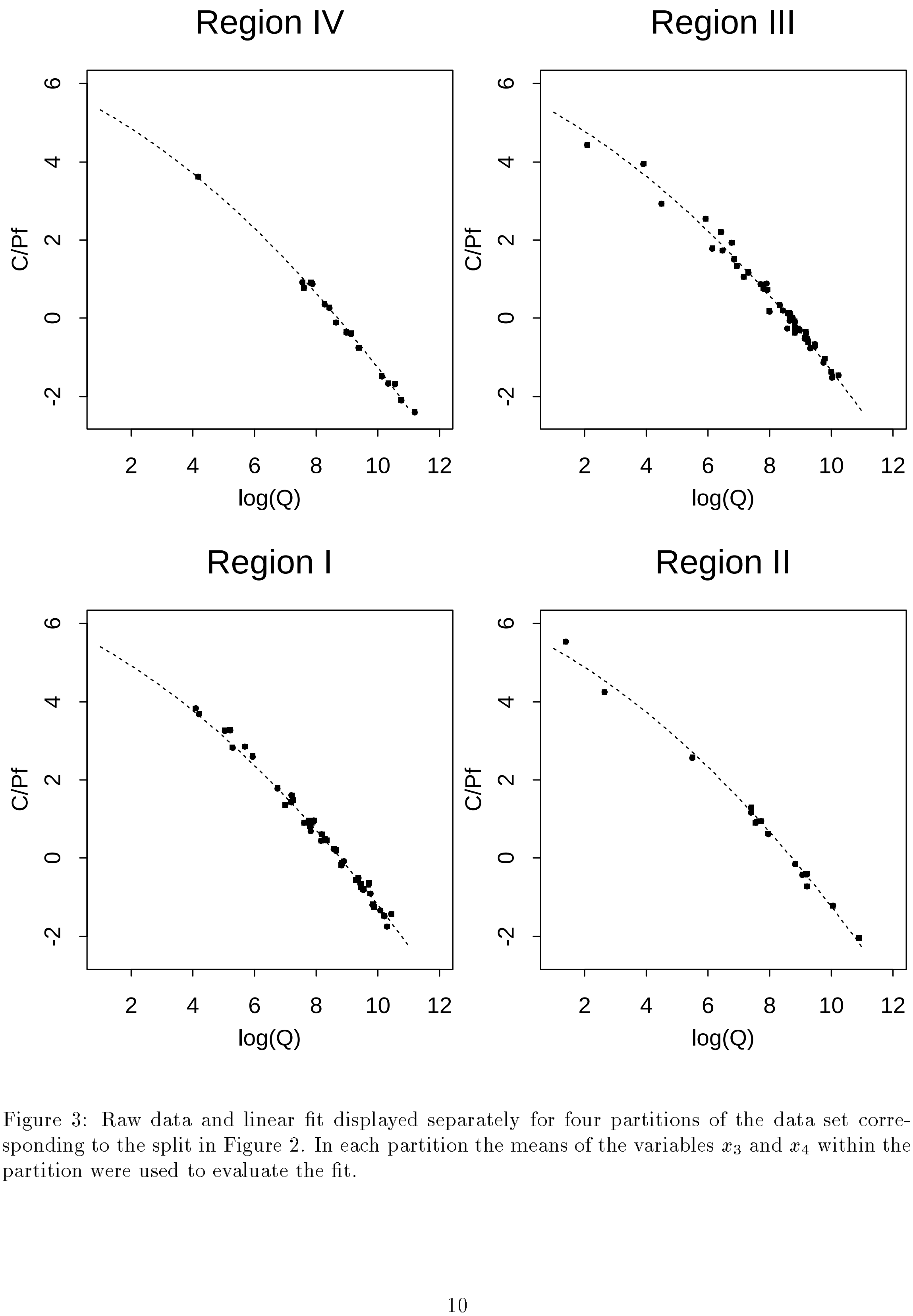

Figure 3: Raw data and linear fit displayed separately for four partitions of the data set corresponding to the split in Figure 2. In each partition the means of the variables x_3 and x_4 within the partition were used to evaluate the fit.

	OLS		*COLS*	*ALS*	*Ste*	*NE*
	est	(sd)	est	est	est	est
β_0	7.2075	(0.3372)	7.2082	7.408	7.501	7.5254
β_1	-0.3858	(0.0383)	-0.3858	-0.4078	-0.4292	-0.4343
β_2	-0.0316	(0.0027)	-0.0316	-0.0306	-0.0295	-0.0292
β_3	-0.2470	(0.0670)	-0.2470	-0.2445	-0.2501	-0.2560
β_4	-0.0784	(0.0617)	-0.0784	-0.0587	-0.0422	-0.0336
σ^2	0.0210		0.0207	0.0115	0.00732	0.01240
λ			0.3943			0.0989
α			0.00168			1.0000
ρ	0%		1.2%	43%	45%	44%
$log - lik$	65.68		65.72	66.14	66.91	67.09

Table 1: Estimates and standard errors of the model parameters for the ordinary least squares model and estimates of a selection of composed error stochastic frontier models. The parameter σ^2 stands for the variance of the normal part of the error (the fluctuation of the frontier), λ and α denote the scale and shape parameters of the density of the inefficiencies (if applicable), and ρ denotes the percentage of the total variance explained by the inefficiency terms.

	N	$N-1$
m_2	0.02013	0.02029
m_3	-0.00020	-0.00021
m_4	0.00146	0.00148
m_4^*	0.00025	0.00024
$\hat{\lambda}^{-1}$	2.4358	2.5364
$\hat{\alpha}$	0.00147	0.00168
$\hat{\sigma}^2$	0.01988	0.02003

Table 2: The moments were obtained by summing the corresponding powers of the residuals and dividing N or $N-1$, where N is the number of observations.

	$\alpha = .1$	$\hat{\alpha} \approx .20$	$\alpha = .5$	$\alpha = 1$	$\alpha = 2$	$\alpha = 3$	$\alpha = 4$	ols
β_0	7.6113	7.5907	7.5523	7.5254	7.5150	7.5218	7.5340	7.2075
β_1	-0.4632	-0.4558	-0.4440	-0.4343	-0.4249	-0.4198	-0.4164	-0.3858
β_2	-0.0274	-0.0279	-0.0286	-0.0292	-0.0297	-0.0300	-0.0301	-0.0316
β_3	-0.2610	-0.2599	-0.2560	-0.2521	-0.2487	-0.2472	-0.2465	-0.2470
β_4	-0.0280	-0.0288	-0.0336	-0.0391	-0.0453	-0.0490	-0.0515	-0.0784
α	0.1000	0.1995	0.5000	1.0000	2.0000	3.0000	4.0000	
λ	0.3281	0.2229	0.1387	0.0989	0.0712	0.0589	0.0515	
σ^2	0.0134	0.0127	0.0117	0.0110	0.0103	0.0099	0.0096	0.0210
log-lik	67.54	67.64	67.43	67.08	66.71	66.51	66.39	65.68

Table 3: Maximum likelihood estimates and corresponding values of the log-likelihood for several choices of α. Last column: ordinary least squares estimates and corresponding log-likelihood.

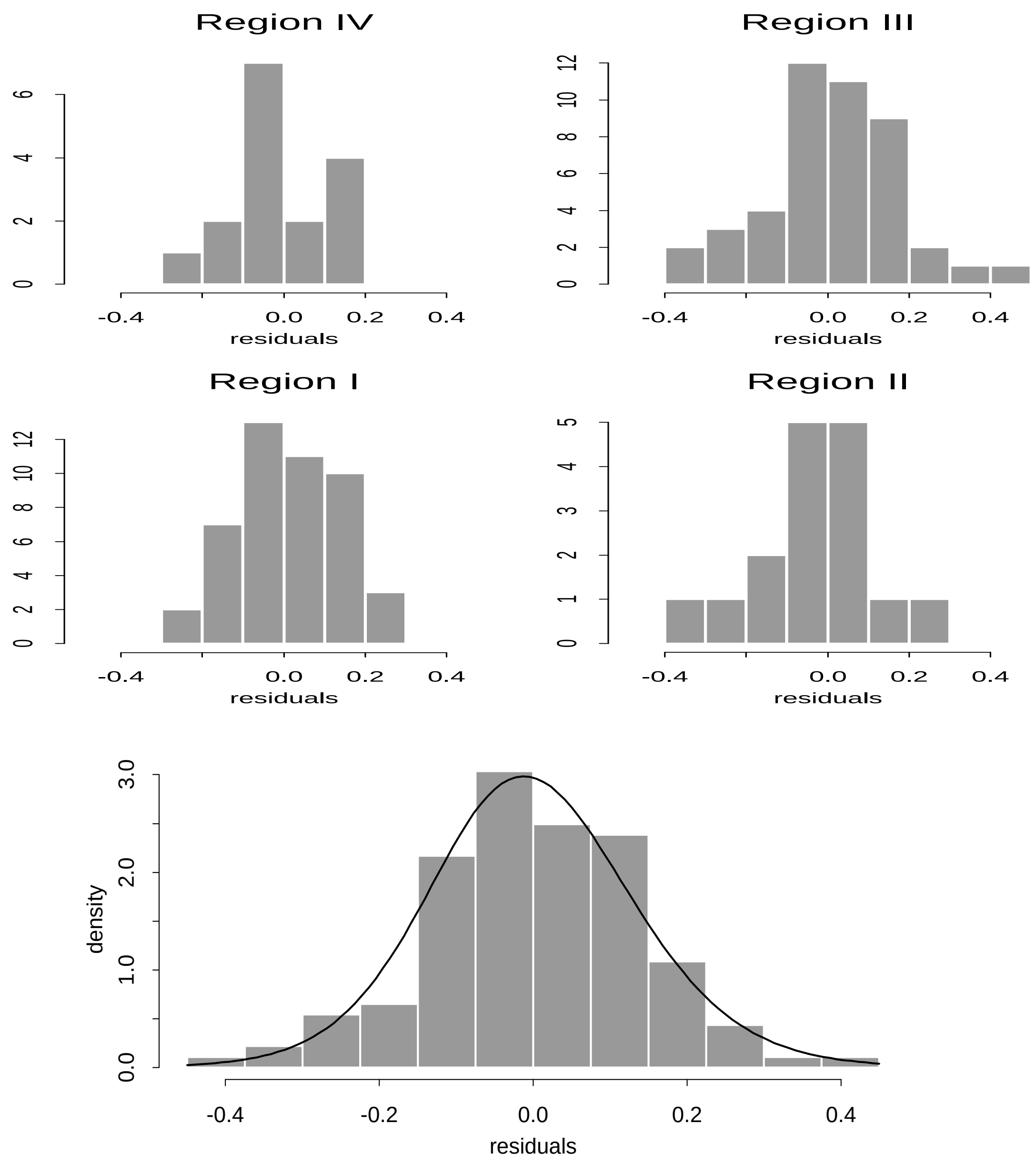

Figure 4: Histogram of residuals in each of the four partitions of the data set corresponding to the split in in Figure 2 and density histogram of all residuals superimposed by a smooth density estimate.